Session : 2025-26 **AS PER NEW NTA SYLLABUS**

Total Questions : 557

MATHS (QUESTION BANK)

6.APPLICATION OF DERIVATIVES

Single Correct Answer Type

1. The maximum value of the function $f(x)$ given by $f(x) = x(x-1)^2, 0 < x < 2$, is
   a) 0 b) 4/27 c) -4 d) 1/4
2. For a given integer k, in the interval $\left[2\pi k + \frac{\pi}{2}, 2\pi k - \frac{\pi}{2}\right]$ the graph of $\sin x$ is
   a) Increasing from -1 to 1
   b) Decreasing from -1 to 0
   c) Decreasing from 0 to 1
   d) None of the above
3. If θ is the semi vertical angle of a cone of maximum volume and given slant height, then $\tan\theta$is given by
   a) 2 b) 1 c) $\sqrt{2}$ d) $\sqrt{3}$
4. The value of b for which the function $f(x) = \sin x - bx + c$ is decreasing in the interval $(-\infty, \infty)$ is given by
   a) $b < 1$ b) $b \geq 1$ c) $b > 1$ d) $b \leq 1$
5. The function $f(x) = 2x^3 + 3x^2 - 12x + 1$ decreases in the interval
   a) (2, 3) b) (1, 2) c) (-2, 1) d) (-3, -2)
6. If $f(x) = 2\,x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$, then $f(x)$
   a) Increases on R
   b) Decreases in $[0, \infty)$
   c) Neither increases nor decreases in $(0, \infty)$
   d) None of these
7. The maximum value of $f(x) = 3\cos^2 x + 4\sin^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}$, is
   a) 4 b) $3 + \sqrt{2}$ c) $4 + \sqrt{2}$ d) $2 + \sqrt{2}$
8. If $a^2x^4 + b^2y^4 = c^6$, then maximum value of xy is
   a) $\frac{c^2}{\sqrt{ab}}$
   b) $\frac{c^3}{ab}$
   c) $\frac{c^3}{\sqrt{2ab}}$
   d) $\frac{c^3}{2ab}$
9. A stone is dropped into a quiet lake. If the waves moves in circles at the rate of 30cm/sec when the radius is 50 m, the rate of increase of enclosed area is
   a) $30\,\pi$ m^2/sec b) 30 m^2/sec c) 3π m^2/sec d) None of these
10. The equation of the tangent to the curve $x = t\cos t$, $y = t\sin t$ at the origin is
    a) $x = 0$ b) $y = 0$ c) $x + y = 0$ d) $x - y = 0$
11. The rate of change of the surface area of a sphere of a sphere of radius$r$,when the radius is increasing at the rate of 2 cm/sis proportional to

a) $\frac{1}{r}$ b) $\frac{1}{r^2}$ c) r d) r^2

12. The maximum value of $(1/x)^x$, is

a) e b) e^e c) $e^{1/e}$ d) $(1/e)^{1/e}$

13. If $f(x) = 2x^3 - 21x^2 + 36x - 30$, then which one of the following is correct

a) $f(x)$has minimum $at\ x = 1$ b) $f(x)$has maximum $at\ x = 6$

c) $f(x)$has maximum $at\ x = 1$ d) $f(x)$has maxima or minima

14. An edge of a variable cube is increasing at the rate of 10cm/s. How fast the volume of the cube will increase when the edge is 5 cm long?

a) 750 cm $^3/_s$ b) 75 cm $^3/_s$ c) 300 cm$^3/_s$ d) 150 cm $^3/_s$

15. The tangents to the curve $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$ at the points $\theta = (2k+1)\pi, k \in Z$ are parallel to:

a) $y = x$ b) $y = -x$ c) $y = 0$ d) $x = 0$

16. The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at $P(0, -3)$ meets the curve again at the point

a) $(-1, 1), (1, 5)$ b) $(1, -1), (-1, -5)$ c) $(-1, -5), (-1, 1)$ d) $(-1, 5), (1, -1)$

17. The normal to the curve represented parametrically by $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ at any point θ, is such that it

a) Makes a constant angle with x-axis

b) Is at a constant distance from the origin

c) Passes through the origin

d) Satisfies all the three conditions

18. If $f(x) = \begin{cases} 3x^2 + 12\,x - 1, -1 \le x \le 2 \\ 37 - x, \quad 2 < x \le 3 \end{cases}$, then

a) $f(x)$ is increasing in $[-1, 2]$

b) $f(x)$ is continuous in $[-1, 3]$

c) $f(x)$ is maximum at $x = 2$

d) All the above

19. The value of c, in the Lagrange's Mean value theorem $\frac{f(b)-f(a)}{b-a} = f'(c)$, for the function $f(x) = x(x-1)(x-2)$ in the interval $[0, 1/2]$, is

a) $\frac{1}{4}$ b) $1 - \frac{\sqrt{21}}{6}$ c) $\frac{9}{8}$ d) $1 + \frac{\sqrt{21}}{6}$

20. If $f(x) = kx - \sin x$ is monotonically increasing, then

a) $k > 1$ b) $k > -1$ c) $k < 1$ d) $k < -1$

21. Let $f(x) = x^3 - 6x^2 + 15x + 3$. Then,

a) $f(x) > 0\ for\ all\ x \in R$

b) $f(x) > f(x+1)\ for\ all\ x \in R$

c) $f(x)$ is invertible

d) None of these

22. The diagonal of a square is changing at the rate of 0.5 cm s^{-1}.Then, the rate of change of area , when the area is 400 cm^2 is equal to

a) $20\sqrt{2}cm^2/s$ b) $10\sqrt{2}cm^2/s$ c) $\frac{1}{10\sqrt{2}}cm^2/s$ d) $\frac{10}{\sqrt{2}}cm^2/s$

23. If $ax^2 + \frac{b}{x} \ge c$ for all positive x, where $a, b, > 0$, then

a) $27ab^2 \ge 4c^3$ b) $27ab^2 < 4c^3$ c) $4ab^2 \ge 27c^3$ d) None of these

24. The equation(s) of the tangent(s) to the curve $y = x^4$ from the point (2, 0) not on the curve is given by

a) $y = \frac{4098}{81}$

b) $y - 1 = 5(x - 1)$

c) $y - \frac{4096}{81} = \frac{2048}{27}\left(x - \frac{8}{3}\right)$

d) $y - \frac{32}{243} = \frac{80}{81}\left(x - \frac{2}{3}\right)$

25. The value of c in Rolle's theorem for the function $f(x) = \frac{x(x+1)}{e^x}$ defined on $[-1,0]$, is

a) 0.5 b) $\frac{1+\sqrt{5}}{2}$ c) $\frac{1-\sqrt{5}}{2}$ d) -0.5

26. The point on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at which the normal is parallel to the x-axis, is

a) $(0, 0)$ b) $(0, a)$ c) $(a, 0)$ d) (a, a)

27. The value of x for which the polynomial $2x^3 - 9x^2 + 12x + 4$ is a decreasing function of x, is

a) $-1 < x < 1$ b) $0 < x < 2$ c) $x > 3$ d) $1 < x < 2$

28. The function $f(x) = 1 - x^3 - x^5$ is decreasing for

a) $1 \le x \le 5$ b) $x \le 1$ c) $x \ge 1$ d) All values of x

29. $y = \{x(x-3)\}^2$ increases for all values of x lying in the interval

a) $0 < x < \frac{3}{2}$ b) $0 < x < \infty$ c) $-\infty < x < 0$ d) $1 < x < 3$

30. If m denotes the slope of the normal to the curve $y = -3\log(9 + x^2)$ at the point $x \neq 0$, then,

a) $m \in [-1, 1]$ b) $m \in R - (-1, 1)$ c) $m \in R - [-1, 1]$ d) $m \in (-1, 1)$

31. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real number x, then the minimum value of $f(x)$

a) Does not exist because f is unbounded

b) Is not attained even though f is bounded

c) Is equal to 1

d) Is equal to -1

32. The function $f(x) = |px - q| + r|x|, x \in (-\infty, \infty)$, where $p > 0, q > 0, r > 0$ assume its minimum value only at one point if

a) $p \neq q$ b) $r \neq q$ c) $r \neq p$ d) $p = q = r$

33. If the function $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of x, then

a) $K < 1$ b) $K > 1$ c) $K < 2$ d) $K > 2$

34. A man of 2m height walks at a uniform speed of 6 km/h away from a lamp post of 6 m height. The rate at which the length of his shadow increase is

a) 2km/h b) 1km/h c) 3km/h d) 6km/h

35. ΔABC is not right angled and is inscribed in a fixed circle. If a, A, b, B be slightly varied keeping c, C fixed, then

$\frac{da}{\cos A} + \frac{db}{\cos B} =$

a) $2R$ b) π c) 0 d) None of these

36. A value of c for which the conclusion of Mean value theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

a) $2\log_3 e$ b) $\frac{1}{2}\log_e 3$ c) $\log_3 e$ d) $\log_e 3$

37. If $ax + \frac{b}{x} \ge c$ for all positive values of x and a, b, c are positive constants, then

a) $ab \ge \frac{c^2}{4}$ b) $ab < \frac{c^2}{4}$ c) $bc \ge \frac{c^2}{4}$ d) $ac \ge \frac{c^2}{4}$

38. Let $f(x) = \int_0^x \frac{\cos t}{t} dt$. Then, at $x = (2n+1)\frac{\pi}{2}, f(x)$ has

a) Maxima when $n = -2, -4, -6, \dots$ and minima when $n = -1, -3, -5, \dots$

b) Maxima when $n = -1, -3, -5, \dots$ and minima when $n = 1, 3, 5, \dots$

c) Minima when $n = 0, 2, 4, \dots$ and maxima when $n = 1, 3, 5, \dots$

d) None of these

39. The line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a,b) for

a) $n=2$ only b) $n=-3$ only c) Any $n\in R$ d) None of these

40. The pressure P and volume V of a gas are connected by the relation $PV^{1/4}=$ constant. The percentage increase in the pressure corresponding to a deminition of 1/2 % in the volume is

a) $\frac{1}{2}\%$ b) $\frac{1}{4}\%$ c) $\frac{1}{8}\%$ d) None of these

41. In the mean value theorem $\frac{f(b)-f(a)}{b-a}=f'(c)$, if $a=0, b=\frac{1}{2}$ and $f(x)=x(x-1)$ $(x-2)$, then value of c is

a) $1-\frac{\sqrt{15}}{6}$ b) $1+\sqrt{15}$ c) $1-\frac{\sqrt{21}}{6}$ d) $1+\sqrt{21}$

42. If $f(x)=\frac{1}{4x^2+2x+1}$, then its maximum value is

a) 4/3 b) 2/3 c) 1 d) 3/4

43. The diameter of a circle is increasing at the rate of 1cm/sec. When its radius is π, the rate of increase of its area is

a) π cm^2/sec b) 2π cm^2/sec c) π^2 cm^2/sec d) $2\pi^2$ cm^2/ sec^2

44. The minimum value of $2x+3y$, when $xy=6$, is

a) 9 b) 12 c) 8 d) 6

45. The equation of the normal to the curve $y^4=ax^3$ at (a, a) is

a) $x+2y=3a$ b) $3x-4y+a=0$ c) $4x+3y=7a$ d) $4x-3y=0$

46. The value of c in Rolle's theorem when
$f(x)=2x^3-5x^2-4x+3, x\in[1/3,3]$, is

a) 2 b) $-1/3$ c) -2 d) 2/3

47. Suppose the cubic x^3-px+q has three distinct real roots where $p>0$ and $q>0$. Then, which one of the following holds?

a) The cubic has maxima at both $\frac{p}{3}$ and $-\frac{p}{3}$
b) The cubic has minima at $\frac{p}{3}$ and maxima at $-\frac{p}{3}$
c) The cubic has minima at $-\frac{p}{3}$ and maxima at $\frac{p}{3}$
d) The cubic has minima at both $\frac{p}{3}$ and $-\frac{p}{3}$

48. The chord joining the points where $x=p$ and $x=q$ on the curve $y=ax^2+bx+c$ is parallel to the tangent at the point on the curve whose abscissa is

a) $\frac{p+q}{2}$ b) $\frac{p-q}{2}$ c) $\frac{pq}{2}$ d) None of these

49. n is a positive integer. If the value of c prescribed in Rolle's theorem for the function $f(x)=2x(x-3)^n$ on the interval $[0,3]$ is $3/4$, then the value of n is

a) 5 b) 2 c) 3 d) 4

50. The shortest distance between the line $y-x=1$ and the curve $x=y^2$ is

a) $\frac{3\sqrt{2}}{8}$ b) $\frac{2\sqrt{3}}{8}$ c) $\frac{3\sqrt{2}}{5}$ d) $\frac{\sqrt{3}}{4}$

51. If the distance s covered by a particle in time t is proportional to the cube root of its velocity, then the acceleration is

a) A constant b) $\propto s^3$ c) $\propto\frac{1}{s^3}$ d) $\propto s^5$

52. The distance travelled s(in meteres) by a particle in t second is given by, $s=t^3+2t^2+t$.The speed of the particle after 18 will be

a) 8 cm/s b) 6 cm/s c) 2 cm/s d) None of these

53. Using differentials, the approximate value of $(627)^{1/4}$ is

a) 5.002 b) 5.003 c) 5.005 d) 5.004

54. The length of the subtangent at any point (x_1,y_1)on the curve $y=a^x, (a>0)$ is

a) $2\log a$ b) $\frac{1}{\log a}$ c) $\log a$ d) $a^{2x_1}\log a$

55. Using differentials the approximate value of $\sqrt{401}$ is

a) 20.100 b) 20.025 c) 20.030 d) 20.125

56. A ladder 10 m long rests against a vertical wall with the lower end on the horizontal ground. The lower end of the ladder is pulled along the ground away from the wall at the rate of 3 cm/s. The height of the upper end while it is descending at the rate of 4cm/s, is

a) $4\sqrt{3}$m b) $5\sqrt{3}$m c) 6m d) 8m

57. A cubic $f(x)$ vanishes at $x=-2$ and has relative minimum/maximum at $x=-1$ and $x=\frac{1}{3}$ such that $\int_{-1}^{1} f(x)dx=\frac{14}{3}$. Then, $f(x)$ is

a) x^3+x^2-x b) x^3+x^2-x+1 c) x^3+x^2-x+2 d) x^3+x^2-x-2

58. The different between the greatest and least values of the function $f(x)=\cos x\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3\,x\ is$

a) $\frac{2}{3}$ b) $\frac{8}{7}$ c) $\frac{3}{8}$ d) $\frac{9}{4}$

59. The number of real roots of the equation $e^{x-1}+x-2=0$

a) 1 b) 2 c) 3 d) 4

60. If $f(x)=\sin^6 x+\cos^6 x$, then which one of the following is false?

a) $f(x)\le 1$ b) $f(x)\le 2$ c) $f(x)>\frac{1}{4}$ d) $f(x)\le\frac{1}{8}$

61. The set $\{x^3-12x:-3\le x\le 3\}$ is equal to

a) $\{x:-16\le x\le 16\}$ b) $\{x:-12\le x\le 12\}$ c) $\{x:-9\le x\le 9\}$ d) $\{x:0\le x\le 10\}$

62. If $xy=a^2$ and $S=b^2x+c^2y$ where a,b and c are constants, then the minimum value of S is

a) abc b) $\sqrt{a}\,bc$ c) $2abc$ d) None of these

63. Let $g(x)=f(x)+f'(1-x)$ and $f''(x)<0, 0\le x\le 1$. Then

a) $g(x)$ increases on [1/2,1] and decreases on [0,1/2]
b) $g(x)$ decreases on [0,1]
c) $g(x)$ increases on [0,1]
d) $g(x)$ increases on [0,1/2] and decreases on [1/2,1]

64. Select the correct statement from (a), (b), (c), (d) The function $f(x)=xe^{1-x}$

a) Strictly increasing in the interval $\left(\frac{1}{2},2\right)$ b) Increasing in the interval $(0,\infty)$
c) Decreases in the interval $(0,2)$ d) Strictly decreasing in the interval $(1,\infty)$

65. If $\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+\cdots+\frac{a_{n-1}}{2}+a_n=0$. Then the function $f(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n$ has in $(0,1)$

a) At least one zero b) At most one zero c) Only 3 zeros d) Only 2 zeros

66. A particle is moving along the curve $x=at^2+bt+c$. If $ac=h^2$, then the particle would be moving with uniform

a) Rotation b) Velocity c) Acceleration d) Retardation

67. The approximate value of $(33)^{1/5}$ is

a) 2.0125 b) 2.1 c) 2.01 d) None of these

68. At an instant the diagonal of a square is increasing at the rate of 0.2cm/sec and the area is increasing at the rate of 6cm^2/sec. At that moment its side is

a) $\frac{30}{\sqrt{2}}$cm b) $30\sqrt{2}$ cm c) 30 cm d) 15 cm

69. A missile is fired from the ground level rises x metres vertically upwards in t seconds where $x=100t-\frac{25}{2}t^2$.The maximum height reached is

a) 200 m b) 125 m c) 160 m d) 190 m

70. The intercepts made by the tangent to the curve $y=\int_0^x |t|\,dt$, which is parallel to the line $y=2x$, on y-axis are equal to

a) $1,-1$ b) $-2,2$ c) 3 d) -3

71. The function $f(x) = \tan x - x$
a) Always increases
b) Always decreases
c) Never decreases
d) Some times increases and some times decreases

72. The maximum value of xy subject to $x + y = 8$, is
a) 8 b) 16 c) 20 d) 24

73. The tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$ at the point
a) (3, 9) b) (2, -1) c) (2, 1) d) (1, 2)

74. The point P of the curve $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line $4x - 3y + 2 = 0$ is given by
a) (2, 4) b) $(1, \sqrt{2})$ c) $(1/2, -1/2)$ d) $(1/8, -1/16)$

75. If the parametric equation of a curve given by $x = e^t \cos t, y = e^t \sin t$, then the tangent to the curve at the point $t = \pi/4$ makes with axis of x the angle
a) 0 b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

76. All points on the curve $y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ at which the tangents are parallel to the axis of x lie on a
a) Circle b) Parabola c) Line d) None of these

77. The point of inflexion for the curve $y = x^{5/2}$ is
a) (1, 1) b) (0, 0) c) (1, 0) d) (0, 1)

78. The minimum value of $2x + 3y$, when $xy = 6$, is
a) 12 b) 9 c) 8 d) 6

79. If $(x) = x^2 - 2x + 4$ on [1, 5], then the value of a constant c such that $\frac{f(5)-f(1)}{5-1} = f'(c)$, is
a) 0 b) 1 c) 2 d) 3

80. Let a, b be two distinct roots of a polynomial $f(x)$. Then there exists at least one root lying between a and b of the polynomial
a) $f(x)$ b) $f'(x)$ c) $f''(x)$ d) None of these

81. A population $p(t)$ of 1000 bacteria introduced into nutrient medium grows according to the relation$p(t) = 1000 + \frac{1000t}{100+t^2}$. The maximum size of this bacterial population is
a) 1100 b) 1250 c) 1050 d) 5250

82. If $f'(x) = (x-a)^{2n}(x-b)^{2m+1}$ where $m, n \in N$, then
a) $x = b$ is a point of minimum
b) $x = b$ is a point of maximum
c) $x = b$ is a point of inflexion
d) None of these

83. A point is moving on$y = 4 - 2x^2$. The x −coordinate of the point is decreasing at the rate of 5 unit per second. Then, the rate at which y-coordinate of the point is changing when the point is at (1,2)is
a) 5 units b) 10 units c) 15 units d) 20 units

84. The point of the curve $y^2 = 2(x-3)$ at which the normal is parallel to line $y - 2x + 1=0$
a) (5, 2) b) $\left(-\frac{1}{2}, -2\right)$ c) $(5, -2)$ d) $\left(\frac{3}{2}, 2\right)$

85. The function $f(x) = \frac{x}{1+|x|}$ is
a) Strictly increasing b) Strictly decreasing
c) Neither increasing nor decreasing d) Not differential at $x = 0$

86. The function $f(x) = 2x^3 - 3x^2 + 90x + 174$ is increasing in the interval
a) $\frac{1}{2} < x < 1$ b) $\frac{1}{2} < x < 2$ c) $3 < x < \frac{59}{4}$ d) $-\infty < x < \infty$

87. Let $f(x) = \begin{cases} |x|, & for\ 0 < |x| \le 2 \\ 1, & for\ \ x = 0 \end{cases}$, then at $x = 0, f$ has

a) A local maximum b) A local minimum c) No local extremum d) No local maximum

88. The set of values of a for which the function $f(x) = x^2 + ax + 1$ is an increasing function on $[1, 2]$ is
a) $(-2, \infty)$ b) $[-4, \infty)$ c) $[-\infty, -2)$ d) $(-\infty, 2]$

89. A particle moves along the curve$y = x^2 + 2x$. Then, The point on the curve such that x and y coordinates of the particle change with the same rate is
a) (1,3) b) $\left(\frac{1}{2}, \frac{5}{2}\right)$ c) $\left(-\frac{1}{2}, -\frac{3}{4}\right)$ d) $(-1, -1)$

90. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0. If\ P(-1) < P(1)$,then in the interval$[-1,1]$
a) $P(-1)$ is the minimum and $P(1)$is the maximum of P.
b) $P(-1)$ is not minimum but $P(1)$is the maximum of P.
c) $P(-1)$ is the minimum and $P(1)$is not the maximum of P.
d) Neither $P(-1)$ is the minimum nor $P(1)$is not the maximum of P.

91. If the equation $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x = 0$ has a positive root α, then the equation $n\, a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1 = 0$ has
a) A positive root less than α
b) A positive root larger than α
c) A negative root
d) No positive root

92. If the error committed in measuring the radius of the circle is 0.05%, then the corresponding error in calculating the area is
a) 0.05% b) 0.0025% c) 0.25% d) 0.1%

93. The edge of a cube is equal to the radius of the sphere. If the rate at which the volume of the cube is increasing is equal to λ, then the rate of increase of volume of the sphere is
a) $\frac{4\pi\lambda}{3}$ b) $4\pi\lambda$ c) $\frac{\lambda}{3}$ d) None of these

94. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where$\theta \in (0, \pi/2)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is
a) $\pi/3$ b) $\pi/6$ c) $\pi/8$ d) $\pi/4$

95. Roll's theorem is not applicable to the function $f(x) = |x|$ for $-2 \le x \le 2$ because
a) f is continuous for $-2 \le x \le 2$ b) f is not derivable for $x = 0$
c) $f(-2) = f(2)$ d) f is not a constant function

96. The abscissa of the point on the curve
$y = a(e^{x/a} + e^{-x/a})$
Where the tangent is parallel to the x-axis, is
a) 0 b) a c) $2a$ d) $-2a$

97. The value of a in order that $f(x) = \sin x - \cos x - ax + b$ decreases for all real values of x is given by
a) $a \ge \sqrt{2}$ b) $a < \sqrt{2}$ c) $a \ge 1$ d) $a < 1$

98. Let $f(x) = 1 + 2x^2 + 2^2x^4 + \ldots\ldots + 2^{10}x^{20}$.Then, $f(x)$ has
a) More than one minimum b) Exactly one minimum
c) At least one maximum d) None of the above

99. If the subnormal at any point on $y = a^{1-n}x^n$ is of constant length, then the value of n, is
a) 1 b) 1/2 c) 2 d) -2

100. The normal to the curve $x = a(1 + \cos\theta), y = a\sin\theta$ at θ always passes through the fixed point
a) $(a, 0)$ b) $(0, a)$ c) $(0, 0)$ d) (a, a)

101. If tangent to the curve $x = at^2, y = 2at$ is perpendicular to x-axis, then its point of contact is
a) (a, a) b) $(0, a)$ c) $(0, 0)$ d) $(a, 0)$

102. If $y = 4x - 5$ is tangent to the curve $y^2 = px^3 + q$ at $(2, 3)$ then (p, q) is

a) $(2,7)$ b) $(-2,7)$ c) $(-2,-7)$ d) $(2,-7)$

103. A particle is moving in a straight line. At time t, the distance between the particle from its starting point is given by $x = t - 6t^2 + t^3$. Its acceleration will be zero at

a) $t = 1$ unit time b) $t = 2$ units time c) $t = 3$ units time d) $t = 4$ units time

104. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at $(2, 3)$, then

a) $p = 2, q = -7$ b) $p = -2, q = 7$ c) $p = -2, q = -7$ d) $p = 2, q = 7$

105. Let the function $g: (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is

a) Even and is strictly increasing in $(0, \infty)$

b) Odd and is strictly decreasing in $(-\infty, \infty)$

c) Odd and is strictly increasing in $(-\infty, \infty)$

d) Neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

106. The tangent to the curve $y = 2x^2 - x + 1$ at a point P is parallel to $y = 3x + 4$, then the coordinates of $P$are

a) $(2, 1)$ b) $(1, 2)$ c) $(-1, 2)$ d) $(2, -1)$

107. Let a, b, c be positive real numbers and $ax^2 + b/x^2 \geq 2$ for all $x \in R^+$. Then,

a) $4ab \geq c^2$ b) $4ac \geq b^2$ c) $4bc \geq a^2$ d) $4ac < b^2$

108. The function $f(x) = x^4 - 62\,x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$ at $x = 1$. Then, the value of a is

a) 120 b) -120 c) 52 d) 60

109. The point on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$, the normal at which is parallel to the x-axis, is

a) $(0, 0)$ b) $(0, a)$ c) $(a, 0)$ d) (a, a)

110. The equation of the tangent to curve $y(2x-1)e^{2(1-x)}$ at the points its maximum, is

a) $y - 1 = 0$ b) $x - 1 = 0$ c) $x + y - 1 = 0$ d) $x - y + 1 = 0$

111. If for a function $f(x), f'(a) = 0, f''(a) = 0, f'''(a) > 0$, then at $x = a, f(x)$ is

a) Minimum b) Maximum c) Not an extreme point d) Extreme point

112. The function $f(x) = x + \sin x$ has

a) A minimum but no maximum

b) A maximum but no minimum

c) Neither maximum nor minimum

d) Both maximum and minimum

113. Gas is being pumped into a spherical balloon at the rate of $30ft^3$/min.Then, the rate at which the radius increases when it reaches the value 15ft is

a) $\frac{1}{15\pi}$ft/min b) $\frac{1}{30\pi}$ft/min c) $\frac{1}{20}$ft/min d) $\frac{1}{25}$ft/min

114. The equation of tangent to the curve $\frac{x^2}{3} - \frac{y^2}{2} = 1$, which is parallel to $y = x$, is

a) $y = x \pm 1$ b) $y = x - 1/2$ c) $y = x + 1/2$ d) $y = 1 - x$

115. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ cut each other orthogonally, then

a) $a^2 + b^2 = l^2 + m^2$ b) $a^2 - b^2 = l^2 - m^2$ c) $a^2 - b^2 = l^2 + m^2$ d) $a^2 + b^2 = l^2 - m^2$

116. A point moves along the curve $12y = x^3$ in such a way that the rate of increase of its ordinate is more than the rate of increase of abscissa. The abscissa of the point lies in the interval

a) $(-2, 2)$ b) $(-\infty, -2) \cup (2, \infty)$ c) $[-2, 2]$ d) None of these

117. The smallest circle with centre on y-axis and passing through the point(7,3)has radius

a) $\sqrt{58}$ b) 7 c) 3 d) 4

118. The point in the interval $[0, 2\pi]$, where $f(x) = e^x\ \sin x$ has maximum slope, is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) $\frac{3\pi}{2}$

119. The perimeter of a sector is$p$. The area of the sector is maximum, when its radius is

a) $\sqrt{p}$ b) $\frac{1}{\sqrt{p}}$ c) $\frac{p}{2}$ d) $\frac{p}{4}$

120. The normal at point (1,1) of the curve $y^2 = x^3$is parallel to the line

a) $3x - y - 2 = 0$ b) $2x + 3y - 7 = 0$ c) $2x - 3y + 1 = 0$ d) $2y - 3x + 1 = 0$

121. A particle moves in a straight line so that$s=\sqrt{t}$,then its acceleration is proportional to

a) $(\text{velocity})^3$ b) velocity c) $(\text{velocity})^2$ d) $(\text{velocity})^{3/2}$

122. If PQ and PR are the two sides of a triangle, then the angle between them which gives maximum area of the triangle, is

a) π b) $\pi/3$ c) $\pi/4$ d) $\pi/2$

123. The function $f(x)=a\cos x+b\tan x+x$ has extreme values at $x=0$ and $x=\frac{\pi}{6}$, then

a) $a=-\frac{2}{3}, b=-1$ b) $a=\frac{2}{3}, b=-1$ c) $a=-\frac{2}{3}, b=1$ d) $a=\frac{2}{3}, b=1$

124. The distance between the origin and the normal to the curve $y=e^{2x}+x^2$ at $x=0$ is

a) 2 b) $\frac{2}{\sqrt{3}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{1}{2}$

125. The function $f(x)=x\,e^{1-x}$ strictly

a) Increases in the interval $(0,\infty)$

b) Decreases in the interval $(0,2)$

c) Increases in the interval $(1/2,2)$

d) Decreases in the interval $(1,\infty)$

126. If f and g are two decreasing functions such that gof exists, then gof, is

a) An increasing function

b) A decreasing function

c) Neither increasing nor decreasing

d) None of these

127. The length of subnormal of parabola $y^2=4ax$ at any point is equal to

a) $\sqrt{2}a$ b) $2\sqrt{2}a$ c) $\frac{a}{\sqrt{2}}$ d) $2a$

128. If tangent to the curve $x=at^2, y=2at$ is perpandicular to x-axis, then its point of contact is

a) (a,a) b) $(0,a)$ c) $(0,0)$ d) $(a,0)$

129. The abscissa of the point on the curve $ay^2=x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

a) $2\,a/9$ b) $4\,a/9$ c) $-4\,a/9$ d) $-2\,a/9$

130. The point on the curve $y=2x^2-6x-4$ at which the tangent is parallel to the x-axis, is

a) $\left(\frac{3}{2},\frac{13}{2}\right)$ b) $\left(-\frac{5}{2},-\frac{17}{2}\right)$ c) $\left(\frac{3}{2},\frac{17}{2}\right)$ d) $\left(\frac{3}{2},-\frac{17}{2}\right)$

131. If the function $f(x)=x^3-6x^2+ax+b$ satisfies Rolle's theorem in the interval $[1,3]$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right)=0$, then

a) $a=-11$ b) $a=-6$ c) $a=6$ d) $a=11$

132. Let$g(x)=\begin{cases} 2e & if\ x\le 1 \\ \log(x-1), & if\ \ x>1 \end{cases}$. The equation of the normal to y=$g(x)$at the point (3,log 2), is

a) $y-2x=6+\log 2$ b) $y+2x=6+\log 2$ c) $y+2x=6-\log 2$ d) $y+2x=-6+\log 2$

133. If f is an increasing function and g is a decreasing function on an interval I such that fog exists, then

a) fog is an increasing function on I

b) fog is a decreasing function on I

c) fog is neither increasing nor decreasing on I

d) None of these

134. N characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha+\beta x^2$ seconds, α and β are constants. The optical value of x for fast processing is,

a) α/β b) β/α c) $\sqrt{\alpha/\beta}$ d) $\sqrt{\beta/\alpha}$

135. The longest distance of the point $(a,0)$ from the curve $2x^2+y^2-2\,x=0$, is given by

a) $\sqrt{1-2\,a+a^2}$ b) $\sqrt{1+2\,a+2a^2}$ c) $\sqrt{1+2\,a-a^2}$ d) $\sqrt{1-2\,a+2\,a^2}$

136. The coordinates of the point on the curve $y = x^2 - 3x + 2$ where the tangent is perpendicular to the straight line $y = x$ are

a) (0.2) b) (1.0) c) (−1,6) d) (2,−2)

137. The tangent and normal at the point $P(at^2, 2\,at)$ to the parabola $y^2 = 4\,ax$ meet the x-axis in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through T, P, G is

a) $\tan^{-1} t^2$ b) $\cot^{-1} t^2$ c) $\tan^{-1} t$ d) $\cot^{-1} t$

138. The normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that

a) It is at a constant distance from the origin
b) It passes through $\left(\frac{a\pi}{2}, -a\right)$
c) It makes angle $\frac{\pi}{2} - \theta$ with the x-axis
d) It passes through the origin

139. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

a) Increasing on $\left[-\frac{1}{2}, 1\right]$ b) Decreasing on R c) Increasing on R d) Decreasing on $\left[-\frac{1}{2}, 1\right]$

140. The function $f(x) = 2x^3 - 15x^2 + 36x + 4$ is maximum at

a) $x = 2$ b) $x = 4$ c) $x = 0$ d) $x = 3$

141. A particle moves on the parabola $y^2 = 4ax$ in such a way that its projection on the y-axis has a constant velocity. Then its projection on x-axis moves with

a) Constant velocity b) Constant acceleration c) Variable velocity d) Variable acceleration

142. The points of extremum of the function $\phi(x) = \int_1^x e^{-t^2/2}(1 - t^2)dt$, are

a) $x = 0, 1$ b) $x = 1, -1$ c) $x = 1/2$ d) $x = -1/2$

143. A stone is thrown vertically upwards and the height x ft reached by the stone in t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in

a) 2s b) 2.5s c) 3s d) 1.5s

144. If $ax^2 + bx + 4$ attains its minimum value -1 at $x = 1$, then the values of $a$and b are respectively

a) $5, -10$ b) $5, -5$ c) $5, 5$ d) $10, -5$

145. The function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing on

a) $(0, \infty)$ b) $(-\infty, 0)$ c) $(-\infty, \infty)$ d) None of these

146. Let $f(x) = e^x \sin x$, slope of the curve $y = f(x)$ is maximum at $x = a$, if 'a' equals

a) 0 b) $\pi/4$ c) $\pi/2$ d) None of these

147. The slope of the tangent to the curve $y = \sqrt{9 - x^2}$ at the point where ordinate and abscissa are equal, is

a) 1 b) -1 c) 0 d) None of these

148. If $0 < x < \frac{\pi}{2}$, then

a) $\cos(\sin x) > \cos x$
b) $\cos(\sin x) < \cos x$
c) $\cos(\sin x) = \sin(\cos x)$
d) $\cos(\sin x) < \sin(\cos x)$

149. In the mean value theorem $f(b) - f(a) = (b - a)f'(c)$, if $a = 4, b = 9$ and $f(x) = \sqrt{x}$, then the value of c is

a) 8.00 b) 5.25 c) 4.00 d) 6.25

150. If the function $f(x) = (2a - 3)(x + 2\sin 3) + (a - 1)(\sin^4 x + \cos^4 x) + \log 2$ does not possess critical points, then

a) $a \in (-\infty, 4/3) \cup (2, \infty)$
b) $a \in (4/3, 2)$
c) $a \in (4/3, \infty)$
d) $a \in (2, \infty)$

151. If $s = ae^t + be^{-t}$ is the equation of motion of a particle, then its acceleration is equal to

a) s b) $2s$ c) $3s$ d) $4s$

152. The angle of intersection of the curves $y = x^2$ and $x = y^2$ is

a) $\tan^{-1}\left(\frac{4}{3}\right)$ b) $\tan^{-1}(1)$ c) $90°$ d) $\tan^{-1}\left(\frac{3}{4}\right)$

153. A spherical balloon is expanding. If the radius is increasing at the rate of 2 cm/min, the rate at which the volume increase (in cubic centimeters per minute)when the radius is 5 cm, is

a) 10π b) 100π c) 200π d) 50π

154. If the radius of a circle be increasing at a uniform rate of 2cm/s. The area of increasing of area of circle ,at the instant when the radius is 20 cm, is

a) $70\pi cm^2/s$ b) $70cm^2/s$ c) $80\pi cm^2/s$ d) $80\ cm^2/s$

155. The abscissa of the points, where the tangent to curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to x − axis , are

a) $x = 0\ and\ 0$ b) $x = 1\ and - 1$ c) $x = 1\ and\ - 3$ d) $x = -1\ and\ 3$

156. If $f(x) = x^3 - 6x^2 + 9x + 3$ be a decreasing function, then x lies in

a) $(-\infty, -1) \cap (3, \infty)$ b) $(1, 3)$ c) $(3, \infty)$ d) None of these

157. If the curve $y = ax^3 + bx^2 + cx$ is inclined at 45° to x-axis at $(0, 0)$ but touches x-axis at $(1, 0)$, then

a) $a = 1, b = -2, c = 1$ b) $a = 1, b = 1, c = -2$ c) $a = -2, b = 1, c = 1$ d) $a = -1, b = 2, c = 1$

158. The value of x for which

$1 + x \log_e\left(x + \sqrt{x^2 + 1}\right) \geq \sqrt{x^2 + 1}\ are$

a) $x \leq 0$ b) $0 \leq x \leq 1$ c) $x \geq 0$ d) None of these

159. The function $f(x) = x^{-x}, (x \in R)$ attains a maximum, value at x which is

a) 2 b) 3 c) $\frac{1}{e}$ d) 1

160. The value of c in $(0, 2)$ satisfying the Mean value theorem for the function $f(x) = x(x - 1)^2, x \in [0, 2]$ is equal to

a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$

161. If the ratio of base radius and height of a cone is 1 ∶ 2 and percentage error in radius is λ %, then the error in its volume is

a) λ % b) 2λ % c) $3\ \lambda$% d) None of these

162. The values of a in order that $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of x, is given by

a) $a < 1$ b) $a \geq 1$ c) $a \leq \sqrt{2}$ d) $a < \sqrt{2}$

163. The function $f(x) = \cos(\pi/x)$ is increasing in the interval

a) $(2n + 1, 2\,n), n \in N$

b) $\left(\frac{1}{2\,n + 1}, 2\,n\right), n \in N$

c) $\left(\frac{1}{2\,n + 2}, \frac{1}{2\,n + 1}\right), n \in N$

d) None of these

164. If the curves $\frac{x^2}{a^2} + \frac{y^2}{12} = 1$ and $y^3 = 8x$ intersect at right angle then the value of $a^2$is equal to

a) 16 b) 12 c) 8 d) 4

165. Let $f(x)$ and $g(x)$ be differentiable for $0 \leq x \leq 1$, such that $f(0) = 2, g(0) = 0, f(1) = 6$. Let there exist a real number c in $[0, 1]$ such that $f'(c) = 2g'(c)$, then the value of $g(1)$ must be

a) 1 b) 2 c) -2 d) -1

166. If a differentiable function $f(x)$ has a relative minimum at $x = 0$, then the function $\phi(x) = f(x) + ax + b$ has a relative minimum at $x = 0$ for

a) All a and all b b) All b if $a = 0$ c) All $b > 0$ d) All $a > 0$

167. The point at which the tangent to the curve $y = 2x^2 - x + 1$ is parallel to $y = 3x + 9$, will be

a) $(2, 1)$ b) $(1, 2)$ c) $(3, 9)$ d) $(-2, 1)$

168. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

a) 5 b) -5 c) $1/5$ d) None of these

169. For the curve $y^n = a^{n-1}x$ if the subnormal at any point is a constant, then n is equal to

a) 1 b) 2 c) -2 d) -1

170. On which of the following intervals is the function $f(x) = 2x^2 - \log|x|, x \neq 0$ increasing?

a) $(1/2, \infty)$

b) $(-\infty, -1/2) \cup (1/2, \infty)$

c) $(-\infty, -1/2) \cup (0, 1/2)$

d) $(-1/2, 0) \cup (1/2, \infty)$

171. The abscissa of the point on the curve $ay^2 - x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

a) $\frac{2a}{9}$ b) $\frac{4a}{9}$ c) $-\frac{4a}{9}$ d) $-\frac{2a}{9}$

172. If $f(x) = \sin x - \cos x$, the interval in which function is decreasing in $0 \leq x \leq 2\pi$, is

a) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$ b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ d) None of these

173. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has local minimum at

a) $x = -2$ b) $x = 0$ c) $x = 1$ d) $x = 2$

174. The least value of the $f(x)$ given by $f(x) = \tan^{-1} x - \frac{1}{2}\log_e x$ in the interval $[1/\sqrt{3}, \sqrt{3}]$, is

a) $\frac{\pi}{6} + \frac{1}{4}\log_e 3$ b) $\frac{\pi}{3} - \frac{1}{4}\log_e 3$ c) $\frac{\pi}{6} - \frac{1}{4}\log_e 3$ d) $\frac{\pi}{3} + \frac{1}{4}\log_e 3$

175. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

a) $a > 0, b > 0$ b) $a > 0, b < 0$ c) $a < 0, b < 0$ d) Data is insufficient

176. Let y be the number of people in a village at time t. Assume that the rate of change of the population is proportional to the number of people in the village at any time and further assume that the population never increase in time. Then, the population of the village at any fixed t is given by

a) $y = e^{kt} + c$, for some constants $c \leq 0$ and $k \geq 0$ b) $y = ce^{kt}$, for some constants $c \geq 0$ and $k \leq 0$

c) $y = e^{ct} + k$, for some constants $c \leq 0$ and $k \geq 0$ d) $y = k\, e^{ct}$, for some constants $c \geq 0$ and $k \leq 0$

177. The equation of the tangent to the curve $x^2 - 2xy + y^2 + 2x + y - 6 = 0$at $(2,2)$ is

a) $2x + y - 6 = 0$ b) $2y + x - 6 = 0$ c) $x + 3y - 8 = 0$ d) $3x + y - 8 = 0$

178. The length of the normal to the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta) at\ \theta = \frac{\pi}{2} is$

a) 2a b) $\frac{a}{2}$ c) $\frac{a}{\sqrt{2}}$ d) $\sqrt{2a}$

179. The maximum value of function$f(x) = \sin x\,(1 + \cos x), x \in R$ is

a) $\frac{3^{3/2}}{4}$ b) $\frac{3^{5/3}}{4}$ c) $\frac{3}{2}$ d) $\frac{3^{7/5}}{4}$

180. The minimum value of $2x^2 + x - 1$ is

a) $-\frac{1}{4}$ b) $\frac{3}{2}$ c) $-\frac{9}{8}$ d) $\frac{9}{8}$

181. If $x = t^2$ and $y = 2t$, then equation of the normal at $t = 1$, is

a) $x + y - 3 = 0$ b) $x + y - 1 = 0$ c) $x + y + 1 = 0$ d) $x + y + 3 = 0$

182. The side of an equilateral triangle is $'a'$ units and is increasing at the rate of λ units/sec. The rate of increase of its area is

a) $\frac{2}{\sqrt{3}}\lambda\, a$ b) $\sqrt{3}\lambda\, a$ c) $\frac{\sqrt{3}}{2}\lambda\, a$ d) None of these

183. If a and b are positive numbers such that $a > b$, then the minimum value of a sec $\theta - b\tan\theta\left(0 < \theta < \frac{\pi}{2}\right)$ is

a) $\frac{1}{\sqrt{a^2 - b^2}}$ b) $\frac{1}{\sqrt{a^2 + b^2}}$ c) $\sqrt{a^2 + b^2}$ d) $\sqrt{a^2 - b^2}$

184. If $y = x^n$, then the ratio of relative errors in y and x is

a) $1 : 1$ b) $2 : 1$ c) $1 : n$ d) $n : 1$

185. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
a) 5 b) 7 c) 1 d) 3

186. The function $f(x) = x^3 + ax^2 + bx + c, a^2 \leq 3b\ has$
a) One maximum value
b) One minimum value
c) No extreme value
d) One maximum and one minimum value

187. The fixed point P on the curve $y = x^2 - 4x + 5$ such that the tangent at P is perpendicular to the line $x + 2y - 7 = 0$ is given by
a) (3, 2) b) (1, 2) c) (2, 1) d) None of these

188. If the area of the triangle, included between the axes and any tangent to the curve $xy^n = a^{n+1}$ is constant, then the value of n is
a) -1 b) -2 c) 1 d) 2

189. The radius of a circular plate is increasing at the rate of 0.01 cm/s when the radius is 12 cm. Then , The rate at which the area increase, is
a) $0.24\,\pi$ sq cm/s b) $60\,\pi$ sq cm/s c) $24\,\pi$ sq cm/s d) $1.2\,\pi$ sq cm/s

190. If $g(x) = \min(x, x^2)$ where x is real number, then
a) $g(x)$ is an increasing function
b) $g(x)$ is a decreasing function
c) $g(x)$is a constant function
d) $g(x)$ is a continuous function except at $x = 0$

191. The angle between the curves $y = a^x$ and $y = b^x$ is equal to
a) $\tan^{-1}\left(\left|\frac{a-b}{1+ab}\right|\right)$
b) $\tan^{-1}\left(\left|\frac{a+b}{1-ab}\right|\right)$
c) $\tan^{-1}\left(\left|\frac{\log b+\log a}{1+\log a \log b}\right|\right)$
d) $\tan^{-1}\left(\left|\frac{\log a-\log b}{1+\log a \log b}\right|\right)$

192. The function which is neither decreasing nor increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, is
a) $\operatorname{cosec} x$ b) $\tan x$ c) x^2 d) $|x - 1|$

193. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point
a) 0 b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

194. A function f is defined by $f(x) = e^x \sin x$ in$[0, \pi]$. Which of the following is not correct?
a) f is continous in $[0, \pi]$
b) f is differentiable in $[0, \pi]$
c) $f(0) = f(\pi)$
d) Rolle's theorem is not true in $[0, \pi]$

195. If $xy = c^2$, then minimum value of $ax + by$ is
a) $c\sqrt{ab}$ b) $2c\sqrt{ab}$ c) $-c\sqrt{ab}$ d) $-2c\sqrt{ab}$

196. If $x - 2y = 4$, the minimum value of xy is
a) -2 b) 0 c) 0 d) -3

197. The function $f(x) = (9 - x^2)^2$ increasing in
a) $(-3, 0) \cup (3, \infty)$ b) $(-\infty, -3) \cup (3, \infty)$ c) $(-\infty, -3) \cup (0, 3)$ d) $(-3, 3)$

198. The real number x when added to its inverse gives the minimum value of the sum at x equals to
a) 2 b) 1 c) -1 d) -2

199. The points on the curve $12y = x^3$ whose ordinate and abscissa change at the same rate, are
a) $(-2, -2/3), (2, 2/3)$ b) $(-2/3, -2), (2/3, 2)$ c) $(-2, -2/3)$ only d) $(2/3, 2)$ only

200. Let $P(2, 2)$ and $Q(1/2, -1)$ be two points on the parabola $y^2 = 2x$. The coordinates of the point R on the parabola $y^2 = 2x$, where the tangent to the curve is parallel to the chord PQ, are
a) $(2, -1)$ b) $(1/8, 1/2)$ c) $(\sqrt{2}, 1)$ d) $(-\sqrt{2}, 1)$

201. If $f(x) = \frac{1}{x+1} - \log(1 + x)\,, x > 0$, then f is
a) an increasing function
b) a decreasing function
c) both increasing and decreasing function
d) None of the above

202. The equation of the tangent to the curve $x = 2\cos^3\theta$ and $y = 3\sin^3\theta$ at the point, $\theta = \pi/4$ is

a) $2x + 3y = 3\sqrt{2}$ b) $2x - 3y = 3\sqrt{2}$ c) $3x + 2y = 3\sqrt{2}$ d) $3x - 2y = 3\sqrt{2}$

203. The length of the subtangent to the curve $x^2 + xy + y^2 = 7$ at $(1, -3)$ is

a) 3 b) 5 c) $\frac{3}{5}$ d) 15

204. The line $(x/a) + (y/b) = 2$, touches the curve $\frac{x^n}{a^n} + \frac{y^n}{b^n} = 2$, at

a) (b, a) b) $(-b, -a)$ c) (a, b) d) None of these

205. The tangent to the curve $y = x^3 - 6x^2 + 9x + 4, 0 \leq x \leq 5$ has maximum slope at x which is equal to

a) 2 b) 3 c) 4 d) None of these

206. If ST and SN are the lengths of the subtangent and the subnormal at the point $\theta = \frac{\pi}{2}$ on the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta), a \neq 1$, then

a) $ST = SN$ b) $ST = 2SN$ c) $ST^2 = aSN^3$ d) $ST^3 = aSN$

207. Any tangent to the curve $y = 2x^5 + 4x^3 + 7x + 9$

a) Is parallel to x-axis

b) Is parallel to y-axis

c) Makes an acute angle with the x-axis

d) Makes an obtuse angle with x-axis

208. A circular metal plate is heated so that its radius increases at a rate of 0.1 mm per minute. Then the rate at which the plate's area is increasing when the radius is 50 cm is

a) $10\,\pi$ mm/minute b) $100\,\pi$ mm/minute c) π mm/minute d) $-\pi$ mm/minute

209. The two tangents to the curve $ax^2 + 2hxy + by^2 = 1, a > 0$ at the points where it crosses x-axis, are

a) Parallel

b) Perpendicular

c) Inclined at an angle of $\pi/4$

d) None of these

210. The maximum value of xy when $x + 2y = 8$ is

a) 20 b) 16 c) 24 d) 8

211. $f(x)$ satisfies the conditions of Rolle's theorem in [1, 2] and $f(x)$ is continuous in [1, 2], then $\int_1^2 f'(x)\,dx$ is equal to

a) 3 b) 0 c) 1 d) 2

212. The angle between the tangents to the curve $y^2 = 2ax$ at the point where $x = \frac{a}{2}$, is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

213. If the velocity v of a particle moving along a straight line and its distance s from a fixed point on the line are related by $v^2 = a^2 + s^2$, then its acceleration equals

a) as b) s c) s^2 d) $2s$

214. The value of x for which the polynomial $2x^3 - 9x^2 + 12x + 4$ is a decreasing function of x, is

a) $-1 < x < 1$ b) $0 < x < 2$ c) $x > 3$ d) $1 < x < 2$

215. The sum of two numbers is 20. If the product of the square of one number and cube of the other is maximum, then the numbers are

a) 12, 8 b) 3, 4 c) 9, 12 d) 15, 18

216. Coordinates of a point on the curve $y = x\log x$ at which the normal is parallel to the line $2x - 2y = 3$, are

a) $(0, 0)$ b) (e, e) c) $(e^2, 2e^2)$ d) $(e^{-2}, -2e^{-2})$

217. Which of the following functions is not decreasing on $(0, \pi/2)$?

a) $\cos x$ b) $\cos 2x$ c) $\cos^2 x$ d) $\tan x$

218. If $f(x) = (ab - b^2 - 2)x + \int_0^x (\cos^4\theta + \sin^4\theta)d\theta$ is decreasing function of x for all $x \in R$ and $b \in R$, b being independent of x, then

a) $a \in (0, \sqrt{6})$ b) $a \in (-\sqrt{6}, \sqrt{6})$ c) $a \in (-\sqrt{6}, 0)$ d) None of these

219. Consider the following statements S and R :

S: Both $\sin x$ and $\cos x$ are decreasing function in $\left(\frac{\pi}{2}, \pi\right)$

R: If a differentiating function decreases in (a, b) then its derivative also decreases in (a, b)

Which of the following is true?

a) Both S and R are wrong

b) Both S and R are correct but R is not the correct explanation for S

c) S is correct and R is the correct explanation for S

d) S is correct, R is wrong

220. Let $f(x) = \int e^x (x-1)(x-2)dx$. Then, f decreases in the interval

a) $(-\infty, -2)$ b) $(-2, -1)$ c) $(1, 2)$ d) $(2, \infty)$

221. A spherical balloon is being inflated at the rate of $30cc$ /min. The rate of increase of the surface area of the balloon when its diameter is 14cm, is

a) 7 sq cm/min b) 10 sq cm/min c) 17.5 sq cm/min d) 28 sq cm/min

222. A man 2 metres tall walks away from a lamp post 5 metres height at the rate of 4.8 km/hr. The rate of increase of the length of his shadow is

a) 1.6 km/hr b) 6.3 km/hr c) 5 km/hr d) 3.2 km/hr

223. The equation of the tangents at the origin to the curve $y^2 = x^2(1+x)$ are

a) $y = \pm x$ b) $x = \pm y$ c) $y = \pm 2x$ d) None of these

224. A particle moves along a straight line with the law of motion given by $s^2 = at^2 + 2bt + c$. then the acceleration varies are

a) $\frac{1}{s^3}$ b) $\frac{1}{s}$ c) $\frac{1}{s^4}$ d) $\frac{1}{s^2}$

225. If a particle moving along a line follows the law $t = as^2 + bs + c$, then the retardation of the particle is proportional to

a) Square of the velocity

b) Cube of the displacement

c) Cube of the velocity

d) Square of the displacement

226. Let $f(x) = x^3 - 6x^2 + 12x - 3$. Then at $x = 2$, $f(x)$ has

a) A maximum

b) A minimum

c) Both a maximum and a minimum

d) Neither a maximum nor a minimum

227. The set of all values of a for which the function

$$f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$$

Decreases for all real x is

a) $(-\infty, \infty)$

b) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup [1, \infty)$

c) $\left(-3, 5 - \frac{\sqrt{27}}{2}\right) \cup (2, \infty)$

d) $[1, \infty)$

228. A sphere of radius 100 mm shrinks to radius 98 mm, then the approximate decrease in its volume is

a) $12000\,\pi$ mm^3 b) $800\,\pi$ mm^3 c) $80000\,\pi$ mm^3 d) $120\,\pi$ mm^3

229. The value of c in Lagrange's mean value theorem for the function $f(x) = x(x-2)$ when $x \in [1,2]$, is

a) 1 b) 1/2 c) 2/3 d) 3/2

230. If the curves $x^2 = 9A(9-y)$ and $x^2 = A(y+1)$ intersect orthogonally, then the value of A is

a) 3 b) 4 c) 5 d) 7

231. The length of subtangent to the curve $x^2y^2 = a^4$ at the point $(-a, a)$ is

a) $3a$ b) $2a$ c) a d) $4a$

232. The real number x when added to its inverse gives the minimum value of the sum at x which is equal to

a) -2 b) 2 c) 1 d) -1

233. The equation of the normal line to the curve $y = x \log_e x$ parallel to $2x - 2y + 3 = 0$ is

a) $x + y = 3e^{-2}$ b) $x - y = 6e^{-2}$ c) $x - y = 3e^{-2}$ d) $x - y = 6e^2$

234. Let $f(x)$ and $g(x)$ are defined and differentiable for $x \geq x_0$ and $f(x_0) = g(x_0), f'(x) > g'(x)$ for $x > x_0$, then

a) $f(x) < g(x)$ for some $x > x_0$

b) $f(x) = g(x)$ for some $x > x_0$

c) $f(x) > g(x)$ for all $x > x_0$

d) None of these

235. If the semi-vertical angle of a cone is 45°, then the rate of change of volume of the cone is

a) Curved surface area times the rate of change of r

b) Base area times the rate of change of l

c) Base area times the rate of change of r

d) None of these

236. For the curve $y = x\,e^x$, the point

a) $x = -1$ is a point of minimum

b) $x = 0$ is a point of minimum

c) $x = -1$ is a point of maximum

d) $x = 0$ is a point of maximum

237. The function $f(x) = \log_e\left(x^3 + \sqrt{x^6 + 1}\right)$ is of the following types:

a) Even and increasing b) Odd and increasing c) Even and decreasing d) Odd and decreasing

238. The slope of the tangent to the curve $x = 3t^2 + 1, y = t^3 - 1$, at $x = 1$is

a) 0 b) $\frac{1}{2}$ c) ∞ d) -2

239. If the function $f: R \to R$ be defined by $f(x) = \tan x - x$, then $f(x)$

a) Increases b) Decreases c) Remains constant d) Becomes zero

240. The extreme values of

$4\cos(x^2)\cos\left(\frac{\pi}{3} + x^2\right)\cos\left(\frac{\pi}{3} - x^2\right)$ over R, are

a) -1, 1 b) -2, 2 c) -3, 3 d) -4, 4

241. If the rate of change of sine of an angle θ is k, then the rate of change of its tangent is

a) k^2 b) $\frac{1}{k^2}$ c) k d) $\frac{1}{k}$

242. The maximum and minimum values of $y = \frac{ax^2+2bx+c}{Ax^2+2Bx+C}$ are those for which

a) $ax^2 + 2\,bx + c - y(Ax^2 + 2\,B\,x + C)$ is equal to zero

b) $ax^2 + 2\,bx + c - y(Ax^2 + 2\,Bx + C)$ is a perfect square

c) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{d\,x^2} = 0$

d) $ax^2 + 2\,bx + c - y(Ax^2 + 2\,B\,x + C)$ is not a perfect square

243. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

244. Let the function $f: R \to R$ be defined by $f(x) = 2x + \cos x$, then f

a) Has a minimum at $x = \pi$

b) Has a maximum at $x = 0$

c) Is decreasing on R

d) Is increasing function on R

245. The slope of the tangent to the curves $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

a) 22/7　　b) 6/7　　c) -6　　d) -7

246. The function $x\sqrt{1-x^2}, (x > 0)$ has

a) A local maxima　　b) A local minima
c) Neither a local maxima nor a local minima　　d) None of the above

247. The attitude of a right circular cone of minimum volume circumscribed about a sphere of radius r is

a) $2\,r$　　b) $3\,r$　　c) $5\,r$　　d) $3/2\,r$

248. If a particle moves according to the law $s = 6t^2 - \frac{t^3}{2}$, then the time at which it is momentarily at rest is

a) $t = 0$ only　　b) $t = 8$ only　　c) $t = 0, 8$　　d) None of these

249. If $f(x) = x^2 + 4x + 1$, then

a) $f(x) = f(-x)$, for all x　　b) $f(x) \neq 1$, for all x=0
c) $f''(x) > 0$,for all x　　d) $f(x) > 1$,for $x \leq 4$

250. The function $f(x) = \cot^{-1} x + x$ increasing in the interval

a) $(1, \infty)$　　b) $(-1, \infty)$　　c) $(-\infty, \infty)$　　d) $(0, \infty)$

251. If the normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{2}$ with the positive x-axis, then $f'(3)$ is equal to

a) -1　　b) $-\frac{3}{4}$　　c) $\frac{4}{3}$　　d) 1

252. A minimum value of $\int_0^x te^{t^2} dt$ is

a) 0　　b) 1　　c) 2　　d) 3

253. The equation of motion of a particle moving along a straight line is s=$2t^3 - 9t^2 + 12t$, where the units of s and t are centimeter and second. The acceleration of the particle will be zero after

a) $\frac{3}{2}s$　　b) $\frac{2}{3}s$　　c) $\frac{1}{2}s$　　d) 1s

254. The minimum value of $x^2 + \frac{1}{1+x^2}$ is at

a) $x = 0$　　b) $x = 1$　　c) $x = 4$　　d) $x = 3$

255. The edge of a cube is equal to the radius of a sphere. If the edge and the radius increase at the same rate, then the ratio of the increases in surface areas of the cube and sphere is

a) $2\pi : 3$　　b) $3 : 2\pi$　　c) $6 : \pi$　　d) None of these

256. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-5)^5 dt$ has local minimum at $x =$

a) 0　　b) 1　　c) 2　　d) 3

257. The slope of the tangent to the curve $x = 3t^2 + 1, y = t^3 - 1$ at $x = 1$ is

a) $\frac{1}{2}$　　b) 0　　c) -2　　d) ∞

258. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope is

a) $\frac{\pi}{4}$　　b) $\frac{\pi}{2}$　　c) π　　d) None of these

259. The equation of the normal to the curve $y = e^{-2|x|}$ at the point where the curve cuts the line $x = 1/2$, is

a) $2e(ex + 2y) = e^2 - 4$
b) $2e(ex - 2y) = e^2 - 4$
c) $2e(ey - 2x) = e^2 - 4$
d) None of these

260. In [0, 1] Lagranges Mean value theorem is NOT applicable to

a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$　　b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

c) $f(x) = x|x|$　　d) $f(x) = |x|$

261. If θ is the angle between the curves $xy = 2$ and $x^2 + 4y = 0$, then $\tan\theta$ is equal to

a) 1 b) -1 c) 2 d) 3

262. A stone thrown upwards, has equation of motion $s = 490t - 49t^2$.Then, the maximum height reached by it ,is

a) 24500 b) 12500 c) 12250 d) 25400

263. If $f(x)$ is a function given by

$$f(x) = \begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix}, \text{where } 0 < a < b < \frac{\pi}{2}$$

Then the equation $f'(x) = 0$

a) Has at least one root in (a, b)

b) Has at most one root in (a, b)

c) Has exactly one root in (a, b)

d) Has no root in (a, b)

264. If $\phi(x)$ is continuous at $x = \alpha$ such that $\phi(\alpha) < 0$ and $f(x)$ is a function such that $f'(x) = (ax - a^2 - x^2)\phi(x)$ for all x, then $f(x)$ is

a) Increasing in the neighbourhood of $x = \alpha$

b) Decreasing in the neighbourhood of $x = \alpha$

c) Constant in the neighbourhood of $x = \alpha$

d) Minimum at $x = \alpha$

265. The point on the curve $y = x^3$ at which the tangent to the curve is parallel to the x-axis, is

a) (2, 2) b) (3, 3) c) (4, 4) d) (0, 0)

266. The function $f(x) = \cot^{-1} x + x$ increases in the interval

a) $(1, \infty)$ b) $(-1, \infty)$ c) $(-\infty, \infty)$ d) $(0, \infty)$

267. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on $[1, 3]$, satisfies the Rolle's theorem for $c = \frac{2\sqrt{3}+1}{\sqrt{3}}$, then

a) $a = 11, b = 6$ b) $a = -11, b = 6$ c) $a = 11, b \in R$ d) None of these

268. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function is

a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ c) $\left(0, \frac{\pi}{2}\right)$ d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

269. The height of a cylinder is equal to the radius. If an error of $\alpha\%$ is made in the height, then percentage error in its volume is

a) α % b) 2α % c) 3α % d) None of these

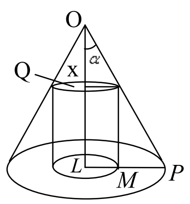
270. The condition $f(x) = x^3 + px^2 + qx + r (x \in R)$ to have no extreme value, is

a) $p^2 < 3q$ b) $2p^2 < q$ c) $p^2 < \frac{1}{4}q$ d) $p^2 > 3q$

271. Given that f is a real valued differentiable function such that $f(x)f''(x) < 0$ for all $x \in R$. It follows that

a) $f(x)$ is increasing b) $f(x)$ is decreasing c) $|f(x)|$ is increasing d) $|f(x)|$ is decreasing

272. A given right circular cone has volume p and the largest right circular cylinder that can be inscribed in the cone has a volume$q$. Then, $p: q$ is



a) 9: 4 b) 8: 3 c) 7: 2 d) None of the above

273. The function $f(x) = \frac{\log x}{x}$ is increasing in the interval

a) $(1, 2e)$ b) $(0, e)$ c) $(2, 2e)$ d) $(1/e, 2e)$

274. The maximum value $x^3 - 3x$ in the interval $[0, 2]$ is

a) -2 b) 0 c) 2 d) 1

275. The function $f(x) = ax + \frac{b}{x}, b, x > 0$ takes the least value at x equal to

a) b b) $\sqrt{a}$ c) $\sqrt{b}$ d) $\sqrt{\frac{b}{a}}$

276. The maximum area of the rectangle that can be inscribed in a circle of radius r, is

a) πr^2 b) r^2 c) $\pi r^2/4$ d) $2r^2$

277. If $f(x) = k\,x^3 - 9\,x^2 + 9\,x + 3$ is increasing on R, then

a) $k < 3$ b) $k > 3$ c) $k \leq 3$ d) None of these

278. A spherical iron ball of radius 10 cm, coated with a layer of ice of uniform thickness, melts at a rate of 100π cm^3/min.The rate at which the thickness of decreases when the thickness of ice is 5 cm, is

a) 1cm/min b) 2 cm/min c) $\frac{1}{376}$cm/min d) 5 cm/min

279. The minimum value of $\left(1 + \frac{1}{\sin^n \alpha}\right)\left(1 + \frac{1}{\cos^n \alpha}\right)$, is

a) 1 b) 2 c) $\left(1 + 2^{n/2}\right)^2$ d) 4

280. The number of critical points of $f(x) = |x|(x-1)(x-2)(x-3)$ is

a) 1 b) 2 c) 3 d) 4

281. A particle moves on a line according to the law $s = at^2 + bt + c$. If the displacement after 1 sec is 16 cm, the velocity after 2 sec is 24 cm/sec and acceleration is 8cm/ sec^2 , then

a) $a = 4, b = 8, c = 4$ b) $a = 4, b = 4, c = 8$ c) $a = 8, b = 4, c = 4$ d) None of these

282. At what point on the curve$x^3 - 8a^2y = 0$, the slope of the normal is $-\frac{2}{3}$?

a) (a, a) b) $(2a, -a)$ c) $(2a, a)$ d) None of these

283. The equation of tangent to the curve $y = be^{-x/a}$ at the point where it crosses y –axis, is

a) $ax + by = 1$ b) $ax - by = 1$ c) $\frac{x}{a} - \frac{y}{b} = 1$ d) $\frac{x}{a} + \frac{y}{b} = 1$

284. The function $f(x) = x^2e^{-x}$ increases in the interval

a) (0, 2) b) (2, 3) c) (3, 4) d) (4, 5)

285. The slope of the tangent to the curve $y = \cos^{-1}(\cos x)\ at\ x = -\frac{\pi}{4}$, is

a) 1 b) 0 c) 2 d) -1

286. Let $f: R \longrightarrow R$ be defined by $f(x) = \begin{cases} k - 2x, & if\ x \leq -1 \\ 2x + 3, & if x > -1 \end{cases}$

If $f$has a local minimum at $x = -1$, then a possible value of k is

a) 1 b) 0 c) $-\frac{1}{2}$ d) -1

287. The minimum value of $2\log_{10} x - \log_x 0.01, x > 1$, is

a) 1 b) -1 c) 2 d) 1/2

288. Let $f(x) = \sqrt{x-1} + \sqrt{x + 24 - 10\sqrt{x-1}}, 1 \leq x \leq 26$ be real valued function, then $f'(x)$ for $1 < x < 26$ is

a) 0 b) $\frac{1}{\sqrt{x-1}}$ c) $2\sqrt{x-1} - 5$ d) None of these

289. A function f is defined by $f(x) = 2 + (x-1)^{2/3}$ in [0, 2]. Which of the following is not correct?

a) f is not derivable in (0, 2) b) f is continuous in [0, 2]
c) $f(0) = f(2)$ d) Rolle's Theorem is true in [0, 2]

290. The number of values of x where $f(x) = \cos x + \cos\sqrt{2}x$ attains its maximum is

a) 1 b) 0 c) 2 d) Infinite

291. The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is

a) $\frac{1}{14}$ b) 0.01 c) $\frac{1}{7}$ d) None of these

292. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$, is

a) Increasing on $[0, \infty)$
b) Decreasing on $[0, \infty)$
c) Increasing on $[0, \pi/e)$ and decreasing on $[\pi/e, \infty)$
d) Decreasing on $[0, \pi/e)$ and increasing on $[\pi/e, \infty)$

293. Let $f(x) = x^3$ use Mean value theorem to write $\frac{f(x+h)-f(x)}{h} = f'(x+\theta h)$ with $0 < \theta < 1$. If $x \neq 0$, then $\lim_{h \to 0} \theta$ is equal to
a) -1 b) -0.5 c) 0.5 d) 1

294. For the parabola $y^2 = 4ax$, the ratio of the subtangent to the abscissa is
a) 1 : 1 b) 2 : 1 c) $x : y$ d) $x^2 : y$

295. If $ab = 2a + 3b,\ a > 0,\ b > 0$, then the minimum value of ab is
a) 12 b) 24 c) $\frac{1}{4}$ d) None of these

296. Let $f(x)$ be a function such that $f'(a) \neq 0$. Then, at $x = a, f(x)$
a) Cannot have a maximum
b) Cannot have a minimum
c) Must have neither a maximum nor a minimum
d) None of these

297. The equation of the one of the tangents to the curve $y = \cos(x + y), -2\pi \leq x \leq 2\pi$ that is parallel to the line $x + 2y = 0$, is
a) $x + 2y = 1$ b) $x + 2y = \frac{\pi}{2}$ c) $x + 2y = \frac{\pi}{4}$ d) None of these

298. The function $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has maximum value at $x = \frac{\pi}{3}$. The value of a is
a) 3 b) $\frac{1}{3}$ c) 2 d) $\frac{1}{2}$

299. The length of tangent, subtangent, normal and subnormal for the curve $y = x^2 + x - 1$ at (1, 1) are A, B, C and D respectively, then their increasing order is
a) B, D, A, C b) B, A, C, D c) A, B, C, D d) B, A, D, C

300. If $P(1, 1), Q(3, 2)$ and R is a point on x-axis is, then the value of $PR + RQ$ will be minimum at
a) $\left(\frac{5}{3}, 0\right)$ b) $\left(\frac{1}{3}, 0\right)$ c) (3,) d) (1, 0)

301. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval
a) (0, 1) b) (1, 2) c) (2, 3) d) (1, 3)

302. If the rate of change of area of a square plate is equal to that of the rate of change of its perimeter, then length of the side is
a) 1 unit b) 2 units c) 3 units d) 4 units

303. If $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$, then $f(x)$ is
a) Increasing in $(-\infty, -2)$ and in (0, 1)
b) Increasing in (-2, 0) and in $(1, \infty)$
c) Decreasing in (-2, 0) and in (0, 1)
d) Decreasing in $(-\infty, -2)$ and in $(1, \infty)$

304. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for all $x \in [1, 6]$, then
a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

305. $f(x) = x^3 - 6x^2 - 36x + 2$ is decreasing function, then $x \in$
a) $(6, \infty)$ b) $(-\infty, -2)$ c) $(-2, 6)$ d) None of these

306. Tangents are drawn from the origin to the curve $y = \cos\ x$. Their points of contact lie on
a) $x^2y^2 = y^2 - x^2$ b) $x^2y^2 = x^2 + y^2$ c) $x^2y^2 = x^2 - y^2$ d) None of these

307. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\, dt$, then $f(x)$ increases on
a) $(-2, 2)$ b) $(0, \infty)$ c) $(-\infty, 0)$ d) None of these

308. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0,1]is
a) 0 b) 1 c) 2 d) -1

309. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\ cm^2$/min. When the thickness of ice is 15 cm. then the rate at which the thickness of ice decreases, is

a) $\frac{5}{6\pi}$ cm/min
b) $\frac{5}{54\pi}$ cm/min
c) $\frac{5}{18\pi}$ cm/min
d) $\frac{1}{36\pi}$ cm/min

310. If the curve $y = 2x^3 + ax^2 + bx + c$ passes through the origin and the tangents drawn to it at $x = -1$ and $x = 2$ are parallel to the x-axis, then the values of a, b and $c$are respectively

a) 12, -3 and 0
b) -3, -12 and 0
c) -3, 12 and 0
d) 3, -12 and 0

311. The length of the smallest intercept made by the coordinate axes on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

a) $a + b$
b) $\frac{a+b}{2}$
c) $\frac{a+b}{4}$
d) None of these

312. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point

a) $(1, 0)$
b) at no point
c) $(0, 1)$
d) $(0, 0)$

313. The interval in which the function $f(x) = x\ e^{2-x}$ increases is

a) $(-\infty, 0)$
b) $(2, \infty)$
c) $(0, 2)$
d) None of these

314. If $f(x)$ satisfies the conditions for Rolle's theorem in [3, 5], then $\int_3^5 f(x)\ dx$ equals

a) 2
b) -1
c) 0
d) $-\frac{4}{3}$

315. While measuring the side of an equilateral triangle an error of k% is made, the percentage error in its area is

a) k %
b) $2k$ %
c) $\frac{k}{2}$%
d) $3k$ %

316. An object is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$.
As it passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$,its y —coordinate is decreasing at the rate of 3unit per second.The rate at which the$x$ —coordinate changes at this point is (in unit per second)

a) 2
b) $3\sqrt{3}$
c) $\sqrt{3}$
d) $2\sqrt{3}$

317. The point of parallel $2y = x^2$, which is nearest to the point (0, 3) is

a) $(\pm 4, 8)$
b) $(\pm 1, 1/2)$
c) $(\pm 2, 2)$
d) None of these

318. The function $y = x - \cot^{-1} x - \log\left(x + \sqrt{x^2+1}\right)$ is increasing on

a) $(-\infty, 0)$
b) $(-\infty, 0)$
c) $(0, \infty)$
d) $(-\infty, \infty)$

319. The maximum distance from the origin of a point on the curve $x = a \sin t - \sin\left(\frac{at}{b}\right)$, $y = a \cos t - b \cos\left(\frac{at}{b}\right)$, both $a, b > 0$, is

a) $a - b$
b) $a + b$
c) $\sqrt{a^2 + b^2}$
d) $\sqrt{a^2 - b^2}$

320. For what value of a, $f(x) = -x^3 + 4ax^2 + 2x - 5$ is decreasing$\forall\ x$?

a) $(1, 2)$
b) $(3, 4)$
c) R
d) No value of a

321. If $4a + 2b + c = 0$, then the equation $3ax^2 + 2bx + c = 0$ has at least one real root lying in the interval

a) $(0, 1)$
b) $(1, 2)$
c) $(0, 2)$
d) None of these

322. The minimum value of $e^{(x^4 - x^3 + x^2)}$, is

a) e
b) e^2
c) 1
d) e^{-1}

323. If the Mean value theorem is
$f(b) - f(a) = (b - a)f'(c)$
Then, for the function $x^2 - 2x + 3$ in $\left[1, \frac{3}{2}\right]$, the value of c is

a) 6/5
b) 5/4
c) 4/3
d) 7/6

324. If y=$\frac{\sin(x+a)}{\sin(x+b)}$, $a \neq b$,then y is

a) Minima at x=0
b) Maxima at x=0
c) Neither minima nor maxima at x=0
d) None of the above

325. If $a < 0$, the function $f(x) = a^{ax} + e^{-ax}$ is a monotonically decreasing function for values of x given by

a) $x > 0$ b) $x < 0$ c) $x > 1$ d) $x < 1$

326. If θ is the angle between the curves $x\,y = 2$ and $x^2 + 4y = 0$, then $\tan\theta$ is equal to

a) 1 b) -1 c) 2 d) 3

327. The equation of the tangents to $2x^2 - 3y^2 = 36$ which are parallel to the straight line $x + 2y - 10 = 0$ are

a) $x + 2y = 0$ b) $x + 2y + \sqrt{\frac{288}{15}} = 0$ c) $x + 2y + \sqrt{\frac{1}{15}} = 0$ d) None of these

328. If the function $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x - 1$ has positive points of extremum then

a) $a \in (3, \infty) \cup (-\infty, -3)$

b) $a \in (-\infty, -3) \cup (3, 29/7)$

c) $(-\infty, 7)$

d) $(-\infty, 29/\,7)$

329. The minimum value of $(x-a)(x-b)$ is

a) ab b) $\frac{(a-b)^2}{4}$ c) 0 d) $-\frac{(a-b)^2}{4}$

330. The function $f(x) = x + \frac{1}{x}$ has

a) A local maxima at x=1 and a local minima at $x = -1$

b) A local minima at x=1 and a local maxima at $x = -1$

c) Absolute maxima at $x = 1$ and absolute minima at $x = -1$

d) Absolute minima at $x = 1$ and absolute maxima at $x = -1$

331. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinates axes, lies in the first quadrant. If its area is 2, then the value of b is

a) -1 b) 3 c) -3 d) 1

332. The length of the longest interval, in which the function $3\sin x - 4\sin^3 x$ is increasing, is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) π

333. If $f(x) = x^\alpha, \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1] is

a) -2 b) -1 c) 0 d) 1/2

334. If $2\,a + 3\,b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

a) $(0, 1)$ b) $(1, 2)$ c) $(2, 3)$ d) None of these

335. If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is

a) 1% b) 2% c) 3% d) 4%

336. If $g(x) = \min(x, x^2)$, where x is real number, then

a) $g(x)$ is an increasing function b) $g(x)$ is a decreasing function

c) $f(x)$ is a constant function d) $g(x)$ is a continuous function except at $x = 0$

337. The slope of the tangent to the curve $y = \sin^{-1}(\sin x)$ at $x = \frac{3\pi}{4}$, is

a) 1

b) -1

c) 0

d) Non-existent

338. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

a) 0 b) 1 c) 2 d) None of these

339. Maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is

a) 0 b) 12 c) 16 d) 32

340. If $a^2x^4 + b^2y^4 = c^6$, then maximum value of xy is

a) $\frac{c^2}{\sqrt{ab}}$ b) $\frac{c^3}{ab}$ c) $\frac{c^3}{\sqrt{2ab}}$ d) $\frac{c^3}{2ab}$

341. If the volume of a sphere is increasing at a constant rate, then the rate at which its radius is increasing,is

a) a constant b) Proportional to the radius

c) Inversely proportional to the radius d) Inversely proportional to the surface area

342. If $1° = 0.017$ radians, then the approximate value of $\sin 46°$ is

a) 0.7194 b) $\frac{0.017}{\sqrt{2}}$ c) $\frac{1.017}{2}$ d) None of these

343. A particle moves along a straight line according to the law $s = 16 - 2t + 3t^3$, where s a metre is the distance of the particle from a fixed point at the end of t seconds. The acceleration of the particle at the end of 2 s is

a) $36m/s^2$ b) $34m/s^2$ c) 36m d) None of these

344. The line(s) parallel to the normal to the curve $xy = 1$ is/are

a) $3x + 4y + 5 = 0$ b) $3x - 4y + 5 = 0$ c) $4x + 3y + 5 = 0$ d) $3x - 4y - 5 = 0$

345. The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2 cm/s is proportional to

a) $\frac{1}{r^2}$ b) $\frac{1}{r}$ c) r^2 d) r

346. The angle of intersection of the curves $y = x^2, 6y = 7 - x^3$ at (1, 1) is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) None of these

347. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

a) Cut at right angel b) Touch each other c) Cut at an angle $\frac{\pi}{3}$ d) Cot at an angle $\frac{\pi}{4}$

348. If a particle moves such that the displacement is proportional to the square of the velocity acquired , then its acceleration is

a) Proportional to s^2 b) Proportional to $\frac{1}{s^2}$ c) Proportional to $\frac{1}{s}$ d) A constant

349. The point (0, 3) is nearest to the curve $x^2 = 2y$ at

a) $(2\sqrt{2}, 0)$ b) (0, 0) c) (2, 2) d) None of these

350. The two curves $x^3 - 3\,xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

a) 45° b) 60° c) 90° d) 30°

351. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa, is

a) (2,4) b) (2,-4) c) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

352. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

a) $2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

353. $(1 + x)^n \leq 1 + x^n$, where

a) $n > 1$ b) $0 \leq n \leq 1$ and $x > 0$ c) $n > 1$ and $x > 0$ d) $x < 0$

354. The function $f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\}\,dt$ attains its maximum at $x =$

a) 1 b) 2 c) 3 d) 4

355. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line $x = 1$, is

a) $x + y = e$ b) $e(x + y) = 1$ c) $y + ex = 1$ d) None of these

356. The angle between the tangents at those points on the curve $x = t^2 + 1$ and $y = t^2 - t - 6$ where it meets x-axis is

a) $\pm\tan^{-1}\left(\frac{4}{29}\right)$ b) $\pm\tan^{-1}\left(\frac{5}{49}\right)$ c) $\pm\tan^{-1}\left(\frac{10}{49}\right)$ d) $\pm\tan^{-1}\left(\frac{8}{29}\right)$

357. The equation of normal to the curve $x^2y = x^2 - 3x + 6$ at the point with abscissa $x = 3$ is

a) $3x + 27y = 79$ b) $27x - 3y = 79$ c) $27x + 3y = 79$ d) $3x - 27y = 79$

358. If f and g are two increasing functions such that fog is defined, then

a) gof is an increasing function
b) gof is a decreasing function
c) gof is neither increasing nor decreasing
d) None of these

359. The function x^x is increasing, when

a) $x > \frac{1}{e}$ b) $x < \frac{1}{e}$ c) $x < 0$ d) For all real x

360. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$ that is parallel to the x-axis, is

a) Y=0 b) Y=1 c) Y=2 d) Y=3

361. The angle of intersection of the curves $y = x^2, y = 7 - x^3$ at (1, 1) is

a) $\pi/4$ b) $\pi/3$ c) $\pi/2$ d) None of these

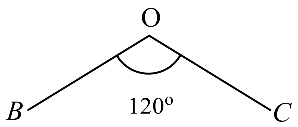
362. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real number x, then minimum value of $f(x)$

a) Does not exist b) Is equal to 1 c) Is equal to 0 d) I equal to -1

363. If $f(x) = |x| + |x-1| + |x-2|$, then which one of the following is not correct?

a) $f(x)$ has a minimum at $x = 1$
b) $f(x)$ has a maximum at $x = 1$
c) $f(x)$ has neither a maximum nor a minimum at $x = 0$
d) $f(x)$ has neither a maximum nor a minimum at $x = 2$

364. OB And OC are two roads enclosing an angle of120°. X And Y start from 'O' at the same time. X Travels along OB with a speed of 4 km/h and Y travels along OC with a speed of 3 km/h. The rate at which the shortest distance between X and Y is increasing after 1 h is



a) $\sqrt{37} km/h$ b) 37 km/h c) 13 km/h d) $\sqrt{13} km/h$

365. The function $y = a(1 - \cos x)$ is maximum when x is equal to

a) π b) $\pi/2$ c) $-\pi/2$ d) $-\pi/6$

366. The minimum value of $f(x) = e^{(x^4 - x^3 + x^2)}$ is

a) e b) $-e$ c) 1 d) -1

367. If$f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1], is

a) -2 b) -1 c) 0 d) 1/2

368. If $\log_e 4 = 1.3868$, then $\log_e 4.01 =$

a) 1.3968 b) 1.3898 c) 1.3893 d) None of these

369. There is an error of$\pm$0.04cm in the measurement of the diameter of a sphere. when the radius is 10 cm, the percentage error in the volume of the sphere is

a) ± 1.2 b) ± 1.0 c) ± 0.8 d) ± 0.6

370. If $f(x) = a \log_e |x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$, then

a) $a = -3/4, b = -1/8$ b) $a = 3/4, b = -1/8$ c) $a = -3/4, b = 1/8$ d) None of these

371. A variable triangle ABC is inscribed in a circle of diameter x units. If at a particular instant the rate of change of side $'a'$ is $x/2$ times the rate of change of the opposite angle A, then $A =$

a) $\pi/6$ b) $\pi/3$ c) $\pi/4$ d) $\pi/2$

372. The equation of the tangent to the curve $y = 4xe^x$at $\left(-1, \frac{-4}{e}\right)$is

a) $Y = -1$ b) $y = -\frac{4}{e}$ c) $x = -1$ d) $x = \frac{-4}{e}$

373. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in

a) (2, 2) b) No value of x c) $(0, \infty)$ d) $(-\infty, 0)$

374. If [0, 1], Lagrange's mean value theorem is NOT applicable to

a) $f(x)=\begin{cases}\frac{1}{2}-x, & x<\frac{1}{2}\\ \left(\frac{1}{2}-x\right)^2, & x\geq\frac{1}{2}\end{cases}$
b) $f(x)=\begin{cases}\frac{\sin x}{x}, & x\neq 0\\ 1, & x=0\end{cases}$
c) $f(x)=x|x|$
d) $f(x)=|x|$

375. $f(x)=\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$ is
a) An increasing
b) A decreasing
c) An even
d) None of these

376. The curve $x=y^2$ and $xy=k$ cut at right angles, if
a) $2k^2=1$
b) $4k^2=1$
c) $6k^2=1$
d) $8k^2=1$

377. If the sum of the squares of the intercepts on the axes cut off by the tangent to the curve $x^{1/3}+y^{1/3}=a^{1/3}$ (with $a>0$) at $P(a/8,a/8)$ is 2, then $a=$
a) 1
b) 2
c) 4
d) 8

378. If normal to the curve $y=f(x)$ is parallel to x-axis, then
a) $\frac{dy}{dx}=0$
b) $\frac{dy}{dx}=1$
c) $\frac{dx}{dy}=0$
d) None of these

379. the speed v of particle moving along a straight line is given by $a+by^2=x^2$(where x is its distance from the origin).The acceleration of the particle is
a) bx
b) $\frac{x}{a}$
c) $\frac{x}{b}$
d) $\frac{x}{ab}$

380. A right circular cylinder which is open at the top and has a given surface area, will have the greatest volume if its height h and radius r are related by
a) $2h=r$
b) $h=4r$
c) $h=2r$
d) $h=r$

381. The point on the curve $x^2+y^2=a^2, y\geq 0$ at which the tangent is parallel to x-axis is
a) (a,0)
b) (-a,0)
c) $\left(\frac{a}{2},\frac{\sqrt{3}}{2}a\right)$
d) (0,a)

382. The normal to the parabola $y^2=4\,ax$ at $(at_1^2, 2at_1)$ meets it again at $(at_2^2, 2at_2)$, then
a) $t_1t_2=-1$
b) $t_2=-t_1-\frac{2}{t_1}$
c) $2t_1=t_2$
d) None of these

383. $f(x)=\frac{e^{2x}-1}{e^{2x}+1}$, is
a) An increasing function on R
b) A decreasing function on R
c) An even function on R
d) None of these

384. The interval of increase of the function $F(x)=x-e^x+\tan\left(\frac{2\pi}{7}\right)$ is
a) $(0,\infty)$
b) $(-\infty,0)$
c) $(1,\infty)$
d) $(-\infty,-1)$

385. Divide 12 into two parts such that the product of the square on one part and the fourth power of the second port is maximum, are
a) 6, 6
b) 5, 7
c) 4, 8
d) 3, 9

386. The position of a point in time 't' is given by $x=a+bt-ct^2, y=at+bt^2$.Its acceleration at time 't' is
a) $b-c$
b) $b+c$
c) $2b-2c$
d) $2\sqrt{b^2+c^2}$

387. If $f(x)=\frac{a\sin x+b\cos x}{c\sin x+d\cos x}$ is decreasing for all x, then
a) $ad-bc>0$
b) $ad-bc<0$
c) $ab-cd>0$
d) $ab-cd<0$

388. The value of c in Lagrange's theorem for the function $f(x)=\log_e\sin x$ in the interval $[\pi/6, 5\,\pi/6]$ is
a) $\frac{\pi}{4}$
b) $\frac{\pi}{2}$
c) $\frac{2\pi}{3}$
d) None of these

389. The largest value of $2x^3-3x^2-12x+5$ for $-2\leq x\leq 4$ occurs at x is equal to
a) -4
b) 0
c) 1
d) 4

390. The denominator of a fraction is greatest then 16 of the square of numerator, then least value of fraction is

a) $-1/4$ b) $-1/8$ c) $1/12$ d) $1/16$

391. A particle moves in a straight line so that $s = \sqrt{t}$, then its acceleration is proportional to

a) (velocity) ^{3} b) Velocity c) (velocity)^2 d) (velocity)$^{3/2}$

392. The greatest value of $\sin^3 x + \cos^3 x$ is

a) 1 b) 2 c) $\sqrt{2}$ d) $\sqrt{3}$

393. A cone of maximum volume is inscribed in given sphere, then ratio of the height of the cone to diameter of the sphere is

a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

394. The maximum value of $f(x) = \frac{x}{4+x+x^2}$ on $[-1, 1]$ is

a) $-\frac{1}{4}$ b) $-\frac{1}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{5}$

395. The length of the subtangent at (2,2) to the curve $x^5 = 2y^4$ is

a) $\frac{5}{2}$ b) $\frac{8}{5}$ c) $\frac{2}{5}$ d) $\frac{5}{8}$

396. If a point is moving in a line so that its velocity at time t is proportional to the square of the square of the distance covered, then its acceleration at time t varies as

a) Cube of the distance
b) The distance
c) Square of the distance
d) None of these

397. Define$f(x) = \int_0^x \sin t \, dt, x \geq 0$. Then,

a) f is decreasing only in the interval $\left(0, \frac{\pi}{2}\right)$ b) f is decreasing in the interval $[0, \pi]$

c) f attains maximum at $x = \frac{\pi}{2}$ d) f attains minimum at $x = \pi$

398. The distance covered by a particle in t second is given by $x = 3 + 8t - 4t^2$. After 1s its velocity will be

a) 0 unit b) 3 units c) 4 units d) 7 units

399. The angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ is

a) 30° b) 45° c) 60° d) 90°

400. If the slope of the tangent to the curve $y = e^x \cos x$ is minimum at $x = a, 0 \leq a \leq 2\pi$, then the value of a is

a) 0 b) π c) 2π d) $3\pi/2$

401. If $f(x) = 2x^3 + 9x^2 + \lambda x + 20$ is a decreasing function of x in the largest possible interval $(-2, -1)$, then $\lambda =$

a) 12 b) -12 c) 6 d) None of these

402. Let $f(x) = \begin{cases} |x^3 + x^2 + 3x + \sin x| \left(3 + \sin\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then number of points (where $f(x)$ attains its minimum value) is

a) 1 b) 2 c) 3 d) Infinite many

403. In the interval (-3, 3) the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is

a) increasing b) Decreasing
c) Neither increasing nor decreasing d) Party increasing and party decreasing

404. A particle is moving in a straight line such that the distance described ‘s’ and the time taken ‘t’ are given by $t = as^2 + bs + c, a > 0$. If v is the velocity of the particle at any time t, then acceleration is

a) $-2av$ b) $-2av^2$ c) $-2av^3$ d) None of these

405. In the interval [0, 1], the function $x^2 - x + 1$ is

a) Increasing b) Decreasing
c) Neither increasing nor decreasing d) Do not say anything

406. A spherical balloon is being inflated so that its volume increase uniformly at the rate of $40\text{cm}^3/\text{minute}$. The rate of increase in its surface area when the radius is 8 cm is

a) $10\text{ cm}^3/\text{minute}$ b) $20\text{ cm}^3/\text{minute}$ c) $40\text{ cm}^3/\text{minute}$ d) None of these

407. The radius of a circle is increasing at the rate of 0.1 cm/s. When the radius of the circle is 5 cm, the rate of change of its area, is

a) $-\pi cm^2/s$ b) $10\pi cm^2/s$ c) $0.1\pi cm^2/s$ d) $\pi cm^2/s$

408. The product of the lengths of subtangent and subnormal at any point of a curve is

a) Square of the abscissa b) Square of the ordinate c) Constant d) None of these

409. The function $f(x) = x^{1/x}$ is

a) Increasing in $(1, \infty)$ b) Decreasing in $(1, \infty)$ c) Increasing in $(1, e)$ and decreasing in (e, ∞) d) Decreasing in $(1, e)$ and increasing in (e, ∞)

410. If the function $f(x) = \frac{ax+b}{(x-1)(x-4)}$ has an extremum at $P(2, -1)$, then

a) $a = 0, b = 1$ b) $a = 0, b = -1$ c) $a = 1, b = 0$ d) $a = -1, b = 0$

411. Moving along the x –axis there are two points with $x = 10 + 6t, x = 3 + t^2$.The speed with which they are reaching from each other at the time of encounter is
in cm and t is in seconds)

a) 16 cm/s b) 20 cm/s c) 8 cm/s d) 12 cm/s

412. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}, g(x) = xe^{x^2}+e^{-x^2}$ and$h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote respectively, the absolute maximum of f, g and h on [0, 1], then

a) $a = b$ and $c \neq b$ b) $a = c$ and $a \neq b$ c) $a \neq b$ and $c \neq b$ d) $a = b = c$

413. The points of contact of the tangents drawn from the origin to the curve $y = \sin x$ lie on the curve

a) $x^2 - y^2 = xy$ b) $x^2 + y^2 = x^2y^2$ c) $x^2 - y^2 = x^2y^2$ d) None of these

414. The angle between the curves , $y = x^2$and $y^2 - x = 0$at the point (1,1), is

a) $\frac{\pi}{2}$ b) $\tan^{-1}\frac{4}{3}$ c) $\frac{\pi}{3}$ d) $\tan^{-1}\frac{3}{4}$

415. If sum of two numbers is 6, the minimum value of the sum of their reciprocals is

a) $\frac{6}{5}$ b) $\frac{3}{4}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

416. For what values of$x$, the function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotonic decreasing?

a) $0 < x < 1$ b) $1 < x < 2$ c) $2 < x < 3$ d) $4 < x < 5$

417. The point on the curve $y^2 = x$, the tangent at which makes an angle 45° with x-axis is

a) $\left(\frac{1}{4}, \frac{1}{2}\right)$ b) $\left(\frac{1}{2}, \frac{1}{4}\right)$ c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

418. If the function $f(x) = \cos|x| - 2\,ax + b$ increases along the entire number scale, the range of a is given by

a) $a \leq b$ b) $a = \frac{b}{2}$ c) $a < -\frac{1}{2}$ d) $a > -\frac{3}{2}$

419. The distances moved by a particle in time t seconds is given by $s = t^3 - 6t^2 - 15t + 12$. The velocity of the particle when acceleration becomes zero is

a) 15 b) −27 c) 6/5 d) None of these

420. The radius and height of a cylinder are equal. If the radius of the sphere is equal to the height of the cylinder, then the ratio of the rates of increase of the volume of the sphere and the volume of the cylinder is

a) 4 : 3 b) 3 : 4 c) $4 : 3\pi$ d) $3\pi : 4$

421. For the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$, the value of c for the Lagrange's mean value theorem, is

a) 1 b) $\sqrt{3}$ c) 2 d) None of these

422. The minimum value of $4e^{2x} + 9e^{-2x}$ is

a) 11 b) 12 c) 10 d) 14

423. The abscissa of the points, where the tangent to the curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to x-axis, are

a) $x = 0$ and 0 b) $x = 1$ and -1 c) $x = 1$ and -3 d) $x = -1$ and 3

424. Let$f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$. Then, $f(x)$ has

a) More than one minimum b) Exactly one minimum
c) at least one maximum d) None of the above

425. Let $y = x^2e^{-x}$, then the interval in which y increases with respect to x is

a) $(-\infty, \infty)$ b) $(-2, 0)$ c) $(2, \infty)$ d) $(0, 2)$

426. The function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ has maximum value at$x = \frac{\pi}{3}$. The value of a is

a) 3 b) 1/3 c) 2 d) 1/2

427. If the distance 's' metres traversed by a particle in t seconds is given by $s = t^3 - 3t^2$, then the velocity of the particle when the acceleration is zero, in metre/second is

a) 3 b) -2 c) -3 d) 2

428. If m and M respectively denote the minimum and maximum of $f(x) = (x - 1)^2 + 3 \; for \; x \in [-3,1]$, then the ordered pair (m, M) is equal to

a) $(-3,19)$ b) $(3,19)$ c) $(-19,3)$ d) $(-19, -3)$

429. The function $f(x) = x^{1/x}$ is increasing in the interval

a) (e, ∞) b) $(-\infty, e)$ c) $(-e, e)$ d) None of these

430. The equation of the tangent to the curve $y = 4e^{--\frac{x}{4}}$ at the point where the curve crosses y-axis is equal to

a) $3x + 4y = 16$ b) $4x + y = 4$ c) $x + y = 4$ d) $4x - 3y = -12$

431. In interval $[1, e]$, the greatest value of $x^2 \log x$ is

a) e^2 b) $\frac{1}{e} \log \frac{1}{\sqrt{e}}$ c) $e^2 \log \sqrt{e}$ d) None of these

432. The line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ an angle of 45°, is

a) $y = \frac{1}{4}$ b) $y = \frac{1}{2}$ c) $y = 1$ d) $y = 4$

433. The angle between the curves $y = \sin x$ and $y = \cos x$ is

a) $\tan^{-1}(2\sqrt{2})$ b) $\tan^{-1}(3\sqrt{2})$ c) $\tan^{-1}(3\sqrt{3})$ d) $\tan^{-1}(5\sqrt{2})$

434. For a particle moving in a straight line, if time t be regarded as a function of velocity v, then the rate of change of the acceleration a is given by

a) $a^2 \frac{d^2t}{dv^2}$ b) $a^3 \frac{d^2t}{dv^2}$ c) $-a^3 \frac{d^2t}{dv^2}$ d) None of these

435. The minimum value of $27^{\cos 2x} \cdot 81^{\sin 2x}$, is

a) 1/243 b) -5 c) 1/5 d) 1/3

436. The minimum value of $px + qy$ when $xy = r^2$, is

a) $2r\sqrt{pq}$ b) $2pq\sqrt{r}$ c) $-2r\sqrt{pq}$ d) None of these

437. If $f'(x) = (x - a)^{2n}(x - b)^{2p+1}$ where n and p are positive integers, then

a) $x = a$ is a point of minimum
b) $x = a$ is a point of maximum
c) $x = a$ is not a point of maximum or minimum
d) None of these

438. Let $f(x) = (x - 4)(x - 5)(x - 6)(x - 7)$ then

a) $f'(x) = 0$ has four roots
b) Three roots of $f'(x) = 0$ lie in $(4, 5) \cup (5, 6) \cup (6, 7)$
c) The equation $f'(x) = 0$ has only one root
d) Three roots of $f'(x) = 0$ lie in $(3, 4) \cup (4, 5) \cup (5, 6)$

439. The set of values of a for which the function
$f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$
is increasing on R, is

a) $(0, \infty)$ b) $(-\infty, 0)$ c) $(-\infty, \infty)$ d) None of these

440. A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$,then particle would bemoving with uniform

a) rotation b) velocity c) acceleration d) retardation

441. Function $f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$ is monotonic increasing, if

a) $\lambda > 1$ b) $\lambda < 1$ c) $\lambda < 4$ d) $\lambda > 4$

442. If $x + y = 8$, then maximum value of $x^2 y$ is

a) $\frac{2048}{9}$ b) $\frac{2048}{81}$ c) $\frac{2048}{3}$ d) $\frac{2048}{27}$

443. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3) then

a) $p = 2, q = -7$ b) $p = -2, q = 7$ c) $p = -2, q = -7$ d) $p = 2, q = 7$

444. The function $f(x) = \tan^{-1} x - x$ is decreasing on the set

a) R b) $(0, \infty)$ c) $R - \{0\}$ d) None of these

445. The function f defined by
$f(x) = x^3 - 6x^2 - 36x + 7$ is increasing, if

a) $x > 2$ and also $x > 6$ b) $x > 2$ and also $x < 6$
c) $x > -2$ and also $x < 6$ d) $x < -2$ and also $x > 6$

446. The length of the normal at point $'t'$ of the curve $x = a(t + \sin t), y = a(1 - \cos t)$ is

a) $a \sin t$ b) $2a \sin^3(t/2) \sec(t/2)$ c) $2a \sin(t/2) \tan(t/2)$ d) $2a \sin(t/2)$

447. The values of $'x'$ for which the function $(a + 2)x^3 - 3\,a\,x^2 + 9ax - 1$ decreases monotonically throughout for all real x are

a) $a < -2$ b) $a > -2$ c) $-3 < a < 0$ d) $-\infty < a \leq -3$

448. The distance travelled by a particle upto time x is given by $f(x) = x^3 - 2x + 1$. The time c at which the velocity of the particle is equal to its average velocity between times $x = 1$ sec and $x = 2$ sec, is

a) 1.5 sec b) $\sqrt{\frac{3}{2}}$ sec c) $\sqrt{3}$ sec d) $\sqrt{\frac{7}{3}}$ sec

449. The number of points on the curve $y = x^3 - 2x^2 + x - 2$ where tangents are parallel to x-axis, is

a) 0 b) 1 c) 2 d) 3

450. The maximum value of $f(x) = \frac{x}{4 + x + x^2}$ on [-1, 1] is

a) $-\frac{1}{3}$ b) $-\frac{1}{4}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

451. The maximum value of $x^{1/x}$ is

a) $1/e^e$ b) e c) $e^{1/e}$ d) $1/e$

452. The sides of an equilateral triangle are increasing at the rate of 2cm/s.The rate at Which the area increases, when the side is 10cm is

a) $\sqrt{3}\text{cm}^2/\text{s}$ b) $10\text{cm}^2/\text{s}$ c) $10\sqrt{3}\text{cm}^2/\text{s}$ d) $\frac{10}{\sqrt{3}}\text{cm}^2/\text{s}$

453. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

a) $f(x)$ is strictly increasing function b) $f(x)$ has a local maxima
c) $f(x)$ is strictly decreasing function d) $f(x)$ is bounded

454. Twenty two meters are available to fence a flower bed in the form of a circular sector. If the flower bed should have the greatest possible surface area, the radius of the circle must be

a) 4 m b) 3 m c) 6 m d) 5 m

455. The greatest value of the function $f(x) = xe^{-x}$ in $[0, \infty]$, is

a) 0 b) $1/e$ c) $-e$ d) e

456. Consider the following statements :

I. The function $x + \frac{1}{x} (x \neq 0)$ is a non-increasing function in the interval $[-1, 1]$

II. The maximum and minimum values of the function $|\sin 4x + 3|$ are 2, 4

III. The function $x^2 \log x$ in the interval has a point of maxima
Which of the statement given above is/are correct?
a) (1)only b) Only (2) c) Only (3) d) All (1), (2) and (3)

457. If α and $\beta(\alpha < \beta)$ are two distinct roots of the equation $ax^2 + bx + c = 0$, then
a) $\alpha > -\frac{b}{2a}$ b) $\beta < -\frac{b}{2a}$ c) $\alpha < -\frac{b}{2a} < \beta$ d) $\beta < -\frac{b}{2a} < \alpha$

458. The function f defined by $f(x) = 4x^4 - 2x + 1$ in increasing for
a) $x < 1$ b) $x > 0$ c) $x < \frac{1}{2}$ d) $x > \frac{1}{2}$

459. The function $f(x) = x^2 e^{-2x}, x > 0$.Then, the maximum value of $f(x)$ is
a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

460. The function $f(x) = 1 - x^3$
a) Increases everywhere
b) Decreases in $(0, \infty)$
c) Increases in $(0, \infty)$
d) None of these

461. If the function $f(x) = \frac{a}{x} + x^2$ has a maximum at $x = -3$, then $a =$
a) -1 b) 16 c) 1 d) 4

462. When the tangent to the curve $y = x \log x$ is parallel to the chord joining the points $(1, 0)$ and (e, e), the value of x is
a) $e^{1/1-e}$ b) $e^{(e-1)(2e-1)}$ c) $e^{\frac{2e-1}{e-1}}$ d) $\frac{e-1}{e}$

463. If the normal to curve $y = f(x)$at the point (3,4) makes an angle $3\pi/4$ with the positive x-axis , then $f'(3)$ is
a) 1 b) -1 c) $-\frac{3}{4}$ d) $\frac{3}{4}$

464. The tangent and the normal drawn to the curve $y = x^2 - x + 4$ at $P(1, 4)$ cut the x-axis at A and B respectively. If the length of the subtangent drawn to the curve at P is equal to the length of the subnormal, then the area of the triangle PAB in sq units is
a) 4 b) 32 c) 8 d) 16

465. The distance $'s'$meters covered by a body in t seconds,is given by $s = 3t^2 - 8t + 5$.
The body will stop after
a) $1\ s$ b) $\frac{3}{4} s$ c) $\frac{4}{3} s$ d) $4\ s$

466. Let f be continuous on [1, 5] and differentiable in (1, 5). If $f(1) = -3$ and $f'(x) \geq 9$ for all $x \in (1, 5)$, then
a) $f(5) \geq 33$ b) $f(5) \geq 36$ c) $f(5) \leq 36$ d) $f(5) \geq 9$

467. Let P be the point (other than the origin) of intersection of the curves $y^2 = 4ax$ and $ay^2 = 4x^3$ such that the normals to the two curves meet x-axis at G_1 and G_2 respectively. Then, $G_1 G_2 =$
a) $2a$ b) $4a$ c) a d) None of these

468. The function $y = x^3 - 3x^2 + 6x - 17$
a) Increases everywhere
b) Decreases everywhere
c) Increases for positive x and decreases for negative x
d) Increases for negative x and decreases for positive x

469. If the function $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies the condition of Rolle's theorem in [1, 3] and $f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$, then the values of a, b are respectively
a) -1, 6 b) -2, 1 c) 1, -6 d) $-1, \frac{1}{2}$

470. If the distance $'s'$metres traversed by a particle in t seconds is given by $s = t^3 - 3t^2$,then the velocity of the particle when the acceleration is zero inm/s is
a) 3 b) -2 c) -3 d) 2

471. The function $f(x)$ given by

$f(x)\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix}$ is increasing on

a) R b) $(-2, 0)$ c) $R - [-2, 0]$ d) None of these

472. Let $g(x) = f(x) - 2\{f(x)\}^2 + 9\{f(x)\}^3$ for all $x \in R$. Then,

a) $g(x)$ and $f(x)$ increases and decrease together

b) $g(x)$ increases whenever $f(x)$ decreases and vice-versa

c) $g(x)$ increases for all $x \in R$

d) $g(x)$ decreases for all $x \in R$

473. If $y = \frac{x+c}{1+x^2}$ where c is a constant, then when y is stationary, xy is equal to

a) 1/2 b) 3/4 c) 5/8 d) 1

474. The equation $x \log_e x = 3 - x$ has, in the interval $(1, 3)$

a) Exactly one root b) At most one root c) At least one root d) No root

475. If there is 2% error in measuring the radius of sphere, then… will be the percentage error in the surface area.

a) 3% b) 1% c) 4% d) 2%

476. The function $f(x) = x + \cos x$ is

a) always increasing b) always decreasing

c) Increasing for certain range of x d) None of the above

477. If the curve $y = ax^2 + bx + c$ passes through the point (1, 2) and the line $y = x$ touches it at the origin, then

a) $a = 1, b = -1, c = 0$ b) $a = 1, b = 1, c = 0$ c) $a = -1, b = 1, c = 0$ d) None of these

478. The function $f(x) = \sin^4 x + \cos^4 x$ increases, if

a) $0 < x < \pi/8$ b) $\pi/4 < x < 3\pi/8$ c) $3\pi/8 < x < 5\pi/8$ d) $5\pi/8 < x < 3\pi/4$

479. The length of the normal at any point on the catenary $y = c \cos h\left(\frac{x}{c}\right)$ varies as

a) $(\text{abscissa})^2$ b) $(\text{ordinate})^2$ c) Abscissa d) Ordinate

480. The normal to a curve at $P(x, y)$ meets the x-axis at$G$. If the distance of G from the origin is twice the abscissa of P, then the curve is a

a) Ellipse b) Parabola c) Circle d) Hyperbola

481. Let $p(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_nx^{2^n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 \dots < a_n$. The function $p(x)$ has

a) Neither a maximum nor a minimum b) Only one maximum

c) Only one minimum d) Only one maximum and only one minimum

482. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$, is

a) 1 b) -1 c) 3/2 d) 1/3

483. The minimum radius vector of the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$ is of length

a) $a - b$ b) $a + b$ c) $2a + b$ d) None of these

484. The first and second order derivatives of a function $f(x)$ exist at all points in (a, b) with $f'(c) = 0$, where $a < c < b$. Furthermore, if $f'(x) < 0$ at all points on the immediate left of c and $f'(x) > 0$ for all points on the immediate right of c, then at $x = c, f(x)$ has a

a) Local maximum b) Local minimum c) Point of inflexion d) None of these

485. Let $f(x) = \cos x \sin 2x$. Then, min $|f(x): -\pi \le x \le \pi|$ is greater than

a) $-9/7$ b) $9/7$ c) $-1/9$ d) $-2/9$

486. In $(-4, 4)$ the function $f(x) = \int_{-10}^{x} (t^4 - 4)\, e^{-4t}\, dt$ has

a) No extrema b) One extremum c) Two extrema d) Four extrema

487. If the polynomial equation

$a_0x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2 + a_1x + a_0 = 0$

n positive integer, has two different real roots α and β, then between α and β, the equation $n\, a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1 = 0$ has

a) Exactly one root b) Almost one root c) At least one root d) No root

488. The function $f(x) = \sin^4 x + \cos^4 x$ increasing, if

a) $0 < x < \frac{\pi}{8}$ b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

489. The point of the curve $y = x^2$ which is closest to $\left(4, -\frac{1}{2}\right)$, is

a) $(1, 1)$ b) $(2, 4)$ c) $\left(\frac{2}{3}, \frac{4}{9}\right)$ d) $\left(\frac{4}{3}, \frac{16}{9}\right)$

490. The equation of the tangent to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x) at\ x = 0$ is

a) $x - y + 1 = 0$ b) $x + y + 1 = 0$ c) $2x - y + 1 = 0$ d) $x + 2y + 2 = 0$

491. A circular sector of perimeter 60 m with maximum area is to be constructed .The radius the circular are in metre must be

a) 20 b) 5 c) 15 d) 10

492. If $f(x)$ and $g(x)$ are differentiable functions for $0 \le x \le 1$ such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$, then in the interval $(0, 1)$

a) $f'(x) = 0$ for all x

b) $f'(x) = 2\, g'(x)$ for at least one x

c) $f'(x) = 2\, g'(x)$ for at most one x

d) None of these

493. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has

a) No maxima and minima b) One maximum and one minimum

c) Two maxima d) Two minima

494. The function $f(x) = \frac{|x-1|}{x^2}$ is monotonically decreasing on

a) $(2, \infty)$ b) $(0, 1)$ c) $(-\infty, 1) \cup (2, \infty)$ d) $(-\infty, \infty)$

495. The tangent drawn at the point $(0, 1)$ on the curve $y = e^{2x}$, meets x-axis at the point

a) $\left(\frac{1}{2}, 0\right)$ b) $\left(-\frac{1}{2}, 0\right)$ c) $(2, 0)$ d) $(0, 0)$

496. Let $f(x) = \cot^{-1}\{g(x)\}$, where $g(x)$ is an increasing function on the interval $(0, \pi)$. Then, $f(x)$ is

a) Increasing on $(0, \pi)$

b) Decreasing on $(0, \pi)$

c) Increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$

d) None of these

497. The radius of a cylinder is increasing at the rate of 3m/s and its altitude is decreasing at the rate of 4m/s. The rate of change of volume when radius is 4m and altitude is 6m, is

a) $80\pi\ cu\ m/s$ b) $144\pi\ cu\ m/s$ c) $80\ cu\ m/s$ d) $64\ cu\ m/s$

498. The Rolle's theorem is applicable in the interval $-1 \le x \le 1$ for the function

a) $f(x) = x$ b) $f(x) = x^2$ c) $f(x) = 2x^3 + 3$ d) $f(x) = |x|$

499. If $a > b > 0$, then maximum value of $\frac{ab(a^2-b^2)\sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}, x \in (0, \pi/2)$ is

a) $a^2 - b^2$ b) $\frac{a^2 - b^2}{2}$ c) $\frac{a^2 + b^2}{2}$ d) None of these

500. If $f(x)$ satisfies the condition for Rolle's theorem is $[3, 5]$, then $\int_3^5 f(x)\ dx$ equals

a) 2 b) -1 c) 0 d) -4/3

501. If there is an error of a% in measuring the edge of a cube, then percentage error in its surface is

a) $2a$ % b) $\frac{a}{2}$% c) $3a$ % d) None of these

502. The range of values of a for which the function $f(x) = (a^2 - 7a + 12)\cos x + 2(a - 4)x + 3e^5$ does not possess critical points, is

a) $(1, 5)$ b) $(1, 4) \cup (4, 5)$ c) $(1, 4)$ d) None of these

503. Rolle's theorem is applicable in case of $\phi(x) = a^{\sin x}, a > 0$ in
a) Any interval b) The interval $[0, \pi]$ c) The interval $(0, \pi/2)$ d) None of these

504. The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point
a) $\left(a, \frac{b}{a}\right)$ b) $\left(-a, \frac{b}{a}\right)$ c) $\left(a, \frac{a}{b}\right)$ d) None of these

505. If the line $ax + by + c = 0$ *is* a tangent to the curve $xy = 4$, then
a) $a < 0, b > 0$ b) $a \leq 0, b > 0$ c) $a < 0, b < 0$ d) $a \leq 0, b < 0$

506. The length of the subtangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point (4, 1) is
a) 2 b) 1/2 c) 3 d) 4

507. The maximum value of $\frac{\log x}{x}$ in $(2, \infty)$,is
a) 1 b) $2/e$ c) e d) $1/e$

508. The maximum value of $\frac{\log x}{x}$, is
a) $1/e$ b) e c) $2/e$ d) 1

509. If an error of k % is made in measuring the radius of a sphere, then percentage error in its volume is
a) k% b) $3k$ % c) $2k$ % d) $k/3$ %

510. The point (s) on the curve $y^3 - 3x^2 = 12y$, where the tangent is vertical (parallel to y-axis), is (are)
a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ b) $\left(\pm\frac{\sqrt{11}}{3}, 1\right)$ c) $(0, 0)$ d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

511. If $y = a \log x + bx^2 + x$ has its extremum at $x = -1$ and $x = 2$, then
a) $a = 2, b = \frac{1}{2}$ b) $a = 2, b = -\frac{1}{2}$ c) $a = \frac{1}{2}, b = 2$ d) $a = -\frac{1}{2}, b = 2$

512. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to
a) 1 b) 2 c) 3/2 d) 1/2

513. For the curve $xy = c^2$ the subnormal at any point varies as
a) x^3 b) x^2 c) y^3 d) ∞

514. Gas is being pumped into a spherical ballon at the rate of $30 ft^3$/min.Then,the rate at which the radius increases when it reaches the value 15 ft, is
a) $\frac{1}{30\pi}$ft/min b) $\frac{1}{15\pi}$ft/min c) $\frac{1}{20}$ft/min d) $\frac{1}{15}$ft/min

515. A ladder 10 m long rests against a vertical wall with the lower end on the horizontal ground. The lower end of the ladder is pulled along the ground away from the wall at the rate of 3 cm/s. The height of the upper end while it is descending at the rate of 4 cm/s, is
a) $4\sqrt{3}m$ b) $5\sqrt{3}m$ c) $5\sqrt{2}m$ d) 6 m

516. The number of values of k for which the equation $x^3 - 3x + k = 0$ has two distinct roots lying in the interval $(0, 1)$ are
a) Three
b) Two
c) Infinitely many
d) New value of k satisfies the requirement

517. The critical points of the function
$f(x) = 2\sin^2\left(\frac{x}{6}\right) + \sin\left(\frac{x}{3}\right) - \left(\frac{x}{3}\right)$
Whose coordinates satisfy the inequality $x^2 - 10 < -19.5x$, is
a) -6π b) 6π c) $\frac{9\pi}{2}$ d) -4π

518. The points of extrema of $f(x) = \int_0^x \frac{\sin t}{t} dt$ in the domain $x > 0$ are

a) $(2n+1)\frac{\pi}{2}, n = 1, 2, \ldots$
b) $(4n+1)\frac{\pi}{2}, n = 1, 2, \ldots$
c) $(2n+1)\frac{\pi}{4}, n = 1, 2, \ldots$
d) $n\pi, n = 1, 2, \ldots$

519. The distance travelled by a motor car in t seconds after the brakes are applied is s feet, where $s = 22t - 12t^2$. The distance travelled by the car before it stops, is
a) 10.08 ft
b) 10 ft
c) 11 ft
d) 11.5 ft

520. If $a < 0$, the function $(e^{ax} + e^{-ax})$ is a decreasing function for all values of x, where
a) $x < 0$
b) $x > 0$
c) $x < 1$
d) $x > 1$

521. A condition for a function $y = f(x)$ to have an inverse is that it should be
a) Defined for all x
b) Continuous everywhere
c) Strictly monotone and continuous in the domain
d) An even function

522. Which one of the following is correct?
If $y = x^5 - 5x^4 + 5x^3 - 1$, then
a) y is maximum at $x = 3$ and minimum at $x = 1$
b) y is minimum at $x = 1$
c) y is neither maximum nor minimum at $x = 0$
d) None of the above

523. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real x, then the maximum value of f
a) Does not exist because f is unbounded
b) Is not attained even though f is bounded
c) Is equal to 1
d) Is equal to -1

524. If $f(x) = x + \frac{1}{x}, x > 0$, then its greatest value is
a) -2
b) 0
c) 3
d) None of these

525. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$, is
a) e
b) $\sqrt[e]{e}$
c) 1
d) $e^{1/e}$

526. The length of the subtangent to the curve $x^2 + xy + y^2 = 7$ at $(1, -3)$ is
a) 3
b) 5
c) 3/5
d) 15

527. The absolute maximum of $x^{40} - x^{20}$ on the interval [0, 1] is
a) $-1/4$
b) 0
c) 1/4
d) 1/2

528. The function $f(x) = x\sqrt{ax - x^2}, a > 0$
a) Increases on the interval $(0, 3a/4)$
b) Decreases on the interval $(3a/4, a)$
c) Decreases on the interval $(0, 3a/4)$
d) Increases on the interval $(3a/4, a)$

529. If there is an error of 0.01 cm in the diameter of a sphere, then percentage errors in surface area when the radius = 5 cm, is
a) 0.005%
b) 0.05%
c) 0.1%
d) 0.2%

530. The side of a square is equal to the diameter of a circle. If the side and radius change at the same rate, then the ratio of the change of their area is
a) $1 : \pi$
b) $\pi : 1$
c) $2 : \pi$
d) $1 : 2$

531. The value of a for which the function $f(x) = \begin{cases} \tan^{-1} a - 3x^2, 0 < x < 1 \\ -6x, \ x \geq 1 \end{cases}$ has a maximum at $x = 1$, is
a) 0
b) 1
c) 2
d) -1

532. Tangent of the angle at which the curves $y = a^x$ and $y = b^x$ $(a \neq b > 0)$ intersect, is given by
a) $\frac{\log ab}{1 + \log ab}$
b) $\frac{\log \frac{a}{b}}{1 + (\log a)(\log b)}$
c) $\frac{\log ab}{1 + (\log a)(\log b)}$
d) None of these

533. In a right triangle $BAC, \angle A = \frac{\pi}{2}$ and $a + b = 8$. The area of the triangle is maximum when $\angle C$, is

a) $\pi/3$ b) $\pi/4$ c) $\pi/6$ d) $\pi/2$

534. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Then area of the triangle is maximum when $\theta =$

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

535. If $s = e^t(\sin t - \cos t)$ is the equation of motion of a moving particle, then acceleration at time t is given by

a) $2\, e^t(\cos t + \sin t)$ b) $2\, e^t(\cos t - \sin t)$ c) $e^t(\cos\, t - \sin t)$ d) $e^t(\cos\, t + \sin t)$

536. If from Lagrange's mean value theorem, we have

$f'(x_1) = \dfrac{f'(b) - f(a)}{b - a}$, then

a) $a < x_1 \le b$ b) $a \le x_1 < b$ c) $a < x_1 < b$ d) $a \le x_1 \le b$

537. The slope of the tangent to the curve $y = x^2 - x$ at the point where the line $y = 2$ cuts the curve in the first quadrant is

a) 2 b) 3 c) -3 d) None of these

538. The function $f(x) = x(x + 3)e^{-1(1/2)x}$ satisfies the conditions of Rolle's therorem in [-3, 0]. The value of c is

a) 0 b) -1 c) -2 d) -3

539. The length of the subtangent to the curve $x^2y^2 = a^4$ at $(-a, a)$ is

a) $\dfrac{a}{2}$ b) $2a$ c) a d) $\dfrac{a}{3}$

540. The equation of the tangent top the curve $y = 1 - e^{x/2}$ at the point on intersection with the y-axis is

a) $x + 2y = 0$ b) $2x + y = 0$ c) $x - y = 2$ d) None of these

541. If the volume of a sphere is increasing at a constant rate, then the rate at which its radius is increasing is

a) A constant
b) proportional to the radius
c) Inversely proportional to the radius
d) Inversely proportional to the surface area

542. The function $f(x)$ given by $f(x) = \begin{vmatrix} x - 1 & x + 1 & 2x + 1 \\ x + 1 & x + 3 & 2x + 3 \\ 2x + 1 & 2x - 1 & 4x + 1 \end{vmatrix}$ has

a) One point of maximum and one point of minimum
b) One point of maximum only
c) One point of minimum only
d) None of above

543. The perimeter of a sector is a constant. If its area is to be maximum, the sectorial angle is

a) $\dfrac{\pi^c}{6}$ b) $\dfrac{\pi^c}{4}$ c) 4^c d) 2^c

544. The interval in which the function x^3 increases less rapidly than $6\, x^2 + 15\, x + 5$, is

a) $(-\infty, -1)$ b) $(-5, 1)$ c) $(-1, 5)$ d) $(5, \infty)$

545. If a particle is moving such that the velocity acquired is proportional to the square root of the distance covered ,then its acceleration is

a) a constant b) $\propto s^2$ c) $\propto \dfrac{1}{s^2}$ d) $\propto s$

546. The coordinates of the point on the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ where tangent is inclined at an angle $\frac{\pi}{4}$ to the x-axis are

a) (a, a) b) $(a(\pi/2 - 1), a)$ c) $(a(\pi/2 + 1), a)$ d) $(a, a(\pi/2 + 1))$

547. For the function $f(x) = xe^x$the point

a) $x = 0$ is a maximum
b) $x = 0$ is a minimum
c) $x = -1$is a maximum
d) $x = -1\ is\ a$ minimum

548. The circumference of a circle is measured as 56 cm with an error 0.02 cm. The percentage error in its area is

a) 1/7 b) 1/28 c) 1/14 d) 1/56

549. If $f(x) = \sin x / e^x$ in $[0, \pi]$, then $f(x)$

a) Satisfies Rolle's Theorem and $c = \frac{\pi}{4}$, So that $f'^{\left(\frac{\pi}{4}\right)} = 4$

b) Does not satisfy Rolle's Theorem but $f'\left(\frac{\pi}{4}\right) > 0$

c) Satisfies Rolle's Theorem but $f'\left(\frac{\pi}{4}\right) = 0$

d) Satisfies Lagrange's Mean Value Theorem but $f'\left(\frac{\pi}{4}\right) \neq 0$

550. For the curve $xy = c^2$, the subnormal at any point varies

a) x^2 b) x^3 c) y^2 d) y^3

551. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is

a) 0 b) 1 c) 2 d) 3

552. A stone is thrown vertically upwards from the top of a tower 64 m high according to the law s=48t-$16t^2$.The greatest height attained by the stone above ground is

a) 36 m b) 32 m c) 100 m d) 64 m

553. $2x^3 - 6x + 5$ is an increasing function, if

a) $0 < x < 1$ b) $-1 < x < 1$ c) $0 < -1$ or $x > 1$ d) $-1 < x < -\frac{1}{2}$

554. The set of all values of the parameter a for which the points of minimum of the function $y = 1 + a^2x - x^3$ satisfy the inequality $\frac{x^2+x+2}{x^2+5x+6} \leq 0$, is

a) An empty set
b) $(-3\sqrt{3}, -2\sqrt{3})$
c) $(2\sqrt{3}, 3\sqrt{3})$
d) $(-3\sqrt{3}, -2\sqrt{3}) \cup (2\sqrt{3}, 3\sqrt{3})$

555. If m be the slope of the tangent to the curve $e^{2y} = 1 + 4x^2$, then

a) $m < 1$ b) $|m| \leq 1$ c) $|m| > 1$ d) None of these

556. The minimum value of $2^{(x^2-3)^3+27}$, is

a) 2^{27} b) 2 c) 1 d) 4

557. The equation of the tangent to the curve $(1 + x^2)y = 2 - x$, where it crosses the x-axis, is

a) $x + 5y = 2$ b) $x - 5y = 2$ c) $5x - y = 2$ d) $5x + y - 2 = 0$

Session : 2025-26 **AS PER NEW NTA SYLLABUS**

Total Questions : 557

MATHS (QUESTION BANK)

6.APPLICATION OF DERIVATIVES

: ANSWER KEY :

1)	b	2)	a	3)	c	4)	c	141)	b	142)	b	143)	b	144)	a
5)	c	6)	a	7)	c	8)	c	145)	a	146)	c	147)	b	148)	a
9)	a	10)	b	11)	c	12)	c	149)	d	150)	a	151)	a	152)	d
13)	c	14)	a	15)	c	16)	b	153)	c	154)	c	155)	d	156)	b
17)	b	18)	d	19)	b	20)	a	157)	a	158)	c	159)	c	160)	b
21)	c	22)	b	23)	a	24)	c	161)	c	162)	b	163)	d	164)	d
25)	c	26)	b	27)	d	28)	d	165)	b	166)	b	167)	b	168)	a
29)	a	30)	b	31)	d	32)	c	169)	b	170)	d	171)	b	172)	d
33)	d	34)	c	35)	c	36)	a	173)	d	174)	b	175)	b	176)	b
37)	a	38)	b	39)	c	40)	c	177)	a	178)	d	179)	a	180)	c
41)	c	42)	a	43)	c	44)	b	181)	a	182)	c	183)	d	184)	d
45)	c	46)	a	47)	b	48)	a	185)	c	186)	c	187)	a	188)	c
49)	c	50)	a	51)	d	52)	a	189)	a	190)	a	191)	d	192)	a
53)	d	54)	b	55)	b	56)	c	193)	b	194)	d	195)	b	196)	a
57)	c	58)	d	59)	a	60)	d	197)	a	198)	b	199)	a	200)	b
61)	a	62)	c	63)	b	64)	d	201)	b	202)	c	203)	d	204)	c
65)	a	66)	c	67)	a	68)	a	205)	d	206)	a	207)	c	208)	b
69)	a	70)	b	71)	a	72)	b	209)	a	210)	d	211)	b	212)	d
73)	d	74)	d	75)	d	76)	b	213)	b	214)	d	215)	a	216)	d
77)	b	78)	a	79)	b	80)	b	217)	d	218)	b	219)	d	220)	c
81)	c	82)	a	83)	d	84)	c	221)	b	222)	d	223)	a	224)	a
85)	a	86)	d	87)	a	88)	a	225)	c	226)	d	227)	b	228)	c
89)	c	90)	b	91)	a	92)	d	229)	d	230)	b	231)	c	232)	c
93)	a	94)	b	95)	b	96)	a	233)	c	234)	c	235)	b	236)	a
97)	a	98)	b	99)	b	100)	a	237)	b	238)	a	239)	a	240)	a
101)	c	102)	d	103)	b	104)	a	241)	b	242)	b	243)	a	244)	d
105)	c	106)	b	107)	a	108)	a	245)	b	246)	a	247)	d	248)	c
109)	b	110)	a	111)	c	112)	b	249)	c	250)	c	251)	d	252)	a
113)	b	114)	a	115)	c	116)	b	253)	a	254)	a	255)	b	256)	d
117)	b	118)	b	119)	d	120)	b	257)	b	258)	b	259)	b	260)	a
121)	a	122)	d	123)	a	124)	c	261)	d	262)	c	263)	a	264)	a
125)	d	126)	a	127)	d	128)	c	265)	d	266)	c	267)	c	268)	b
129)	b	130)	d	131)	d	132)	b	269)	c	270)	a	271)	d	272)	a
133)	b	134)	c	135)	d	136)	b	273)	b	274)	c	275)	d	276)	d
137)	c	138)	a	139)	a	140)	a	277)	b	278)	a	279)	c	280)	d

281)	a	282)	c	283)	d	284)	a
285)	d	286)	d	287)	d	288)	a
289)	d	290)	a	291)	a	292)	b
293)	c	294)	b	295)	b	296)	d
297)	b	298)	c	299)	d	300)	a
301)	a	302)	b	303)	b	304)	d
305)	c	306)	c	307)	c	308)	c
309)	c	310)	b	311)	a	312)	a
313)	d	314)	d	315)	b	316)	b
317)	c	318)	d	319)	b	320)	d
321)	c	322)	c	323)	b	324)	c
325)	b	326)	d	327)	d	328)	b
329)	d	330)	b	331)	c	332)	a
333)	d	334)	a	335)	a	336)	a
337)	b	338)	c	339)	b	340)	c
341)	d	342)	c	343)	a	344)	b
345)	c	346)	c	347)	a	348)	d
349)	c	350)	c	351)	d	352)	c
353)	b	354)	a	355)	d	356)	c
357)	b	358)	a	359)	a	360)	d
361)	c	362)	d	363)	b	364)	a
365)	a	366)	c	367)	d	368)	c
369)	d	370)	a	371)	b	372)	b
373)	d	374)	d	375)	a	376)	d
377)	c	378)	c	379)	c	380)	d
381)	d	382)	b	383)	d	384)	b
385)	c	386)	d	387)	b	388)	b
389)	d	390)	b	391)	a	392)	a
393)	a	394)	c	395)	b	396)	a
397)	a	398)	a	399)	d	400)	b
401)	a	402)	a	403)	b	404)	c
405)	c	406)	a	407)	d	408)	b
409)	c	410)	c	411)	c	412)	d
413)	c	414)	d	415)	c	416)	b
417)	a	418)	c	419)	b	420)	a
421)	b	422)	b	423)	d	424)	a

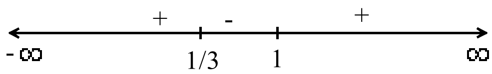
425)	d	426)	c	427)	c	428)	b
429)	d	430)	c	431)	a	432)	b
433)	a	434)	c	435)	a	436)	a
437)	c	438)	b	439)	a	440)	c
441)	d	442)	d	443)	a	444)	c
445)	d	446)	c	447)	d	448)	c
449)	c	450)	d	451)	c	452)	c
453)	a	454)	d	455)	b	456)	a
457)	c	458)	d	459)	c	460)	b
461)	d	462)	a	463)	a	464)	d
465)	c	466)	a	467)	b	468)	a
469)	c	470)	c	471)	c	472)	a
473)	a	474)	c	475)	c	476)	a
477)	b	478)	b	479)	b	480)	d
481)	c	482)	a	483)	b	484)	b
485)	a	486)	c	487)	c	488)	b
489)	a	490)	a	491)	c	492)	b
493)	b	494)	c	495)	b	496)	b
497)	a	498)	b	499)	b	500)	d
501)	a	502)	b	503)	b	504)	d
505)	c	506)	b	507)	d	508)	a
509)	b	510)	d	511)	b	512)	a
513)	c	514)	a	515)	d	516)	d
517)	b	518)	d	519)	a	520)	a
521)	c	522)	c	523)	d	524)	d
525)	b	526)	d	527)	b	528)	a
529)	a	530)	c	531)	d	532)	b
533)	a	534)	a	535)	a	536)	c
537)	b	538)	c	539)	c	540)	a
541)	d	542)	d	543)	d	544)	c
545)	a	546)	c	547)	d	548)	c
549)	c	550)	d	551)	b	552)	c
553)	c	554)	c	555)	b	556)	c
557)	a						

MATHS (QUESTION BANK)

6.APPLICATION OF DERIVATIVES

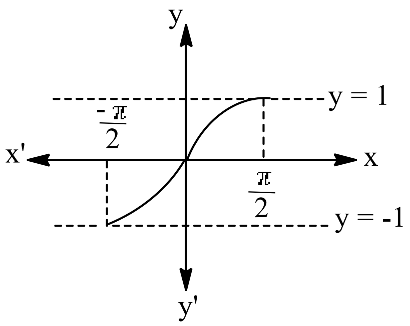
: HINTS AND SOLUTIONS :

1 **(b)**
We have,
$f(x) = x(x-1)^2$
$\Rightarrow f''(x) = (x-1)^2 + 2x(x-1)$
$\Rightarrow f''(x) = (x-1)(3x-1)$
The changes in the signs of $f''(x)$ are shown in diagram



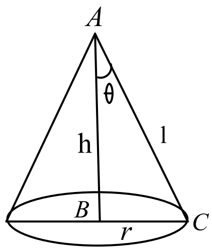
Clearly, $f(x)$ attains a local maximum at $x = \frac{1}{3}$ and a local minimum at $x = 1$
$\therefore$ Maximum value of $f(x) = f\left(\frac{1}{3}\right) = \frac{4}{27}$

2 **(a)**
Since, $2\pi k - \frac{\pi}{2} \leq \sin x \leq 2\pi k + \frac{\pi}{2}$



For $k = 0$
$-\frac{\pi}{2} < \sin x < \frac{\pi}{2}$
Which increase from -1 *to* 1.
Similarly, for other values of k it is increase from -1 *to* 1.

3 **(c)**
Volume of cone, $V = \frac{\pi}{3} r^2 h$



$\Rightarrow \quad V = \frac{\pi}{3} r^2 \sqrt{l^2 - r^2}$
On differentiating w.r.t. r, we get
$$\frac{dV}{dr} = \frac{\pi}{3}\left[2r\sqrt{l^2-r^2} + \frac{r^2}{2\sqrt{l^2-r^2}}(-2r)\right]$$
Put $\frac{dV}{dr} = 0$
$$\Rightarrow \quad 2r\sqrt{(l^2-r^2)} - \frac{r^3}{\sqrt{l^2-r^2}} = 0$$
$$\Rightarrow \quad r[2(l^2-r^2) - r^2] = 0$$
$$\Rightarrow \quad r = \pm\, l\sqrt{\frac{2}{3}}$$
$\therefore$ At $\quad r = l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2} < 0$, maxima
$$\therefore \quad h = \sqrt{l^2 - \frac{2}{3}l^2} = \frac{l}{\sqrt{3}}$$
In $\Delta ABC, \tan\theta = \frac{r}{h} = \frac{l\sqrt{\frac{2}{3}}}{\frac{l}{\sqrt{3}}} = \sqrt{2}$

4 **(c)**
Given that $f(x) = \sin x - bx + c$
$\therefore f'(x) = \cos x - b$
For decreasing, $f'(x) < 0$, for all $x \in R$.
$\Rightarrow \cos x < b$ *for all* $x \in R \Rightarrow b > 1$.

5 **(c)**
Given, $f(x) = 2x^3 + 3x^2 - 12x + 1$
$\Rightarrow f'(x) = 6x^2 + 6x - 12$
For $f(x)$*to* be decreasing, $f'(x) < 0$

$\Rightarrow 6(x^2+x-2)<0$
$\Rightarrow (x+2)(x-1)<0$
$\Rightarrow x \in (-2,1)$

6 **(a)**

We have,

$$f(x)=2\,x+\cot^{-1}x+\log\left(\sqrt{1+x^2}-x\right)$$

$$\therefore f'(x)=2-\frac{1}{1+x^2}+\frac{1}{\sqrt{1+x^2}-x}\left(\frac{x}{\sqrt{1+x^2}}-1\right)$$

$$\Rightarrow f'(x)=\frac{1+2\,x^2}{1+x^2}-\frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow f'(x)=\frac{1+2\,x^2}{1+x^2}-\frac{\sqrt{(1+x^2)}}{1+x^2}$$

$$\Rightarrow f'(x)=\frac{x^2+\sqrt{1+x^2}\{\sqrt{1+x^2}-1\}}{1+x^2}$$
$$>0 \text{ for all } x$$

Hence, $f(x)$ is an increasing function on $(-\infty,\infty)$ and in particular on $(0,\infty)$

7 **(c)**

We have,

$$f(x)=3\cos^2 x+4\sin^2 x+\cos\frac{x}{2}+\sin\frac{x}{2}$$

$$\Rightarrow f(x)=4-\cos^2 x+\cos\frac{x}{2}+\sin\frac{x}{2}$$

$$\Rightarrow f'(x)=\sin 2x-\tfrac{1}{2}\left(\sin\tfrac{x}{2}-\cos\tfrac{x}{2}\right) \quad ....\text{(i)}$$

$$\Rightarrow f'(x)=2\sin x\cos x-\frac{1}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x)=2\sin x\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)+\frac{1}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x)=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left\{2\sin x\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\frac{1}{2}\right\}$$

$$\Rightarrow f'(x)=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left\{2\sqrt{2}\sin x\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{1}{2}\right\}$$

For local maximum or minimum, we must have

$f'(x)=0$

$$\Rightarrow \cos\frac{x}{2}-\sin\frac{x}{2}=0$$

$$\Rightarrow \cos\frac{x}{2}=\sin\frac{x}{2}\Rightarrow \tan\frac{x}{2}=1\Rightarrow\frac{x}{2}=\frac{\pi}{4}\Rightarrow x=\frac{\pi}{2}$$

Now,

$$f''(x)=2\cos 2x-\tfrac{1}{4}\left(\cos\tfrac{x}{2}+\sin\tfrac{x}{2}\right) \quad \text{[Using (i)]}$$

$$\Rightarrow f''\left(\frac{\pi}{2}\right)=2\cos\pi-\frac{1}{4}\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)$$
$$=-2-\frac{1}{2\sqrt{2}}<0$$

Thus, $f(x)$ attains a local maximum at $x=\frac{\pi}{2}$

Local maximum value $=f\left(\frac{\pi}{2}\right)=4+\frac{2}{\sqrt{2}}=4+\sqrt{2}$

8 **(c)**

$$\because \quad y=\left(\frac{c^6-a^2x^4}{b^2}\right)^{\frac{1}{4}}$$

Let $f(x)=xy=\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{\frac{1}{4}}$

$$\Rightarrow \quad f'(x)=\frac{1}{4}\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{-3/4}\left(\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}\right)$$

Put $f'(x)=0$

$$\Rightarrow \quad x=\pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$$

$$\therefore \quad f\left(\frac{c^{3/2}}{c^{1/4}\sqrt{a}}\right)=\frac{c^3}{\sqrt{2ab}}$$

9 **(a)**

Let the radius of the circular wave ring by r cm at any time t. Then, $\frac{dr}{dt}=30$ cm/sec (given)

Let A be the area of the enclosed ring. Then,

$A=\pi r^2$

$$\Rightarrow\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$\Rightarrow\frac{dA}{dt}=2\pi\times 50\times\frac{30}{100}\text{m}^2\text{sec}=30\pi^2\ \text{m}^2/\text{sec}$$

10 **(b)**

We have,

$x=t\cos t$ and $y=t\sin t$

$\therefore \frac{dx}{dt}=\cos t-t\sin t$ and $\frac{dy}{dx}=\sin t+t\cos t$

At the origin, we have

$x=0, y=0\Rightarrow t\cos t=0$ and $t\cos t=0\Rightarrow t=0$

The slope of the tangent at $t=0$ is

$$\frac{dy}{dx}=\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)_{t=0}=\left(\frac{\sin t+t\cos t}{\cos t-t\sin t}\right)_{t=0}=0$$

So, the equation of the tangent at the origin is

$t-0=0(x-0)\Rightarrow y=0$

11 **(c)**

Surface area of sphere $S=4\pi r^2$ and $\frac{dr}{dt}=2$

$$\therefore\ \frac{dS}{dt}=4\pi\times 2r\frac{dr}{dt}=8\pi r\times 2=16\pi r$$

$$\Rightarrow\frac{dS}{dt}\propto r$$

12 **(c)**

Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x} = e^{-x \log x}$. Then,

$f'(x) = -\left(\frac{1}{x}\right)^x (\log x + 1) = -x^{-x}(\log x + 1)$

Now,

$f'(x) = 0$

$\Rightarrow -x^{-x}(\log x + 1) = 0$

$\Rightarrow \log x + 1 = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1}$

Clearly, $f''(x) < 0$ at $x = e^{-1}$

Hence, $f(x) = x^{-x}$ is maximum for $x = e^{-1}$. The maximum value is $e^{1/e}$

13 **(c)**

Given, $f(x) = 2x^3 - 21x^2 + 36x - 30$

$\Rightarrow \quad f'(x) = 6x^2 - 42x + 36$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow \; 6x^2 - 42x + 36 = 0 \quad \Rightarrow \quad x = 6, 1$

And $\quad f''(x) = 12x - 42$

$f''(1) = -30$ and $f''(6) = 30$

Hence, $f(x)$ has maxima at $x = 1$ and minima at $x = 6$

14 **(a)**

Let $l$be the length of an edge and Vbe the volume of cue at any time t.

$\because \quad V = t^3$

$\because \; \frac{dV}{dt} = 3l^2 \frac{dl}{dt}$

$= 3 \times 5^2 \times 10 cm^3/s$

$= 750 cm^3/s$.

15 **(c)**

We have, $\frac{dy}{dx} = \frac{-\sin\theta}{1-\cos\theta}$

Clearly, $\frac{dy}{dx} = 0$ for $\theta = (2k+1)\pi$

So, the tangent is parallel to x-axis i.e. $y = 0$

16 **(b)**

We have,

$5x^5 - 10x^3 + x + 2y + 6 = 0 \quad$ ...(i)

Differentiating with respect to x, we get

$25x^4 - 30x^2 + 1 + 2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}(25x^4 - 30x^2 + 1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,-3)} = -\frac{1}{2}$

The equation of the normal at $(0, -3)$ is

$y + 3 = 2(x - 0) \Rightarrow 2x - y - 3 = 0$

Solving (i) and (ii), we obtain the coordinates of their points of intersection as $P(0, -3)$, $(1, -1)$ and $(-1, -5)$

Hence, the normal at $P(0, -3)$ meets the curve again at $(1, -1)$ and $(-1, -5)$

17 **(b)**

We have,

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\,\theta\cos\theta$

and, $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\,\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \tan\theta \Rightarrow -\frac{1}{\frac{dy}{dx}} = -\cot\theta$

Hence, the slope of the normal varies as θ

The equation of the normal at any point is

$y - a(\sin\theta - \theta\cos\theta)$

$= -\cot\theta\,\{x - a(\cos\theta + \theta\sin\theta)\}$

$\Rightarrow x\cos\theta + y\sin\theta = a$

Clearly, it is a line at a constant distance $|a|$ from the origin

18 **(d)**

We have,

$f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$

Clearly, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

So, $f(x)$ is continuous at $x = 2$

Hence, it is continuous on $[-1, 3]$

Thus, option (b) is correct

We find that

$f'(x) = 6x + 12 > 0$ for all $x \in [-1, 2]$

$\Rightarrow f(x)$ is increasing on $[-1, 2]$

Thus, option (a) is correct

Also,

$f'(x) < 0$ for all $x \in (2, 3]$

$\Rightarrow f(x)$ is decreasing on $(2, 3]$

Hence, $f(x)$ is attains the maximum value at $x = 2$

So, option (c) is correct

19 **(b)**

Given, $f(x) = x^3 - 3x^2 + 2x$

$\Rightarrow f'(x) = 3x^2 - 6x + 2$

Now, f(a)=f(0)=0

And $f(b) = f\left(\frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right) = \frac{3}{8}$

By Lagrange's Mean Value Theorem

$\frac{f(b) - f(a)}{b - a} = f'(c)$

$\Rightarrow \frac{\frac{3}{8} - 0}{\frac{1}{2} - 0} = 3c^2 - 6c + 2$

$\Rightarrow 12c^2 - 24x + 5 = 0$

This is a quadratic equation in c.

$c = \frac{24 \pm \sqrt{576 - 240}}{24}$

$= 1 \pm \frac{\sqrt{21}}{6}$

But c lies between 0 to$\frac{1}{2}$

$\therefore$ we take, $c = 1 - \frac{\sqrt{21}}{6}$

20 **(a)**

Since,$f(x) = kx - \sin x$is monotonically increasing for all $x \in R$.Therefore,

$f'(x) > 0$for all $x \in R$

$\Rightarrow K - \cos x > 0$

$\Rightarrow K > \cos x$

$\Rightarrow K > 1$ [$\therefore$ maximum value $of \cos x\ is$ 1]

21 **(c)**

We have, $f(-1) < 0$

So, $f(x) > 0$ for all $x \in R$ is not true

Now, $f'(x) = x^3 - 6x^2 + 15x + 3$

$\Rightarrow f'(x) = 3x^2 - 12x + 15$

$\Rightarrow f'(x) = 3(x^2 - 4x + 5) = 3\{(x-2)^2 + 1\} > 0$ for all $x \in R$

$\Rightarrow f(x)$ is strictly increasing on R

$\Rightarrow f(x)$ is invertible on R

22 **(b)**

Let D denotes the diagonal of the square.

Given,

$\frac{dD}{dt} = \frac{0.5cm}{s} \quad \ldots..(i)$

Since, A=400$= \frac{1}{2}D^2 \Rightarrow D = 20\sqrt{2} \quad \left(\therefore = \frac{D^2}{2}\right)$

On differentiating w. r. t. t, we get

$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{D^2}{2}\right)$

$= 20\sqrt{2} \times 0.5$

$=10\sqrt{2}cm^2/s$

23 **(a)**

Consider the function $f(x)$ given by

$f(x) = ax^2 + \frac{b}{x} - c, x > 0$

Clearly, $f(x) \geq 0$ for all $x > 0$ $\quad [\because ax^2 + \frac{b}{x} \geq c$ (Given)]

Also, $f'(x) = 2ax - \frac{b}{x^2}$ and $f''(x) = 2a + \frac{2b}{x^3}$

For points of local maximum/minimum, we must have

$f'(x) = 0 \Rightarrow 2ax^3 = b \Rightarrow x = \left(\frac{b}{2a}\right)^{1/3}$

Now, $f''\left\{\left(\frac{b}{2a}\right)^{1/3}\right\} = 2a + 4a = 6a > 0$

Therefore, $f(x)$ attains a local minimum at $x = \left(\frac{b}{2a}\right)^{1/3}$

Now,

$f\left\{\left(\frac{b}{2a}\right)^{1/3}\right\} = a\left(\frac{b}{2a}\right)^{2/3} + b\left(\frac{2a}{b}\right)^{1/3} - c$

$\Rightarrow f\left\{\left(\frac{b}{2a}\right)^{1/3}\right\} = \frac{b^{2/3}a^{1/3}}{2^{2/3}} + b^{2/3}(2a)^{1/3} - c$

But, $f(x) \geq 0$ for all $x > 0$

$\frac{b^{2/3}a^{1/3}}{2^{2/3}} + b^{2/3}(2a)^{1/3} - c \geq 0$

$\Rightarrow (b^2a)^{1/3}\left(\frac{1}{2^{2/3}} + 2^{1/3}\right) \geq c$

$\Rightarrow 3(b^2a)^{1/3} \geq 2^{2/3}c \Rightarrow 27ab^2 \geq 4c^3$

24 **(c)**

Suppose the tangent from the point (2, 0) to $y = x^4$ touches the curve at (x_1, y_1). The equation of the tangent at (x_1, y_1) is

$y - y_1 = 4x_1^3(x - x_1)$

If it passes through (2, 0), then

$0 - y_1 = 4x_1^3(2 - x_1)$

$\Rightarrow y_1 = 4x_1^3(x_1 - 2)$

$\Rightarrow x_1^4 = 4x_1^3(x_1 - 2) \quad [\because (x_1, y_1)$ lies on $y = x^4 \therefore y_1 = x_1^4]$

$\Rightarrow 3x_1^4 - 8\,x_1^3 = 0$

$\Rightarrow x_1^3(3x_1 - 8) = 0$

$\Rightarrow x_1 = 0$ or, $x_1 = 8/3$

Now, $x_1 = 0$, and $y_1 = x_1^4 \Rightarrow y_1 = 0$

$x_1 = 8/3$, and $y_1 = x_1^4 \Rightarrow y_1 = \left(\frac{8}{3}\right)^4 = \frac{4096}{81}$

Thus the points of tangency are (0, 0) and (8/3, 4096/81)

Hence, the equations of the tangents are

$y = 0$ and $y = -\frac{4096}{81} = \frac{2048}{27}\left(x - \frac{8}{3}\right)$

26 **(b)**

The equation of given curve is $\sqrt{x} + \sqrt{y} = \sqrt{a}$

$\therefore \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

The normal is parallel to x-axis, if

$\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \Rightarrow x_1 = 0$

$\therefore$ From equation of curve, $y_1 = a$

$\therefore$ Required point is $(0, a)$

27 **(d)**

Let $f(x) = 2x^3 - 9x^2 + 12x + 4$

$\Rightarrow \quad f'(x) = 6x^2 - 18x + 12$

$f'(x) < 0$ for function to be decreasing

$\Rightarrow \quad 6(x^2 - 3x + 2) < 0$

$\Rightarrow \quad (x^2 - 2x - x + 2) < 0$

$\Rightarrow \quad (x-2)(x-1) < 0$

$\Rightarrow \quad 1 < x < 2$

28 **(d)**

Given that, $f(x) = 1 - x^3 - x^5$

On differentiating w.r.t. x, we get

$f'(x) = -3x^2 - 5x^4$

$\Rightarrow \quad f'(x) = -(3x^2 + 5x^4)$

$\Rightarrow \quad f'(x) < 0$for all values of x

29 **(a)**

We have, $y = \{x(x-3)\}^2$

$\therefore \dfrac{dy}{dx} = 2x(x-3)(2x-3)$

Clearly, $\dfrac{dy}{dx} > 0$ for $0 < x < \dfrac{3}{2}$

Hence, y is increasing for $0 < x < 3/2$

30 **(b)**

We have,

$y = -3\log_e(9 + x^2)$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{6x}{9 + x^2}$

$\Rightarrow -\dfrac{dy}{dx} = \dfrac{9 + x^2}{6x} \Rightarrow m = \dfrac{9 + x^2}{6x} \Rightarrow |m| = \dfrac{9 + |x|^2}{6|x|}$

Now,

A. M. $\geq$ G. M.

$\Rightarrow \dfrac{9 + |x|^2}{2} \geq \sqrt{9|x|^2}$

$\Rightarrow \dfrac{9 + |x|^2}{2} \geq 3|x|$

$\Rightarrow \dfrac{9 + |x|^2}{6|x|} \geq 1 \Rightarrow |m| \geq 1 \Rightarrow m \in R - (-1, 1)$

31 **(d)**

We have,

$f(x) = \dfrac{x^2 - 1}{x^2 + 1} = 11 - \dfrac{2}{x^2 + 1}$

Clearly, $f(x)$ will be minimum when $\frac{2}{x^2+1}$ is maximum i.e. when $x^2 + 1$ is minimum.

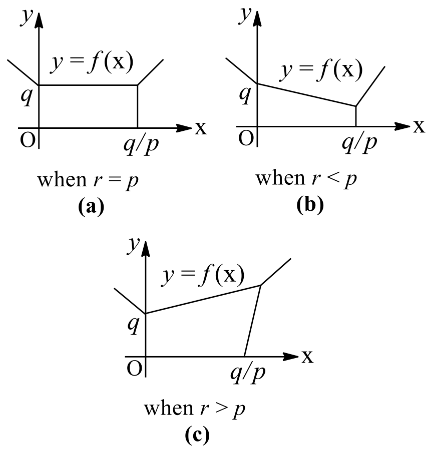
Obviously, $x^2 + 1$ is minimum at $x = 0$

$\therefore$ Minimum value of $f(x)$ is $f(0) = -1$

32 **(c)**

We have, $f(x) = |px - q| + r|x|, x \in (-\infty, \infty)$

$$= \begin{cases} -px + q - rx, & x \leq 0 \\ -px + q + rx, & 0 < x < q/p \\ px - q + rx, & q/p < x \end{cases}$$



Thus, f has infinite points if minimum, if $r = p$

In case, $p \neq r$, then $x = 0$ is point of minimum, if $r > p$ and $x = \frac{q}{p}$ is point of minimum, if $r < p$

33 **(d)**

Since $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all x

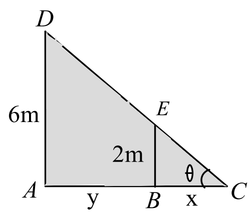
$\therefore f'(x) > 0$ for all x

$\Rightarrow \dfrac{K - 2}{(\sin x + \cos x)} > 0$ for all x

$\Rightarrow K - 2 > 0 \Rightarrow K > 2$

34 **(c)**

In ΔADC , $\qquad \tan\theta = \frac{6}{x+y}$



And in ΔBCE, $\qquad \tan\theta = \frac{2}{x}$

$\therefore \dfrac{2}{x} = \dfrac{6}{x + y} \Rightarrow y = 2x$

On differentiating w. r. t. t,we get

$\frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 3km/h \quad \left[\because \frac{dy}{dt} = 6, \text{given}\right]$

36 **(a)**

Using Mean value theorem,

$f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$

$\Rightarrow \dfrac{1}{c} = \dfrac{\log_e 3 - \log_e 1}{1}$

$\Rightarrow c = \dfrac{2}{\log_e 3} = 2\log_3 e$

37 **(a)**

Let $f(x) = ax + \frac{b}{x}$. Then,

$ax + \frac{b}{x} \geq c$ for all $x > 0$

$\Rightarrow f(x) \geq c$ for all $x > 0$

$\Rightarrow c$ is the smallest value of $f(x)$

Now,

$$f'(x) = 0 \Rightarrow a - \frac{b}{x^2} = 0 \Rightarrow x = \pm\sqrt{\frac{b}{a}}$$

and, $f''(x) = \frac{2b}{x^3} > 0$ for $x = \sqrt{\frac{b}{a}}$

Thus, $f(x)$ attains a local minimum at $x = \sqrt{\frac{b}{a}}$

$\Rightarrow f\left(\sqrt{\frac{b}{a}}\right) \geq c$ [$\because c$ is the smallest value of $f(x)$]

$\Rightarrow \sqrt{ab} + \sqrt{ab} \geq c$

$\Rightarrow 2\sqrt{ab} \geq c \Rightarrow ab \geq \frac{c^2}{4}$ [$\because a, b, c$ are all positive]

38 **(b)**

We have,

$$f(x) = \int_0^x \frac{\cos t}{t} dt, x > 0 \Rightarrow f'(x) = \frac{\cos x}{x}, x > 0$$

$$\therefore f'(x) = 0 \Rightarrow \frac{\cos x}{x} = 0$$

$$\Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}, n \in Z$$

Now,

$$f'(x) = \frac{\cos x}{x}$$

$$\Rightarrow f''(x) = -\frac{x \sin x - \cos x}{x^2}$$

$$\Rightarrow f''\left((2\,n+1)\frac{\pi}{2}\right) = -\frac{2}{(2\,n+1)\pi}(-1)^n$$

$$= \frac{2(-1)^{n+1}}{(2\,n+1)\pi}$$

$$\Rightarrow f''\left((2\,n+1)\frac{\pi}{2}\right) > 0 \text{ for } n = -2, -4, -6, \ldots$$

< 0 for $n = 0, 2, 4, 6, \ldots$

> 0 for $n = 1, 3, 5, \ldots$

< 0 for $n = -1, -3, -5, \ldots$

Hence, $f(x)$ attains respectively minima, maxima, minima and maxima

39 **(c)**

We have,

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

$$\Rightarrow \frac{n\,x^{n-1}}{a^n} + \frac{n\,y^{n-1}}{b^n}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b^n\,x^{n-1}}{a^n\,y^{n-1}} \Rightarrow \left(\frac{dy}{dx}\right)_{(a,b)} = -\frac{b}{a} \text{ for all } n$$

The equation of the tangent at (a, b) is

$$y - a = -\frac{b}{a}(x - a) \Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

Thus, $\frac{x}{a} + \frac{y}{b} = 2$ touches the curve (i) at (a, b) for all n

40 **(c)**

We have,

$PV^{1/4} = \lambda$ (constant)

$$\Rightarrow \log P + \frac{1}{4}\log V = \log \lambda$$

$$\Rightarrow \frac{1}{P}\frac{dP}{dV} + \frac{1}{4V} = 0$$

$$\Rightarrow \frac{dP}{dV} = -\frac{1}{4}\frac{P}{V}$$

$$\therefore \Delta P = \frac{dP}{dV}\Delta V$$

$$\Rightarrow \Delta P = -\frac{1}{4}\frac{P}{V}\Delta V$$

$$\Rightarrow \frac{\Delta P}{P} \times 100 = -\frac{1}{4}\frac{\Delta V}{V} \times 100$$

$$\Rightarrow \frac{\Delta P}{P} \times 100 = -\frac{1}{4} \times -\frac{1}{2}$$

$$= \frac{1}{8}\left[\because \frac{\Delta V}{V} \times 100 = \frac{1}{2} \text{ (given)}\right]$$

41 **(c)**

From mean value theorem $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given, $a = 0 \Rightarrow f(a) = 0$

and $b = \frac{1}{2} \Rightarrow f(b) = \frac{3}{8}$

Now, $f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$

$\therefore f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$

$= c^2 - 3c + 2 + c^2 - 2c + c^2 - c$

$\Rightarrow f'(c) = 3c^2 - 6c + 2$

By definition of mean value theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{\left(\frac{3}{8}\right) - 0}{\left(\frac{1}{2}\right) - 0} = \frac{3}{4}$$

$$\Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$$

This is a quadratic equation in c

$$\therefore c = \frac{6 \pm \sqrt{36 - 15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$$

Since, 'c' lies between $\left[0, \frac{1}{2}\right]$

$$\therefore c = 1 - \frac{\sqrt{21}}{6} \quad \left(\text{neglecting } c = 1 + \frac{\sqrt{21}}{6}\right)$$

42 **(a)**

$$\because f(x) = \frac{1}{4x^2 + 2x + 1}$$

On differentiating w.r.t. x, we get

$f'(x) = \frac{-(8x+2)}{(4x^2+2x+1)^2}$ ...(i)

For maxima or minima put $f'(x) = 0$

$\Rightarrow \quad 8x + 2 = 0 \quad \Rightarrow \quad x = -\frac{1}{4}$

Again differentiating w.r.t. x of Eq. (i), we get

$$f''(x) = -\frac{\left[\begin{array}{c}(4x^2+2x+1)^2(8)-(8x-2)2\times \\ (4x^2+2x+1)(8x+2)\end{array}\right]}{(4x^2+2x+1)^4}$$

At $x = -\frac{1}{4}, f''\left(-\frac{1}{4}\right) = -\text{ve}$

$f(x)$ is maximum at $x = -\frac{1}{4}$

$\therefore$ maximum value of $f(x)$

$$f\left(-\frac{1}{4}\right)_{\max} = \frac{1}{4 \times \frac{1}{16} - 2 \times \frac{1}{4} + 1}$$

$$= \frac{1}{\frac{1}{4} - \frac{2}{4} + 1}$$

$$= \frac{4}{1 - 2 + 4} = \frac{4}{3}$$

44 **(b)**

Let $f(x) = 2x + 3y$

$\therefore \quad f(x) = 2x + \frac{18}{x} \qquad [\because xy = 6, \text{given}]$

$\Rightarrow \quad f'(x) = 2 - \frac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow \quad 0 = 2 - \frac{18}{x^2} \quad \Rightarrow \quad x = \pm 3$

And $\quad f''(x) = \frac{36}{x^3} \quad \Rightarrow f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum.

The minimum value is $f(3) = 12$

45 **(c)**

On differentiating given curve w. r. t. x, we get

$4\, y^3 \frac{dy}{dx} = 3ax^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$

$\therefore$ Equation of normal at point (a, a) is

$y - a = -\frac{4}{3}(x - a)$

$\Rightarrow \quad 4x + 3y = 7a$

47 **(b)**

Let $f(x) = x^3 - px + q$

Then $\quad f'(x) = 3x^2 - p$

Put $\quad f'(x) = 0 \quad \Rightarrow \quad x = \sqrt{\frac{p}{3}}, -\sqrt{\frac{p}{3}}$

Now, $f''(x) = 6x$

$\therefore$ At $\quad x = \sqrt{\frac{p}{3}, \ f''(x) = 6\sqrt{\frac{p}{3}} > 0,}$ minima

And at $x = -\sqrt{\frac{p}{3}, \ f''(x)} < 0$, maxima

48 **(a)**

For $x = p, y = ap^2bp + c$

And for $x = q, \ y = aq^2 + bq + c$

Slope $= \frac{aq^2+bq+c-ap^2-bp-c}{q-p}$

$= a(q + p) + b$

$\frac{dy}{dx} = 2ax + b = a(q + p) + b$

(according to the equation)

$\therefore \quad x = \frac{q + p}{2}$

49 **(c)**

Clearly, $f(x)$ is continuous and differentiable on the intervals $[0, 3]$ and $(0, 3)$ respectively for all $n \in N$

Also, $f(0) = f(3) = 0$

It is given that Rolle's theorem for the function $f(x)$ defined on $[0, 3]$ is applicable with $c = \frac{3}{4}$

$\therefore f'(c) = 0$

$\Rightarrow 2(c - 3)^n + 2nc(c - 3)^{n-1} = 0$

$[\because f(x) = 2x(x - 3)^n \therefore f'(x)$

$\qquad = 2(x - 3)^n + 2nx(x - 3)^{n-1}]$

$\Rightarrow 2(c - 3)^{n-1}(c - 3 + nc) = 0$

$\Rightarrow \frac{3}{4} - 3 + \frac{3}{4}n = 0 \Rightarrow n = 3 \quad \left[\because c = \frac{3}{4}\right]$

50 **(a)**

Given,

$y - x = 1$

$\Rightarrow \quad y = x + 1$

$\frac{dy}{dx} = 1$

And $y^2 = x$

$\Rightarrow \quad 2y\frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$

$\therefore \quad 1 = \frac{1}{2y}$

$\Rightarrow \quad 2y = 1$

$\Rightarrow \quad y = \frac{1}{2}$

$\therefore$ point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ required shortest distance

$$= \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

51 **(d)**

Since, $s^3 \propto v \Rightarrow \frac{ds}{dt} = ks^3$.......(i)

$\Rightarrow \quad \frac{d^2s}{dt^2} = 3ks^2\frac{ds}{dt}$

$\Rightarrow \quad \frac{d^2s}{dt} = 3k^2s^5 \quad [from\ Eq.(i)]$

Hence, acceleration of particle is proportional to s^5.

52 **(a)**

Given, $s = t^3 + 2t^2 + t$

$\Rightarrow \quad v = \frac{ds}{dt} = 3t^2 + 4t + 1$

Speed of the particle after 1 s

$v_{(t=1)} = \left(\frac{ds}{dt}\right)_{(t=1)}$

$= 3 \times 1^2 + 4 \times 1 + 1 = 3 + 5 = 8$ cm/s

54 **(b)**

Given, $y = a^x \quad \Rightarrow \quad \frac{dy}{dx} = a^x \log a$

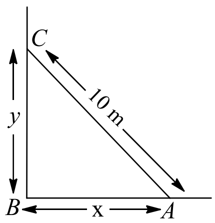
Now, length of subtangent at any point (x_1, y_1)

$= \frac{y}{dy/dx} = \frac{a^{x_1}}{a^{x_1}\log a} = \frac{1}{\log a}$

56 **(c)**

Let $AB = x$m, $BC = y$m and $AC = 10$m

$\therefore \quad x^2 + y^2 = 100 \quad$ ...(i)



On differentiating w.r.t t,m we get

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \quad 2x(3) - 2y(4) = 0$

$\Rightarrow \quad x = \frac{4y}{3}$

On putting this value in Eq. (i), we get

$\frac{16}{9}y^2 + y^2 = 100$

$\Rightarrow \quad y^2 = \frac{100\times 9}{25} = 36 \quad \Rightarrow \quad y = 6$m

57 **(c)**

Since $f(x)$, which is of degree 3, has relative minimum/maximum at $x = -1$ and $x = \frac{1}{3}$.

Therefore $x = -1, x = \frac{1}{3}$ are roots of $f'(x) = 0$.

Thus, $x + 1$ and $3x - 1$ are factors of $f'(x)$

Consequently, we have

$f'(x) = \lambda(x+1)(3x-1) = \lambda(3x^2 + 2x - 1)$

$\Rightarrow f(x) = \lambda(x^3 + x^2 - x) + c$

Now, $f(-2) = 0 \qquad$ [Given]

$\Rightarrow c = 2\lambda \qquad$ ...(i)

We have,

$\int_{-1}^{1} f(x)dx = \frac{14}{3}$

$\Rightarrow \int_{-1}^{1}[\lambda(x^3 + x^2 - x) + c]dx = \frac{14}{3}$

$\Rightarrow \lambda\int_{-1}^{1} x^2 + \int_{-1}^{1} c\,dx = \frac{14}{3} \Rightarrow \frac{2\lambda}{3} + 2\,c = \frac{14}{3} \Rightarrow \lambda + 3\,c = 7$ ...(ii)

Solving (i) and (ii), we get $\lambda = 1, c = 2$

Hence, $f(x) = x^3 + x^2 - x + 2$

58 **(d)**

$f(x) = cosx + \frac{1}{2}cos2x - \frac{1}{3}cos3x$

$\Rightarrow \quad f'(x) = -\sin x - \sin 2x + \sin 3x$

Put $\quad f'(x) = 0$

$\Rightarrow \ 2\sin\frac{3x}{2}\cos\frac{x}{2} = 2\sin\frac{3x}{2}\cos\frac{3x}{2}$

$\Rightarrow \ \sin\frac{3x}{2} = 0, \cos\frac{3x}{2} = \cos\frac{x}{2}$

$\Rightarrow \quad x = \frac{2n\pi}{3}, \frac{3x}{2} = 2n\pi \pm \frac{x}{2}$

$\Rightarrow$ At $x = 0, \frac{2\pi}{3}$,

At $x = 0$

$f(x) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$

At $\quad x = \frac{2\pi}{3}$,

$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3} = -\frac{13}{12}$

$\therefore$ difference $= \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$

59 **(a)**

Let $f(x) = e^{x-1} + x - 2$. Then,

$f(1) = e^0 + 1 - 2 = 0$

$\Rightarrow x = 1$ is a real roots of the equation $f(x) = 0$

Let $x = \alpha$ be other real root of $f(x) = 0$ such that $\alpha > 1$

Consider the interval $[1, \alpha]$

Clearly, $f(1) = f(\alpha) = 0$

So, by Rolle's theorem $f'(x) = 0$ has a root in $(1, \alpha)$

But, $f'(x) = e^{x-1} + 1 > 1$ for all x

$\therefore f'(x) \neq 0$ for any $x \in (1, \alpha)$

This is a contradiction

Hence, $f(x) = 0$ has no real root other than 1

60 **(d)**

We have,

$f(x) = \sin^6 x + \cos^6 x$

$\Rightarrow f(x) = (\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x + \cos^2 x)(\sin^2 x \cos^2 x)$

$\Rightarrow f(x) = 1 - 3(\sin^2 x \cos^2 x) = 1 - \frac{3}{4}\sin^2 2x$

$\therefore f(x) \le 1$, for all x $\left[\because -\frac{3}{4}\sin^2 2x < 0\right]$

and, $f(x) > 1 - \frac{3}{4} = \frac{1}{4}$ for all x $\left[\because -\frac{3}{4}\sin^2 2x \ge -\frac{3}{4}\right]$

61 **(a)**

Let $y = x^3 - 12x \quad \Rightarrow \frac{dy}{dx} = 3x^2 - 12$

Put $\frac{dy}{dx} = 0, \quad 3x^2 - 12 = 0$

$\Rightarrow \quad x = \pm 2$

At $x = 2, \quad y = 2^3 - 12(2) = -16$

At $x = -2, \quad y = (-2)^3 - 12(-2) = 16$

Hence, option (a) is correct

62 **(c)**

We have,

$xy = a^2$ and $S = b^2x + c^2y$

$\Rightarrow S = b^2x + \frac{c^2a^2}{x}$

$\Rightarrow \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2}$ and $\frac{d^3S}{dx^2} = \frac{2c^2a^2}{x^3}$

For local maximum or minimum, we must have

$\frac{dS}{x} = 0 \Rightarrow b^2 - \frac{c^2a^2}{x^2} = 0 \Rightarrow x^2 = \frac{c^2a^2}{b^2} \Rightarrow x = \pm\frac{ca}{b}$

Clearly, $\frac{d^2S}{dx^2} > 0$ for $x = \frac{ca}{b}$

So, $x = \frac{ca}{b}$ is the point of local minimum

Local minimum value of $S = b^2\left(\frac{ca}{b}\right) + c^2\left(\frac{a^2b}{ca}\right) = 2abc$

63 **(b)**

We have,

$g(x) = f(x) + f(1-x)$

$\therefore g'(x) = f'(x) - f'(1-x)$ for all $x \in [0,1]$

Now, $f''(x) < 0$ for $0 \le x \le 1$

$\Rightarrow f'(x)$ is a decreasing function on $[0, 1]$

$\Rightarrow f'(x) > f'(1-x)$ if $x < 1-x$

and,

$f'(x) < f'(1-x)$ if $x > 1-x$

$\Rightarrow f'(x) - f'(1-x) > 0$ if $x < \frac{1}{2}$

and,

$f'(x) - f'(1-x) < 0$ if $x > \frac{1}{2}$

$\Rightarrow g'(x) > 0$ if $x \in (0, 1/2)$

and,

$\Rightarrow g'(x) < 0$ if $x \in (1/2, 1)$

$\Rightarrow g(x)$ decreases on $[1/2, 1]$ and increases on $[0, 1/2]$

64 **(d)**

Since, $\quad f(x) = x\, e^{1-x}$

$f'(x) = -xe^{1-x} + e^{1-x}$

$= e^{1-x}(1-x)$

$\Rightarrow \quad f'(x) < 0, \ \forall\ (1, \infty)$

65 **(a)**

Consider the function

$$\phi(x) = a_0\frac{x^{n+1}}{n+1} + a_1\frac{x^n}{n} + a_2\frac{x^{n-1}}{n-1} + \cdots + a_{n-1}\frac{x^2}{2} + a_nx$$

Since $\phi(x)$ is a polynomial. Therefore, it is continuous on $[0, 1]$ and differentiable on $(0, 1)$

Also, $\phi(0) = 0$

and, $\phi(1) = \frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-2} + \cdots + a_n = 0$ [Given]

$\therefore \phi(0) = \phi(1)$

Thus, $\phi(x)$ satisfies conditions of Rolle's theorem on $[0, 1]$

Consequently, there exist $c \in (0, 1)$ such that $\phi'(c) = 0$ i.e. $c \in (0, 1)$ is a zero of $\phi'(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n = f(x)$

66 **(c)**

Given, $x = at^2 + bt + c$

$\Rightarrow$ (*speed*) $\frac{dx}{dt} = 2at + b$

$\Rightarrow$ (acceleration) $\frac{d^2x}{dt^2} = 2a$

$\therefore$ The particle will moving with Uniform acceleration.

69 **(a)**

On differentiating given equation w. r. t. x, we get

$\frac{dx}{dt} = 100 - \frac{25}{2}.(2t) = 100 - 25t$

At maximum height, velocity $\frac{dx}{dt} = 0$

$\therefore \quad 100 - 25t = 0 \quad \Rightarrow \quad t = 4$

$\therefore \quad x = 100 \times 4 - \frac{25 \times 16}{2} = 200m$

70 **(b)**

We have,

$y = \int_0^x |t|\, dt \quad$ ...(i)

$\Rightarrow \frac{dy}{dx} = |x|$

Let $P(x_1, y_1)$ be a point on the curve (i) such that the tangent at P is parallel to the line $y = 2x$

$\therefore$ (Slope of the tangent at P) $= 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 2 \Rightarrow |x_1| = 2 \Rightarrow x_1 = \pm 2$

Now, $y = \int_0^x |t|\, dt$

$$y = \begin{cases} \int_0^x t dt = \frac{x^2}{2}, & \text{if } x \geq 0 \\ -\int_0^x t\, dt = -\frac{x^2}{2}, & \text{if } x < 0 \end{cases}$$

$\therefore x_1 = 2 \Rightarrow y_1 = 2$ and $x_1 = -2 \Rightarrow y_1 = -2$

Thus, the two points on the curve are (2, 2) and (−2, −2)

The equations of the tangents at these two points are

$y - 2 = 2(x - 2)$ and $y + 2 = 2(x + 2)$ respectively

Or, $2x - y - 2 = 0$ and $2x - y + 2 = 0$ respectively

These tangents cut off intercepts −2 and 2 respectively on y-axis

71 **(a)**

We have,

$f'(x) = \sec^2 x - 1 \geq 0$ for all x [$\because |\sec x| \geq 1$ for all x]

Hence, $f(x)$ always increases

72 **(b)**

Let $P = xy$. Then,

$P = x(8 - x)$ [$\because x + y = 8$ (given)]

$\Rightarrow P = 8x - x^2 \Rightarrow \frac{dP}{dx} = 8 - 2x$ and $\frac{d^2P}{dx^2} = -2$

For maximum and minimum, we must have

$\frac{dP}{dx} = 0 \Rightarrow 8 - 2x = 0 \Rightarrow x = 4$

Clearly, $\frac{d^2P}{dx^2} = -1 < 0$ for all x

Hence, P is maximum when $x = y = 4$. The maximum value of P is given by $P = 4 \times 4 = 16$

73 **(d)**

Given curve is $y = 2x^2 - x + 1$ ...(i)

$\Rightarrow \frac{dy}{dx} = 4x - 1$

Since, tangent to the curve is parallel to the given line$y = 3x + 9$. Then, slopes will be equal

$\therefore \quad 4x - 1 = 3$

$\Rightarrow \quad x = 1$

From Eq. (i), $y = 2(1)^2 - 1 + 1 = 2$

Hence, required point is (1, 2)

74 **(d)**

Let $P(x_1, y_1)$ be a point on $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line $4x - 3y + 2 = 0$

$\therefore \left(\frac{dy}{dx}\right)_{(x_1,y_1)} \times \left(\frac{-4}{-3}\right) = -1 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{3}{4}$ ...(i)

Now,

$y^2 = 3x^3$

$\Rightarrow 2y\frac{dy}{dx} = 6x^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{3x_1^2}{y_1}$ ...(ii)

From (i) and (ii), we get

$\frac{3x_1^2}{y_1} = \frac{3}{4} \Rightarrow y_1 = 4x_1^2$ ...(iii)

Since (x_1, y_1) lies on $y^2 = 2x^3$

$\therefore y_1^2 = 2x_1^3$ ...(iv)

Solving (iii) and (iv), we get

$(4x_1)^2 = 2x_1^3 \Rightarrow x_1 = 0, x_1 = 1/8$

Putting the values of x_1 in (iv), we get

$y_1 = 0, y_1 = \pm\frac{1}{16}$

Hence, the required points are (0, 0), (1/8, 1/16), (1/8, −1/16)

75 **(d)**

We have,

$x = e^t \cos t$ and $y = e^t \sin t$

$\Rightarrow \frac{dx}{dt} = e^t(\cos t - \sin t)$ and $\frac{dy}{dt} = e^t(\sin t + \cos t)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t + \cos t}{\cos t - \sin t} \Rightarrow \left(\frac{dy}{dx}\right)_{t=\pi/4} = \infty$

So, tangent at $t = \frac{\pi}{4}$ subtends a right angle with x-axis

76 **(b)**

We have,

$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 4a\left(1 + \cos\frac{x}{a}\right)$

For points at which the tangents are parallel to x-axis, we must have

$\frac{dy}{dx} = 0$

$\Rightarrow 4a\left(1 + \cos\frac{x}{a}\right) = 0 \Rightarrow \cos\frac{x}{a} = -1 \Rightarrow \frac{x}{a} = (2n + 1)\pi$

For these values of x, we have $\sin\frac{x}{a} = 0$

Putting $\sin\frac{x}{a} = 0$ in (i), we get $y^2 = 4ax$

Therefore, all these points lie on the parabola $y^2 = 4ax$

77 **(b)**

Given $y = x^{5/2}$

$\therefore \frac{dy}{dx} = \frac{5}{2}x^{3/2}, \frac{d^2y}{dx^2} = \frac{15}{4}x^{1/2}$

At $x = 0, \frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$

and $\frac{d^3y}{dx^3}$ is not defined

When $x = 0, y = 0$

$\therefore$ $(0, 0)$ is a point of inflexion

78 **(a)**

Let $f(x) = 2x + 3y$ and $xy = 6$

$\Rightarrow \quad f(x) = 2x + \frac{18}{x}$

On differentiating w.r.t. x, we get

$f'(x) = 2 - \frac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow \quad 0 = 2 - \frac{18}{x^2} \Rightarrow x = \pm 3$

and $f''(x) = \frac{36}{x^3}$

$\Rightarrow \quad f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum

The minimum value of $f(x)$ is

$f(3) = 2(3) + 3(2) = 12$

79 **(b)**

Given, $f(x) = x^2 - 2x + 4$

$f'(x) = 2x - 2$

By applying Mean value theorem

$f'(c) = 2c - 2 = 0$

$\Rightarrow c = 1$

80 **(b)**

By the algebraic meaning of Rolle's theorem between any two roots of a polynomial there is always a root of its derivative

81 **(c)**

$p(t) = 1000 + \frac{1000t}{100 + t^2}$

$\Rightarrow \quad p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$

$= 1000\frac{(100 - t^2)}{(100 + t^2)^2}$

Put $p'(t) = 0$ for maxima or minima

$\Rightarrow \quad 100 - t^2 = 0$

$\Rightarrow \quad t = \pm 10$

Now, $p''(t) = 1000$

$\times \left[\frac{(100 + t^2)^2(-2t) - (100 - t^2)2(100 + t^2)2t}{(100 + t^2)^4}\right]$

$= 1000t\frac{[(100 + t^2)(-2) - (100 - t^2)(4)]}{(100 + t^2)^3}$

$= -1000t\,\frac{[600 - 2t^2]}{(100 + t^2)^3}$

At $t = 10$, $p''(t) < 0$

$\therefore$ The maximum value is

$p(10) = 1000 + \frac{10000}{100 + 100}$

$= 1000 + \frac{10000}{200} = 1050$

82 **(a)**

We have,

$f'(x) = (x - a)^{2n}(x - b)^{2m+1}$

$\therefore f'(x) = 0 \Rightarrow x = a, b$

For $x = b - h$, we have

$f'(x) = (b - h - a)^{2n}(-h)^{2m+1} < 0$

and for $x = b + h$, we have

$f'(x) = (b + h - a)^{2n}\,h^{2m+1} > 0$

Thus, as x passes through $b, f'(x)$ changes sign from negative

Hence, $x = b$ is a point of minimum

83 **(d)**

Given equation of curve is

$y = 4 - 2x^2$

$\Rightarrow \frac{dy}{dt} = -4x\frac{dx}{dt}$

Given $\frac{dx}{dt} = -5$, at point (1,2)

$\therefore \frac{dy}{dt} = -4(1)(-5) = 20$unit/s

84 **(c)**

Given $y^2 = 2(x-3)$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}$

Slope of the normal$= \frac{-1}{(dy/dx)} = -y$

Slope of the given line=2

$\therefore y = -2$

From Eq. (i), $x = 5$

$\therefore$ Required point is $(5, -2)$

85 **(a)**

Given, $f(x) = \frac{x}{1+|x|}$

$\therefore f'(x) = \frac{(1+|x|).1 - x.\frac{|x|}{x}}{(1+|x|)^2}$

$= \frac{1}{(1+|x|)^2} > 0 \forall x \in R$

$\Rightarrow f(x)$ is strictly increasing

86 **(d)**

Given, $f(x) = 2x^2 - 3x^2 + 90x + 174$

$\therefore f'(x)6x^2 - 6x + 90$

Now, $D = b^2 - 4ac = 36 - 4 \times 6 \times 90 < 0$

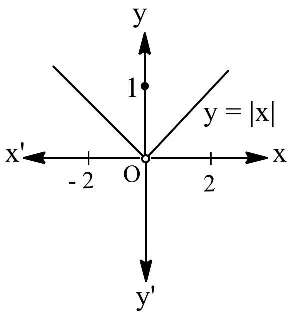
$\therefore f'(x) > 0 \forall x \in (-\infty, \infty)$

87 **(a)**

Given,

$f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \le 2 \\ 1, & \text{for } x = 0 \end{cases}$

It is clear from the graph that $f(x)$ has local maximum.



88 **(a)**

We have,

$f(x) = x^2 + ax + 1$

$\Rightarrow f'(x) = 2x + a$

For $f(x)$ to be increasing on $[1, 2]$, we must have

$f'(x) > 0$ for all $x \in R$

Now,

$f'(x) = 2x + a$

$\Rightarrow f''(x) = 2 > 0$ for all $x \in R$

$\Rightarrow f'(x)$ is increasing for all $x \in R$

$\Rightarrow f'(x)$ is increasing on $[1, 2]$

$\Rightarrow f'(1)$ is the minimum value of $f(x)$ in $[1, 2]$

Thus,

$f'(x) > 0$ for all $x \in [1, 2]$

$\Rightarrow f'(1) > 0$

$\Rightarrow 2 + a > 0 \Rightarrow a > -2 \Rightarrow a \in (-2, \infty)$

89 **(c)**

Since, $\frac{dx}{dt} = \frac{dy}{dt}$ ... (i)

Given equation of curve is

$y = x^2 + 2x$

$\Rightarrow \frac{dy}{dt} = (2x+2)\frac{dx}{dt}$

$\Rightarrow 1 = 2x + 2$ [from Eq.(i)]

$\Rightarrow x = -1/2, \ y = -3/4$

$\therefore$point on the curve is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

90 **(b)**

Given, $p(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow p'(x) = 4x^3 + 3ax^2 + 2bx + c$

$\therefore x = 0$ *is* a solution for p'(x)=0,

$\Rightarrow c = 0$

$\therefore p(x) = x^4 + ax^3 + bx^2 + d$ ...(i)

Also , we have p(−1)<p(1)

$\Rightarrow 1 - a + b + d < 1 + a + b + d$

$\Rightarrow a > 0$

$\therefore p'(x) = 0,$ *only* when x=0

and $p(x)$*is differentiable in* $(-1,1)$, *we should h at the*

points $x = -1,0$ and 1 only

Also ,we have $p(-1) < p(1)$

$\therefore$ Maximum of $p(x) = \max\{p(0), p(1)\}$

And minimum of P(x)=Min {P(-1), P(0)}

In the interval [0,1]

$p'(x) = 4x^3 + 3ax^2 + 2bx$

$= x(4x^2 + 3ax + 2b)$

$\therefore p'(x)$*has only one rootx*

$= 0,$ *then* $4x^2 + 3ax + 2b = 0$ *has*

No real roots.

$\therefore (3a)^2 - 32b < 0$

$\Rightarrow \frac{3a^2}{32} < b$

$\therefore b > 0$

Thus , we have a>0 and b>0

$\therefore p'(x) = 4x^3 + 4ax^2 + 2bx > 0, \forall x \in (0,1)$

Hence ,$p(x) = p(1)$

Similarly,$p(x)$ is decreasing in [-1,0].

Therefore, Minimum $p(x)$does not occur at $x = -1$.

91 **(a)**

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x$. It is given that α is a positive root of $f(x) = 0$. Also, we observe that $f(0) = 0$. Thus, $x = 0$ and $x = \alpha$ are two roots of $f(x)$

By Rolle's theorem, $f'(x) = 0$ has at least one real root between 0 and α

$\therefore n\, a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1 = 0$

Has at least one real root between 0 and α

92 **(d)**

$\because A = \pi r^2$

$\Rightarrow \log A = log\pi + 2\log r$

$\Rightarrow \frac{\Delta A}{A}100 = 2 \times \frac{\Delta r}{100}$

$= 2 \times 0.05$

$= 0.1\%$

94 **(b)**

Equation of tangent at $(3\sqrt{3}, \cos\theta, \sin\theta)$ is

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$$

Thus, sum of intercepts= $\left(3\sqrt{3}\sec\theta + \operatorname{cosec}\theta\right) = f(\theta)$ [say]

$$\Rightarrow \quad f'(\theta) = \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$$

Put $f'(\theta) = 0$

$$\therefore \quad \sin^3\theta = \frac{1}{3^{3/2}}\cos^3\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = \frac{\pi}{6}$$

Also, for $0 < \theta < \frac{\pi}{6}, \frac{dz}{d\theta} < 0$ and for $\frac{\pi}{6} < \theta < \frac{\pi}{2}, \frac{dz}{d\theta} > 0$

$\therefore$Minimum at $\theta = \frac{\pi}{6}$

95 **(b)**

Since, $f(x) - |x|, \forall -2 \le x \le 2$

$$= \begin{cases} -x, & \forall -2 \le x < 0 \\ x, & \forall\, 0 \le x \le 2 \end{cases}$$

This function is not derivable at $x = 0$, therefore Rolle's theorem is not applicable

96 **(a)**

Given equation of curve is

$$y = a(e^{\frac{x}{a}} + e^{-x/a})$$

$$\Rightarrow \quad \frac{dy}{dx} = a\left(e^{x/a}.\frac{1}{a} - e^{-x/a}.\frac{1}{a}\right)$$

Since, the tangent is parallel to x-axis,

$$\therefore \quad \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \left(e^{x/a} - e^{-x/a}\right) = 0$$

$$\Rightarrow \quad e^{2x/a} = 1 \quad \Rightarrow \quad x = 0$$

97 **(a)**

Given, $f(x) = \sin x - \cos x - ax + b$

On differentiating w.r.t. x, we get

$f'(x) = \cos x + \sin x - a$

For decreasing function, $f'(x) < 0$

$\Rightarrow \quad \cos x + \sin x - a < 0$

$\Rightarrow \quad a > \cos x + \sin x$

Since, $-\sqrt{2} \le \cos x + \sin x \le \sqrt{2}$

$\therefore \quad a \ge \sqrt{2}$

98 **(b)**

$f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$

$f'(x) = 4x + 4.2^2x^3 + \ldots + 20.2^{10}x^{19}$

$= x(4 + 4.2^2x^2 + \ldots + 20.2^{10}x^{18})$

For a maximum or minimum, put $f'(x) = 0$

$\Rightarrow \quad x = 0$

But $4 + 12.2^2 + x^2 + \cdots + 20.19.2^{10}x^{18} > 0$

For $x < 0 \Rightarrow f'(x) < 0$ and $x > 0 \Rightarrow f'(x) > 0$

$\therefore$ Exactly one minimum.

99 **(b)**

We have,

Length of the subnormal $= y\frac{dy}{dx}$

Now,

$$y = a^{1-n}x^n \Rightarrow \frac{dy}{dx} = n\, a^{1-n}x^{n-1}$$

$\therefore$ Length of the subnormal $= y\, na^{1-n}x^{n-1} = n(a^{1-n})^2x^{2\,n-1}$

Since the subnormal is of constant length

$\therefore 2n - 1 = 0 \Rightarrow n = 1/2$

100 **(a)**

Given, $x = a(1 + \cos\theta)$, $y = a\sin\theta$

$$\Rightarrow \quad \frac{dx}{d\theta} = a(-\sin\theta), \quad \frac{dy}{d\theta} = a\cos\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{-\cos\theta}{\sin\theta}$$

Equating of normal at the given point is

$$y - a\sin\theta = \frac{\sin\theta}{\cos\theta}[x - a](1 + \cos\theta)]$$

It is clear that in the given options normal passes through the point $(a, 0)$

101 **(c)**

We have, $x = at^2 \Rightarrow \frac{dx}{dt} = 2at$

and $y = 2at \Rightarrow \frac{sy}{st} = 2a$

$\therefore$ Slope of tangent $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$$\Rightarrow \quad \frac{1}{t} = \infty$$

$\Rightarrow \quad t = 0 \Rightarrow$ Point of contact is $(0, 0)$

102 **(d)**

Curve is $y^2 = px^3 + q$

$\therefore\ 2y\dfrac{dy}{dx} = 3px^2$

$\Rightarrow\ \left(\dfrac{dy}{dx}\right)_{(2,3)} = \dfrac{3p.4}{2.3}$

$\Rightarrow\ 4 = 2p$

$\Rightarrow\ p = 2$

Also, curve is passing through (2, 3)

$\therefore\ 9 = 8p + q$

$\Rightarrow\ q = -7$

$\therefore (p, q)$ is $(2, -7)$

103 **(b)**

Given, $x = t - 6t^2 + t^3$

On differentiating, w. r. t. x, we get

$\frac{dx}{dt} = 1 - 12t + 3t^2$

Again, differentiating, we get

$\dfrac{d^2x}{dt^2} = -12 + 6t$

When $\frac{d^2x}{dt^2} = 0 \Rightarrow t = 2$units time

104 **(a)**

Given equation of curve is

$y^2 = px^3 + q$

$\therefore\ 2y\dfrac{dy}{dx} = 3px^2$

$\therefore\ \left(\dfrac{dy}{dx}\right)_{(2,3)} = \dfrac{3p\ (2)^2}{2.3} = 2p$

The equation of tangent at (2, 3) is

$(y - 3) = 2p(x - 2)$

$\Rightarrow\ y = 2px - (4p - 3)$

This is similar to $y = 4x - 5$

$\therefore\ 2p = 4$ and $4p - 3 = 5$

$\Rightarrow\ p = 2$ and $p = 2$

Since, the point (2, 3) lies on the curve

$\therefore\ 9 = 8p + q \quad \Rightarrow \quad q = -7 \quad [\because p = 2]$

105 **(c)**

Given, $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}, for\ u \in (-\infty, \infty)$

Now, $g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2}$

$= 2(\cot^{-1}(e^u)) - \dfrac{\pi}{2}$

$= 2\left(\dfrac{\pi}{2} - \tan^{-1}(e^u)\right) - \dfrac{\pi}{2}$

$= -g(u)$

$\therefore\ g(u)$ Is an odd function.

Also, $g'(u) = 2\frac{1}{1+(e^u)^2}.e^u - 0 > 0$

Which is strictly increasing in $(-\infty, \infty)$

106 **(b)**

Given curve is $y = 2x^2 - x + 1$

Let the coordinate of P are (h, k)

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 4x - 1$

At the point (h, k), the slope

$= \left(\dfrac{dy}{dx}\right)_{(h,k)} = 4h - 1$

Since, the tangent is parallel to the given line $y = 3x + 4$

$\Rightarrow\ 4h - 1 = 3 \ \Rightarrow\ h = 1, k = 2$

$\therefore$ Coordinates of point P are (1, 2)

107 **(a)**

Let $f(x) = ax^2 + \frac{b}{x^2} - c$, where $a, b, c > 0$ and $x > 0$

Then, $f'(x) = 2ax - \frac{2b}{x^3}$ and $f''(x) = 2a + \frac{6b}{x^4}$

For local maximum or minimum, we must have

$f'(x) = 0 \Rightarrow x^4 = \dfrac{b}{a} \Rightarrow x = \pm\left(\dfrac{b}{a}\right)^{1/4}$

Clearly, $f''\left\{\pm\left(\frac{b}{a}\right)^{1/4}\right\} = 2a + 6a > 0$

Therefore, $x = \pm\left(\frac{b}{a}\right)^{1/4}$ are points of local minimum

Local minimum value of $f(x)$ is given by

$f\left\{\left(\dfrac{b}{a}\right)^{1/4}\right\} = a\left(\dfrac{b}{a}\right)^{1/2} + b\left(\dfrac{a}{b}\right)^{1/2} - c = 2\sqrt{ab} - c$

But, it is given that

$f(x) \geq 0$ for all $x \quad \left[\because ax^2 + \frac{b}{x^2} \geq c\right]$

$\therefore 2\sqrt{ab} - c \geq 0 \Rightarrow 4ab \geq c^2$

108 **(a)**

Since $f(x) = x^4 - 62\,x^2 + ax + 9$ attains its maximum at $x = 1$

$\therefore f'(x) = 0$ at $x = 1$

$\Rightarrow f'(x) = 4\,x^3 - 124\,x + a = 0$ at $x = 1$

$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

109 **(b)**

On differentiating given curve w.r.t. x, we get

$\dfrac{1}{2\sqrt{x}} + \dfrac{1}{2\sqrt{y}}.\dfrac{dy}{dx} = 0$

$\Rightarrow\ \dfrac{dy}{dx} = -\dfrac{\sqrt{y}}{\sqrt{x}}$

The normal is parallel to x-axis, if

$-\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \quad \Rightarrow \quad \sqrt{\frac{x_1}{y_1}} = 0 \ \Rightarrow \quad x_1 = 0$ ...(ii)

Since, the point (x_1, y_1) lies on a curve

$\therefore\ \sqrt{x_1} + \sqrt{y_1} = \sqrt{a}$

$\Rightarrow\ 0 + \sqrt{y_1} = \sqrt{a} \qquad$ [from Eq. (ii)]

$\Rightarrow \quad y_1 = a$

$\therefore$ Required point is $(0, a)$

110 **(a)**

$y = (2x-1)e^{2(1-x)}$

$\Rightarrow \quad \frac{dy}{dx} = 2e^{2(1-x)} - 2(2x-1)e^{2(1-x)}$

$= 2e^{2(1-x)}(2-2x)$

$= 4e^{2(1-x)}(1-x)$

Put, $\frac{dy}{dx} = 0 \Rightarrow \quad x = 1$

Now, $\frac{d^2y}{dx^2} = -8e^{2(1-x)}(1-x) - 4e^{2(1-x)}$

$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)_{x=1} = -4 < 0$

So, y is maximum at $x = 1$, when $x = 1, y = 1$.

Thus, the point of maximum is (1,1).

The equation of the tangent at (1,1) is

$y - 1 = 0(x-1) \Rightarrow y = 1$

111 **(c)**

If $f'''(a) > 0$, then it is not an extreme point

112 **(b)**

Given function is $f(x) = x + \sin x$

On differentiating w.r.t. x, we get

$f'(x) = 1 + \cos x$

For maxima or minima put $f'(x) = 0$

$\Rightarrow \quad 1 + \cos x = 0 \quad \Rightarrow \cos x = -1 \quad \Rightarrow \quad x = \pi$

Again differentiating w.r.t. x, we get

$f''(x) = -\sin x, at \;\; x = \pi f''(\pi) = 0$

Again differentiating w.r.t. x, we get $f'''(x) = -\cos x$,

$f'''(\pi) = 1$

At $x = \pi, f(x)$ is minimum

113 **(b)**

Let $V = \frac{4}{3}\pi r^3 \;\Rightarrow\; \frac{dV}{dt} 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{30}{4 \times \pi \times 15 \times 15}$

$= \frac{1}{30\pi} ft/\min \left[\because \frac{dV}{dt} = 30, r = 15\right]$

114 **(a)**

Given, $\frac{x^2}{3} - \frac{y^2}{2} = 1 \qquad$ ...(i)

$\Rightarrow \quad \frac{2x}{3} - y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{3y}$

Now, slope of the line $y = x$ is 1

Since, tangent is parallel to given line, then

$\frac{2x}{3y} = 1 \quad \Rightarrow \quad x = \frac{3y}{2}$ ...(ii)

From Eqs. (i) and (ii), we get $y = \pm 2$

$\therefore$ Points of contact are $(3, 2)$ and $(-3, -2)$

$\therefore$ equations of tangents are

$y - 2 = (x - 3) \quad \Rightarrow \quad x - y - 1 = 0$

and $y + 2 = x + 3 \quad \Rightarrow \quad x - y + 1 = 0$

115 **(c)**

The two curves are

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i) and, $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ ...(ii)

Differentiating with respect to x, we get

$\left(\frac{dy}{dx}\right)_{C_1} = -\frac{b^2}{a^2}\frac{x}{y}, \left(\frac{dy}{dx}\right)_{C_2} = \frac{m^2}{l^2}\frac{x}{y}$

The two curves intersect orthogonally, iff.

$\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2} = -1$

$\Rightarrow -\frac{b^2}{a^2}\frac{x}{y} \times \frac{m^2}{l^2}\frac{x}{y} = -1$

$\Rightarrow m^2b^2x^2 = a^2l^2y^2$ ...(iii)

Subtracting (ii) from (i), we get

$x^2\left(\frac{1}{a^2} - \frac{1}{l^2}\right) + y^2\left(\frac{1}{b^2} + \frac{1}{m^2}\right) = 0$ ...(iv)

From (iii) and (iv), we get

$\frac{1}{m^2b^2}\left(\frac{1}{a^2} - \frac{1}{l^2}\right) = -\frac{1}{a^2l^2}\left(\frac{1}{b^2} + \frac{1}{m^2}\right)$

$\Rightarrow l^2 - a^2 = -b^2 - m^2 \Rightarrow a^2 - b^2 = l^2 + m^2$

116 **(b)**

We have,

$\frac{dy}{dt} > \frac{dx}{dt}, \frac{dy}{dt} > 0$ and $\frac{dx}{dt} > 0$

Now, $12y = x^3$

$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 3\left(x^2\frac{dx}{dt} - 4\frac{dy}{dt}\right) = 0$

$\Rightarrow x^2\frac{dx}{dt} = 4\frac{dy}{dt}$

We have,

$\frac{dy}{dt} > \frac{dx}{dt}$

$\Rightarrow \frac{x^2}{4}\frac{dx}{dt} > \frac{dx}{dt}$

$\Rightarrow (x^2 - 4)\frac{dx}{dt} > 0 \Rightarrow x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

117 **(b)**

Let the centre of circle on y-axis be (0.k).

Let $d = \sqrt{(7-0)^2 + (3-k)^2}$

$\Rightarrow \qquad d^2 = 7^2 + (3-k)^2 = D \qquad$ (say)

On differentiating w. r. t. k , we get

$\frac{dD}{dk} = 0 + 2(3-k)(-1)$

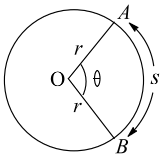
Put $\frac{dD}{dk} = 0 \Rightarrow k = 3$

Now, $\frac{d^2D}{dk^2} = 2 > 0 \; minima,$

$\therefore$ *Minimum* value at k=3 is d=7
Hence, minimum circle of radius 7.

118 **(b)**
$f(x) = e^x \sin x$
$f'(x) = e^x cosx + sin\, xe^x$
$f''(x) = -e^x sin\, x + \cos x e^x + e^x \cos x + e^x \sin x$
$2e^x cosx$
For maximum slope
$f''(x) = 0$
$\Rightarrow \; 2e^x \cos x = 0$
$\Rightarrow \; \cos x = 0$
$\Rightarrow \; x = \frac{\pi}{2}, \frac{3\pi}{2}, \forall x \in [0, 2\pi]$
$f'''(x) = 2 \quad [-e^x \sin x + e^x \cos x]$
$at \;\; x = \frac{\pi}{2} \;\; , f'''(x) < 0$
$and \;\; at \;\; x = \frac{3\pi}{2} f'''(x) > 0$
$\therefore$ slop is maximum at $x = \frac{\pi}{2}$

119 **(d)**
$\because$ Perimeter of a sector= p
Let AOB be the sector with radius r



If angle of the sector be θ radius, then area of sector,
$A = \frac{1}{2} r^2 \theta$ ...(i)
And length of arc, $s = r\theta$
$\Rightarrow \; \theta = \frac{s}{r}$
$\therefore$ perimeter of the sector
$p = r + s + r = 2r + s$ ...(ii)
On subtracting $\theta = \frac{s}{r}$ in Eq. (i), we get
$A = \left(\frac{1}{2} r^2\right)\left(\frac{s}{r}\right) = \frac{1}{2} rs$
$\Rightarrow \; s = \frac{2A}{r}$
Now, on substituting the value of s in Eq. (ii), we get
$p = 2r + \left(\frac{2A}{r}\right) \;\; \Rightarrow \;\; 2A = pr - 2r^2$
On differentiating w.r.t. r, we get
$2\frac{dA}{dr} = p - 4r$
For the maximum area, put
$2\frac{dA}{dr} = 0 \;\Rightarrow\; p - 4r = 0 \;\Rightarrow\; r = \frac{p}{4}$

120 **(b)**
Given, $y^2 = x^3$
On differentiating w.r.t x, we get
$2y\frac{dy}{dx} = 3x^2$
$\Rightarrow \; \frac{dy}{dx} = \frac{3x^2}{2y} \;\; \Rightarrow \;\; \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{3}{2}$
$\therefore$ Equation of normal at point (1, 1) is
$y - 1 = -\frac{2}{3}(x - 1)$
$\Rightarrow \;\; 2x + 3y = 5$
Hence, the equation of line parallel to above line will be
in option (b), *ie*, $\;\; 2x + 3y = 7$

121 **(a)**
Given that, $s = \sqrt{t}$
$\therefore \; \frac{ds}{dt} = \frac{1}{2\sqrt{t}}$
$\Rightarrow \; v = \frac{1}{2\sqrt{t}} \;\; \Rightarrow \;\; \frac{dv}{dt} = -\frac{1}{2.2t^{3/2}}$
$\Rightarrow \; a = -\frac{2}{(2\sqrt{t})^3}$
$\Rightarrow \; a = -2v^3$
$\Rightarrow \; a \propto v^3$

122 **(d)**
Let $PQ = a$ and $PR = b$, then $\Delta = \frac{1}{2} ab \sin\theta$
$\because \; -1 \le \sin\theta \le 1$
$\therefore$ Area is maximum when $\sin\theta = 1 \Rightarrow \;\; \theta = \frac{\pi}{2}$

123 **(a)**
$f'(x) = -\text{a}\sin x + b\sec^2 x + 1$
Now, $f'(0) = 0$ and $f'\left(\frac{\pi}{6}\right) = 0$
$\Rightarrow \; b + 1 = 0$ and $-\frac{a}{2} + \frac{4b}{3} + 1 = 0$
$\Rightarrow \; b = -1, a = -\frac{2}{3}$

124 **(c)**
Given curve is
$y = e^{2x} + x^2$
At $\;x = 0, y = 1$
$\therefore$ Any point on the curve is
$\frac{dy}{dx} = 2e^{2x} + 2x$
Slope of normal at (0,1)$= -\frac{1}{2+0} = -\frac{1}{2}$
$\therefore$ Equation of normal is
$y - 1 = -\frac{1}{2}(x - 0)$
$\Rightarrow \; 2y - 2 = -x$
$\Rightarrow \;\; x + 2y - 2 = 0$

Required distance $=\left|\frac{0+0-2}{\sqrt{1+4}}\right|$

$= \frac{2}{\sqrt{5}}$

125 **(d)**

We have, $f(x) = x\, e^{1-x}$

$\Rightarrow f'(x) = e^{1-x}(1-x) < 0$ for all $x \in (1, \infty)$

So, $f(x)$ strictly decreases in $(1, \infty)$

126 **(a)**

Let $x_1, x_2 \in R$ such that $x_1 < x_2$. Then,

$x_1 < x_2$

$f(x_1) > f(x_2)$ [$\because f$ is a decreasing function]

$\Rightarrow g(f(x_1)) < g(f(x_2))$ [$\because$ g is a decreasing function]

$\Rightarrow gof(x_1) < gof(x_2)$

127 **(d)**

$\because \quad y^2 = 4ax$

$\therefore \frac{dy}{dx} = \frac{2a}{y}$

Length of subnormal $= y\frac{dy}{dx} = y\frac{2a}{y} = 2a$

128 **(c)**

Given, $x = at^2$ and $y = 2at$

$\therefore \quad \frac{dx}{dt} = 2at$ and $\frac{dy}{dt} = 2a$

$\therefore$ Slope of tangent, $\left(\frac{dy}{dx}\right) = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{1}{t} = \infty, \quad \Rightarrow \quad t = 0$ [given]

$\therefore$ Point of contact is (0, 0)

129 **(b)**

We have,

$ay^2 = x^3$

$\Rightarrow 2\,ay\frac{dy}{dx} = 3\,x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3\,x^2}{2ay}$

Let (x_1, y_1) be a point on $ay^2 = x^3$. Then,

$ay_1^2 = x_1^3$ ...(i)

The equation of the normal at (x_1, y_1) is

$y - y_1 = -\frac{2ay_1}{3x_1^2}(x - x_1)$

This meets the coordinate axes at

$A\left(x_1 + \frac{3x_1^2}{2a}, 0\right)$ and $B\left(0, y_1 + \frac{2ay_1}{3x_1}\right)$

Since the normal cuts off equal intercepts with the coordinate axes

$\therefore x_1 + \frac{3\,x_1^2}{2\,a} = y_1 + \frac{2\,ay_1}{3\,x_1}$

$\Rightarrow x_1\frac{(2a + 3x_1)}{2a} = y_1\frac{(3x_1 + 2a)}{3x_1}$

$\Rightarrow 3x_1^2 = 2ay_1$

$\Rightarrow 9x_1^4 = 4a^2y_1^2$ ...(ii)

From (i) and (ii), we get

$9x_1^4 = 4a^2\left(\frac{x_1^3}{a}\right) \Rightarrow x_1 = \frac{4a}{9}$

130 **(d)**

Given, $y = 2x^2 - 6x - 4 \quad \Rightarrow \quad \frac{dy}{dx} = 4x - 6$

Since, $\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = 4x - 6 = 0 \quad \Rightarrow \quad x = \frac{3}{2}$

$\Rightarrow \quad y = 2.\frac{9}{4} - 6.\frac{3}{2} - 4 = -\frac{17}{2}$

$\therefore$ Required point is $\left(\frac{3}{2}, -\frac{17}{2}\right)$

131 **(d)**

$\because f(x) = x^3 - 6x^2 + ax + b$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + a$

By the definition of Rolle's theorem

$f'(c) = 0 \quad \Rightarrow \quad f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow \quad 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 = -12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$

$\Rightarrow \quad 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) - 12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$

$\Rightarrow \quad 12 + 1 + 4\sqrt{3} - 24 - 4\sqrt{3} + a = 0$

$\Rightarrow \quad a = 11$

132 **(b)**

For the point $(3, \log 2)$, we take $y = g(x) = \log(x - 1)$

$\frac{dy}{dx} = \frac{1}{(x-1)} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(3, \log 2)} = \frac{1}{2}$

$\therefore$ Equation of normal is

$y - \log 2 = -2(x - 3)$

$\Rightarrow \quad y + 2x = 6 + \log 2$

134 **(c)**

Let T be the processing time. The number of batches is $\frac{N}{x}$ and the processing time of one batch is $\alpha + \beta\, x^2$ seconds

$\therefore T = \frac{N}{x}(\alpha + \beta\, x^2) = N\left(\frac{\alpha}{x} + \beta\, x\right) \Rightarrow \frac{dT}{dx}$

$= N\left(-\frac{\alpha}{x^2} + \beta\right)$

For fast processing T must be least for which $\frac{dT}{dx} = 0$

$\therefore N\left(-\frac{\alpha}{x^2} + \beta\right) = 0 \Rightarrow x = \sqrt{\frac{\alpha}{\beta}}$

Clearly, $\frac{d^2T}{dx^2} = N\frac{2\alpha}{x^3} > 0$ for $x = \sqrt{\frac{\alpha}{\beta}}$

Hence, T is least when $x = \sqrt{\frac{\alpha}{\beta}}$

135 **(d)**

Let (x, y) be the point on the curve $2x^2 + y^2 - 2x = 0$. Then its distance from $(a, 0)$ is given by

$S = \sqrt{(x-a)^2 + y^2}$

$\Rightarrow S^2 = x^2 - 2\,ax + a^2 + 2\,x - 2\,x^2$ [Using $2\,x^2 + y^2 - 2\,x = 0$]

$\Rightarrow S^2 = -x^2 + 2\,x(1-a) + a^2$ ...(i)

$\Rightarrow 2\,S\,\frac{dS}{dx} = -2\,x + 2(1-a)$

For S to be maximum, we must have,

$\frac{dS}{dx} = 0 \Rightarrow -2x + 2(1-a) = 0 \Rightarrow x = 1-a$

It can be easily checked that $\frac{d^2S}{dx^2} < 0$ for $x = 1-a$

Hence, S is maximum for $x = 1-a$

Putting $x = 1-a$ in (i), we get $S = \sqrt{1 - 2\,a + 2\,a^2}$

136 **(b)**

Given curve is

$y = x^2 - 3x + 2$

$\frac{dy}{dx} = 2x - 3$

$(2x-3) \times 1 = -1$

$\Rightarrow \quad 2x - 3 = -1$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

$\therefore \quad y = 0$

$\Rightarrow$ Required point is(1,0).

137 **(c)**

The equations of the tangent and normal to $y^2 = 4ax$ at $P(at^2, 2at)$ are

$ty = x + at^2$ ...(i)

and, $y + tx = 2\,at + at^3$ ...(ii)

Liens (i) and (ii) meet the x-axis at $T(-at^2, 0)$ and $G(2\,a + at^2, 0)$ respectively

Since PT is perpendicular to PG. Therefore, TG is the diameter of the circle through P, T, G

Hence, the equation of the circle is

$(x + at^2)(x - 2a - at^2) + (y-0)(y-0) = 0$

$\Rightarrow x^2 + y^2 - 2ax - at^2(2a + at^2) = 0$

$\Rightarrow 2x + 2y\,\frac{dy}{dx} - 2a = 0$

$\Rightarrow \frac{dy}{dx} = \frac{a-x}{y}$

$\Rightarrow m_1 = \left(\frac{dy}{dx}\right)_P = \frac{a - at^2}{2at} = \frac{1-t^2}{2t}$ ...(i)

$\Rightarrow y^2 = 4ax$

$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{2a}{y}$

$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_P = \frac{2a}{2at} = \frac{1}{t}$ ...(ii)

Let θ be the angle between the tangents at P to the parabola and the circle. Then,

$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{1}{t} - \frac{1-t^2}{2t}}{1 + \frac{1-t^2}{2t^2}} = t \Rightarrow \theta = \tan^{-1} t$

138 **(a)**

Given, $x = a(\cos\theta + \theta\sin\theta)$

And $y = a(\sin\theta - \theta\cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\theta\cos\theta$

And $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$

$\therefore \frac{dy}{dx} = \tan\theta$

So, equation of normal is

$\Rightarrow \quad y - a\sin\theta + a\theta\cos\theta = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$

$y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta$

$= -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$

$\Rightarrow \quad x\cos\theta + y\sin\theta = a$

It is always at a constant distance 'a' from origin.

139 **(a)**

$\because \; f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$f'(x) = e^{x(1-x)+x.e^{x(1-x)}}.(1-2x)$

$= e^{x(1-x)}\{1 + x(1-2x)\}$

$= e^{x(1-x)}.(-2x^2 + x + 1)$

It is clear that $e^{x(1-x)} > 0$ for all x

Now, by sign rule for $-2x^2 + x + 1$

$f'(x) > 0$, if $x \in \left[-\frac{1}{2}, 1\right]$

So, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$

140 **(a)**

$\because \; f(x) = 2x^3 - 15x^2 + 36x + 4$

On differentiating w.r.t. x, we get

$f'(x) = 6x^2 - 30x + 36$ ...(i)

For maxima or minima $f'(x) = 0$

$\Rightarrow \; 6x^2 - 30x + 36 = 0$

$\Rightarrow \; x^2 - 5x + 6 = 0$

$\Rightarrow \; (x-2)(x-3) = 0 \quad \Rightarrow \quad x = 2, 3$

Again differentiating Eq. (i), we get

$f''(x) = 12x - 30$

$\Rightarrow \quad f''(2) = 24 - 30 = -6 < 0$

Therefore, $f(x)$ is maximum at, $x = 2$

141 **(b)**

We have,

$y^2 = 4ax$

$\Rightarrow 2y\frac{dy}{dt} = 4a\frac{dx}{dt}$

$\Rightarrow y\frac{dy}{dt} = 2a\frac{dx}{dt}$

$\Rightarrow y\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2 = 2a\frac{d^2x}{dt^2}$

$\Rightarrow y \times 0 + (\text{Constant})^2$

$$= 2a\frac{d^2x}{dt^2} \quad \left[\because \frac{dy}{dt} = \text{Constant}\right]$$

$\Rightarrow \frac{d^2x}{dt^2} = \text{Constant}$

$\Rightarrow$ Projection $(x, 0)$ of any point (x, y) on X-axis moves with constant acceleration

142 **(b)**

We have,

$$\phi(x) = \int_1^x e^{-t^2/2}(1 - t^2)dt \Rightarrow \phi'(x)$$

$$= e^{-x^2/2}(1 - x^2)$$

Now,

$\phi'(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

Hence, $x = \pm 1$ are points of extremum of $\phi(x)$

143 **(b)**

Given, $x = 80t - 16t^2$

$\Rightarrow \quad \frac{dx}{dt} = 80 - 32t$

At maximum height$\frac{dx}{dt} = 0$

$\therefore \quad t = 25s$

144 **(a)**

Let $f(x) = ax^2 + bx + 4$

On differentiating w. r. t., we get

$f'(x) = 2ax + b$

For minimum, put $f'(x) = 0 \quad \Rightarrow \quad x = -\frac{b}{2a}$

Since, it is given that at $x = 1$ minimum value is -1

$\therefore \quad 1 = -\frac{b}{2a} \quad \Rightarrow \quad 2a + b = 0 \qquad$ ...(i)

And $\quad f(1) = a + b + 4 = -1$

$\Rightarrow \quad a + b + 5 = 0 \qquad$ ...(ii)

On solving Eqs. (i) and (ii), we get $a = 5, b = -10$

145 **(a)**

Given, $g(x)1 + x) - \frac{2x}{2+x}$

$$\therefore f'(x) = \frac{1}{1+x} - \frac{(2+x)2 - 2x}{(2+x)^2}$$

$$= \frac{x^2}{(1+x)(x+2)^2}$$

Clearly $f'(x) > 0$ for all $x > 0$.

146 **(c)**

Let m be the slope of the curve $y = f(x)$. Then,

$m = \frac{dy}{dx}$

$\Rightarrow m = e^x(\sin x + \cos x)$

$\Rightarrow \frac{dm}{dx} = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$\Rightarrow \frac{dm}{dx} = 2e^x \cos x$

$\Rightarrow \frac{d^2m}{dx^2} = 2e^x(\cos x - \sin x)$

For maximum/minimum value of m, we must have

$\frac{dm}{dx} = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

Clearly, $\frac{d^2m}{dx^2} < 0$ for $x = \frac{\pi}{2}$

Hence, m is maximum when $x = \frac{\pi}{2}$

147 **(b)**

We have,

$y = \sqrt{9 - x^2} \qquad$ ...(i)

Clearly, y is positive and defined for $x \in [-3, 3]$

For the points whose ordinates and abscissae are same i.e. $y = x$, we have

$x = \sqrt{9 - x^2} \Rightarrow 2x^2 = 9 \Rightarrow x = \pm\frac{3}{\sqrt{2}}$

$\therefore y = \pm\frac{3}{\sqrt{2}} \qquad [\because y = x]$

But, $y > 0$. Therefore, $xy = y = \frac{3}{\sqrt{2}}$

So, the point is $P\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

Now,

$y = \sqrt{9 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{9 - x^2}} \Rightarrow \left(\frac{dy}{dx}\right)_p = -1$

148 **(a)**

We know that $\cos x$ is decreasing on $(0, \pi/2)$ and $\sin x < x$ for $0 < x < \frac{\pi}{2}$

$\therefore \cos(\sin x) > \cos x \;$ for $0 < x < \frac{\pi}{2}$

Also,

$0 < \cos x < 1 < \frac{\pi}{2}$ for $0 < x < \frac{\pi}{2}$

and, $\sin x < x \;$ for $0 < x < \frac{\pi}{2}$

$\therefore \sin(\cos x) < \cos x \;$ for $0 < x < \frac{\pi}{2}$

Hence, $\cos(\sin x) > \cos x$ and $\sin(\cos x) < \cos x$ for

$0 < x < \frac{\pi}{2}$

149 **(d)**

Given, $f(b) - f(a) = (b-a)f'(c)$

$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{3-2}{9-4}$

$\Rightarrow f'(c) = \frac{1}{5}$

$\Rightarrow \frac{1}{2}c^{-1/2} = \frac{1}{5}$

$\Rightarrow c = \left(\frac{5}{2}\right)^2 = 6.25$

150 **(a)**

We have,

$f(x) = (2a-3)(x + 2\sin 3) + (a-1)(\sin^4 x + \cos^4 x) + \log 2$

$\Rightarrow f'(x) = 2a - 3 + 4(a-1)\sin x \cos x(\sin^2 x - \cos^2 x)$

$\Rightarrow f'(x) = 2a - 3 - (a-1)\sin 4x$

If $f(x)$ does not have critical points, then $f'(x) = 0$ must not have any solution in R

$\Rightarrow (2a-3) - (a-1)\sin 4x = 0$ must have no solution in R

$\Rightarrow \sin 4x = \frac{2a-3}{a-1}$ must have no solution in R

$\Rightarrow \left|\frac{2a-3}{a-1}\right| > 1$

$\Rightarrow \frac{2a-3}{a-1} < -1$ or, $\frac{2a-3}{a-1} > 1$

$\Rightarrow \frac{3a-4}{a-1} < 0$ or, $\frac{a-2}{a-1} > 0$

$\Rightarrow a \in (1, 4/3)$ or, $a \in (-\infty, 1) \cup (2, \infty)$

$\Rightarrow a \in (-\infty, 1) \cup (1, 4/3) \cup (2, \infty)$

For $a = 1$, we have

$f'(x) = -1 \neq 0$

$\Rightarrow f(x)$ has no critical point for $a = 1$

Hence, $a \in (-\infty, 4/3) \cup (2, \infty)$

152 **(d)**

Given, curve is $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2 = m_1$ (say)

And $x = y^2 \Rightarrow 1 = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2} = m_2$ (say)

∴ Angle of intersection at the point (1, 1) is given by

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

153 **(c)**

Let volume of sphere $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2.(2)$ $\left[\because \frac{dr}{dt} = 2\right]$

$\therefore \frac{dV}{dt} = 8\pi(5)^2 = 200\pi\ cm^3/\min$ $[\because r = 5cm]$

154 **(c)**

Let area of circle, $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi.20.2$

$\Rightarrow \frac{dA}{dt} = 80\pi\ cm^2/s$

155 **(d)**

Given, $y = x^3 - 3x^2 - 9x + 5$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9$

We know that, this equation gives the slope of the tangent to the curve. The tangent is parallel to x-axis.

$\therefore \frac{dy}{dx} = 0 \Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1, 3$

156 **(b)**

$\because f(x) = x^3 - 6x^2 + 9x + 3$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + 9$

$\Rightarrow f'(x) = 3(x^2 - 4x + 3)$

For decreasing, $f'(x) < 0$

$\Rightarrow (x-3)(x-1) < 0,$

$\therefore x \in (1, 3)$

157 **(a)**

We have,

$y = ax^3 + bx^2 + cx$ ...(i)

$\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c$ ...(ii)

It is given that

$\left(\frac{dy}{dx}\right)_{(0,0)} = \tan 45° \Rightarrow c = 1$

Also,

$\left(\frac{dy}{dx}\right)_{(1,0)} = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow 3a + 2b + 1 = 0$ $[\because c = 1]$

Clearly, $a = 1$ and $b = -2$ satisfy this equation

Hence, $a = 1, b = -2$ and $c = 1$

158 **(c)**

Let $f(x) = 1 + x\log_e(x + \sqrt{x^2+1}) - \sqrt{1+x^2}$

Clearly, $f(x)$ is defined for all $x \in R$

Now,

$f'(x) = \log_e\left(x+\sqrt{x^2+1}\right) + \frac{x}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}}$

$\Rightarrow f'(x) = \log_e\left(x+\sqrt{x^2+1}\right)$

$\Rightarrow f'(x) > 0$ for all $x \in R$ [$\because x+\sqrt{x^2+1} \geq 1$ for all $x \in R$]

$\Rightarrow f(x)$ is increasing on R

$\Rightarrow f(x) \geq f(0)$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x+\sqrt{x^2+1}\right) - \sqrt{1+x^2} \geq 0$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x+\sqrt{x^2+1}\right) \geq \sqrt{1+x^2}$ for all $x \geq 0$

159 **(c)**

Given, $f(x) = x^{-x}$

$\Rightarrow \log f(x) = -x\log x$

On differentiating w.r.t. x, we get

$\frac{1}{f(x)}.f'(x) = -\log x - 1$

$\Rightarrow f'(x) = -f(x)(1+\log x)$

Put $f'(x) =)0$

$\Rightarrow \log x = -1 \quad \Rightarrow \quad x = e^{-1}$

$\therefore \quad f''(x) = -f'(x)(1+\log x) - f(x)\frac{1}{x}$

$= f(x)(1+\log x)^2 - \frac{f(x)}{x}$

At $x = \frac{1}{e}$,

$f''(x) = -ef\left(\frac{1}{e}\right) < 0$, maxima

Hence, at $x = \frac{1}{e}$, $f(x)$ is maximum.

160 **(b)**

$f(x) = x(x-1)^2$

$f'(x) = 2x(x-1) + (x-1)^2$

$= (x-1)(2x+x-1) = (x-1)(3x-1)$

$\therefore \quad f'(c) = \frac{f(2)-f(0)}{2-0}$

$\Rightarrow \quad (c-1)(3c-1) = \frac{2-0}{2} = 1$

$\Rightarrow \quad 3c^2 - 4c = 0$

$\Rightarrow \quad c(3c-4) = 0$

$\Rightarrow \quad c = 0 \ \ or \ \ c = \frac{4}{3}$

$\therefore$ The value of c in $(0,2)$ is $\frac{4}{3}$

161 **(c)**

Let r be the base radius and h be the height of the cone. Then, $2r = h$. Let V be the volume of the cone. Then,

$V = \frac{1}{3}\pi r^2 h = \frac{4\pi r^3}{3}$

$\Rightarrow \frac{dV}{dr} = 4\pi r^2$

$\therefore \Delta V = \frac{dV}{dr}\Delta r$

$\Rightarrow \Delta V = 4\pi r^2\,\Delta r$

$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{4\pi r^2\,\Delta r}{\frac{4}{3}\pi r^3} \times 100 = 3\left(\frac{\Delta r}{r} \times 100\right)$

$= 3\lambda$

162 **(b)**

Given, $f'(x) < 0, \forall\, x \in R$

$\Rightarrow \quad \sqrt{3}\cos x + \sin x - 2a < 0, \forall\, x \in R$

$\Rightarrow \quad \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a, \forall\, x \in R$

$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a, \forall\, x \in R$

$\Rightarrow \quad a \geq 1 \left[\because \sin\left(x+\frac{\pi}{3}\right) \leq 1\right]$

163 **(d)**

We have,

$f(x) = \cos\left(\frac{\pi}{x}\right) \Rightarrow f'(x) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right)$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow 2\,n\,\pi < \frac{\pi}{x} < (2\,n+1)\,\pi$

$\Rightarrow \frac{1}{2\,n} > x > \frac{1}{2\,n+1} \Rightarrow x \in \left(\frac{1}{2\,n+1}, \frac{1}{2\,n}\right)$

164 **(d)**

Given curves are $\frac{x^2}{a^2} + \frac{y^2}{12} = 1$

$\Rightarrow \quad \frac{2x}{a^2} + \frac{2y}{12}\cdot\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{12x}{a^2y} = m_1 \qquad$ (say)

And $y^3 = 8x \Rightarrow 3y^2\frac{dy}{dx} = 8$

$\Rightarrow \quad \frac{dy}{dx} = \frac{8}{3y^2} = m_2 \qquad$ (say)

For $\theta = \frac{\pi}{2}, 1 + m_1m_2 = 0$

$\Rightarrow \quad 1 + \left(\frac{-12x}{a^2y}\right)\left(\frac{8}{3y^2}\right) = 0$

$\Rightarrow \quad 3a^2(8x) - 96x = 0$

$\Rightarrow \quad a^2 = 4$

165 **(b)**

Consider the function $\phi(x) = f(x) - 2\,g(x)$ defined on $[0,1]$

As $f(x)$ and $g(x)$ are differentiable for $0 \leq x \leq 1$. Therefore, $\phi(x)$ is differentiable on $(0,1)$ and continuous on $[0,1]$

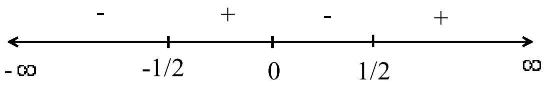
We have,
$\phi(0) = f(0) - 2\,g(0) = 2 - 0 = 2$
$\phi(1) = f(1) - 2\,g(1) = 6 - 2\,g(1)$
Now, $\phi'(x) = f'(x) - 2\,g'(x)$
$\Rightarrow \phi'(c) = f'(c) - 2\,g'(c) = 0$ [Given]
Thus, $\phi(x)$ satisfies Rolle's theorem on $[0, 1]$
$\therefore \phi(0) = \phi(1)$
$\Rightarrow 2 = 6 - 2\,g(1) \Rightarrow g(1) = 2$

166 **(b)**
$\phi'(x) = f'(x) + a$
$\because\ \phi'(0) = 0$
$\Rightarrow\ f'(0) + a = 0$
$\Rightarrow\ a = 0\quad (\because f'(0) = 0)$
Also, $\phi'(0) > 0\quad (\because f''(0) > 0)$
$\Rightarrow\ \phi'(x)$ has relative minimum at $x = 0$ for all b if $a = 0$

167 **(b)**
Given curve is $y = 2x^2 - x + 1$
On differentiating w.r.t. x, we get
$\frac{dy}{dx} = 4x - 1$
Since, this is parallel to the given curve $y = 3x + 9$
$\therefore$ These slopes are equal
$\Rightarrow\ 4x - 1 = 3\ \Rightarrow\ x = 1$
At $x = 1,\ y = 2(1)^2 - 1 + 1\ \Rightarrow y = 2$
Thus, the point is (1, 2).

168 **(a)**
Given, $y = -x^3 + 3x^2 + 2x - 27$
$\Rightarrow\ \frac{dy}{dx} = -3x^2 + 6x + 2$
Let slope $z = \frac{dy}{dx} = -3x^2 + 6x + 2$
Then, $\frac{dz}{dx} = -6x + 6$
For maximum or minimum put $\frac{dz}{dx} = 0$
$\Rightarrow\ -6x + 6 = 0$
$\Rightarrow\ x = 1$
Now, $\frac{d^2z}{dx^2} = -6 < 0$, maxima
The maximum value of z at $x = 1$, is given by
$z = -3 + 6 + 2 = 5$

169 **(b)**
For the curve $y^n = a^{n-1}x$
$ny^{n-1}y.\frac{dy}{dx} = a^{n-1}$
$\therefore$Length of subnormal$=y = \frac{dy}{dx}$
$= y \times \frac{a^{n-1}}{ny^{n-1}} = \frac{a^{n-1}}{ny^{n-2}}$
For constant subnormal, n should be 2

170 **(d)**
We have,
$f(x) = 2\,x^2 - \log|x|$
$\Rightarrow f(x) = \begin{cases} 2\,x^2 - \log x, & x > 0 \\ 2\,x^2 - \log(-x), & x < 0 \end{cases}$
$\Rightarrow f''(x) = 4\,x - \frac{1}{x}$ for all $x \neq 0$
For $f(x)$ to be increasing, we must have
$f''(x) > 0$
$\Rightarrow 4\,x - \frac{1}{x} > 0$
$\Rightarrow \frac{4\,x^2 - 1}{x} > 0$


$\Rightarrow \frac{(2x - 1)(2x + 1)}{x} > 0$
$\Rightarrow x(2x - 1)(2x + 1) > 0$
$\Rightarrow x \in (-1/2, 0) \cup (1/2, \infty)$

171 **(b)**
Let $P(x_1, y_1)$ be the point on the curve $ay^2 = x^3$ where the normal cuts off equal intercepts from the coordinate axes. Therefore,
Slope of the normal at $P = \pm 1$
$\Rightarrow -\frac{1}{\left(\frac{dy}{dx}\right)_P} = \pm 1$
$\Rightarrow \left(\frac{dy}{dx}\right)_P = \pm 1$
$\Rightarrow \frac{3x_1^2}{2ay_1} = \pm 1\qquad \left[\because ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2\right]$
$\Rightarrow 9x_1^4 = 4a^2y_1^2$
$\Rightarrow 9x_1^4 = 4ax_1^3\quad [\because (x_1, y_1)$ lies on $ay^2 = x^3 = 0$
$\therefore ay_1^2 = x_1^3]$
$\Rightarrow x_1 = 0, x_1 = \frac{4a}{9}$
At $(x_1 = 0, y_1 = 0)$, the normal is y-axis
So, the required point is (x_1, y_1), where $x_1 = \frac{4a}{9}$

172 **(d)**
$\because\ f(x) = \sin x - \cos x$
On differentiating w.r.t. x, we get
$f'(x) = \cos x + \sin x$
$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$
$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right)$
$= \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right]$
For decreasing, $f'(x) < 0$

$\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < \frac{3\pi}{2}$(within $0 \le x \le 2\pi$)

$\Rightarrow \frac{\pi}{2} + \frac{\pi}{4} < \left(x - \frac{\pi}{4} + \frac{\pi}{4}\right) < \frac{3\pi}{2} + \frac{\pi}{4}$

$\Rightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$

173 **(d)**

Since, $f(x) = \frac{x}{2} + \frac{2}{x}$

$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$

For maxima or minima, put $f'(x) = 0$

$\therefore \quad \frac{1}{2} - \frac{2}{x^2} = 0 \quad \Rightarrow \quad x = \pm 2$

Now, $f''(x) = \frac{4}{x^3}$

$\Rightarrow \quad f''(2) = \frac{4}{8} = \frac{1}{2} > 0$, minima

And $f''(-2) = -\frac{4}{8} = -\frac{1}{2} < 0$, maxima

Hence, $f(x)$ has local minimum at $x = 2$

174 **(b)**

We have,

$f(x) = \tan^{-1} x - \frac{1}{2}\log_e x$

$\Rightarrow f'(x) = \frac{1}{1+x^2} - \frac{1}{2x}$

$\Rightarrow f'(x) = \frac{2x - 1 - x^2}{2x(1+x^2)}$

$\therefore f'(x) = 0 \Rightarrow 2x - 1 - x^2 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$

Now,

$f(1) = \tan^{-1} 1 = \frac{\pi}{4}, f\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} - \frac{1}{2}\log_e\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6} + \frac{1}{4}\log_e 3$

and, $f(\sqrt{2}) = \frac{\pi}{3} - \frac{1}{4}\log_e 3$

Hence, the least value of $f(x)$ is $\frac{\pi}{3} - \frac{1}{4}\log_e 3$

175 **(b)**

The given equation of curve is $xy = 1$

$\Rightarrow \quad x\frac{dy}{dx} = -y \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$

Let $\left(t, \frac{1}{t}\right)$ be any point on the curve at which normal to the curve can be drawn

$\therefore \quad \left(\frac{dy}{dx}\right)_{\left(t,\frac{1}{t}\right)} = -\frac{1}{t^2}$

So, slope of normal= t^2

Therefore, the given line $ax + bx + c = 0$ will be normal to the curve, if

$t^2 = \frac{-b}{a}$

Since, $\quad t^2 > 0$

$\therefore$ Either $\quad b > 0, \quad a < 0$

or $\quad a > 0, \quad b < 0$

176 **(b)**

Given, $\frac{dy}{dt} \propto y, where$ y is the position of village

$\Rightarrow \frac{1}{y}dy = k\,dt$

$\Rightarrow \quad \log y = \log c + kt$ [on integrating]

$\Rightarrow \quad \log\frac{y}{c} = kt \Rightarrow y = ce^{kt}$

177 **(a)**

Given , $x^2 - 2xy + y^2 + 2x + y - 6 = 0$

On differentiating w.r.t.x, we get

$2x - 2\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} + 2 + \frac{dy}{dx} = 0$

At (2,2)

$4 - 2\left(2 + 2\frac{dy}{dx}\right) + 4\frac{dy}{dx} + 2 + \frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -2$

$\therefore$ Equation of tangent at (2,2) is

$(y - 2) = -2(x - 2)$

$\Rightarrow \quad 2x + y = 6$

178 **(d)**

Given curves are

$x = a(\theta + sin\theta), y = a(1 - cos\theta)$

$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{a(\sin\theta)}{a(1 + cos\theta)}$

$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

$= tan\frac{\theta}{2}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)} = \tan\frac{\pi}{4} = 1$

At $\theta = \frac{\pi}{2}, y = a\left(1 - cos\frac{\pi}{2}\right) = a$

$\therefore$ length of normal $= y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$= a\sqrt{1 + (1)^2} = \sqrt{2}a$

179 **(a)**

Given,

$f(x) = \sin x\,(1 + cosx)$

$\Rightarrow \quad f(x) = \sin x + \frac{1}{2}\sin 2x$

On differentiating w.r.t. x, we get

$f'(x) = \cos x + \cos 2x$

Put $f'(x) = 0$

$\cos x + \cos 2x = 0$

$\Rightarrow \quad 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right) = 0$

$\Rightarrow \quad \cos\frac{x}{2} = 0, \quad \cos\frac{3x}{2} = 0$

$\Rightarrow \quad x = \pi, x = \frac{\pi}{3}$

Now, $f''(x) = -\sin x - 2\sin 2x$

At $x = \frac{\pi}{3}, f''(x) = -\sin\frac{\pi}{3} - 2\sin\frac{2\pi}{3}$

$= -\frac{\sqrt{3}}{2} - \sqrt{3} < 0$, maxima

$\therefore$ maximum value

$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right)$

$= \frac{3\sqrt{3}}{4} = \frac{3^{3/2}}{4}$

180 **(c)**

Let $y = 2x^2 + x - 1$

$y = 4x + 1$

For maxima or minima, put $y' = 0$

$\Rightarrow \quad x = -\frac{1}{4}$

Now, $y'' = 4 = +$ ve

y is minimum at $x = -\frac{1}{4}$

Thus, minimum value $= 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) - 1 = -\frac{9}{8}$

Alternate

Here $a > 0$

$\therefore$ Minimum value$= \frac{4ac - b^2}{4a}$

$= \frac{4 \times 2(-1) - 1}{4 \times 2} = -\frac{9}{8}$

181 **(a)**

$\because \quad x = t^2$ and $y = 2t$

$\therefore$ At $t = 1$, $x = 1$ and $y = 2$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{2t} = \frac{1}{t}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,2)} = 2$

$\therefore$ Equation of normal is

$y - 2 = -1(x - 1)$

$\Rightarrow \quad x + y - 3 = 0$

183 **(d)**

Let $y = a\sec\theta - b\tan\theta$

$\Rightarrow \quad \frac{dy}{d\theta} = a\, sec\theta tan\theta - bsec^2\theta$

Put $\frac{dy}{d\theta} = 0 \Rightarrow sec\theta(a\, tan\theta - bsec\theta) = 0$

$\Rightarrow sin\theta = \frac{b}{a} \quad (\because sec\theta \neq 0)$

Now, $\frac{d^2y}{d\theta^2} > 0, at\ sin\theta = \frac{b}{a}$

$\therefore$minimum value is

$y = a\frac{a}{\sqrt{a^2 - b^2}} - b\frac{b}{\sqrt{a^2 - b^2}}$

$=\sqrt{a^2 - b^2}$

184 **(d)**

We have,

$y = x^n$

$\Rightarrow \log y = n\log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{n}{x} \Rightarrow \frac{dy}{dx} = \frac{ny}{x}$

$\therefore \Delta y = \frac{dy}{dx}\Delta x$

$\Rightarrow \Delta y = \frac{ny}{x}\Delta x \Rightarrow \frac{\Delta y}{y} = \left(\frac{\Delta x}{x}\right) \times n \Rightarrow \frac{\Delta y}{y} \div \frac{\Delta x}{x} = n$

185 **(c)**

Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \forall x \in R$

$\therefore f(x)$ is increasing

$\therefore \quad f(x) = 0$ has only one solution

186 **(c)**

Given, $f(x) = x^3 + ax^2 + bx + c, a^2 \leq 3b$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 + 2ax + b$

Put $f'(x) = 0$

$\Rightarrow \quad 3x^2 + 2ax + b = 0$

$\Rightarrow \quad x = \frac{-2a \pm \sqrt{4a^2 - 12b}}{2 \times 3}$

$= \frac{-2a \pm 2\sqrt{a^2 - 3b}}{6}$

Since, $a^2 \leq 3b$,

$\therefore \quad x$ Has an imaginary value.

Hence, no extreme value of x exist.

188 **(c)**

We have,

$xy^n = a^{n+1} \Rightarrow y^n + n\,xy^{n-1}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{nx}$

Let (x_1, y_1) be a point on $x\,y^n = a^{n+1}$. The equation of the tangent at (x_1, y_1) is

$y = y_1 = -\frac{y_1}{nx_1}\ (x - x_1)$

This meets with the coordinate axes at $A\big((n + 1)x_1, 0\big)$ and $B\left(0, \frac{(n+1)y_1}{n}\right)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}(n + 1)x_1 \cdot \left(\frac{n+1}{n}\right)y_1$

$\Rightarrow$ Area of $\Delta OAB = \frac{1}{2}\frac{(n+1)^2}{n}x_1y_1 \quad$ ...(i)

Since (x_1, y_1) lies on $xy^n = a^{n+1}$

$\therefore x_1 y_1^n = a^{n+1}(x_1, y_1) \Rightarrow x_1 = \dfrac{a^{n+1}}{y_1^n}$

Putting the value of x_1 in (i), we get

Area of $\Delta OAB = \frac{1}{2}\frac{(n+1)^2}{n} a^{n+1} y_1^{-n+1}$

This will be a constant, if $n = 1$

189 **(a)**

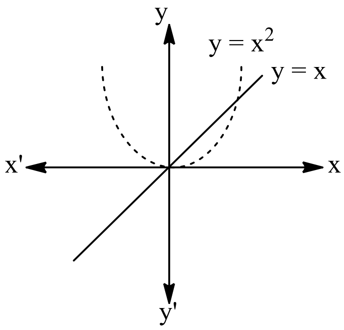
The area of circular plate is $A = \pi r^2$

$\Rightarrow \dfrac{dA}{dt} = 2\pi r \cdot \dfrac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi(12)(0.01) = 0.24\pi$ sq cm/s

190 **(a)**

$\therefore g(x) = \min(x, x^2)$



It is clear from the graph that g(x) is an increasing function

191 **(d)**

The point of intersection of given curve is (0, 1).

On differentiating given curves, we get

$\frac{dy}{dx} = a^x \log a, \frac{dy}{dx} = b^x \log b$

$\Rightarrow \quad m_1 = a^x \log a, m_2 = b^x \log b$

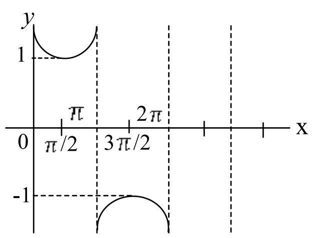
At (0,1) $m_1 = \log a, m_2 = \log b$

$\therefore \quad \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \quad \theta = \tan^{-1}\left|\dfrac{\log a - \log b}{1 + \log a \log b}\right|$

192 **(a)**

The graph of cosec x is opposite in interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



193 **(b)**

Given, $f(x) = x^{25}(1-x)^{75}$

$\Rightarrow \quad f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$

$= 25x^{24}(1-x)^{74}(1-4x)$

Put, $f'(x) = 0 \quad \Rightarrow \quad x = 0, 1, \frac{1}{4}$

If $x < \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{74}(1-4x) > 0$

And if $x > \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{24}(1-4x) < 0$

Thus, $f'(x)$ changes its sign from positive to negative as x passes through $1/4$ from left to right.

Hence, $f(x)$ attains its maximum at x=1/4

194 **(d)**

Given, $f(x) = e^x \sin x, \ x \in [0, \pi]$

At $x = 0, \quad f(0) = 0$

And at $x = \pi, \ f(\pi) = 0$

Also, it is continuous and differentiable in the given interval.

Hence, it satisfies the Rolle's theorem.

Hence, option (d) is the required answer

195 **(b)**

Given curve is

$xy = c^2 \quad \Rightarrow y = \dfrac{c^2}{x^2}$

Let $f(x) = ax + by = ax + \frac{bc^2}{x^2}$

On differentiating w.r.t. x, we get

$f'(x) = a - \dfrac{bc^2}{x^2}$

For a maximum or minima, put $f'(x) = 0$

$\Rightarrow \ ax^2 - bc^2 = 0$

$\Rightarrow \ x^2 = \dfrac{bc^2}{a} \ \Rightarrow \ x = \pm c\sqrt{\dfrac{b}{a}}$

Again on differentiating, we get $f''(x) = \frac{2bc^2}{x^3}$

At6 $x = c\sqrt{\frac{b}{a}}, \ f''(x) > 0$

$\therefore \quad f(x)$ is minimum at $x = c\sqrt{\frac{b}{a}}$

The minimum value at $x = c\sqrt{\frac{b}{a}}$ is

$\therefore f\left(c\sqrt{\dfrac{b}{a}}\right) = a.c\sqrt{\dfrac{b}{a}} + \dfrac{bc^2}{c}.\sqrt{\dfrac{a}{b}}$

$= \dfrac{abc + abc}{\sqrt{ab}} = \dfrac{2abc}{\sqrt{ab}} = 2c\sqrt{ab}$

196 **(a)**

Given, $\quad x - 2y = 4$

Let $\quad A = xy \quad \Rightarrow \quad A = 2y^2 + 4y$

$\Rightarrow \quad \dfrac{dA}{dy} = 4 + 4y$

For extremum value, $\frac{dA}{dy} = 0$

$\Rightarrow \quad y = -1$

Now, $\frac{d^2A}{dy^2} = 4 > 0$, minima

At $y = -1$,

$x = 4 + 2(-1) = 2$

$\therefore \quad A = xy = 2(-1) = -2$

$\therefore$ Minimum value of xy is -2

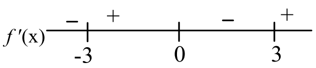
197 **(a)**

Given, $f(x) = (9 - x^2)^2$

$\Rightarrow \quad f''(x) = 2(9 - x^2)(-2x)$

Now, put $f''(x) = 0$

$\Rightarrow \quad 2(9 - x^2)(-2x) = 0 \quad \Rightarrow \quad x = 0, \pm 3$



$\therefore \quad f''(x)$ is increasing in $(-3, 0) \cup (3, \infty)$

198 **(b)**

Let $f(x) = x + \frac{1}{x} \quad \Rightarrow \quad f'(x) = 1 - \frac{1}{x^2}$

For maxima and minima, put $f'(x) = 0$

$$\Rightarrow \quad 1 - \frac{1}{x^2} = 0 \quad \Rightarrow \quad x = \pm 1$$

Now, $f''(x) = \frac{2}{x^3}$

At $x = 1$, $f''(x) = \frac{2}{x^3}$

At $x = 1$, $f''(x) = +$ ve, minima

And at $x = -1$, $f''(x) = -$ve, maxima

Thus, $f(x)$ attains minimum value at $x = 1$

199 **(a)**

We have,

$12y = x^3$

$$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$\Rightarrow 12 = 3x^2 \qquad \left[\because \frac{dy}{dt} = \frac{dx}{dt}\right]$$

$\Rightarrow x = \pm 2$

$$\therefore 12y = x^3 \Rightarrow y = \pm\frac{2}{3}$$

Hence, the points are $(2, 2/3)$ and $(-2, -2/3)$

200 **(b)**

Let $R(x_1, y_1)$ be the point on the parabola $y^2 = 2x$ such that tangent at R is parallel to the chord PQ

$$\therefore \left(\frac{dy}{dx}\right)_R = \text{Slope of } PQ$$

$$\Rightarrow \frac{1}{y_1} = \frac{-1-2}{\frac{1}{2} - 2} \qquad \left[\because y^2 = 2x \Rightarrow 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}\right]$$

$$\Rightarrow \frac{1}{y_1} = 2 \Rightarrow y_1 = \frac{1}{2}$$

Since (x_1, y_1) lies on $y^2 = 2x$

$$\therefore y_1^2 = 2x_1 \Rightarrow x_1 = \frac{1}{8} \qquad [\because y = 1/2]$$

Hence, the required point is $(1/8, 1/2)$

201 **(b)**

Given curve is $f(x) = \frac{1}{x+1} - \log(1 + x)$

On differentiating w.r.t. x, we get

$$f'(x) = -\frac{1}{(x+1)^2} - \frac{1}{1+x}$$

$$\Rightarrow \quad f'(x) = -\left[\frac{1}{x+1} + \frac{1}{(x+1)^2}\right]$$

$\Rightarrow \quad f'(x) = -$ve, when $x > 0$

$\therefore \quad f(x)$ is a decreasing function

202 **(c)**

Given curves are $x = 2\cos^3\theta$ and $y = 3\sin^3\theta$

$\therefore \quad \frac{dx}{d\theta} = -6\cos^2\theta\sin\theta$,

$$\frac{dy}{d\theta} = 9\sin^2\theta\cos\theta$$

$$\therefore \quad \frac{dy}{dx} = -\frac{9\sin^2\theta\cos\theta}{6\cos^2\theta\sin\theta} = -\frac{3}{2}\tan\theta$$

At $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{3}{2}\tan\frac{\pi}{4} = -\frac{3}{2}$

Also, at $\theta = \frac{\pi}{4}$, $x = 2\cos^3\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

And $y = 3\sin^2\theta = 3\left(\frac{1}{\sqrt{2}}\right)^3$

$$= \frac{3\sqrt{2}}{4}$$

Equation of tangent at $\left(\frac{1}{\sqrt{2}}, \frac{3\sqrt{2}}{4}\right)$ is

$$\left(y - \frac{3\sqrt{2}}{4}\right) = -\frac{3}{2}\left(x - \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \quad 4y - 3\sqrt{2} = -6x + 3\sqrt{2}$

$\Rightarrow \quad 3x + 2y = 3\sqrt{2}$

203 **(d)**

Given curve is $x^2 + xy + y^2 = 7$

$$\Rightarrow \qquad 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} = -\frac{(2x + y)}{x + 2y}$$

$$\Rightarrow \qquad \left(\frac{dy}{dx}\right)_{(1,-3)} = \frac{-(2-3)}{(1-6)} = -\frac{1}{5}$$

$\therefore$ Length of subtangent$= \frac{y}{\frac{dy}{dx}} = \frac{-3}{\frac{1}{-5}} = 15$

204 **(c)**

Let the point of contact be (x_1, y_1)

The equation of curve is

$$\frac{x^n}{a^n} + \frac{y^n}{b^n} = 2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{b^n}{a^n} \cdot \frac{x_1^{n-1}}{y_1^{n-1}} \qquad \text{...(i)}$$

The equation of line is $\frac{x}{a}+\frac{y}{b}=2$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=-\frac{b}{a}$ ...(ii)

From Eqs. (i) and (ii), we get

$-\frac{b}{a}=-\frac{b^n}{a^n}\cdot\frac{x_1^{n-1}}{y_1^{n-1}} \quad \Rightarrow \quad x_1=\frac{ay_1}{b}$ ...(iii)

Also, (x_1,y_1) lies on the given line

$\therefore \ \frac{ay_1}{ab}+\frac{y_1}{b}=2$

$\Rightarrow \quad y_1=b$ and $x_1=a$

205 **(d)**

$\because \ y=x^3-6x^2+9x+4$

Now, $\frac{dy}{dx}=3x^2-12x+9$

Let $u=\frac{dy}{dx}=3x^2-12x+9$

Now, $\frac{du}{dx}=6x-12$

Put $\frac{du}{dx}=0$ for maximum or minimum

$\therefore \ 6x-12=0$

$\Rightarrow \ x=2$

Now, at $x=0,\ u=9$

at $x=2,\ u=-3$

and at $x=5,\ u=24$

Thus, the maximum of $u(x), 0\le x\le 5$ is $u(5)$

Hence, $x=5$

206 **(a)**

Given,

$x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$

$\Rightarrow \ \frac{dx}{d\theta}=a(1+\cos\theta)$ and $\frac{dy}{d\theta}=a\sin\theta$

$\therefore \ \frac{dy}{dx}=\frac{a\sin\theta}{a(1+\cos\theta)}=\tan\frac{\theta}{2}$

Now, length of subtangent= $\left|\frac{y}{dy/dx}\right|$

$\therefore \ ST=\frac{a(1-\cos\theta)}{\tan(\theta/2)}=a\sin\theta$

$\Rightarrow$ Length of suntangent at $\theta=\frac{\pi}{2}$,

$ST=a\sin\frac{\pi}{2}=a$

And length of subnormal= $\left|y\frac{dy}{dx}\right|$

$\Rightarrow \quad SN=a(1-\cos\theta).\tan\frac{\theta}{2}$

$=a.2\sin^2\frac{\theta}{2}\tan\frac{\theta}{2}$

$\Rightarrow$ Length of subnormal at $\theta=\frac{\pi}{2}$,

$SN+a.2.1.2=a$

Hence, $SN=ST$

207 **(c)**

We have,

$y=2x^2+4x^3+7x+9$

$\Rightarrow \frac{dy}{dx}=10x^4+12x^2+7>0$ for all $x\in R$

Thus, any tangent to the given curve makes an acute angle with x-axis

209 **(a)**

At the point where the given curve crosses x-axis, we have

$y=0$

$\Rightarrow ax^2=1$ [Putting $y=0$ in $ax^2+2hxy+by^2=1$]

$\Rightarrow x=\pm\frac{1}{\sqrt{a}}$

Thus, the given curve cuts x-axis at $P[(1/\sqrt{a},0)$ and $Q(-1/\sqrt{a},0)$

Now,

$ax^2+2hxy+by^2=1$

$\Rightarrow 2ax+2h\left(x\frac{dy}{dx}+y\right)+2by\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{ax+hy}{hx+by}$

$\Rightarrow \left(\frac{dy}{x}\right)_P=-\frac{a}{h}$ and $\left(\frac{dy}{dx}\right)_Q=-\frac{a}{h}\Rightarrow\left(\frac{dy}{dx}\right)_P$

$=\left(\frac{dy}{dx}\right)_Q$

Hence, the tangents are parallel

210 **(d)**

Given, $y=\frac{8-x}{2}$

Let $p=xy=\frac{8x-x^2}{2}$

$\Rightarrow \quad \frac{dp}{dx}=\frac{1}{2}(8-2x)$

For maxima or minima, put $\frac{dp}{dx}=0$

$\therefore \quad 8-2x=0$

$\Rightarrow \quad x=4$

Again, $\frac{d^2p}{dx^2}=-1$

$\Rightarrow \quad \left(\frac{d^2p}{dx^2}\right)=-1<0$

Thus, function is maximum at $x=4$ and $y=2$

Therefore, maximum value of $p=4\times 2=8$

211 **(b)**

$\int_1^2 f'(x)\,dx=[f(x)]_1^2=f(2)-f(1)=0$

[$\because f(x)$ satisfies the condition of Rolle's theorem]

$\therefore \ f(2)=f(1)$

212 **(d)**

We have,

$y^2=2ax$ ...(i)

$\Rightarrow \frac{dy}{dx} = \frac{a}{y}$

Putting $x = \frac{a}{2}$ in (i), we get $y = \pm a$

Thus, the two points on the curve (i) are $P(a/2, a)$ and $Q(a/2, -a)$

$\therefore \left(\frac{dy}{dx}\right)_P = 1$ and $\left(\frac{dy}{dx}\right)_Q = -1$

Clearly, $\left(\frac{dy}{dx}\right)_P \times \left(\frac{dy}{dx}\right)_Q = -1$

Hence, the angle between the tangents at P and Q is a right angle

213 **(b)**

We have,

$v^2 = a^2 + s^2$

$\Rightarrow 2v\frac{dv}{ds} = 2s \Rightarrow v\frac{dv}{ds} = s \Rightarrow$ Acceleration $= s$

214 **(d)**

Since, $f(x) = 2x^3 - 9x^2 = 12x + 4$

$\Rightarrow f'(x) = 6x^2 - 18x + 12$

For function to be decreasing $f'(x) < 0$

$\Rightarrow 6(x^2 - 3x + 2) < 0$

$\Rightarrow (x-2)(x-1) < 0$

$\Rightarrow 1 < x < 2$

215 **(a)**

Let the numbers be x, y.,

$\therefore \quad x + y = 20 \quad \Rightarrow \quad y = 20 - x$

Let $\quad S = x^3 (20-x)^2$

$\Rightarrow \quad \frac{dS}{dx} = 2x^3 (20-x)(-1) + 3x^2(20-x)^2$

For maximum or minimum, put $\frac{dS}{dx} = 0$

$\Rightarrow \quad x^2(20-x)(-2x + 60 - 3x) = 0$

$\Rightarrow \quad x = 0,\ x = 12$ or $x = 20$

x cannot be 0 or 20

Now $\frac{dS}{dx} = x^2(20-x)^2[60-5x]$

For $0 < x < 12, \frac{dS}{dx} > 0$ and $12 < x < 20, \frac{dS}{dx} < 0$

$\therefore \quad S$ is maximum at $x = 12$

For $x = 12$, then $y = 20 - 12 = 8$

The required numbers are 12, 8

216 **(d)**

Given curve is $y = x\log x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 1 + \log x$

The slope of the normal $= -\frac{1}{(dy/dx)} = \frac{-1}{1+\log x}$

The slope of the given line $2x - 2y = 3$ is1

Since, these lines are parallel

$\therefore \quad \frac{-1}{1+\log x} = 1 \quad \Rightarrow \quad \log x = -2 \quad \Rightarrow \quad x = e^{-2}$

and $y = -2e^{-2}$

$\therefore$ Coordinates of the point are $(e^{-2} - 2, -2e^{-2})$

218 **(b)**

$f'(x) = (ab - b^2 - 2) + \cos^4 x + \sin^4 x < 0$

$= ab - b^2 - 2 + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x < 0$

$\Rightarrow \quad ab - b^2 - 1 < \left(\frac{1}{2}\right)\sin^2 2x < \frac{1}{2}$

$\Rightarrow \quad 2ab - 2b^2 - 2 < 0$

$\Rightarrow \quad 2b^2 - 2ab + 3 > 0$

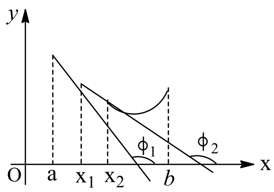
$\therefore (-2a^2) - 4.2.3 < 0$

$\Rightarrow a^2 < 6$

$\Rightarrow \quad -\sqrt{6} < a < \sqrt{6}$

219 **(d)**

We know that, $\sin x$ and $\cos x$ decrease in $\frac{\pi}{2} < x < \pi$, so the statement S is correct



The statement R is incorrent which is clear from the graph that $f(x)$ is differentiable in (a, b)

Also, $a < x_1 < x_2 < b$

But $f''(x_1) = \tan \phi_1 < \tan \phi_2 = f''(x_2)$

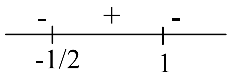
$\Rightarrow$ Derivate is increasing

220 **(c)**

Given that, $f(x) = \int e^x(x-1)(x-2)dx$

On differentiating w.r.t. x, we get

$f''(x) = e^x(x-1)(x-2)$



Since, $f(x)$ is decreasing

$\therefore \quad f''(x) < 0 \quad \Rightarrow \quad e^x(x-1)(x-2) < 0$

$\Rightarrow (x-1)(x-2) < 0 \quad \Rightarrow \quad 1 < x < 2$

221 **(b)**

Let volume of sphere(V)=$\frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} 4\pi r^2 \frac{dr}{dt} \quad \Rightarrow \frac{dr}{dt} = \frac{35}{4\pi r^2}$

Let surface area of sphere(S)= $4\pi r^2$

$\Rightarrow \frac{dS}{dt} = 2 \times a\pi r \frac{dr}{dt}$

$= 2 \times 4\pi r \times \frac{35}{4\pi r^2}$

$\therefore \quad \frac{dS}{dt} = 10sq$ cm/min

223 **(a)**

The equations of the tangents at the origin can be obtained by equating the lowest degree term to zero. So, the required tangents given by

$y^2 - x^2 = 0 \Rightarrow y = \pm x$

224 **(a)**

Given, s=$\sqrt{at^2 + 2bt + c}$

$\Rightarrow \frac{ds}{dt} = \frac{2at + 2b}{2\sqrt{at^2 + 2bt + c}} = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2s}{dt^2} = \frac{\left[\frac{\sqrt{at^2+2bt+c+a} \\ -(at+b)\frac{(2at+2b)}{2\sqrt{at^2+2bt+c}}}{\left(\sqrt{at^2+2bt+c}\right)^2}\right]}{\left(\sqrt{at^2 + 2bt + c}\right)^2}$$

$\Rightarrow \frac{d^2}{dt^2} = \frac{ac - b^2}{s^3} \Rightarrow \text{Acceleration} \propto \frac{1}{s^3}$

225 **(c)**

We have,

$t = as^2 + bs + c$

$\Rightarrow 1 = (2as + b)\frac{ds}{dt}$

$\Rightarrow \frac{ds}{dt} = \frac{1}{2as + b} \Rightarrow \frac{d^2s}{dt^2} = -\frac{2a}{(2as + b)^2}\frac{ds}{dt}$

$= -2a\left(\frac{ds}{dt}\right)^3$

226 **(d)**

We have,

$f(x) = x^3 - 6x^2 + 12x - 3$

$\Rightarrow f'(x) = 3x^2 - 12x + 12, f'(x) = 6x - 12$ and $f'''(x) = 6$

We observe that $f'(2) = 0, f''(2) = 0$ but $f'''(2) \neq 0$

So, $x = 2$ is neither a point of local maximum nor a point of local minimum. In fact, it is a point of inflexion

227 **(b)**

For $f(x)$ to be decreasing for all x, we must have $f'(x) < 0$ for all x

$\Rightarrow 5\left(\frac{\sqrt{a + 4}}{1 - a} - 1\right)x^4 - 3 < 0$ for all x

$\Rightarrow \left(\frac{\sqrt{a + 4}}{1 - a} - 1\right)x^4 < \frac{3}{5}$ for all x

$\Rightarrow \left(\frac{\sqrt{a + 4}}{1 - a} - 1\right) \leq 0 \Rightarrow \frac{\sqrt{a + 4}}{1 - a} \leq 1$

This inequality is trivially true for all a satisfying $a > 1$ i.e. $a \in (1, \infty)$

So let us take $a < 1$

Since $\sqrt{a + 4}$ is real, therefore $a > -4$

Thus, we have $-4 \leq a < 1$

For these values of a, we must have

$\frac{\sqrt{a + 4}}{1 - a} \leq 1$

$\Rightarrow \sqrt{a + 4} \leq 1 - a$

$\Rightarrow (a + 4) \leq 1 + a^2 - 2\,a$

$\Rightarrow 0 \leq a^2 - 3\,a - 3$

$\Rightarrow a \geq \frac{3 + \sqrt{21}}{2}$ or, $a \leq \frac{3 - \sqrt{21}}{2}$

$\Rightarrow -4 \leq a \leq \frac{3 - \sqrt{21}}{2}$ $\quad [\because a \geq -4]$

Hence, $a \in \left[-4, \frac{3 - \sqrt{21}}{2}\right] \cup (1, \infty)$

228 **(c)**

Let x be the radius and V be the volume of the sphere. Then, $V = \frac{4}{3}\pi x^3$

$\therefore \frac{dV}{dx} = 4\pi x^2$

We have, $x = 100$ and $x + \Delta x = 98$

$\therefore \Delta x = -2$

Now, $\Delta V = \frac{dV}{dx}\Delta x$

$\Rightarrow \Delta V = 4\pi \times 100^2$

$\times -2 \left[\because \left(\frac{dV}{dx}\right)_{x=100} = 4\pi \times 100^2\right]$

$\Rightarrow \Delta V = -80000\pi$

Hence, decrease in volume is 80000π mm^3

230 **(b)**

On differentiating given curves respectively

$2x = -9A\frac{dy}{dx} \Rightarrow \left(\frac{dy}{dx}\right)_{c_1} = -\frac{2x}{9A}$

And $2x = A\frac{dy}{dx}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{c_2} = \frac{2x}{A}$

For orthogonally,

$\left(\frac{dy}{dx}\right)_{c_1}\left(\frac{dy}{dx}\right)_{c_2} = -1$

$\Rightarrow \left(-\frac{2x}{9A}\right)\left(\frac{2x}{A}\right) = -1$ ...(i)

From given curves,

$9A(9 - y) = A(y + 1) \Rightarrow y = 8$

Since, $x^2 = 9A(9 - y)$

$\Rightarrow x^2 = 9A(9 - 8) = 9A$

From Eq. (i), $\frac{4x^2}{9A^2} = 1$

$\Rightarrow \frac{4.9\,A}{9A^2} = 1 \Rightarrow A = 4$

231 **(c)**

Equation of the curve is $x^2y^2 = a^4$

On differentiating w.r.t. x, we get

$$x^2 2y\frac{dy}{dx} + y^2 2x = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(-a,a)} = -\left(\frac{a}{-a}\right) = 1$$

Therefore, length of subtangent at the point $(-a, a)$

$$= \frac{y}{\left(\frac{dy}{dx}\right)} = \frac{a}{1} = a$$

232 **(c)**

Let $A = x + \frac{1}{x} \Rightarrow \frac{dA}{dx} = 1 - \frac{1}{x^2}$

Put $\frac{dA}{dx} = 0$ for maxima or minima

$$1 - \frac{1}{x^2} = 0 \Rightarrow x = -1, 1$$

Again on differentiating w.r.t. x, we get

$$\frac{d^2A}{dx^2} = \frac{2}{x^3} \Rightarrow \left(\frac{d^2A}{dx^2}\right)_{x=1} = 2 > 0$$

$\therefore$ A is minimum at $x = 1$

233 **(c)**

Given, $y = x\log_e x$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{x} + \log x = 1 + \log x$$

$\therefore$ Slope of normal$= \frac{-1}{\frac{dy}{dx}} = \frac{-1}{1+\log x}$

And given line is $2x - 2y + 3 = 0 \Rightarrow \frac{dy}{dx} = 1$

Since, normal line is parallel to the given line

$$\therefore -\frac{1}{1 + \log x} = 1 \Rightarrow x = e^{-2}$$

Now, intersecting point of given curve and $x = e^{-2}$ is $(e^{-2}, -2e^{-2})$

$\therefore$ Required equation of line is

$$y + 2e^{-2} = 1(x - e^{-2}) \Rightarrow x - y = 3e^{-2}$$

234 **(c)**

Consider the function $\phi(x) = f(x) - g(x)$ on the interval $[x_0, x]$. Clearly, $\phi(x)$ satisfies conditions of Lagrange's theorem on $[x_0, x]$. Therefore, there exists $c \in (x_0, x)$ such that

$$\phi(x) - \phi(x_0) = \phi'(c)(x - x_0)$$

$$\Rightarrow \phi(x) = \phi'(c)(x - x_0) \quad \text{...(i)}$$

Now,

$$\phi(x) = f(x) - g(x)$$

$$\Rightarrow \phi'(x) = f'(x) - g'(x)$$

$\Rightarrow \phi'(c) = f'(c) - g'(c) > 0$ $[\because f'(x) > g'(x)$ for $x > x_0]$

From (i), we have

$\phi(x) > 0$ for all $x > x_0$ $[\because \phi'(c) > 0$ and $x - x_0 > 0]$

$\Rightarrow f(x) - g(x) > 0$ for all $x > x_0$

$\Rightarrow f(x) > g(x)$ for all $x > x_0$

235 **(b)**

Let r be the radius of the base, h the height and l slant height of the cone with semi-vertical angle 45°. Then,

$r = h$ and $l^2 = r^2 + h^2 = 2r^2$

Let V be the volume. Then,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$$

$$\Rightarrow \frac{dv}{dt} = \pi r^2\frac{dr}{dt} = \text{Area of the base} \times \frac{dr}{dt}$$

236 **(a)**

We have,

$$y = x\,e^x \Rightarrow \frac{dy}{dx} = e^x + x\,e^x$$

For maximum or minimum, we must have

$$\frac{dy}{dx} = 0 \Rightarrow e^x(1 + x) = 0 \Rightarrow x = -1$$

Now, $\frac{d^2y}{dx^2} = 2e^x + x\,e^x$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=-1} = e^{-1}(2 - 1) > 0$$

Hence, $x = -1$ is a point of local minimum

237 **(b)**

We have,

$f(x) + f(-x) = \log 1 = 0 \Rightarrow f(-x) = -f(x)$

So, $f(x)$ is an odd function

Now,

$$f(x) = \log\left(x^3 + \sqrt{x^6 + 1}\right)$$

$$\Rightarrow f'(x) = \frac{1}{x^3 + \sqrt{x^6 + 1}}\left\{3x^2 + \frac{6x^5}{2\sqrt{x^6 + 1}}\right\}$$

$$= \frac{3x^2}{\sqrt{x^6 + 1}} > 0$$

$\Rightarrow f(x)$ is increasing

238 **(a)**

Given curve is $x = 3t^2 + 1, y = t^3 - 1$

For $x = 1$, $3t^2 + 1 = 1 \Rightarrow t = 0$

$$\therefore \frac{dx}{dt} = 6t, \frac{dy}{dx} = 3t^2$$

Now, $\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right) = \frac{3t^2}{6t} = \frac{t}{2}$

$$\therefore \left(\frac{dy}{dx}\right)_{(t=0)} = \frac{0}{2} = 0$$

239 **(a)**

$\because f(x) = \tan x - x$

On differentiating w.r.t. x, we get

$f'(x) = \sec^2 x - 1 = \frac{1}{\cos^2 x} - 1 = \frac{1-\cos^2 x}{\cos^2 x}$

Since, $0 \le \cos^2 x \le 1$ for all values of x

$\therefore f'(x) > 0$for all values of$x$. Thus, $f(x)$ always increases

240 **(a)**

Let $f(x) = 4\cos(x^2)\cos\left(\frac{\pi}{3}+x^2\right)\cos\left(\frac{\pi}{3}-x^2\right)$

$= 2\cos(x^2)\left[\cos\left(\frac{2\pi}{3}\right)+\cos(2x^2)\right]$

$[\because \quad 2\cos A\cos B = \cos(A+B)+\cos(A-B)]$

$= 2\cos(x^2)\left[-\frac{1}{2}+\cos(2x^2)\right]$

$= -\cos(x^2) + 2\cos(x^2)\cos(2x^2)$

$= -\cos(x^2) + \cos(3x^2) + \cos(x^2)$

$\Rightarrow \quad f(x) = \cos(3x^2)$ ...(i)

$\Rightarrow \quad f'(x) = -[\sin(3x^2)](6x)$

For extremum, put $f'(x) = 0$

$\Rightarrow \quad -\sin(3x^2)(6x) = 0$

$\Rightarrow \quad x = 0, \sqrt{\pi}$

Put $x = 0, \sqrt{\pi}$ in Eq. (i), we get

$f(0) = \cos(0) = 1$

And $f(\pi) = \cos(3\pi) = -1$

241 **(b)**

$\frac{d}{d\theta}(\sin\theta) = k \Rightarrow \cos\theta = k$

$\therefore \frac{d}{d\theta}(\tan\theta) = \sec^2\theta = \frac{1}{k^2}$

242 **(b)**

Let $f(x) = \frac{ax^2+2bx+c}{Ax^2+2Bx+C}$

Let $P(x_1, y_1)$ be a point on the curve $y = f(x)$ where y is maximum or minimum. Then, at P

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} \ne 0$

The equation of the tangent at $P(x_1, y_1)$ is

$y - y_1 = \left(\frac{dy}{dx}\right)_P (x - x_1) \Rightarrow y$

$= y_1 \quad \left[\because \left(\frac{dy}{dx}\right)_P = 0\right]$

Putting $y = y_1$ in $y = \frac{ax^2+2bx+c}{Ax^2+2Bx+C}$, we get the x-coordinate of P. This means that $y = \frac{ax^2+2bx+c}{Ax^2+2Bx+C}$ gives only one value of x

This is possible only when $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is a perfect square

243 **(a)**

Since, $\quad y = x^2 - 5x + 6$

$\therefore \quad \frac{dy}{dx} = 2x - 5$

Now, $\quad m_1 = \left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1$

And $m_2 = \left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$

Now, $m_1 m_2 = -1 \times 1 = -1$

Hence, angle between the tangents is $\frac{\pi}{2}$

244 **(d)**

We have,

$f(x) = 2x + \cos x$

$\Rightarrow f'(x) = 2 - \sin x > 0$ for all $x \in R$

$\Rightarrow f(x)$ is an increasing function

245 **(b)**

Given curve are

$x = t^2 - 3t - 8, \quad y = 2t^2 - 2t - 5$ ...(i)

When $x = 2$, then

$2 = t^2 + 3t - 8$

$\Rightarrow \quad t^2 + 3t - 10 = 0 \quad \Rightarrow \quad t = 2, -5$ ...(ii)

When $y = -1$, then

$-1 = 2t^2 - 2t - 5$

$\Rightarrow \quad 2t^2 - 2t - 4 = 0 \quad \Rightarrow \quad t = -1, 2$ ...(iii)

From Eqs. (ii) and (iii), $t = 2$

On differentiating Eq. (i) w.r.t. t respectively, we get

$\frac{dx}{dt} = 2t + 3$ and $\frac{dy}{dt} = 4t - 2$

$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(t=2)} = \frac{8-2}{4+3} = \frac{6}{7}$

246 **(a)**

Let $f(x) = x\sqrt{1-x^2}$

$\Rightarrow \quad f'(x) = \frac{1-2x^2}{\sqrt{1-x^2}}$

$\Rightarrow$ For maxima or minima, put $\frac{dy}{dx} = 0$

$x = \pm\frac{1}{\sqrt{2}}$

But $x > 0$, therefore we have $x = \frac{1}{\sqrt{2}}$

Now, $f''(x) = \frac{\sqrt{1-x^2}(-4x)-(1-2x^2)\frac{-x}{\sqrt{1-x^2}}}{(1-x^2)}$

$= \frac{2x^3-3x}{(1-x^2)^{3/2}}$

$\Rightarrow \quad f''\left(\frac{1}{\sqrt{2}}\right)$=−ve, maximum.

248 **(c)**

We have,

$s = 6t^2 - \frac{t^3}{2} \Rightarrow \frac{ds}{dt} = 12t - \frac{3t^2}{2}$ and $\frac{d^2s}{dt^2} = 12 - 3t$

When the particle is momentarily at rest, we have $\frac{ds}{dt} = 0$ and $\frac{d^2s}{dt^2} \ne 0$

Now,

$$\frac{ds}{dt} = 0 \Rightarrow t = 0, t = 8$$

Clearly, $\frac{d^2s}{dt^2} \neq 0$ for $t = 0, 8$

249 **(c)**

Given, $f(x) = x^2 + 4x + 1$

$\Rightarrow \quad f'(x) = 2x + 4$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow \quad 2x + 4 = 0$

$\Rightarrow \quad x = -2$

Now, $\quad f''(x) = 2 > 0$

250 **(c)**

$f(x) = \cot^{-1} x + x$

$$\Rightarrow f'(x) = -\frac{1}{1 + x^2} + 1$$

$= \frac{x^2}{1+x^2}$, clearly, $f'(x) >$for all$x$.

So, $f(x)$ increases in $(-\infty, \infty)$.

251 **(d)**

Slope of the curve at an angle $\theta = \frac{3\pi}{4}$ is

$$\frac{dy}{dx} = \tan\frac{3\pi}{4} = -1$$

Slope of the normal $= \frac{-1}{dy/dx}$

$$\therefore \left(\frac{dy}{dx}\right)_{(3,4)} = 1$$

$\Rightarrow \quad f'(3) = 1$

252 **(a)**

Let $\quad f(x) = \int_0^x te^{t^2}\, dt$

$\Rightarrow \quad f'(x) = [xe^{x^2}] \qquad$ [by Leibnitz rule]

For maxima or minima, put $f'(x) = 0 \Rightarrow \quad x = 0$

Now, $\quad f''(x) = e^{x^2} + 2x\, e^{x^2}$

$f''(0) = 1 > 0$

$\therefore f(x)$ is minimum at $x = 0$

$\therefore \quad f(0) = 0$

253 **(a)**

Given $s = 2t^3 - 9t^2 + 12t \Rightarrow \frac{ds}{dt} = 6t^2 - 18t + 12$

Again , $\frac{d^2s}{dt^2} = 12t - 18 =$ acceleration

If acceleration becomes zero, then

$$0 = 12t - 18 \Rightarrow t = \frac{3}{2}s$$

Hence, acceleration will be zero after $=\frac{3}{2}s$.

254 **(a)**

Let $\quad f(x) = x^2 + \frac{1}{1+x^2}$

$$f'(x) = 2x - \frac{1}{(1 + x^2)^2}.2x$$

For a minimum, put $f'(x) = 0 \Rightarrow \quad x = 0$

So, the function has minimum value at $x = 0$.

255 **(b)**

Let the length of an edge of the cube be x units.

Let S_1 and S_2 be the surface areas of the cube and sphere respectively. Then,

$S_1 = 6x^2$ and $= S_2 = 4\pi x^2$

$$\Rightarrow \frac{dS_1}{dt} = 12x\frac{dx}{dt} \text{ and } S_2 = 8\pi x\frac{dx}{dt} \Rightarrow \frac{\frac{dS_1}{dt}}{\frac{dS_2}{dt}} = \frac{3}{2\pi}$$

256 **(d)**

We have,

$$f(x) = \int_{-1}^{x} t(e^t - 1)\,(t - 1)(t - 2)^3(t - 3)^5 dt$$

$\Rightarrow f'(x) = x(e^x - 1)(x - 1)(x - 2)^3(x - 3)^5$

Now, $f'(x) = 0 \Rightarrow x = 0, 1, 2, 3$

Clearly, $f'(x)$ changes its sign from negative to positive in the neighbourhood of $x = 3$

So, $x = 3$ is a point of local minimum

257 **(b)**

Given, $\quad x = 3t^2 + 1$ and $\quad y = t^3 - 1$

$\therefore \quad \frac{dx}{dt} = 6t$ and $\frac{dy}{dt} = 3t^2$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t}{2}$$

Since, $\quad x = 1 \quad \Rightarrow \quad 1 - 3t^2 + 1$

$\Rightarrow \quad t = 0$

$$\therefore \quad \frac{dy}{dx} = 0$$

258 **(b)**

(Slope) $f'(x) = e^x \cos x + \sin x\, e^x$

$= e^x\sqrt{2} \sin(x + \pi/4)$

$f''(x) = \sqrt{2}\, e^x\{\sin x + \pi/4\} + \cos\{x + \pi/4\}$

$= 2\, e^x. \sin(x + \pi/2)$

For maximum slope put, $f''(x) = 0$

$\Rightarrow \sin(x + \pi/2) = 0$

$\Rightarrow \ \cos x = 0$

$\therefore \ \ x = \pi/2, 3\pi/2$

$f'''(x) = 2e^x \cos(x + \pi/2)$

$f'''(\pi/2) = 2e^x. \cos\pi = -$ve

Maximum slope is at $x = \pi/2$

259 **(b)**

We have,

$$y = e^{-2|x|} = \begin{cases} e^{2x}, x < 0 \\ e^{-2x}, x \geq 0 \end{cases}$$

The line $x = 1/2$ cuts this curve at the point $P(1/2, e^{-1})$

$$\text{Also, } \frac{dy}{dx} = \begin{cases} 2e^{2x}, x < 0 \\ -2e^{-2x}, x > 0 \end{cases}$$

$\therefore \left(\frac{dy}{dx}\right)_{x=1/2} = -\frac{2}{e}$

The equation of the normal at P is

$y - \frac{1}{e} = \frac{e}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow 4(ey-1) = e^2(2x-1) \Rightarrow 2e(ex-2y)$
$= e^2 - 4$

260 **(a)**

For Lagrange's Mean value theorem we know, $f(x)$ should be continuous in [a, b] and differentiable in] a, b [.

(a) Given, $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

Which is clearly not differentiable at $x = \frac{1}{2}$; *as* RHD at $(x = 1/2) = -1$ *and* LHD at $(x = 1/2) = 0$ ⇒Lagrange's Mean Value is not applicable.

Where option, (b), (c), (d) are continuous and differentiable.

261 **(d)**

The point of intersection of given curves are (2, -1)

On differentiating the given curves respectively

$x\frac{dy}{dx} + y = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \quad m_1 = \left(\frac{dy}{dx}\right)_{(2,-1)} = \frac{1}{2}$

And $\quad 2x + 4\frac{dy}{dx} = 0$

$\Rightarrow \quad m_2 = \left(\frac{dy}{dx}\right)_{(2,-1)} = -1$

$\because \quad \tan\theta = \left|\frac{\frac{1}{2}+1}{1-\frac{1}{2}}\right| = \left|\frac{\frac{3}{2}}{\frac{1}{2}}\right| = 3$

262 **(c)**

Given , $\quad s = 490t - 4.9t^2$

$\Rightarrow \quad \frac{ds}{dt} = 490 - 9.8t$

A stone is reached the maximum height when $\frac{ds}{dt} = 0$

$\Rightarrow \quad 490 - 9.8t = 0 \Rightarrow t = 50$

∴ Maximum height at $t = 50$

$s = 490(50) - 4.9(50)^2 = 12250$

263 **(a)**

We know that $\sin x, \cos x$ and $\tan x$ are continuous on $[a,b]$ and differentiable on (a,b). Therefore, $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b)

Also, $f(a) = f(b) = 0$

Therefore, by Rolle's theorem there exists at least one $c \in (a,b)$ such that $f'(c) = 0$

Hence, $f'(x) = 0$ has at least one root in (a,b)

264 **(a)**

Since $\phi(x)$ is continuous at $x = \alpha$ such that $\phi(\alpha) < 0$. Therefore,

$\phi(x) < 0$ for all x in the neighbourhood of $x = \alpha$

Now,

$f'(x) = (ax - a^2 - x^2)\phi(x)$ for all x in the *nbd* of $x = \alpha$

$\Rightarrow f'(x) = -(x^2 - ax + a^2)\phi(x)$ for all x in the *nbd* of $x = \alpha$

$\Rightarrow f'(x) > 0$ for all x in the *nbd* of $x = \alpha$

$[\because x^2 - ax + a^2 > 0$ for all $x]$

$\Rightarrow f(x)$ is increasing in the *nbd* of $x = \alpha$

265 **(d)**

Given curve is $y = x^3$

$\Rightarrow \quad \frac{dy}{dx} = 3x^2$

Since, the tangent is parallel to x-axis

$\therefore \quad \frac{dy}{dx} = 0 \quad \Rightarrow \quad 3x^2 = 0 \quad \Rightarrow \quad x = 0, y = 0$

266 **(c)**

We have,

$f(x) = \cot^{-1}x + x \Rightarrow f'(x) = \frac{-1}{1+x^2} + 1$

$= \frac{x^2}{1+x^2}$

Clearly, $f'(x) > 0$ for all x

So, $f(x)$ increases in $(-\infty, \infty)$

267 **(c)**

Since $f(x)$ satisfies conditions of Rolle's Theorem on. Therefore,

$f(1) = f(3)$

$\Rightarrow 1 - 6 + a + b = 27 - 54 + 3\,a + b$

$\Rightarrow 2\,a = 22 \Rightarrow a = 11$

As $f(1) = f(3)$ is independent of b. Therefore, $a = 11$ and $b \in R$

268 **(b)**

Since, $f(x) = \tan^{-1}(\sin x + \cos x)$

$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$

$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$

For $f(x)$ to be increasing,

$-\frac{\pi}{2} < x\frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Hence, option (b) is correct, which lies in the above interval.

269 **(c)**

Let h be the height of the cylinder. Then, its volume V is given by

$V = \pi h^3 \qquad [\because r = h]$

$\Rightarrow \frac{dV}{dh} = 3\pi h^2$

$\therefore \Delta V = \frac{dV}{dh}\Delta h$

$\Rightarrow \Delta V = 3\pi h^2\, \Delta h$

$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{3\pi h^2}{\pi h^3}\Delta h \times 100 = 3\left(\frac{\Delta h}{h} \times 100\right)$

$= 3\alpha$

270 **(a)**

Given, $f(x) = x^3 + px^2 + qx + r$

$\Rightarrow \quad f'(x) = 3x^2 + 2xp + q$ or $f'(x) < 0$

Clearly, $\quad f'(x) > 0$

Now, $b^2 - 4ac < 0$

$\Rightarrow \quad 4p^2 - 4 \times 3 \times q < 0$

$\Rightarrow \quad p^2 < 3q$

271 **(d)**

We have, $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\, dt$

$\Rightarrow f'(x) = 2xe^{-(x^2+1)^2} - 2xe^{-x^4}$

$\Rightarrow f'(x) = 2x\left\{-e^{(x^2+1)^2} - e^{-x^4}\right\}$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow 2x\left\{-e^{(-x^2+1)^2} - e^{-x^4}\right\} > 0$

$\Rightarrow x < 0 \qquad [\because e^{-(x^2+1)^2} < e^{-x^4}]$

$\Rightarrow x \in (-\infty, 0)$

Hence, $f(x)$ is increasing on $(-\infty, 0)$

272 **(a)**

Let H be the height of the cone and α be its semi-vertical angle. Suppose that x is the radius of the inscribed cylinder and h be its height

$\therefore \quad h = QL = OL - OQ$

$= H - x\cot\alpha$

V =Volume of the cylinder

$= \pi x^2(H - x\cot\alpha)$

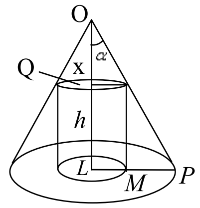
Also, $p = \frac{1}{3}\pi(H\tan\alpha)^2 H \quad$ ...(i)

$\frac{dV}{dx} = \pi(2Hx - 3x^2\cot\alpha)$

$\therefore \frac{dV}{dx} = 0 \quad \Rightarrow \quad x = 0,$

$x = \frac{2}{3}H\tan\alpha$

$\Rightarrow \left.\frac{d^2V}{dx^2}\right|_{x=\frac{2}{3}H\tan\alpha} = -2\pi H < 0$



$\therefore \quad V$ is maximum when $x = \frac{2}{3}H\tan\alpha$

And $q = V_{\max} = \pi\frac{4}{9}H^2\tan^2\alpha\frac{1}{3}H$

$= \frac{4}{9}p \quad$ [from Eq. (i)]

Hence, $p : q = 9 : 4$

273 **(b)**

Clearly, $f(x)$ is defined for all $x > 0$

We have,

$f'(x) = -\frac{\log x}{x^2} + \frac{1}{x^2} = \frac{1 - \log x}{x^2}$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow \frac{1 - \log x}{x^2} > 0 \Rightarrow 1 - \log x > 0 \Rightarrow \log x < 1 \Rightarrow x < e^1$

Also $f(x)$ is defined for $x > 0$. Hence, $f(x)$ is increasing in the interval $(0, e)$

274 **(c)**

Given, $f(x) = x^3 - 3x$

$\therefore \quad f'(x) = 3x^2 - 3$

For maxima, $\quad f'(x) = 6x$

$\Rightarrow \quad 3x^2 - 3 = 0 \quad \Rightarrow \quad x = \pm 1$

$\because \quad x = 1 \in [0, 2]$

At $x = 1$, $f''(x) > 0$, minima

$f(0) = 0$, $f(1) = -2$ and $f(2) = 2$

Hence, maximum value is 2

275 **(d)**

Given, $f(x) = ax + \frac{b}{x}$

On differentiating w.r.t. x, we get

$f'(x) = a - \frac{b}{x^2}$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow \quad x = \sqrt{\frac{b}{a}}$

Again, differentiating w.r.t. x, we get

$$f''(x) = \frac{2b}{x^3}$$

At $x = \sqrt{\frac{b}{a}}$, $f''(x) = +ve$

$\Rightarrow$ $f(x)$ is minimum at $x = \sqrt{\frac{b}{a}}$

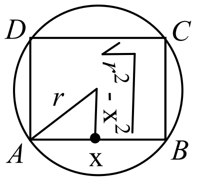
$\therefore$ $f(x)$ has the least value at $x = \sqrt{\frac{b}{a}}$

276 **(d)**

Area of rectangle, $A = 2x.2\sqrt{r^2 - x^2}$

$= 4x\sqrt{r^2 - x^2}$

$$\frac{dA}{dx} = \frac{4(r^2 - 2x^2)}{\sqrt{r^2 - x^2}}$$



For maximum or minimum put $\frac{dA}{dx} = 0$

$\Rightarrow \quad x = \frac{r}{\sqrt{2}}$

It can be easily checked that $\frac{d^2A}{dx^2} < 0$ for $x = \frac{r}{\sqrt{2}}$

The maximum value of A is given by

$$A = 4\frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}} = 2r^2$$

277 **(b)**

Since $f(x) = k\,x^3 - 9\,x^2 + 9\,x + 3$ is increasing on R

$\therefore f'(x) > 0$ for all $x \in R$

$\Rightarrow 3\,k\,x^2 - 18\,x + 9 > 0$ for all $x \in R$

$\Rightarrow k\,x^2 - 6\,x + 3 > 0$ for all $x \in R$

$\Rightarrow k > 0$ and $36 - 4(k)(3) < 0 \Rightarrow k > 0$ and $k > 3 \Rightarrow k > 3$

278 **(a)**

Given, $\frac{dV}{dt} = 100\pi cm^3/min$

$$\therefore \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 100\pi \Rightarrow 3r^2\frac{dr}{dt} = \frac{300\pi}{4\pi}$$

$$\Rightarrow \left(\frac{dr}{dt}\right)_{r=5} = \frac{300}{4 \times 3 \times 25} = 1cm/\text{min}$$

279 **(c)**

Let $f(\alpha) = \left(1 + \frac{1}{\sin^n \alpha}\right)\left(1 + \frac{1}{\cos^n \alpha}\right)$

$$\Rightarrow f(\alpha) = 1 + \frac{1}{\sin^n \alpha} + \frac{1}{\cos^n \alpha} + \frac{1}{\sin^n \alpha \cos^n \alpha}$$

$$\Rightarrow f'(\alpha) = -\frac{n\cos\alpha}{\sin^{n+1}\alpha} + \frac{n\sin\alpha}{\cos^{n+1}\alpha} - \frac{n\{\cos^2\alpha - \sin^2\alpha\}}{\sin^{n+1}\alpha\cos^{n+1}\alpha}$$

Now, $f'(\alpha) = 0 \Rightarrow \cos\alpha = \sin\alpha \Rightarrow \alpha = \pi/4$

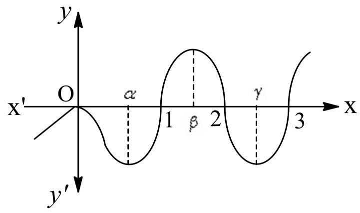
Clearly, $f(\alpha)$ is maximum at $\alpha = 0$ and $\alpha = \pi/2$ and between two maxima there is one minima

Hence, $\alpha = \pi/4$ gives the minimum value of $f(\alpha)$ and is given by $f\left(\frac{\pi}{4}\right) = \left(1 + 2^{n/2}\right)^2$

280 **(d)**

We have, $f(x) = |x|(x-1)(x-2)(x-3)$

$$= \begin{cases} x(x-1)(x-2)(x-3), & x \geq 0 \\ -x(x-1)(x-2)(x-3), & x < 0 \end{cases}$$



It is clear from the figure that, there are four critical points

$ie, 0, \alpha, \beta, \gamma$

281 **(a)**

We have,

$s = at^2 + bt + c$ ...(i)

$\Rightarrow \frac{ds}{dt} = 2at + b$ ...(ii)

$\Rightarrow \frac{d^2s}{dt^2} = 2a$ ....(iii)

At $t = 1$, we have $s = 16$

$\therefore a + b + c = 16$

At $t = 2$, we have

$\frac{ds}{dt} = 24$ and $\frac{d^2s}{dt^2} = 8 \Rightarrow 2a + b = 24$ and $2a = 8$

Solving these equations, we get

$a = 4, b = 8$ and $c = 4$

282 **(c)**

Given curve is $x^3 - 8a^2\,y = 0$.

On differentiating w.r.t. x, we get

$$3x^2 - 8a^2\,\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{8a^2}$$

$\therefore$ Slope of the normal $= -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{1}{\frac{3x^2}{8a^2}} = -\frac{8a^2}{3x^2}$

Given, $\frac{-8a^2}{3x^2} = \frac{-2}{3} \Rightarrow x^2 = 4a^2 \Rightarrow x = \pm 2a$

At $x = \pm 2a$, $y = \pm a$

$\therefore (x, y) = (2a, a)$

283 **(d)**

Given equation of curve is$y = be^{-x/a}$

Since, the curve crosses y-axis $ie, x = 0$

$\Rightarrow \quad y = be^{-0} \quad \Rightarrow \quad y = b$

On differentiating Eq. (i) w.r.t. x, we get

$\frac{dy}{dx} = \frac{-b}{a}\ e^{-x/a}$

At point $(0, b)$, $\left(\frac{dy}{dx}\right)_{(0,b)} = \frac{-b}{a} e^{-0/a} = \frac{-b}{a}$

$\therefore$ Required equation of tangent is

$y - b = \frac{-b}{a}(x - 0)$

$\Rightarrow \quad \frac{y}{b} - 1 = -\frac{x}{a}$

$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 1$

284 **(a)**

Given, $f(x) = x^2 e^{-x}$

$\Rightarrow f'(x) = 2xe^{-x} - x^2 e^{-x}$

For $f(x)$ To be increasing, $f'(x) > 0$

$\Rightarrow 2xe^{-x} - x^2 e^{-x} > 0$

$\Rightarrow e^{-x}(2 - x) > 0$

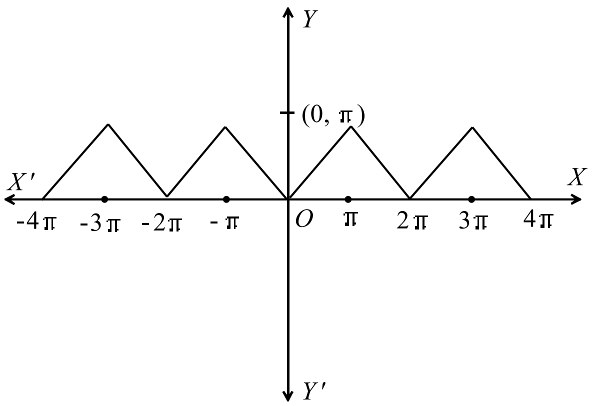
$\Rightarrow x \in (0,2)$

285 **(d)**

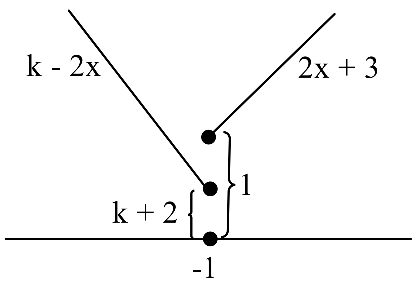
The graph of $y = \cos^{-1}(\cos x)$ is shown diagram

Clearly, $y = \cos^{-1}(\cos x) = -x$ for $-\pi < x < 0$

$\therefore \frac{dy}{dx} = -1$ at $x = -\frac{\pi}{4}$



286 **(d)**



$\because k + 2 \geq 1$

$\therefore k \leq -1$

287 **(d)**

Let $y = 2\log_{10} x - \log_x 0.01$. Then,

$y = 2\log_{10} x - \log_x 10^{-2}$

$\Rightarrow y = 2\log_{10} x + \frac{2}{\log_{10} x} = 2\left\{\log_{10} x + \frac{1}{\log_{10} x}\right\}$

$\geq 4[\because x > 1]$

288 **(a)**

From Rolle's theorem in given interval, [1,26], $f(1) = (26) = 5$ function satisfy all the condition of Rolle's theorem, therefore in [1,26], at least, there is one point for which $f'(x) = 0$

289 **(d)**

Given, $f(x) = 2 + (x - 1)^{2/3}$

By using Rolle's theorem, the given function is not differentiable at $x = 1$, which cannot satisfy Rolle's theorem.

290 **(a)**

Given, $f(x) = \cos x + \cos\sqrt{2}x$

$\Rightarrow \quad f'(x) = -\sin x - \sqrt{2}\sin\sqrt{2}\,x$

$\Rightarrow \quad f''(x) = -\cos x - 2\cos\sqrt{2}\,x$

For extremum value, put $f'(x) = 0$

$\Rightarrow \quad -\sin x - \sqrt{2}\sin\sqrt{2}\,x = 0 \quad \Rightarrow \quad x = 0$

$\therefore$ At $x = 0, f''(x) < 0$, maxima

$\therefore$ $f(x)$ Is maximum only once.

292 **(b)**

We have, $f(x) = \frac{\log(\pi+x)}{\log(e+x)}$

Clearly, domain of $f(x)$ is $(-e, \infty)$

Now,

$$f'(x) = \frac{\frac{\log(e+x)}{\pi+x} - \frac{\log(\pi+x)}{e+x}}{\{\log(e+x)\}^2}$$

$\Rightarrow f'(x)$

$$= \frac{(e+x)\log(e+x) - (\pi+x)\log(\pi+x)}{(\pi+x)(e+x)\{\log(e+x)\}^2} \quad \text{... (i)}$$

Let $g(x) = x\log x$ for $x > 0$. Then, $g'(x) = (1 + \log x)$

Now,

$g'(x) > 0 \Rightarrow 1 + \log x > 0 \Rightarrow \log x > -1 \Rightarrow x > e^{-1} = \frac{1}{e}$

Thus,

$g(x)$ is increasing on $(1/e, \infty)$

$\Rightarrow g(\pi + x) > g(e + x)$ for all $x > 0$

$\left[\because \pi + x > e + x > 1/e \text{ for all } x > 0\right]$

$\Rightarrow (\pi + x)\log(\pi + x) > (e + x)\log(e + x)$ for all $x > 0$ ...(ii)

From (i) and (ii), we find that

$f'(x) < 0$ for all $x \in [0, \infty)$

Hence, $f(x)$ is decreasing on $[0, \infty)$

293 **(c)**

Given, $f(x) = x^3$

$\therefore \quad f(x+h) = (x+h)^3$

Now, $f(x) = 3x^2$

$\therefore \quad f(x+\theta h) = 3(x+\theta h)^2$

Given, $\frac{f(x+h)-f(x)}{h} = f'(x+\theta h)$

$\Rightarrow \quad \dfrac{(x+h)^3 - x^3}{h} = 3(x+\theta h)^2$

$\Rightarrow \quad \dfrac{x^3 + h^3 + 3xh(x+h) - x^3}{h}$

$= 3(x^2 + \theta^2 h^2 + 2x\theta h)$

$\Rightarrow \quad h^2 + 3x^2 + 3xh = 3x^2 + 3\theta^2 h^2 + 6x\theta h$

$\Rightarrow \quad h + 3x = 3\theta^2 h + 6x\theta$

Taking limit on both sides, we get

$\lim\limits_{h\to 0}(h+3x) = \lim\limits_{h\to 0}(3\theta^2 h + 6x\theta)$

$\Rightarrow \quad 3x = 6x \lim\limits_{h\to 0}\theta$

$\Rightarrow \quad \lim\limits_{h\to 0}\theta = \dfrac{1}{2} = 0.5$

294 **(b)**

We have,

$y^2 = 4ax \Rightarrow 2y\dfrac{dy}{dx} = 4a \Rightarrow \dfrac{dy}{dx} = \dfrac{2a}{y}$

$\therefore$ Length of the subtangent $= \frac{y}{\frac{dy}{dx}} = \frac{y}{\frac{2a}{y}} = \frac{y^2}{2a} =$

$\frac{4ax}{2a} = 2x$

So, subtangent : Abscissa $= 2x : x = 2 : 1$

295 **(b)**

Given,

$ab = 2a + 3b \quad \Rightarrow \quad (a-3)b = 2a \quad \Rightarrow \quad b = \dfrac{2a}{a-3}$

Now, let $z = ab = \frac{2a^2}{a-3}$

On differentiating w.r.t. x, we get

$\dfrac{dz}{sa} = \dfrac{2[(a-3)3a - a^2]}{(a-3)^2} = \dfrac{2[a^2 - 6a]}{(a-3)^2}$

For a minimum, put $\frac{dz}{da} = 0$

$\Rightarrow \quad a^2 - 6a = 0 \quad \Rightarrow \quad a = 0, 6$

At $a = 6$, $\frac{d^2z}{da^2} = +ve$

When, $a = 6$, $b = 4$

$\therefore \ (ab)_{\min} = 6 \times 4 = 24$

296 **(d)**

For the function $f(x) = |x|, f'(0) \neq 0$ but $f(x)$ has a minimum at $x = 0$. So, none of the options is correct

297 **(b)**

We have,

$y = \cos(x+y)$

$\Rightarrow \frac{dy}{dx} = -\sin(x+y)\left(1 + \frac{dy}{dx}\right) \quad$ ...(i)

Since the tangent is parallel to $x + 2y = 0$

$\therefore$ Slope of the tangent $= -\frac{1}{2}$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{2}$

Putting $\frac{dy}{dx} = -\frac{1}{2}$ in (i), we get

$-\dfrac{1}{2} = -\sin(x+y)\left(1 - \dfrac{1}{2}\right)$

$\Rightarrow \sin(x+y) = 1 \quad$ ....(ii)

Now,

$y = \cos(x+y)$ and $1 = \sin(x+y)$

$\Rightarrow y^2 + 1 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0$

Putting $y = 0$ in $y = \cos(x+y)$ and $\sin(x+y) = 1$, we get

$\sin x = 1$ and $\cos x = 0 \Rightarrow x = -\frac{\pi}{2}, -\frac{3\pi}{2}$

Thus, the points on the curve $y = \cos(x+y)$ where tangents are parallel to $x + 2y = 0$ are $(\pi/2, 0), (-3\pi/2, 0)$

The equation of the tangent at $(\pi/2, 0)$ is

$y - 0 = -\dfrac{1}{2}\left(x - \dfrac{\pi}{2}\right) \Rightarrow x + 2y = \dfrac{\pi}{2}$

298 **(c)**

Given, $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has minimum value at $x = \frac{\pi}{3}$, then $f'(x) = 0$ at $x = \frac{\pi}{3}$

$\Rightarrow \quad f'(x) = a\cos x + \cos 3x$

$\Rightarrow f'\left(\dfrac{\pi}{3}\right) = a\cos\dfrac{\pi}{3} + \cos\left(3 \times \dfrac{\pi}{3}\right) = 0$

$\Rightarrow \quad a \times \dfrac{1}{2} + \cos(\pi) = 0$

$\Rightarrow \dfrac{a}{2} - 1 = 0 \Rightarrow a = 2$

299 **(d)**

Given, $y = x^2 + x - 1$

$\therefore \quad \dfrac{dy}{dx} = 2x + 1$

At $(1, 1)$, $\quad \frac{dy}{dx} = 3 = m$

$\therefore$ Length of tangent,

$A = \left|\dfrac{y_1\sqrt{1+m^2}}{m}\right| = \left|\dfrac{1\sqrt{1+9}}{3}\right| = \dfrac{\sqrt{10}}{3}$

Length of subtangent, $B = \left|\frac{y_1}{m}\right| = \frac{1}{3}$

Length of normal,

$C = \left|y_1\sqrt{1+m^2}\right| = \left|1\sqrt{1+9}\right| = \sqrt{10}$

And length of subnormal, $D = |y_1 m| = 3$

Now, increasing order isB, A, D, C.

301 **(a)**

Given, $2a + 3b + 6c = 0$

Let $f'(x) = ax^2 + bx + c$

$f(x) = \dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx$

$\Rightarrow f(x) = \dfrac{2ax^3 + 3bx^2 + 6cx + 6d}{6}$

Now, $f(1) = \frac{2a+3b+6c}{6} = 0$

And $f(0) = 0$

$\therefore f(0) = f(1)$

There 0 and 1 are the roots of the polynomial $f(x)$.So by Rolle's theorem, there exists at least one root of the polynomial $f'(x)$ lying between 0 and 1.

302 **(b)**

Let at any point t, the length of a side of the square be x. Then,

$A = \text{Area} = x^2$ and $P = \text{Perimeter} = 4x$

$\Rightarrow \frac{dA}{dt} = 2x\frac{dx}{dt}$ and $\frac{dP}{dt} = 4\frac{dx}{dt}$

It is given that

$\dfrac{dA}{dt} = \dfrac{dP}{dt} \Rightarrow 2x\dfrac{dx}{dt} = 4\dfrac{dx}{dt} \Rightarrow x = 2$

303 **(b)**

Since, $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

$f'(x) = 12x^3 + 12x^2 - 24x$

$= 12x(x-1)(x+2)$

For above it is clear that $f'(x)$ increasing in (-2,0)and in (1,∞)

304 **(d)**

By Lagrange's mean value theorem there exists $c \in (1,6)$ such that

$f'(c) = \dfrac{f(6) - f(1)}{6 - 1}$

$\Rightarrow \dfrac{f(6) + 2}{5} = f'(c)$

≥ 2 $[\because f'(x) \geq 2 \text{ for all } x \in [1,6]]$

$\Rightarrow f(6) + 2 \geq 10 \Rightarrow f(6) \geq 8$

305 **(c)**

Given, $f(x) = x^3 - 6x^2 - 36x + 2$

$f'(x) = 3x^2 - 12x - 36$

For decreasing, $f'(x) < 0$

$\Rightarrow 3(x^2 - 4x - 12) < 0$

$\Rightarrow (x-6)(x+2) < 0$

$\Rightarrow -2 < x < 6$

$\Rightarrow x \in (-2,6)$

306 **(c)**

Let (x_1, y_1) be one of the points of contact. Then, the equation of the tangent at (x_1, y_1) is

$y - y_1 = \left(\dfrac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1) \Rightarrow y - y_1$

$= -\sin x_1 (x - x_1)$

This passes through the origin

$\therefore 0 - y_1 = -\sin x_1 (0 - x_1) \Rightarrow y_1 = x_1 \sin x_1$ ...(i)

Also point (x_1, y_1) lies on $y = \cos x$

$y_1 = \cos x_1$ ...(ii)

From (i) and (ii), we have

$\sin^2 x_1 + \cos^2 x_1 = \dfrac{y_1^2}{x_1^2} + y_1^2 \Rightarrow x_1^2 = y_1^2 + y_1^2 x_1^2$

Hence, the locus of (x_1, y_1) is $x^2 = y^2 + y^2x^2$

307 **(c)**

We have, $f(x) = \displaystyle\int_{x^2}^{x^2+1} e^{-t^2} dt$

$\Rightarrow f'(x) = (2x+1)e^{-(x^2+1)^2} - 2x\, e^{-x^4}$ [Using Leibnitz's rule]

$\Rightarrow f'(x) > 0$ for all $x \in (-\infty, 0)$

$\Rightarrow f(x)$ is increasing on $(-\infty, 0)$

308 **(c)**

$f(x) = (x+1)^{1/3} - (x-1)^{1/3}$

$f'(x) = \dfrac{1}{3}(x+1)^{-2/3} - \dfrac{1}{3}(x-1)^{-2/3}$

Now, put $f'(x) = 0$

$\Rightarrow \dfrac{1}{3}(x+1)^{-2/3} = \dfrac{1}{3}(x-1)^{-2/3}$

$\Rightarrow (x-1)^2 = (x+1)^2$

$\Rightarrow x = 0$

At $x = 0$, f'' is negative, so it is maximum.

$\therefore$ The greatest value of

$f(x) = (0+1)^{1/3} - (0-1)^{1/3} = 2$

309 **(c)**

Given , $\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -50$

$\Rightarrow \dfrac{dr}{dt} = -\dfrac{50}{4\pi r^2}$

$\Rightarrow \left(\dfrac{dr}{dt}\right)_{r=15} = -\dfrac{50}{4\pi \times 225} = -\dfrac{1}{18\pi}$ cm/min

Hence , the thickness of ice decrease by $1/18\pi$ cm/min

310 **(b)**

Given equation of curve is

$y = 2x^3 + ax^2 + bx + c$ ...(i)

$\Rightarrow 0 = 2(0) + a(0) + b(0) + c$ [passes through (0, 0)]

$\Rightarrow c = 0$ ...(ii)

On differentiating Eq. (i), we get

$\dfrac{dy}{dx} = 6x^2 + 2ax + b$

Since, the tangent at $x = -1$ and $x = 2$ are parallel to x-axis

$\therefore \dfrac{dy}{dx} = 0$

At $x=-1$, $6(-1)^2+2a(-1)+b=0$

$\Rightarrow \quad 6-2a+b=0 \quad$ ...(iii)

At $x=2$, $6(2)^2+2a(2)+b=0$

$\Rightarrow \quad 24+4a+b=0 \quad$ ...(iv)

On solving Eqs. (iii) and (iv), we get

$a=-3, \quad b=-12 \quad$ and $\quad c=0$

311 **(a)**

The equation of the tangent at point $P(\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

This cuts the coordinate axes at $A(a\sec\theta, 0)$ and $B(0, \text{cosec}\,\theta)$

$\therefore AB^2=a^2\sec^2\theta+b^2\,\text{cosec}^2\theta$

Let $Z=a^2\sec^2\theta+b^2\,\text{cosec}^2\theta$. Then,

$$\frac{dZ}{d\theta}=2a^2\sec^2\theta\tan\theta-2b^2\,\text{cosec}^2\theta\cot\theta$$

$$\Rightarrow \frac{dZ}{d\theta}=2a^2\frac{\sin\theta}{\cos^3\theta}-\frac{2b^2\cos\theta}{\sin^3\theta}$$

$$\Rightarrow \frac{dZ}{d\theta}=\frac{2a^2\sin^4\theta-2b^2\cos^4\theta}{\sin^3\theta\cos^3\theta}$$

$$\therefore \frac{dZ}{d\theta}=0 \Rightarrow \tan^2\theta=\frac{b^2}{a^2} \Rightarrow \tan\theta=\pm\sqrt{\frac{b}{a}}$$

Now,

$\frac{d^2Z}{d\theta^2}=2a^2(\sec^4\theta+2\sec^2\theta\tan^2\theta)+$
$2b^2(\text{cosec}^4\theta+2\,\text{cosec}^2\theta\cot^2\theta)>0$ for all θ

$\therefore Z$ is minimum when $\tan\theta=\pm\sqrt{\frac{b}{a}}$

For this value of θ, we have

$$Z=a^2\left(1+\frac{b}{a}\right)+b^2\left(1+\frac{a}{b}\right)=(a+b)^2 \Rightarrow AB=a+b$$

312 **(a)**

Given equation is

$y-e^{xy}+x=0$

$\Rightarrow \quad e^{xy}=x+y$

On taking log on both sides, we get

$\log(x+y)=xy$

On differentiating w.r.t. x, we get

$$\frac{1}{x+y}+\left(1+\frac{dy}{dx}\right)=x\frac{dy}{dx}+y$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y+x}-x\right)=y-\frac{1}{y+x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y(y+x)-1}{1-x(y+x)}$$

Since, the curve has a vertical tangent

$$\therefore \frac{dy}{dx}=\infty \Rightarrow 1-x(x+y)=0$$

Which is satisfied by the point $(1, 0)$

313 **(d)**

We have,

$f(x)=x\,e^{2-x} \Rightarrow f'(x)=e^{2-x}(1-x)$

For $f(x)$ to be increasing, we must have

$f'(x)>0 \Rightarrow e^{2-x}(1-x)>0 \Rightarrow x>1$

Hence, $f(x)$ is increasing on $(-\infty, 1)$

314 **(d)**

Since, $f(x)$ satisfies all the conditions of Rolle's theorem in $[3, 5]$

Let $f(x)=(x-3)(x-5)=x^2-8x+15$

Now, $\int_3^5 f(x)\,dx=\int_3^5(x^2-8x+15)\,dx$

$$=\left[\frac{x^3}{3}-\frac{8x^2}{2}+15x\right]_3^5$$

$$=\left(\frac{125}{3}-100+75\right)-(9-36+45)$$

$$=\frac{50}{3}-18=-\frac{4}{3}$$

315 **(b)**

Let x be the length of a side of the triangle. Then, its area A is given by

$$A=\frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dx}=\frac{\sqrt{3}}{2}x$$

$$\therefore \Delta A=\frac{dA}{dx}\Delta x$$

$$\Rightarrow \Delta A=\frac{\sqrt{3}}{2}x\,\Delta x$$

$$\Rightarrow \frac{\Delta A}{A}\times 100=\frac{\frac{\sqrt{3}}{2}x\,\Delta x}{\frac{\sqrt{3}}{4}x^2}\times 100=2\left(\frac{\Delta x}{x}\times 100\right)=2k$$

316 **(b)**

The equation of given circle is

$x^2+y^2=1$

On differentaing w.r.t. t, we get

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

But we have, $x=\frac{1}{2}, y=\frac{\sqrt{3}}{2}$ and $\frac{dy}{dt}=-3$, then

$$\frac{1}{2}\frac{dx}{dt}+\frac{\sqrt{3}}{2}(-3)=0 \quad \Rightarrow \quad \frac{dx}{dt}=3\sqrt{3}$$

317 **(c)**

Let (x, y) be any point on the parabola $2y=x^2$

Let $\quad f(x)=(x-0)^2+(y-3)^2=x^2+\left(\frac{x^2}{2}-3\right)^2$

$$\Rightarrow \quad f'(x)=2x+2\left(\frac{x^2}{2}-3\right)(x)$$

Put $f'(x)=0 \Rightarrow 2x\left(1+\frac{x^2}{2}-3\right)=0$

$\Rightarrow \quad x=0,\pm 2$

Now, $f''(x)=2+2\left(\frac{x^2}{2}-3\right)+2x(x)$

At $x=0$, $f''(x)=2-6<0$, maximum

At $x=2$, $f''(x)>0$, minimum

At $x=-2$, $f''(x)>0$, minimum

$\therefore$ At $x=\pm 2 \Rightarrow y=2$

$\therefore$ Required point is $(\pm 2,2)$

318 **(d)**

We have,

$y=x-\cot^{-1}x-\log\left(x+\sqrt{1+x^2}\right)$

$$\Rightarrow \frac{dy}{dx}=1+\frac{1}{1+x^2}-\frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1+x^2+1-\sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1+x^2}(\sqrt{1+x^2}-1)}{\sqrt{1+x^2}}+1\geq 0 \text{ for all } x$$

Thus, the given function is increasing for all $x\in(-\infty,\infty)$

319 **(b)**

Let $P(x,y$ be a point of the curve

$x=a\sin t-b\sin\left(\frac{at}{b}\right), y=a\cos t-b\cos\left(\frac{at}{b}\right)$

and, O be the origin. Then,

$$OP^2=\left\{a\sin t-b\sin\left(\frac{at}{b}\right)\right\}^2+\left\{a\cos t-b\cos\left(\frac{at}{b}\right)\right\}^2$$

$$\Rightarrow OP^2=a^2+b^2-2ab\cos\left(\frac{a-b}{b}\right)t$$

Clearly, OP^2 is maximum when $\cos\left(\frac{a-b}{t}\right)t$ minimum $i.e.$ equal to -1

In that case, we have

$OP^2=a^2+b^2+2ab=(a+b)^2 \Rightarrow OP=a+b$

320 **(d)**

$\therefore f(x)=-x^3+4ax^2+2x-5$

$\therefore f'(x)=-3x^2+8ax+2$

Since, $f(x)$ is decreasing. $\forall x$, therefore $f'(x)<0$

$\Rightarrow -3^2+8ax+2<0$

$\Rightarrow 3x^2-8ax-2>0$

Now, $D=(-8a)^2-4(3)(-2)<0$

$\Rightarrow 64a^2+24<0$

$\Rightarrow a^2<-\frac{3}{8}$

Hence, no value of a exist.

322 **(c)**

Clearly, $e^{(x^4-x^3+x^2)}$ will be minimum when $x^4-x^3+x^2$ is minimum

Now,

$x^4-x^3+x^2=x^2(x^2-x+1)$

$=x^2\left\{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right\}\geq 0$ for all $x\in R$

So, the minimum value of $x^4-x^3+x^2$ is 0

Hence, the minimum value of $e^{(x^4-x^3+x^2)}$ is $e^0=1$

323 **(b)**

Let $f(x)=x^2-2x+3$

Since, $f'(c)=\frac{f\left(\frac{3}{2}\right)-f(1)}{\frac{3}{2}-1}$ [given]

$$\Rightarrow \quad 2c-2=\frac{\frac{9}{4}-\frac{6}{2}+3-(1-2+3)}{\frac{3}{2}-1}=\frac{1}{2}$$

$$\Rightarrow \quad c=\frac{5}{4}\in\left[1,\frac{3}{2}\right]$$

324 **(c)**

Given, $y=\frac{\sin(x+a)}{\sin(x+b)}$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin(x+b)\cos(x+a)-\sin(x+b)\cos(x+a)}{\sin^2(a+b)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin(b-a)}{\sin^2(x+b)}$$

For maxima or minima put, $\frac{dy}{dx}=0$

Sin $(b-a)=0$

It means no value of$x$, it is neither maxima nor minima.

325 **(b)**

We have,

$f(x)=e^{ax}+e^{-ax}$

$\Rightarrow f'(x)=a\{e^{ax}-e^{-ax}\}$

$$\Rightarrow f'(x)=2a\left\{ax+\frac{a^3x^3}{3!}+\frac{a^5x^5}{5!}+\cdots\right\}$$

$$\Rightarrow f'(x)=2a^2x\left\{1+\frac{a^2x^2}{3!}+\frac{a^4x^4}{5!}+\cdots\right\}$$

Now,

$$f'(x)<0\Rightarrow x<0\left[\because 2a^2\left\{1+\frac{a^2x^2}{3!}+\frac{a^4x^4}{5!}+\cdots\right\}>0\right]$$

Hence, $f(x)$ is decreasing on $(-\infty,0)$

326 **(d)**

The point of intersection of given curve is $(-2,-1)$

On differentiating the given curve respectively, we get

$x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}$

$\Rightarrow \quad m_1=\left(\frac{dy}{dx}\right)_{(-2,-1)}=-\frac{1}{2}$

And $\quad 2x+4\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{x}{2}$

$\Rightarrow \quad m_2=\left(\frac{dy}{dx}\right)_{(-2,-1)}=1$

$\therefore \quad \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-\frac{1}{2}-1}{1-\frac{1}{2}}\right|=3$

327 **(d)**

We have,

$2x^2-3y^2=36 \Rightarrow 4x-6y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2x}{3y}$

The slope of the given line is $-1/2$

If the tangent is parallel to the given line then

$\frac{dy}{dx}=-\frac{1}{2} \Rightarrow \frac{2x}{3y}=-\frac{1}{2} \Rightarrow x=-\frac{3y}{4}$

Putting $x=-\frac{3y}{4}$ in $2x^2-3y^2=36$, we get

$2\left(\frac{9y^2}{16}\right)-3y^2=36 \Rightarrow y^2=-\frac{288}{15}$

This does not given real values

Hence, the required tangent does not exist

328 **(b)**

We have,

$f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$

$\Rightarrow f'(x)=3x^2+6x(a-7)+3(a^2-9)$

If the function $f(x)$ has a positive point of minimum, then $f'(x)=0$ must have positive roots

$\Rightarrow x=0$ is less than the roots of $f'(x)=0$

$\Rightarrow f'(0)>0$ and Discriminant ≥ 0

$\Rightarrow 4(a-7)^2-4(a^2-9)\geq 0$ and $3(a^2-9)>0$

$\Rightarrow 7a-29\leq 0$ and $a^2-9>0$

$\Rightarrow a<\frac{29}{7}$ and $(a<-3$ or $a>3) \Rightarrow a\in(-\infty,-3)\cup(3,29/7)$

330 **(b)**

Given, $\quad f(x)=x+\frac{1}{x}$

$\Rightarrow \quad f'(x)=1-\frac{1}{x^2}$

$\Rightarrow$ For local maxima or local minima, put $f'(x)=0$

$\Rightarrow \quad 1-\frac{1}{x^2}=0$

$\Rightarrow x=\pm 1$

Now, $f''(x)=\frac{2}{x^3}$

At $x=1$, $\quad f''(x)=\frac{2}{1^3}>0$, local minima

At $x=-1$, $f''(x)=\frac{2}{(-1)^3}$

$=-2<0$, local maxima

331 **(c)**

Given curve is $y=f(x)=x^2+bx-b$

On differentiating, we get $\frac{dy}{dx}=2x+b$

The equation of the tangent at (1, 1) is

$y-1=\left(\frac{dy}{dx}\right)_{(1,1)}=(x-1)$

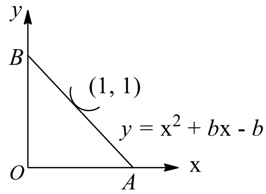
$\Rightarrow y-1=(b+2)(x-1)$

$\Rightarrow \quad (2+b)x-y=1+b$

$\Rightarrow \frac{x}{\frac{(1+b)}{(2+b)}}-\frac{y}{1+b}=1$

So, OA= $\frac{1+b}{2+b}$

and $OB=-(1+b)$



Now, area of $\Delta\, AOB=\frac{1}{2}.\frac{1+b}{2+b}.[-(1+b)]=2$ (given)

$\Rightarrow \quad 4(2+b)+(1+b)^2=0$

$\Rightarrow \quad 8+4b+1+b^2+2b=0$

$\Rightarrow \quad b^2+6b+9=0$

$\Rightarrow \quad (b+3)^2=0 \quad \Rightarrow b=-3$

332 **(a)**

Let $f(x)=3\sin x-4\sin^3 x=\sin 3x$

Since, $\sin x$ is increasing in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore \quad -\frac{\pi}{2}\leq 3x\leq\frac{\pi}{2} \quad \Rightarrow \quad -\frac{\pi}{6}\leq x\leq\frac{\pi}{6}$

Thus, length of interval $=\left|\frac{\pi}{6}\left(-\frac{\pi}{6}\right)\right|=\frac{\pi}{3}$

333 **(d)**

To satisfy Rolle's theorem, it should be continuous in [0, 1].

$\Rightarrow \lim_{n\to 0^+} f(x)=f(0)$

$\Rightarrow \lim_{n\to 0^+}\frac{\log x}{x^{-a}}=0$

$\Rightarrow \lim_{n\to 0^+}\frac{1/x}{-\alpha x^{-a-1}}=0 \quad$ [using L'Hospital's rule]

$\Rightarrow \lim_{n\to 0^+}-\frac{1}{\alpha x^{-\alpha}}=0$

$\Rightarrow \lim_{n\to 0^+}-\frac{1}{\alpha}x^{\alpha}=0$

Which shows $\alpha > 0$otherwise, it would be discontinuous also when

$\alpha > 0, f(x)$ is differentiable in $(0,1)$ and $f(1)$

$= f(0) = 0.$

Clearly $\alpha > 0$, thus $\alpha = \frac{1}{2}$is the possible answer.

334 **(a)**

Consider the function

$$f(x) = \frac{a\,x^3}{3} + \frac{b\,x^2}{2} + cx$$

we have, $f(0) = 0$ and,

$$f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2\,a + 3\,b + 6\,c}{6} = \frac{0}{6}$$

$$= 0\ [\because 2\,a + 3\,b + 6\,c = 0]$$

Therefore, 0 and 1 are roots of the polynomial $f(x)$

Consequently, there exists at least one root of the polynomial $f'(x) = a\,x^2 + bx + c$ lying between 0 and 1

335 **(a)**

The time period T of simple pendulum of length l is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \log T = \log 2\pi + \frac{1}{2}(\log l - \log g)$$

$$\Rightarrow \frac{1}{T}\frac{dT}{dl} = \frac{1}{2l} \Rightarrow \frac{dT}{dl} = \frac{T}{2l}$$

It is given that $\frac{\Delta l}{l} \times 100 = 2$

Now, $\Delta T = \frac{dT}{dl}\Delta l$

$$\Rightarrow \Delta T = \frac{T}{2l}\Delta l$$

$$\Rightarrow \frac{\Delta T}{T} \times 100 = \frac{1}{2}\left(\frac{\Delta l}{l} \times 100\right) = \frac{1}{2} \times 2 = 1$$

336 **(a)**

$\because g(x) = \min(x, x^2)$

$\therefore\ \ g(x)$ is an increasing function

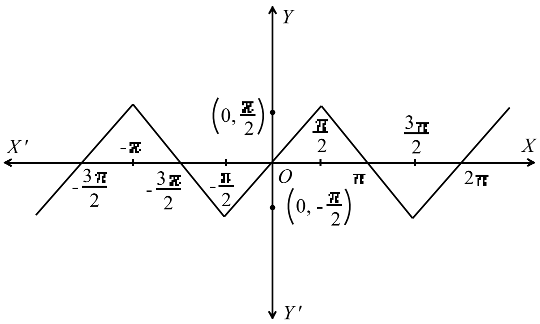
337 **(b)**

The graph of $y = \sin^{-1}(\sin x)$ is shown in diagram

It is evident from the graph that

$\frac{dy}{dx} = -1$ for all $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, the slope of the tangent at $x = \frac{3\pi}{4}$ is -1



338 **(c)**

Given, $f(x) = 2x^3 - 9ax^2 + 12a^2\,x + 1$ attains maximum and minimum at p and q respectively.

$\therefore\quad f'(p) = 0,\ \ f'(q) = 0$

$f''(p) < 0$ and $f''(q) > 0$

Since, $f'(p) = 0$ and $f'(q) = 0$

$\Rightarrow\quad 6p^2 - 18ap + 12a^2 = 0$

And $\ 6q^2 - 18aq + 12a^2 = 0$

$\Rightarrow\quad p^2 - 3ap + 2a^2 = 0$

And $\ q^2 - 3aq + 2a^2 = 0$

$\Rightarrow\quad p = a, 2a; q = a, 2a$ ...(i)

Now, $f''(p) < 0$

$\Rightarrow\quad 12p - 18a < 0\quad \Rightarrow\quad p < \frac{3}{2}a$ ...(ii)

And $f''(q) > 0\quad \Rightarrow\quad 12q - 18a > 0\quad \Rightarrow\quad q > \frac{3}{2}a$ ...(iii)

From Eqs. (i), (ii), (iii), we get

$p = a,\ \ q = 2a$

Now, $\ p^2 = q$

$\Rightarrow\quad a^2 = 2a\quad \Rightarrow\quad a = 0,2$

But for $a = 0, f(x) = 2x^3 + 1$ which does not attains a maximum or minimum for any value of x.

Hence, $\ a = 2$

339 **(b)**

Let $f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve $f'(x) = -3x^2 + 6x + 9$

Let $\ g(x) = f'(x) = -3x^2 + 6x + 9$

On differentiating w.r.t. x, we get

$g'(x) = -6x + 6$

For maxima or minima put $g'(x) = 0 \Rightarrow x = 1$

Now, $g''(x) = -6 < 0$ and hence, at $x = 1$, $g(x)$ (slope) will have maximum value

$\therefore \quad [g(1)]_{\max} = -3 \times 1 + 6(1) + 9 = 12$

340 **(c)**

Given, $a^2x^4 + b^2y^4 = c^6$

$$\Rightarrow \quad y = \left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$$

and let $f(x) = xy = x\left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$

$$\Rightarrow \quad f(x) = \left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{1/4}$$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{1}{4}\left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{1/4}\left(\frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2}\right)$$

For maxima or minimum, put $f'(x) = 0$

$$\Rightarrow \frac{4x^3c^6}{b^2} = \frac{8x^7a^2}{b^2} = 0$$

$$\Rightarrow \frac{4x^3}{b^2}(c^6 - 2a^2x^4) = 0$$

$$\Rightarrow x^4 = \frac{c^6}{2a^2} \Rightarrow \quad \pm\frac{c^{3/2}}{c^{1/4}\sqrt{a}}$$

At $x = \frac{c^{3/2}}{2^{1/4}\sqrt{a}}$, $f(x)$ will be maximum

$$\therefore \quad f\left(\frac{c^{3/2}}{2^{1/4}\sqrt{a}}\right) = \left(\frac{c^{12}}{2a^2b^2} - \frac{c^{12}}{4a^2b^2}\right)^{1/4}$$

$$= \left(\frac{c^{12}}{4a^2b^2}\right)^{1/4} = \frac{c^3}{\sqrt{2ab}}$$

341 **(d)**

Given that, $\frac{dV}{dt} = k$ (say)

$$\because \quad V = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad \frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$$

$$\Rightarrow \quad \frac{dR}{dt} = \frac{k}{4\pi R^2}$$

$\Rightarrow$ Rate of increasing radius is inversely proportional to its surface area

343 **(a)**

Given, $s = 16 - 2t + 3t^3$

$$\Rightarrow \quad \frac{ds}{dt} = -2 + 9t^2 \quad \Rightarrow \frac{d^2s}{dt^2} = 18t$$

At $t = 2s$, acceleration a$= \frac{d^2s}{dt^2} = 18 \times 2 = 36 m/s^2$

344 **(b)**

We have,

$$y = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x^2}$$

$$\Rightarrow \text{Slope of the normal at any point} = \frac{-1}{\frac{dy}{dx}} = x^2 > 0$$

Clearly, slopes of the lines given in options (b) and (d) are positive

345 **(c)**

Let surface area of sphere,

$$S = \frac{4}{3}\pi r^3 \quad \Rightarrow \frac{dS}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 4\pi r^2(2) = 8\pi r^2 \quad \therefore \frac{dS}{dt} \propto r^2$$

346 **(c)**

Given equation of curves are $y = x^2$ and $6y = 7 - x^3$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_{(1,1)} = 2(1) = 2$$

And $m_2 = \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}$

Now, $m_1m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

347 **(a)**

On differentiating the given curves respectively

$\therefore \quad \left(\frac{dy}{dx}\right)_{C_1} = \frac{x^2-y^2}{2xy} = m_1$ (say)

And $\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2xy}{x^2-y^2} = m_2$ (say)

$m_1 \times m_2 = -1$

Hence, the two curves cut at right angles

348 **(d)**

Since, $s \propto v^2 \Rightarrow v = k\sqrt{s}$

Now, acceleration $\frac{dv}{dt} = k\frac{1}{2\sqrt{s}}\frac{ds}{dt}$

$=k\frac{1}{2\sqrt{s}} \cdot k\sqrt{s} \qquad \left[\because v = \frac{ds}{dt} = k\sqrt{s}\right]$

$= \frac{k^2}{2}$ =constant

349 **(c)**

Let $P(x, y)$ be the point on the curve $x^2 = 2y$ such that it is nearest to the point $A(0, 3)$. Then,

$AP^2 = x^2 + (y-3)^2$

$\Rightarrow AP^2 = 2y + (y-3)^2 = y^2 - 4y + 9$

$= (y-2)^2 + 5 \geq 5$

Clearly, AP^2 is minimum when $y = 2$ and the minimum value of AP is $\sqrt{5}$

Putting $y = 2$ in $x^2 = 2y$ we get $x = \pm 2$

Hence, the required points are (2, 2) and (−2, 2)

351 **(d)**

Equation of parabola is $y^2 = 18x$....(i)

$\Rightarrow\ 2y\frac{dy}{dt} = 18\frac{dx}{dt} \Rightarrow 2.2y = 18 \qquad \left[\because \frac{dy}{dt} = 2\frac{dx}{dt}\right]$

$\Rightarrow\ y = \frac{9}{2}$

$\therefore\ From\ Eq.(i), \left(\frac{9}{2}\right)^2 = 18x \Rightarrow x = \frac{9}{8}$

$\therefore Point\ is \left(\frac{9}{8}, \frac{9}{2}\right)$.

352 **(c)**

Given curves are $y^2 = x$ and $x^2 = y$

On differentiating w.r.t. x, we get

$2y\frac{dy}{dx} = 1$ and $2x = \frac{dy}{dx}$

$\Rightarrow\ \frac{dy}{dx} = \frac{1}{2y}$and $\frac{dy}{dx} = 2x$

At (0,0)

$m_1 = \frac{dy}{dx} = \infty$ ad $m_2 = \frac{dy}{dx} = 0$

$\therefore\quad tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \infty$

$\Rightarrow\quad \theta = \frac{\pi}{2}$

353 **(b)**

Let $f(x) = (1+x)^n - (1+x^n)$. Then,

$f'(x) = n(1+x)^{n-1} - nx^{n-1}$

$\Rightarrow f'(x) = n\{(1+x)^{n-1} - x^{n-1}\}$

$\Rightarrow f'(x) = n\left\{\frac{1}{(1+x)^{1-n}} - \frac{1}{x^{1-n}}\right\} < 0,$

if $n \geq 0, 1-n \geq 0$ and $x > 0$

$\Rightarrow f(x)$ is decreasing when $x > 0, 0 \leq n \leq 1$

$\Rightarrow f(x) < f(0)$

$\Rightarrow (1+x)^n - (1+x^n) < 0$

$\Rightarrow (1+x)^n < 1 + x^n$, if $0 \leq n \leq 1$ and $x > 0$

354 **(a)**

We have,

$$f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\}\,dt$$

$$\Rightarrow f(x) = \int_1^x (t-1)\,(t-2)^2(5t-7)dt$$

$\Rightarrow f'(x) = (x-1)(x-2)^2(5x-7)$

For maximum or minimum, we must have

$f'(x) = 0 \Rightarrow x = 1, 2, 7/5$

Now,

$$f''(x) = (x-2)^2(5x-7) + 5(x-1)(x-2)^2 + 2(x-2)(x-1)(5x-7)$$

$\Rightarrow f''(1) < 0, f''(7/5) > 0$ and $f''(2) = 0$

Hence, $f(x)$ attains its maximum at $x = 1$

355 **(d)**

The curve $y = e^{-|x|}$ cuts the line $x = 1$ at $(1, 1/e)$

Now,

$$y = e^{-|x|} = \begin{cases} e^x, x < 0 \\ e^{-x}, x \geq 0 \end{cases}$$

$$\therefore \frac{dy}{dx} = \begin{cases} e^x, x < 0 \\ -e^{-x}, x > 0 \end{cases}$$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = -e^{-1} = -\frac{1}{e}$

The equation of the tangent at $(1, 1/e)$ is

$$y - \frac{1}{e} = -\frac{1}{e}(x-1) \Rightarrow ey - 1 = -x + 1 \Rightarrow x + ey = 2$$

356 **(c)**

Given, $x = t^2 + 1$ and $y = t^2 - t - 6$

$\therefore\ \frac{dx}{dt} = 2t$ and $\frac{dy}{dx} = 2t - 1$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-1}{2t}$

When, it ,meet x-axis, then $y = 0$

$\therefore\quad t^2 - t - 6 = 0$

$\Rightarrow\quad y = 3, -2$

$\therefore\ \left(\frac{dy}{dx}\right)_{at\ t=3} = \frac{6-1}{6} = \frac{5}{6} = m_1$ [say]

And $\left(\frac{dy}{dx}\right)_{at\ t=-2} = \frac{5}{4} = m_1$ [say]

$\therefore$ Required angle$= \pm\tan^{-1}\left\{\left|\frac{\frac{5}{6}-\frac{5}{4}}{1+\frac{25}{24}}\right|\right\}$

$= \pm\tan^{-1}\left\{\frac{10}{49}\right\}$

357 **(b)**

Given curve is $x^2y = x^2 - 3x + 6$ ...(i)

At $x = 3$, $3^2(y) = 3^2 - 3(3) + 6$

$\Rightarrow\ y = \frac{2}{3}$

On differentiating Eq. (i) w.r.t. x, we get

$2xy + x^2\frac{dy}{dx} = 2x - 3$

$\Rightarrow\ \frac{dy}{dx} = \frac{2x - 3 - 2xy}{x^2}$

$\Rightarrow\ \left(\frac{dy}{dx}\right)_{\left(3,\frac{2}{3}\right)} = \frac{6 - 3 - 2 \times 3 \times \frac{2}{3}}{3^2} = -\frac{1}{3^2}$

$\therefore$ Equating of normal is

$y - \frac{2}{3} = 3^2(x-3)$

$\Rightarrow\quad 27x - 3y = 79$

358 **(a)**

Let $x_1, x_2 \in R$ such that $x_1 < x_2$. Then,

$x_1 < x_2$

$\Rightarrow f(x_1) < f(x_2)$ [$\because$ is an increasing function]

$\Rightarrow g(f(x_1)) < g(f(x_2))$ [∵ g is an increasing function]

$\Rightarrow gof(x_1) < gof(x_2)$

Hence, gof is an increasing function

359 **(a)**

Let $y = x^x$

On differentiating w.r.t. x , we get

$$\frac{dy}{dt} = x^x(1 + \log x)$$

For increasing function,

$$\frac{dy}{dx} > 0$$

$\Rightarrow \quad x^x(1 + \log x) > 0) \quad \Rightarrow \quad 1 + \log x > 0$

$\Rightarrow \quad \log_e x > \log_e \frac{1}{e} \quad \Rightarrow \quad x > \frac{1}{e}$

Function is increasing when $x > \frac{1}{e}$

360 **(d)**

$We\ have, y = x + \frac{4}{x^2}$

On differencing with respect to x, we get

$$\frac{dy}{dx} = 1 - \frac{8}{x^3}$$

Since, the tangent is parallel to $x - axis$, therefore

$$\frac{dy}{dx} = 0$$

$\Rightarrow \quad x^3 = 8 \quad \Rightarrow x = 2\ and\ y = 3$

361 **(c)**

The two curves are $y = x^2$ and $6y = 7 - x^3$.

These two intersect at (1, 1)

Now,

$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow m_1 = \left(\frac{dy}{dx}\right)_{(1,1)} = 2$$

and, $y = y - x^3 \Rightarrow \frac{dy}{dx} = -\frac{x^2}{2} \Rightarrow m^2 = \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}$

Clearly, $m_1\, m_2 = -1$

Thus, the angle of intersection of two curves is $\pi/2$

362 **(d)**

$f(x) = \frac{x^2-1}{x^2+1}$ (for every real number x)

$$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)2x}{(x^2+1)^2}$$

$\Rightarrow \quad f'(x) = \frac{4x}{(x^2+1)^2}$ ...(i)

For minima or maxima, put $f'(x) = 0$

$\therefore \quad \frac{4x}{(x^2+1)^2} = 0 \quad \Rightarrow \quad x = 0$

Now, $\quad f''(x) = \frac{(x^2+1)^2 4 - (4x)2(x^2+1)2x}{(x^2+1)^4}$

$$f''(x) = \frac{12x^2 + 4}{(x^2+1)^3}$$

At $\quad x = 0$

$f''(0) = \frac{-12\times 0^2+4}{(0^2+1)^3} > 0$, minima

Then, minimum value of $f(x)$ at $x = 0$ is

$$f(0) = \frac{0^2 - 1}{0^2 + 1} = -1$$

363 **(b)**

We have,

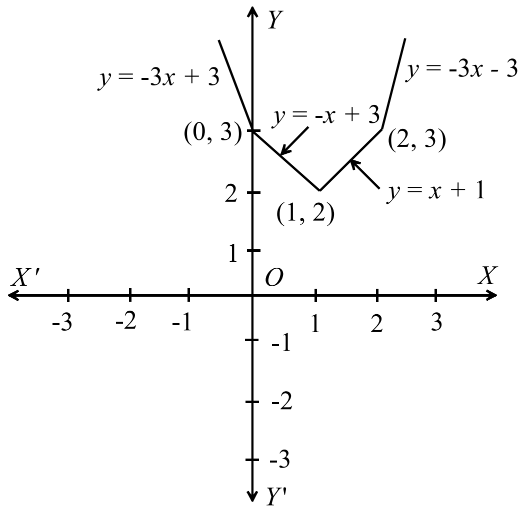
$$f(x) = |x| + |x-1| + |x-2|$$

$$= \begin{cases} -3x + 3, & x < 0 \\ -x + 3, & 0 \le x < 1 \\ x + 1, & 1 \le x < 2 \\ 3x - 3, & x \ge 2 \end{cases}$$

$$\Rightarrow f''(x) = \begin{cases} -3, & x < 0 \\ -1, & 0 < x < 1 \\ 1, & 1 < x < 2 \\ 3, & x > 2 \end{cases}$$

Clearly, $f(x)$ is not differentiable at $x = 0, 1, 2$

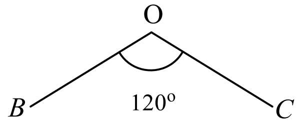
It is evident from the definition that $f(x)$ attains a minimum at $x = 1$ but it does not have a maximum or minimum at $x = 0, 2$



364 **(a)**

After time t the distance covered by X is $4t$ and Y is $3t$.

Let shortest distance between X and Y is A. Then , by cosine law



$$A^2 = (4t)^2 + (3t)^2 - (4t)(3t)2\cos 120°$$

$\Rightarrow \quad A^2 = 16t^2 + 9t^2 - 24t^2\left(-\frac{1}{2}\right) = 37t^2 \quad \ldots.(i)$

$\Rightarrow \quad A = \sqrt{37t}$

If $t = 1h$, then $A = \sqrt{37}km$

Now, differentiating Eq.(i)w. r. t. t,we get

$$2AA'' = 37(2t)$$

After $t = 1h$, we get

$$2\sqrt{37}A'' = 2(37)$$

$\Rightarrow \quad A'' = \sqrt{37} km/h$

365 **(a)**

$\because y = a(1 - \cos x)$

On differentiating w.r.t. x, we get

$y' = a \sin x \quad$ ...(i)

Put $y' = 0$ for maxima or minima

$\Rightarrow \quad \sin x = 0$

$\Rightarrow \quad x = 0, \pi$

Again differentiating w.r.t. x of Eq. (i) we get

$y'' = a \cos x \;\Rightarrow\; y''(0) = 0$ and $y''(\pi) = -a$

Hence, y is maximum when $x = \pi$

366 **(c)**

$x^4 - x^3 + x^2 = x^2(x^2 - x + 1)$

$$= x^2\left[\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}\right]$$

$\therefore \quad f(x) = e^{x^4 - x^3 + x^2}$ will attain minimum

At $\quad x = 0$

$\Rightarrow$ Minimum of $f(x) = 1$

367 **(d)**

To satisfy Rolle's theorem it should be continuous in [0, 1]

ie, RHL$= f(0)$

$$\Rightarrow \lim_{x \to 0^+} \frac{\log x}{x^{-\alpha}} = 0$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\alpha x^{-\alpha - 1}} = 0 \quad \text{(using L'Hospital's rule)}$$

$$\Rightarrow \lim_{x \to 0^+} \frac{1}{\alpha x^{-\alpha}} = 0$$

$$\Rightarrow \lim_{x \to 0^+} \frac{x^\alpha}{\alpha} = 0$$

Which shows $\alpha > 0$ otherwise it would be continuous also

When $\alpha > 0$, $f(x)$ is differentiating in (0, 1) and $f(1) = f(0) = 0$

Clearly, $\alpha > 0$, thus $\alpha = \frac{1}{2}$ is the possible answer

368 **(c)**

Let $y = f(x) = \log_e x, x = 4$ and $x + \Delta x = 4.01$

Then,

$$\frac{dy}{dx} = \frac{1}{x} \text{ and } \Delta x = 0.01 \Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$$

$$\Delta y = \frac{dy}{dx}\Delta x = \frac{1}{4} \times 0.01 = 0.0025$$

$\therefore \log_e 4.01 = y + \Delta y$

$= \log_e 4 + 0.0025$

$= 1.3868 + 0.0025$

$= 1.3893$

369 **(d)**

Given error in diameter$=\pm 0.04$

$\therefore$Error in radius, $\delta r = \pm 0.02$

$\therefore$percent error in the volume of sphere

$$= \frac{\delta V}{V} \times 100 = \frac{\delta\left(\frac{4}{3}\pi r^2\right)}{\frac{4}{3}\pi r^3} \times 100 = \frac{3\delta}{r} \times 100$$

$$= \frac{3 \times \pm 0.02}{10} \times 100 = \pm 0.6$$

370 **(a)**

For $x > 0$ or $x < 0$

$$f'(x) = \frac{a}{x} + 2bx + 1$$

$\because \quad f'(1) = 0 \quad \Rightarrow \quad a + 2b + 1 = 0 \quad$ ...(i)

And $f'(3) = 0 \quad \Rightarrow \quad \frac{a}{3} + 6b + 1 = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

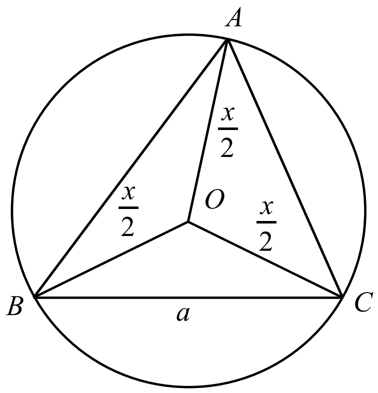
$a = -3/4, \qquad b = -1/8$

371 **(b)**

We have,

$\frac{x}{2} = \frac{a}{2 \sin A} \quad \left[\text{Using} : R = \frac{a}{2 \sin A}\right]$

$\Rightarrow a = x \sin A$



$$\Rightarrow \frac{da}{dt} = x \cos A \frac{dA}{dt}$$

$$\Rightarrow \frac{x}{2}\frac{dA}{dt} = x \cos A \frac{dA}{dt} \quad \left[\frac{da}{dt} = \frac{x}{2}\frac{dA}{dt} \text{(given)}\right]$$

$$\Rightarrow \cos A = \frac{1}{2} \Rightarrow A = \frac{\pi}{3}$$

372 **(b)**

Given curve is $y = 4xe^x$

$$\frac{dy}{dx} = 4e^x + 4xe^x$$

At $\left(-1, -\frac{4}{e}\right), \left(\frac{dy}{dx}\right)_{\left(-1, -\frac{4}{e}\right)} = 4e^{-1} + 4(-1)e^{-1} = 0$

$\therefore$ Equation of tangent is

$$\left(y + \frac{4}{e}\right) = 0(x + 1)$$

$$\Rightarrow \quad y = -\frac{4}{e}$$

373 **(d)**

Given, $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$

$\Rightarrow f'(x) = e^{-(x^2+1)^2} 2x - e^{-(x^2)^2} . 2x$

$= 2xe^{-(x^4+2x^2+1)}\{1 - e^{2x^2+1}\}$

Here, $e^{2x^{2+1}} > 1$

And $e^{-(x^4+2x^2 1)} > 0$for all x

For $f'(x) > 0, x < 0$

374 **(d)**

There is only one function in option (a), whose critical point $\frac{1}{2} \in (0,1)$ but in other parts critical point0 $\notin (0,1)$. Then, we can say that functions in options (b), (c) and (d) are continuous on [0, 1] and differentiable in (0, 1)

Now, for $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

Here, $Lf'\left(\frac{1}{2}\right) = -1$

And $Rf'\left(\frac{1}{2}\right) = 2\left(\frac{1}{2} - \frac{1}{2}\right)(-1) = 0$

$\therefore \ Lf'\left(\frac{1}{2}\right) \neq Rf'\left(\frac{1}{2}\right)$

$\Rightarrow f$ is non-differentiable at $x = \frac{1}{2} \in (0,1)$

$\therefore$ LMVT is NOT applicable to $f(x) in$ [0, 1]

375 **(a)**

Given, $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$

$\therefore \quad f(-x) = \frac{e^{-2x} - 1}{e^{-2x} + 1} = -f(x)$

$\therefore \quad f(x)$ is an odd function

Now, $f'(x) = \frac{4e^{2x}}{(1+e^{2x})^2} > 0, \forall x \in R$

$\Rightarrow \quad f(x)$ is an increasing function

376 **(d)**

The curves $xy = k$ and $x = y^2$ intersect at $P(k^{2/3}, k^{1/3})$

Now, $xy = k \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_p = -\frac{1}{k^{1/3}}$

$x = y^2 \Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow \left(\frac{dy}{dx}\right)_p = \frac{1}{2k^{1/3}}$

If given curves intersect orthogonally, then

$\left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1 \Rightarrow \frac{-1}{k^{1/3}} \times \frac{1}{2k^{1/3}} = -1$

$\Rightarrow 8k^2 = 1$

377 **(c)**

We have,

$x^{1/3} + y^{1/3} = a^{1/3}$

$\Rightarrow \frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-2/3} = -\left(\frac{y}{x}\right)^{2/3} \Rightarrow \left(\frac{dy}{dx}\right)_P = -1$

The equation of the tangent at P is

$y - \frac{a}{8} = -1\left(x - \frac{a}{8}\right) \Rightarrow x + y = \frac{a}{4}$

This cuts intercepts $\frac{a}{4}$ and $\frac{a}{4}$ with each of the coordinate axes

$\therefore \frac{a^2}{16} + \frac{a^2}{16} = 2 \Rightarrow a^2 = 16 \Rightarrow a = 4$

378 **(c)**

Slope of the normal$= -\frac{1}{\left(\frac{dy}{dx}\right)}$

This is parallel to x-axis

$\Rightarrow \quad -\frac{1}{\left(\frac{dy}{dx}\right)} = 0 \quad \Rightarrow \frac{dx}{dy} = 0$

379 **(c)**

Given equation is $a + bv^2 = x^2$

On differentiating with respect to t, we get

$0 + b\left(2v\frac{dv}{dt}\right) = 2x\frac{dx}{dt}$

$\Rightarrow \quad vb\frac{dv}{dt} = x\frac{dx}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{x}{vb} . \frac{dx}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{x}{b}\left(\because \ \frac{dx}{dt} = v\right)$

380 **(d)**

Surface area, S$=2\pi rh + \pi r^2$ ...(i)

And $V = \pi r^2 h$ ...(ii)

From Eq. (i), $h = \frac{S - \pi r^2}{2\pi r}$

$\therefore$ From Eq. (ii), $V = \frac{\text{r}}{2}(\text{S} - \pi\text{r}^2)$

$\Rightarrow \quad \frac{dV}{dr} = \frac{1}{2}(S - 3\pi r^2) = 0$ [say]

$\Rightarrow \quad S - 3\pi r^2 = 0 \Rightarrow S = 3\pi r^2$

On putting the value of S in Eq.(i),we get

$3\pi r^2 = 2\pi rh + +\pi r^2 \Rightarrow r = h$

381 **(d)**

Given curve is

$x^2 + y^2 = a^2$

$\therefore \quad 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$

Given , $\frac{dy}{dx}$=0

$\Rightarrow -\frac{x}{y}$=0

$\Rightarrow \ x = 0$

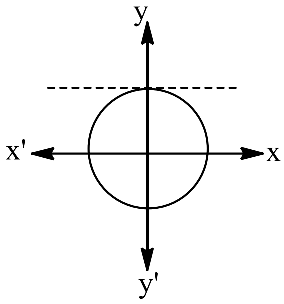
$\therefore \quad y = \pm a$

But $\quad y \geq 0$given

$\therefore \qquad y = a$

$\Rightarrow$ Required point is $(0, a)$

Alternate It is clear from the graph, tangent is parallel to x-axis for $y > 0$ is at (0,a)



383 **(d)**

We have,

$$f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\Rightarrow f'(x) = \frac{2(e^{2x} + 1)e^{2x} - 2(e^{2x} - 1)e^{2x}}{(e^{2x} + 1)^2}$$

$$\Rightarrow f'(x) = \frac{4e^{2x}}{(e^{2x} + 1)^2} > 0 \text{ for all } x \in R$$

So, $f(x)$ is an increasing function on R

384 **(b)**

We have, $\quad f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$

$\Rightarrow \qquad f'(x) = 1 - e^x$

For $f(x)$ to be increasing, we must have

$f'(x) > 0 \quad \Rightarrow \quad 1 - e^x > 0$

$\Rightarrow \quad e^x < 1$

$\Rightarrow \quad x < 0 \quad \Rightarrow \quad x \in (-\infty, 0)$

385 **(c)**

Let x and y be a two parts. Then,

$x + y = 12$

Let $\; P = x^2y^4$

$\Rightarrow \quad \sqrt{p} = xy^2 = L \qquad$ [say]

$\Rightarrow \quad L = x(12 - x)^2$

$$\Rightarrow \quad \frac{dL}{dx} = (12 - x)^2 - 2x(12 - x)$$

For maxima, pot $\frac{dL}{dx} = 0$

$\Rightarrow \quad (12 - x)[12 - 3x] = 0$

$\Rightarrow \quad x = 12, x = 4$

At $x = 4$, it is maximum $(\because x \neq 12)$

Hence, it is maximum, when the parts are 4, 8

386 **(d)**

Given point is $x = a + bt - ct^2$

Acceleration in x direction $= \frac{d^2x}{dt^2} = -2c$

and acceleration in y direction $= \frac{d^2y}{dt^2} = 2b$

$\therefore$ Resulatant acceleration

$$= \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = \sqrt{(-2c)^2 + (2b)^2}$$

$$= 2\sqrt{b^2 + c^2}$$

387 **(b)**

Since $f(x) = \frac{a \sin x + b \cos x}{c \sin x + d \cos x}$ is decreasing for all x

$\therefore f'(x) < 0$ for all x

$$\Rightarrow \frac{ad - bc}{(c \sin x + d \cos x)^2} < 0 \text{ for all } x$$

$\Rightarrow ad - bc < 0$

388 **(b)**

Clearly, $f(x) = \log \sin x$ is continuous on $[\pi/6, 5\pi/6]$ and differentiable on $(\pi/6, 5\pi/6)$.

Therefore, there exists $c \in (\pi/6, 5\pi/6)$ such that

$$f'(c) = \frac{f\left(\frac{5\pi}{6}\right) - f\left(\frac{\pi}{6}\right)}{\frac{5\pi}{6} - \frac{\pi}{6}}$$

$$\Rightarrow \cot c = \frac{-\log_e 2 + \log_e 2}{\frac{2\pi}{3}}$$

$$\Rightarrow \cot c = 0 \Rightarrow c = \frac{\pi}{2} \in (\pi/6, 5\pi/6)$$

389 **(d)**

Let $\quad f(x) = 2x^3 - 3x^2 - 12 + 5$

$\therefore \qquad f'(x) = 6x^2 - 6x - 12$

For maxima or minima, put $f'(x) = 0$

$\therefore \qquad 6x^2 - 6x - 12 = 0$

$\Rightarrow \qquad x = -1, 2$

Now, $\quad f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$, maxima

$\therefore \quad f(x)$ is maximum at $x = -1$

At $\; x = -1, \; f(x) = -2 - 3 + 12 + 5 = 12$

At $\; x = 4, f(x) = 128 - 48 - 48 + 5 = 37$

Hence, largest value of $f(x)$ in the given interval is 4

390 **(b)**

Let the number be x, then $f(x) = \frac{x}{x^2+16}$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{(x^2 + 16).1 - x(2x)}{(x^2 + 16)^2}$$

$$= \frac{x^2+16-2x^2}{(x^2+16)^2} = \frac{16-x^2}{(x^2+16)^2} \quad \ldots(\text{i})$$

Put $f'(x) = 0$ for maxima or minima

$f'(x) = 0 \quad \Rightarrow \quad 16 - x^2 = 0 \quad \Rightarrow \quad x = 4, -4$

Again, on differentiating w.r.t. x, we get

$f''(x)$

$$= \frac{(x^2 + 16)^2 - (-2x) - (16 - x^2)2(x^2 + 16)2x}{(x^2 + 16)^4}$$

At $x = 4, f''(x) < 0$

$\therefore$ $f(x)$ is maximum at $x = 4$

And at $x = -4, f''(x) > 0$ $f(x)$ is minimum

$\therefore$ Least value of $f(x) = \frac{-4}{16+16} = -\frac{1}{8}$

391 **(a)**

Given ,$s = \sqrt{t}$

$\therefore \frac{ds}{dt} = \frac{1}{2\sqrt{t}} \quad \Rightarrow v = \frac{1}{2\sqrt{t}}$

$\Rightarrow \frac{dv}{dt} = -\frac{1}{2.2t^{3/2}} \Rightarrow a = \frac{-2}{(2\sqrt{t})^3}$

$\Rightarrow \quad a = -2v^3 \quad \Rightarrow a \propto v^3$

392 **(a)**

Let y= $\sin^3 x + \cos^3 x$

$\frac{dy}{dx} = 3\sin^2 x \cos x - 3\cos^2 x \sin x$

$= 3 \sin x \cos x \,(\sin x - \cos x)$

Put, $\frac{dy}{dx} = 0, \quad 3\sin x \cos x\,(\sin x - \cos x) = 0$

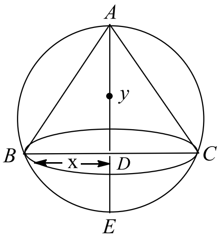
$\Rightarrow \quad x = 0\ \frac{\pi}{2}$ or $\frac{\pi}{4}$

Now, y has its maximum value at $x = 0$ or $\frac{\pi}{2}$ and

$y_{\max} = 1$

393 **(a)**

Let the diameter of the sphere is $AE = 2r$



Let the radius of cone is x and height is y

$\therefore AD = y$

Since, $BD^2 = AD.DE$

$\Rightarrow \ x^2 = y(2r - y)$ ...(i)

Volume of cone,

$V = \frac{1}{3}\pi x^2 y = \frac{1}{3}\pi y(2r - y)y$

$= \frac{1}{3}\pi(2ry^2 - y^3)$

On differentiating w.r.t. y, we get

$\frac{dV}{dy} = \frac{1}{3}\pi(4ry - 3y^2)$

For maxima and minima, put $\frac{dV}{dy} = 0$

$\Rightarrow \frac{1}{3}\pi\,(4ry - 3y^2) = 0$

$\Rightarrow \ y(4r - 3y) = 0$

$\Rightarrow \ \ y = \frac{4}{3}r, 0$

Again on differentiating w.r.t. y, we get

$\frac{d^2V}{dy^2} = \frac{1}{3}\pi(4r - 6y)$

At $y = \frac{4}{3}r$, $\frac{d^2V}{dy^2} = \frac{1}{3}\pi(4r - 8r) = -ve$

$\therefore$ Volume of cone is maximum at $y = \frac{4}{3}r$.

Now, required ratio $= \frac{\text{height of cone}}{\text{Diameter of sphere}}$

$= \frac{y}{2r} = \frac{\frac{4r}{3}}{2r} = \frac{2}{3}$

394 **(c)**

$\because \ f(x) = \frac{x}{4 + x + x^2}$

On differentiating w.r.t. x, we get

$f'(x) = \frac{4 + x + x^2 - x(1 + 2x)}{(4 + x + x^2)^2}$

For maximum, put $f'(x) = 0 \ \Rightarrow \ \frac{4-x^2}{(4+x+x^2)^2} = 0$

$\Rightarrow \ x = 2, -2$

Both the values of x are not in the interval $[-1, 1]$

$\therefore \ f(-1) = \frac{-1}{4 - 1 + 1} = \frac{-1}{4}$

$f(1) = \frac{1}{4+1+1} = \frac{1}{6}$ (maximum)

395 **(b)**

Given, $2y^4 = x^5$

$\Rightarrow \quad 8y^3 \frac{dy}{dx} = 5x^4$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(2,2)} = \frac{5(2)^4}{8(2)^3} = \frac{5}{4}$

$\therefore$ Length of subtangent= $\frac{y}{dy/dx} = \frac{2}{5/4} = \frac{8}{5}$

397 **(a)**

$f(x) = \int_0^x \sin t \, dt, x \geq 0$

$\Rightarrow f'(x) = \sin x$

Now, $f'(x) > 0 \text{in} 0 < x < \frac{\pi}{2}$

$\therefore f(x)$ is increasing in 0< $x < \frac{\pi}{2}$.

398 **(a)**

Given , $x = 3 + 8t - 4t^2$,

On differentiating, w. r. t. x, we get

$v = \frac{dx}{dt} = 8 - 8t$

At $t = 1, v = 8 - 8 = 0$ unit

399 **(d)**

Curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ intersect at $P(8, 6)$ and $Q(8, -6)$

Now, $y^2 = 4x + 4 \Rightarrow 2y\frac{dy}{dx} = 4 \Rightarrow \left(\frac{dy}{dx}\right)_{C_1} \frac{2}{y}$

and $y^2 = 36(9-x) \Rightarrow 2y\frac{dy}{dx} = -36 \Rightarrow \left(\frac{dy}{dx}\right)_{c_2} = -\frac{18}{y}$

$\therefore \left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -\frac{36}{y^2}$

Clearly, this product is -1 at $P(8,6)$ and $Q(8,-6)$

Hence, given curves intersect at right angle

400 **(b)**

Let m be the slope of the tangent to the curve $y = e^x \cos x$. Then,

$m = \frac{dy}{dx} = e^x(\cos x - \sin x)$

$\Rightarrow \frac{dm}{dx} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$

$= -2e^x \sin x$

and,

$\frac{d^2m}{dx^2} = -2e^x(\sin x + \cos x)$

$\therefore \frac{dm}{dx} = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

Clearly, $\frac{d^2m}{dx^2} > 0$ for $x = \pi$

Thus, m is maximum at $x = \pi$. Hence, $a = \pi$

401 **(a)**

Since $f(x)$ is decreasing in the interval $(-2,-1)$

$\therefore f'(x) < 0$ for all $x \in (-2,-1)$

$\Rightarrow 6x^2 + 18x + \lambda < 0$ for all $x \in (-2,-1)$

$\Rightarrow x = -2$ and $x = -1$ must lie between the roots of the polynomial $6x^2 + 18x + \lambda$

But, $(-2,-1)$ is the largest possible interval in which $g(x) < 0$. Therefore, $x = -2$ and $x = -1$ are the roots of polynomial $g(x)$

$\therefore$ Product of the roots $= (-2) \times (-1)$

$\Rightarrow \frac{\lambda}{6} = 2 \Rightarrow \lambda = 12$

402 **(a)**

$f(x) = \begin{cases} |x^3 + x^2 + 3x + \sin x|\left(3 + \sin\left(\frac{1}{x}\right)\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Let $g(x) = x^3 + x^2 + 3x + \sin x$

$g'(x) = 3x^2 + 2x + 3 + \cos x$

$= 3\left(x^2 + \frac{2x}{3} + 1\right) + \cos x$

$= 3\left\{\left(x + \frac{1}{3}\right)^2 + \frac{8}{9}\right\} + \cos x > 0$

And $2 < 3 + \sin\left(\frac{1}{x}\right) < 4$

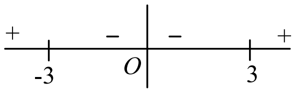
Hence, minimum value of $f(x) is\ 0\ at\ x = 0$

Hence, number of points=1

403 **(b)**

Given, $f(x) = \frac{x}{3} + \frac{3}{x} \Rightarrow f''(x) = \frac{1}{3} - \frac{3}{x^2}$

Put $f''(x) = 0 \Rightarrow x = \pm 3$



Thus, $f(x)$is decreasing in the interval $(-3,3)$.

404 **(c)**

Given, $t = as^2 + bs + c$

$\Rightarrow 1 = 2as\frac{ds}{dt} + b\frac{ds}{dt}$ [differentiating]

$\Rightarrow 1 = 2asv + bv$ ...(i)

$\Rightarrow 0 = 2a\frac{ds}{dt}v + 2as\frac{dv}{dt} + b\frac{dv}{dt}$ [differentiating]

$\Rightarrow \frac{dv}{dt}(2as + b) = -2av^2$

$\Rightarrow \frac{dv}{dt}\left(\frac{1}{v}\right) = -2av^2$ [from Eq.(i)]

$\Rightarrow \frac{dv}{dt} = -2av^3$

405 **(c)**

Let $f(x) = x^2 - x + 1$

$\Rightarrow f'(x) = 2x - 1$

$\Rightarrow f'(0) = -1$

And $f'(1) = 1$

Thus, function is neither increasing nor decreasing.

407 **(d)**

Let area, $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} 2\pi r \frac{dr}{dt}$

$\therefore \left.\frac{dA}{dt}\right|_{r=5} = 10\pi \times 0.1 = \pi \frac{cm^2}{s}$ $[\because \frac{dr}{dt} = 0.1 cm/s]$

408 **(b)**

Length of subtangent $= y\frac{dx}{dy}$

and length of subnormal$= y\frac{dy}{dx}$

$\therefore$ Product $= y^2$

$\Rightarrow$ Required product is the square of the ordinate

409 **(c)**

Let $y = x^{1/x}$

On taking log on both sides, we get

$\log y = \frac{1}{x}\log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2} = \frac{1 - \log x}{x}$

$\Rightarrow \frac{dy}{dx} = x^{1/x}\left(\frac{1 - \log x}{x}\right)$

Now, $x^{1/x} > 0$ for all positive values of x

And $\frac{1-\log x}{x^2} > 0$ in $(1, e)$

And $\frac{1-\log x}{x^2} < 0$in (e, ∞)

$\therefore$ $f(x)$ is increasing in $(1, e)$ and decreasing in (e, ∞)

410 **(c)**

We have,

$f(x) = \frac{ax+b}{(x-1)(x-4)}$ ...(i)

$\Rightarrow f'(x) = \dfrac{a(x-1)(x-4) - (ax+b)(2x-5)}{(x-1)^2(x-4)^2}$

It is given that $f(x)$ has an extremum at $P(2, -1)$. Therefore,

$-1 = \dfrac{2a+b}{(2-1)(2-4)}$ and $0 = \dfrac{-2a+(2a+b)}{(2-1)^2(2-4)^2}$

$\Rightarrow 2a + b = 2$ and $b = 0 \Rightarrow a = 1, b = 0$

411 **(c)**

They will encounter if

$10 + 6t = 3 + t^2 \quad \Rightarrow \quad t^2 - 6t - 7 = 0 \quad \Rightarrow \quad t = 7$

At $t = 7s$, moving in a first point

$v_1 = \frac{d}{dt}(10 + 6t) = 6$ cm/s

At $t = 7s$, moving in a second point

$v_2 = \dfrac{d}{dt}(3 + t^2)$

$= 2t = 2 \times 7 = 14$ cm/s

$\therefore$ Resultant velocity $= v_2 - v_1 = 14 - 6 = 8$ cm/s

412 **(d)**

Given function

$f(x) = e^{x^2} + e^{-x^2}$

$g(x) = xe^{x2} + e^{-x2}$ and

$h(x) = x^2e^{x^2} + e^{-x^2}$ Are strictly increasing on [0, 1]. Hence, at $x = 1$, the given function attains absolute maximum all equal to $e + \frac{1}{e}$

413 **(c)**

Let (h, k) be a point of contact of the tangents drawn from the origin to $y = \sin x$. Then, (h, k) lies on $y = \sin x$

$\therefore k = \sin h$ ...(i)

Now,

$y = \sin x, \Rightarrow \dfrac{dy}{dx} = \cos x \Rightarrow \left(\dfrac{dy}{dx}\right)_{(h,k)} = \cos h$

The equation of the tangent at (h, k) is

$y - k = (\cos h)(x - h)$

This passes through (0, 0)

$\therefore -k = -h\cos h \Rightarrow \frac{k}{h} = \cos h$ ...(ii)

From (i) and (ii), we get

$k^2 + \frac{k^2}{h^2} = 1$ [Squaring and adding]

$\Rightarrow h^2 - k^2 = k^2h^2$

Hence, the locus of (h, k) is $x^2 - y^2 = x^2y^2$

414 **(d)**

Given, $y = x^2$

$\dfrac{dy}{dx} = 2x$

$m_1 = \left(\dfrac{dy}{dx}\right)_{(1,1)}$

and $y^2 = x$

$\Rightarrow 2y\frac{dy}{dx} = 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y}$

$m_2 = \left(\dfrac{dy}{dx}\right)_{(1,1)} = \dfrac{1}{2}$

If θ be the angle between the two curves

$\therefore \tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 + m_2}\right| = \left|\dfrac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}}\right| = \dfrac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{3}{4}\right)$

415 **(c)**

Let two numbers be x and y, then

$x + y = 6$ ...(i)

Let $z = \frac{1}{x} + \frac{1}{y}$ ...(ii)

$\Rightarrow z = \dfrac{1}{x} + \dfrac{1}{6-x}$

On differentiating w. r. t. x, we get

$\dfrac{dz}{dx} = -\dfrac{1}{x^2} + \dfrac{1}{(6-x)^2}$

Put $\frac{dz}{dx} = 0 \Rightarrow -\frac{1}{x^2} + \frac{1}{(6-x)^2} = 0$

$\therefore -(6-x)^2 + x^2 = 0$

$\Rightarrow 12x - 36 = 0$

$\Rightarrow x = 3$

Now, $\frac{d^2z}{dx^2} = \frac{2}{x^3} + \frac{2}{(6-x)^3}$

At $x = 3$, $\frac{d^2z}{dx^2} > 0$, minimum

Hence, minimum value at $x = 3$ is

$z = \dfrac{1}{3} + \dfrac{1}{3} = \dfrac{2}{3}$

416 **(b)**

$f(x) = x^4 - 4x^3 + 4x^2 + 40$

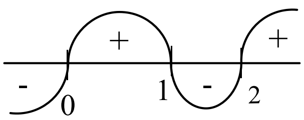
$f''(x) = 4x^3 - 12x^2 + 8x$

For monoatonic decreasing $f''(x) < 0$

$\Rightarrow x(4x^2) - 12x + 8 < 0$

$x(x^2 - 3x + 2) < 0$

$\Rightarrow x(x-1)(x-2) < 0$



$\Rightarrow \quad x \in (-\infty, 0) \cup (1, 2)$

417 **(a)**

Given, $\quad y^2 = x \quad$ ...(i)

$\Rightarrow \quad 2y\frac{du}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$ =slope

Also, slope $\tan 45° = 1$

$\Rightarrow \quad \frac{1}{2y} = 1$

$\Rightarrow \quad y = \frac{1}{2}$

From Eq. (i), if $y = \frac{1}{2}$, then $x = \frac{1}{4}$

418 **(c)**

We have,

$f(x) = \cos|x| - 2ax + b$

$\Rightarrow f(x) = \cos x - 2ax + b \quad [\because \cos(-x) = \cos x]$

$\Rightarrow f'(x) = -\sin x - 2a$

Now,

$f(x)$ is increasing for all $x \in R$

$\Rightarrow f'(x) > 0$ for all $x \in R$

$\Rightarrow f'\left(\frac{\pi}{2}\right) > 0 \Rightarrow -\sin\frac{\pi}{2} - 2a > 0 \Rightarrow -1 - 2a > 0$

$\Rightarrow a < -1/2$

420 **(a)**

Let r be the radius of the cylinder. Let V_1 and V_2 be the volumes of the sphere and cylinder respectively. Then,

$V_1 = \frac{4}{3}\pi r^3$ and $V_2 = \pi r^3 \quad [\because h = r]$

$\Rightarrow \frac{dV_1}{dt} = 4\pi r^2\frac{dr}{dt}$ and $\frac{dV_2}{dt} = 3\pi r^2\frac{dr}{dt} \Rightarrow \frac{\frac{dV_1}{dt}}{\frac{dV_2}{dt}} = 4 : 3$

422 **(b)**

Let $\quad y = 4e^{2x} + 9e^{-2x} \quad \Rightarrow \quad \frac{dy}{dx} = 8e^{2x} - 18e^{-2x}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 16e^{2x} + 36e^{-2x}$

For minimum, put $\frac{dy}{dx} = 0 \quad \Rightarrow \quad 8e^{2x} - 18e^{-2x} = 0$

$\Rightarrow \quad e^{4x} = \frac{9}{4} \quad \Rightarrow \quad x = \frac{1}{4}\log\frac{9}{4}$

At $\quad x = \frac{1}{4}\log\frac{9}{4}, \quad \frac{d^2y}{dx^2} > 0$

$\therefore$ Minimum value at $x = \frac{1}{4}\log\frac{9}{4}$ is

$y = 4e^{2\left(\frac{1}{4}\log\frac{9}{4}\right)} + 9e^{-2\left(\frac{1}{4}\log\frac{9}{4}\right)}$

$= 4e^{\log\left(\frac{9}{4}\right)^{1/2}} + 9e^{\log\left(\frac{9}{4}\right)^{-1/2}}$

$= 4.\frac{3}{2} + 9.\frac{2}{3} = 12$

423 **(d)**

$\because \quad y = x^3 - 3x^2 - 9x + 5$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 3x^2 - 6x - 9$

Since, tangent is parallel to x-axis

$\therefore \quad \frac{dy}{dx} = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$

$\Rightarrow \quad (x+1)(x-3) = 0 \quad \Rightarrow \quad x = -1, 3$

424 **(a)**

$f'(x) = 4x + 16x^3 + \ldots + 2^{10}.20.x^{19}$

$= 4x(1 + 4x^2 + \ldots + 5.2^{10}x^{18}) \quad$ ...(i)

$\Rightarrow \quad f''(x) = 4 + 48x^2 + \cdots + 2^{10}380.x^{18} > 0$

$\therefore f(x)$ is minimum at $x = 0$. [from Eq. (i)]

425 **(d)**

We have,

$y = x^2e^{-x} \Rightarrow \frac{dy}{dx} = 2\,x\,e^{-x} - x^2e^{-x}$

For y to be increasing, we must have

$\frac{dy}{dx} > 0 \Rightarrow e^{-x}(-x^2 + 2\,x) > 0 \Rightarrow -x^2 + 2x > 0$

$\Rightarrow x \in (0, 2)$

Hence, y is increasing on $(0, 2)$

426 **(c)**

Since, $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has maximum at $x = \frac{\pi}{3}$

$\therefore \quad f'\left(\frac{\pi}{3}\right) = 0 \quad \Rightarrow \quad a\cos\frac{\pi}{3} + \cos\pi = 0$

$\Rightarrow \quad a = 2$

427 **(c)**

Given, $\quad s = t^3 - 3t^2$

$\Rightarrow \frac{ds}{dt} = 3t^2 - 6t \quad$ ...(i)

$\Rightarrow \frac{d^2}{dt^2} = 6t - 6 = 0 \quad$ [given]

$\Rightarrow \quad t = 1$

On putting value of $t = 1$ $in\ Eq.(i), we\ get$

$\frac{ds}{dt} = 3 \times 1 - 6 \times 1 = -3m/s$

428 **(b)**

Given,

$f(x) = (x-1)^2 + 3, \quad x \in [-3, 1]$

$\Rightarrow \quad f'(x) = 2(x-1)$

For maxima and minima, put $f'(x) = 0 \Rightarrow \quad x = 1$

Now, $\quad f''(x) = 2$, minima $\forall\, x \in R$

At $\quad x = 1, \quad f(1) = (1-1)^2 + 3 = 3$

At $x = -3, \quad f(-3) = (-3-1)^2 + 3 = 19$

Here, $\quad m = 3 \quad$ and $\quad M = 19$

429 **(d)**

We have, $f(x) = x^{1/x}$

Clearly, $f(x)$ is defined for all $x \in (0, \infty)$

Now,

$f(x) = x^{1/x}$

$\Rightarrow f(x) = e^{(1/x)\log_e x}$

$\Rightarrow f'(x) = x^{1/x}\left(-\frac{1}{x^2}\log_e x + \frac{1}{x^2}\right)$

$= \frac{x^{1/x}(1-\log_e x)}{x^2}$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow \frac{x^{1/x}(1-\log_e x)}{x^2} > 0$

$\Rightarrow 1 - \log_e x > 0$

$\Rightarrow \log_e x < 1 \quad [\because x^{1/x}$ and x^2 are positive$]$

$\Rightarrow x < e \Rightarrow x \in (0, e)$

430 **(c)**

As the curve crosses y-axis *ie*, $x = 0$

$\therefore \quad y = 4e^{-0} \Rightarrow y = 4$

Given, $\quad y = 4e^{-\frac{x}{4}}$

$\Rightarrow \frac{dy}{dx} = 4e^{-\frac{x}{4}}\left(-\frac{1}{4}\right) = -e^{-\frac{x}{4}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,4)} = -e^{-0} = -1$

$\therefore$ Equation of tangent at (0,4) is

$y - 4 = -1(x - 0)$

$\Rightarrow \quad x + y = 4$

431 **(a)**

Given, $f(x) = x^2 \log x$

On differentiating w.r.t. x, we get

$f'(x) = (2\log x + 1)x$

For a maximum put $f'(x) = 0$

$\Rightarrow \quad (2\log x + 1)x = 0 \quad \Rightarrow \quad x = e^{-1/2}, 0$

$\because \quad 0 < e^{-1/2} < 1$

None of these critical points lies in the interval $[1, e]$

So, we only compute the value of $f(x)$ at the end points 1 and e

We have, $f(1) = 0$, $f(e) = e^2$

Hence, greatest value of $f(x) = e^2$

432 **(b)**

Let $y = k$ be a line parallel to x-axis

It crosses the curve $y = \sqrt{x}$ at $P(k^2, k)$

Now, $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)_P = \frac{1}{2k}$

It is given that $y = k$ crosses the curve $y = \sqrt{x}$ at an angle of 45°

$\therefore \left(\frac{dy}{dx}\right)_P = \tan 45° \Rightarrow \frac{1}{2k} = 1 \Rightarrow k = \frac{1}{2}$

Hence, the required line is $y = \frac{1}{2}$

433 **(a)**

Equation of given curves are

$y = \sin x \quad$ ...(i)

and $\; y = \cos x \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = \frac{\pi}{4}$

$\therefore$ Point of intersection of curves is $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

For $y = \sin x, \frac{dy}{dx} = \cos x$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\pi/4} = \frac{1}{\sqrt{2}} = m_1 \qquad$ (say)

For $y = \cos x \quad \Rightarrow \frac{dy}{dx} = -\sin x$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\pi/4} = -\frac{1}{\sqrt{2}} = m_2 \qquad$ (say)

$\therefore \; \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}} = \frac{\frac{2}{\sqrt{2}}}{\frac{1}{2}}$

$\Rightarrow \; \tan\theta = 2\sqrt{2} \; \Rightarrow \; \theta = \tan^{-1}(2\sqrt{2})$

434 **(c)**

Let $t = f(v, then)$

$\frac{dt}{dv} = f'(v) \Rightarrow \frac{dv}{dt} = \frac{1}{f'(v)}$

$\Rightarrow \quad a = \frac{1}{f'(v)} \Rightarrow af'(v) = 1$

$\Rightarrow \quad af''(v)\frac{dv}{dt} + \frac{da}{dt}f'(v) = 0$

$\Rightarrow \quad a^2 f''(v) + \frac{da}{dt} \times \frac{1}{a} = 0 \Rightarrow \frac{da}{dt} = -a^3\frac{d^2t}{dv^2}$

435 **(a)**

Let $f(x) = 27^{\cos 2x} \times 81^{\sin 2x}$. Then,

$f(x) = 3^{3\cos 2x + 4\sin 2x}$

Clearly, $f(x)$ will be minimum when $3\cos\; 2x + 4\sin 2x$ is minimum

We know that

$-\sqrt{a^2 + b^2} \le a\cos x + b\sin x \le \sqrt{a^2 + b^2}$

$\therefore -5 \le 3\cos 2x + 4\sin 2x \le 5$

$\therefore$ The minimum value of $f(x)$ is $3^{-5} = \frac{1}{243}$

436 **(a)**

Let $Z = px + qy$. Then,

$Z = px + \frac{qr^2}{x} \qquad [\because xy = r^2]$

$\Rightarrow \frac{dZ}{dx} = p - \frac{qr^2}{x^2}$

For maximum or minimum, we must have

$\frac{dZ}{dx} = 0 \Rightarrow x = \pm\sqrt{\frac{q\,r^2}{p}}$

For $x=\sqrt{\frac{qr^2}{p}}$, we have

$$\frac{d^2Z}{dx^2}=\frac{2qr^2}{x^3}>0$$

Hence, Z is minimum for $x=\sqrt{\frac{qr^2}{p}}$ with the minimum value given by

$$Z=p\sqrt{\frac{qr^2}{p}}+\frac{qr^2}{\sqrt{\frac{qr^2}{p}}}=2r\sqrt{pq}$$

437 **(c)**

We have,

$f'(x)=(x-a)^{2n}(x-b)^{2p+1}$

$\therefore f'(x)=0\Rightarrow x=a,b$

When, $x=a-h$, we have

$f'(x)=h^{2n}(a+h-b)^{2p+1}$

When $x=a+h$, we have

$f'(x)=h^{2n}(a+h-b)^{2p+1}$

We observe that as x passes through $a, f'(x)$ does not change sign

Hence, there is neither a maximum nor a minimum at $x=a$

438 **(b)**

We have,

$f(x)=(x-4)(x-5)(x-6)(x-7)$

Clearly, $f(4)=f(5)=f(6)=f(7)=0$

By Rolle's theorem, there exist $\alpha_1\in(4,5),\alpha_2\in(5,6),\alpha_3\in(6,7)$ such that

$f'(\alpha_i)=0, i=1,2,3$

Since $f'(x)$ is a cubic polynomial. Therefore, $\alpha_1,\alpha_2,\alpha_3$ are the only roots of $f'(x)=0$

439 **(a)**

We have,

$f(x)=2e^x-ae^{-x}+(2a+1)x-3$

$\Rightarrow f'(x)=2e^x+ae^{-x}+(2a+1)$

For $f(x)$ to be increasing on R, we must have

$f'(x)>0$ for all $x\in R$

$\Rightarrow 2e^x+ae^{-x}+(2a+1)>0$ for all $x\in R$

$\Rightarrow 2(e^x)^2+(2a+1)e^x+a>0$ for all $x\in R$

$\Rightarrow 2y^2+(2a+1)y+a>0$ for all $y=e^x>0$

Thus, the vertex of the parabola given by $y=2x^2(2a+1)x+a$ must be on the left side of the origin and the ordinate at $x=0$ must be positive

$$\therefore -\left(\frac{2a+1}{4}\right)<0 \text{ and } a>0\ \left[\text{Using:} \frac{-b}{2a}>0 \text{ and } f(0)>0\right]$$

$\Rightarrow a>-\frac{1}{2}$ and $a>0\Rightarrow a>0\Rightarrow a\in(0,\infty)$

440 **(c)**

Given curve is $x=at^2+bt+c$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt}=2at+b$$

Again on differentiating, we get $\frac{d^2x}{dt^2}=2a$

441 **(d)**

$$\because\ f(x)=\frac{\lambda\sin x+6\cos x}{2\sin x+3\cos x}$$

On differentiating w.r.t. x, we get

$$f'(x)=\frac{\begin{bmatrix}(2\sin x+3\cos x)(\lambda\cos x-6\sin x)\\-(\lambda\sin x+6\cos x)(2\cos x-3\sin x)\end{bmatrix}}{(2\sin x+3\cos x)^2}$$

The function is monotonic increasing, if $f'(x)>0$

$\Rightarrow 3\lambda(\sin^2x+\cos^2x)-12(\sin^2x+\cos^2x)>0$

$\Rightarrow 3\lambda-12>0\quad(\because \sin^2x+\cos^2x=1)$

$\Rightarrow\ \lambda>4$

442 **(d)**

Let $A=x^2y=x^2(8-x)$ [given, $x+y=8$]

$\Rightarrow\ A=8x^2-x^3$

$$\Rightarrow\ \frac{dA}{dx}=16x-3x^2$$

For maxima or minima, put $\frac{dA}{dx}=0$

$$\Rightarrow\ 16x-3x^2=0\ \Rightarrow\ x=0,\frac{16}{3}$$

Now, $\frac{d^2A}{dx^2}=16-6x$

$\left(\frac{d^2A}{dx^2}\right)_{x=\frac{16}{3}}=16-32<0$, maxima

$\therefore$ Maximum value$=8\left(\frac{16}{3}\right)^2-\left(\frac{16}{3}\right)^3=\frac{2048}{27}$

443 **(a)**

Since (2, 3) lies on $y^2=px^3+q$. Therefore,

$9=8p+q$ ...(i)

Now,

$y^2=px^3+q$

$$\Rightarrow 2y\frac{dy}{dx}=3px^2$$

$$\Rightarrow \frac{dy}{dx}=\frac{3px^2}{2y}\Rightarrow\left(\frac{dy}{dx}\right)_{(2,3)}=\frac{12p}{6}=2p$$

Since $y=4x-5$ is tangent to $y^2=px^3+q$ at (2, 3)

$$\therefore\left(\frac{dy}{dx}\right)_{(2,3)}=\text{Slope of the line } y=4x-5$$

$\Rightarrow 2p=4\Rightarrow p=2$

Putting $p=2$ in (i), we get $q=-7$

444 **(c)**

We have, $f(x)=\tan^{-1}x-x$

$\Rightarrow f'(x)=\frac{1}{1+x^2}-1=\frac{-x^2}{1+x^2}<0$ for all $x\in R-\{0\}$

Hence, $f(x)$ is decreasing on $R-\{0\}$

445 **(d)**

Given $f\,(x)=x^3-6x^2-36x+7$

$\Rightarrow f'(x)=3x^2-12x=36$

For $f(x)$to be increasing, $f'(x)>0$

$\Rightarrow 3(x^2-4x-12)>0$

$\Rightarrow (x-6)(x+2)>0$

$\Rightarrow x>6$

And $x<-2$

446 **(c)**

Given curve is

$x=a(t+\sin t), y=a(1-\cos t)$

$\Rightarrow \dfrac{dx}{dt}=a(1+\cos t), \dfrac{dy}{dt}=a(\sin t)$

$\therefore \dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{a(\sin t)}{a(1+\cos t)}$

$=\dfrac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\cos^2\frac{t}{2}}=\tan\dfrac{t}{2}$

Length of the normal$=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$=a(1-\cos t)\sqrt{1+\tan^2\left(\dfrac{t}{2}\right)}$

$=a(1-\cos t)\sec\left(\dfrac{t}{2}\right)$

$=2a\sin^2\left(\dfrac{t}{2}\right)\sec\left(\dfrac{t}{2}\right)$

$=2a\sin\left(\dfrac{t}{2}\right)\tan\left(\dfrac{t}{2}\right)$

447 **(d)**

If $f(x)=(a+2)x^3-3\,ax^2+9\,ax-1$ decreases monotonically for all $x\in R$, then

$f'(x)\le 0$ for all $x\in R$

$\Rightarrow 3(a+2)x^2-6\,ax+9\,a\le 0$ for all $x\in R$

$\Rightarrow (a+2)x^2-2\,ax+3\,a\le 0$ for all $x\in R$

$\Rightarrow a+2<0$ and Discriminant ≤ 0

$\Rightarrow a<-2$ and $4\,a^2-12\,a(a+2)\le 0$

$\Rightarrow a<-2$ and $-8\,a^2-24\,a\le 0$

$\Rightarrow a<-2$ and $a(a+3)\ge 0$

$\Rightarrow a<-2$ and $a\le -3$ or $a\ge 0$

$\Rightarrow a\le -3\Rightarrow -\infty<a\le -3$

448 **(c)**

By Mechanical interpretation of Lagrange's mean value theorem, the time c at which the velocity of the particle is equal to its average velocity between times $x=1$ sec and $x=2$ sec is given by

$f'(c)=\dfrac{f(2)-f(1)}{2-1}$

$\Rightarrow 3c^2-2=\dfrac{7-0}{2-1}$ $[\because f(x)=x^3-2x+1\ \therefore f'x$
$=3x^2-2]$

$\Rightarrow 3c^2=9\Rightarrow c=\sqrt{3}$ sec

449 **(c)**

We have,

$y=x^3-2x^2+x-2$

$\Rightarrow \dfrac{dy}{dx}=3x^2-4x+1$

At the points where tangents are parallel to x-axis, we must have

$\dfrac{dy}{dx}=0\Rightarrow 3x^2-4x+1\Rightarrow x=1,\dfrac{1}{3}$

Thus, there are two points on the curve where tangents are parallel to x-axis

450 **(d)**

Given, $f(x)=\frac{x}{4+x+x^2}$

Let $f(x)=\frac{1}{u}$, then $u=\frac{4+x+x^2}{x}$

$=\dfrac{4}{x}+1+x$

$\therefore \dfrac{du}{dx}=-\dfrac{4}{x^2}+1,\ \dfrac{d^2u}{dx^2}=\dfrac{8}{x^3}$

For maximum or minimum, put $\frac{du}{dx}=0$

$\Rightarrow\ 1-\dfrac{4}{x^2}=0\ \Rightarrow\ x=\pm 2$

$\therefore$ At $x=-2, \frac{d^2u}{dx^2}=-\frac{8}{(2)^3}<0$, maximum

At $x=-2,\ \frac{d^2u}{dx^2}=1>0$, minima

$\therefore$ At $x=2,\ f(x)$ is maxima

And at $x=-2, f(x)$ is minima

It is increasing function in the given interval

$\therefore$ The maximum value at $x=1$ is

$f(1)=\dfrac{1}{4+1+1}=\dfrac{1}{6}$

451 **(c)**

Given, $y=x^{1/x}$

$\log y=\frac{1}{x}\log x=f(x)$ [say]

$\therefore\ f'(x)=\dfrac{1-\log x}{x^2}$

For maxima and minima, $\frac{1-\log x}{x^2}=0$

$\Rightarrow\ 1-\log x=0$

$\Rightarrow\ x=e$

Now, $f''(x)=\frac{-x-2(1-\log x)x}{(x^2)^2}$

At $x=e$

$f''(x)=-\text{ve}<0$

$f(x)$ is maximum and maximum value of $x^{1/x}=e^{1/e}$.

452 **(c)**

If x is the side of an equilateral triangle and A is its area, then

$$A=\frac{\sqrt{3}}{4}x^2 \quad \Rightarrow \quad \frac{dA}{dt}=\frac{\sqrt{3}}{4}2x\frac{dx}{dt}$$

Here, $x=10\text{cm}$ and $\frac{dx}{dt}=2\text{ cm/s}$

$\therefore \ A=\frac{\sqrt{3}}{4}2(10)2=10\sqrt{3}\text{cm}^2/\text{s}$

453 **(a)**

Here, $f(x)=x^3+bx^2+cx+d$

$\Rightarrow f'(x)=3x^2+2bx+c$

(As we know, if $ax^2+bx+c>0$ for all x

$\Rightarrow a>0 \ and \ D<0$)

Now, $D=4b^2-12c=4(b^2-c)-8c$

(where$b^2-c<0 \ and \ c>0$)

$\therefore D=(-\text{ve}) or \ D<0$.

$\Rightarrow f'(x)=3x^2+2bx+c>0$for all $x\in(-\infty,\infty)$

Hence, $f(x)$is strictly increasing function.

455 **(b)**

We have,

$f(x)=xe^{-x}$

$\Rightarrow f'(x)=e^{-x}(1-x)$

$\therefore f'(x)=0 \Rightarrow x=1$

Now,

$f(0)=0$

and, $\lim_{x\to\infty} f(x)=\lim_{x\to\infty} xe^{-x}=\lim_{x\to\infty}\frac{x}{e^x}=\lim_{x\to\infty}\frac{1}{e^x}=0$

Hence, the greatest value of $f(x)$ is $\frac{1}{e}$

457 **(c)**

By Rolle's theorem, between any two roots of a polynomial $f(x)$ there is a root of its derivative $f'(x)$. Therefore,

$2ax+b=0$ has a root between α and β

$\Rightarrow \alpha<-\frac{b}{2a}<\beta \qquad \left[\because 2ax+b=0 \Rightarrow x=-\frac{b}{2a}\right]$

458 **(d)**

Given, $f(x)=4x^4-2x+1$

$\Rightarrow f'(x)=16x^3-2$

For maxima and minima, put $f'(x)=0$

$\Rightarrow 16x^3-2=0$

$\Rightarrow x=\frac{1}{2}$

In interval $\left(\frac{1}{2},\infty\right)$,put x=1

$f'(1)=16-2=14$ [increasing]

459 **(c)**

Given, $f(x)=x^2e^{-2x}$

$\therefore \ f'(x)=2xe^{-2x}-2x^2e^{-2x}$

$=2x(1-x)e^{-2x}$

For maxima or minima, put $f'(x)=0$

$2x(1-x)e^{-2x}=0$

$\Rightarrow \quad x=0,1$

Now, $f''(x)=2x(-1)e^{-2x}+2(1-x)e^{-2x}-2.2x(1-x)e^{-2x}$

$f''(0)=0+2e^0=2>0$, minima

And $f''(1)=-2e^{-2}+0-0=-\frac{2}{e^2}<0$, maxima

Thus, maximum value is $f(1)=1.e^{-2}=\frac{1}{e^2}$

460 **(b)**

Given, $f(x)=1-x^3$

$f'(x)=-3x^2<0, \forall x\in R$

So, $f(x) is$ decrease in $(-\infty,\infty)$

Hence, option (b) is correct.

461 **(d)**

We have,

$f(x)=\frac{a}{x}+x^2 \Rightarrow f'(x)=-\frac{a}{x^2}+2x$ and $f''(x)=\frac{2a}{x^3}+2$

If $f(x)$ has a maximum at $x=-3$, then

$f'(-3)=0$ and $f''(-3)<0$

Now, $f'(-3)=0 \Rightarrow -\frac{a}{9}=0 \Rightarrow a=-54$

But, $f''(-3)=\frac{-108}{-27}+2>0$

Thus, there is no value of a for which $f(x)$ has a maximum at $x=-3$

464 **(d)**

Given equation of curve is $y=x^2-x+4$

Slope of tangent at $P(1,4)$ is

$\left(\frac{dy}{dx}\right)=2x-1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,4)}=1$

$\therefore$ Equation of tangent is

$y-4=1(x-1) \Rightarrow \quad y-x=3$ ...(i)

And equation of normal at point $P(1,4)$ is

$y-4=-1(x-1) \ \Rightarrow \quad x+y=5$ ...(ii)

Since the tangent cuts x-axis at $A(-3,0)$

And the normal cuts x-axis at $B(5,0)$

$\therefore$ Area of $\Delta PAB=\frac{1}{2}\left|\begin{vmatrix}1 & 4 & 1\\ -3 & 0 & 1\\ 5 & 0 & 1\end{vmatrix}\right|$

$=\frac{1}{2}\ |[-4(-3-5)]|=16$ sq units

465 **(c)**

Given, $s=3t^2-8t+5$

and $v=\frac{ds}{dt}=6t-8$

The body will be stopped when velocity is zero

$\Rightarrow \quad 6t - 8 = 0 \quad \Rightarrow \quad t = \frac{4}{3}s$

466 **(a)**

By Mean value theorem

$$f'(x) = \frac{f(5) - f(1)}{5 - 1} \geq 9$$

$$\Rightarrow \frac{f(5) + 3}{4} \geq 9 \Rightarrow f(5) \geq 33$$

467 **(b)**

We have,

$y^2 = 4ax$ ...(i) and, $ay^2 = 4x^3$ ...(ii)

Solving the two equations, the points of intersection of the two curves are $(0, 0)$, $(a, 2a)$ and $(a, -2a)$

Let us take the point $(a, 2a)$ as point P

Now,

$$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow \left(\frac{dy}{dx}\right)_P = 1$$

So, the equation of the normal to (i) at $P(a, 2a)$ is

$y - 2a = -1(x - a) \Rightarrow x + y - 3a = 0$ ... (iii)

Now,

$$ay^2 = 4x^3 \Rightarrow 2ay\frac{dy}{dx} = 12x^2 \Rightarrow \left(\frac{dy}{dx}\right)_P = \frac{12a^2}{4a^2} = 3$$

So, the equation of the normal to (ii) at $P(a, 2a)$ is

$y - 2a = -\frac{1}{3}(x - a) \Rightarrow x + 3y - 7a = 0$ ...(iv)

Clearly, (iii) and (iv) cut x-axis at $G_1(3a, 0)$ and $G_2(7a, 0)$

$\therefore G_1G_2 = 4a$

468 **(a)**

We have,

$y = x^3 - 3x^2 + 6x - 17$

$$\Rightarrow \frac{dy}{dx} - 3x^2 - 6x + 6 = 3\{(x - 1)^2 + 1\} > 0 \text{ for all } x$$

Hence, y increases for all values of x

469 **(c)**

By Rolle's theorem, $f(1) = f(3)$

$\Rightarrow \quad a + b + 11 - 6 = 27a + 9b + 33 - 6$

$\Rightarrow \quad 13a + 4b + 11 = 0$ ...(i)

Now, $f'(x) = 3ax^2 + 2bx + 11$

$$\Rightarrow \quad f'\left(2 + \frac{1}{\sqrt{3}}\right) = 3a\left(2 + \frac{1}{\sqrt{3}}\right) + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11$$

$$\Rightarrow \quad 0 = 3a\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 4b + \frac{2b}{\sqrt{3}} + 11$$

$$\Rightarrow \quad 13a + 4b + \frac{12a}{\sqrt{3}} + \frac{2b}{\sqrt{3}} + 11 = 0$$

$\Rightarrow \quad -11 + \frac{12a}{\sqrt{3}} + \frac{2b}{\sqrt{3}} + 11 = 0$ .....[from Eq.(i)]

$\Rightarrow \quad 6a + b = 0$ ...(ii)

From Eq. (i) and (ii),

$a = 1, \ b = -6$

470 **(c)**

We have, $s = t^3 - 3t^2$

On differentiating with respect to t, we get

$\frac{ds}{dt} = 3t^2 - 6t$ ...(i)

Again differentiating Eq. (i), we get

$$\frac{d^2s}{dt^2} = 6t - 6 = 0 \quad \Rightarrow \quad t = 1$$

On putting the value of $t = 1$ in Eq. (i), we get

$$\frac{ds}{dt} = 3 \times 1 - 6 \times 1 = 3 - 6 = -3 \text{ m/s}$$

471 **(c)**

We have,

$$f(x) = \begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix}$$

$$\Rightarrow f'(x) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} + \begin{vmatrix} x+1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & x+1 \end{vmatrix} + \begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow f'(x) = 3\{(x + 1)^2 - 1\} = 3(x^2 + 2x)$

For $f(x)$ to be increasing, we must have

$f'(x) > 01$

$\Rightarrow 3(x^2 + 2x) > 0$

$\Rightarrow x^2 + 2x > 0 \Rightarrow x < -2$ or $x > 0 \Rightarrow x \in R - [-2, 0]$

472 **(a)**

We have,

$g(x) = f(x) - 2\{f(x)\}^2 + 9\{f(x)\}^3$ for all $x \in R$

$\Rightarrow g'(x) = f'(x) - 4f(x)f'(x) + 27\{f(x)\}^2 f'(x)$ for all $x \in R$

$\Rightarrow g'(x) = [1 - 4f(x) + 27\{f(x)\}^2]f'(x)$ for all $x \in R$

Clearly, $27\{f(x)\}^2 - 4f(x) + 1$ is a quadratic expression with discriminant less than zero. So, its sign is same as that of the coefficient of $\{f(x)^2\}$ i.e. positive for all $x \in R$

Thus, $g'(x)$ and $f'(x)$ have the same sign

Hence, $g(x)$ and $f(x)$ increase and decrease together

473 **(a)**

We have, $y = \frac{x+c}{1+x^2}$

For stationary values of y, we must have,

$\frac{dy}{dx} = 0$

$\Rightarrow \frac{(1+x^2)-(x+c)(2x)}{(1+x^2)^2} = 0$

$\Rightarrow 1 + x^2 - 2cx - 2x^2 = 0 \Rightarrow x^2 + 2cx - 1 = 0$ ...(i)

Now,

$y = \frac{x+c}{1+x^2}$

$\Rightarrow xy = x\left(\frac{x+c}{1+x^2}\right) = \frac{x^2+cx}{1+x^2}$

$\Rightarrow xy = \frac{1-2cx+cx}{1+1-2cx}$ [Putting x^2 from (i)]

$\Rightarrow xy = \frac{1-cx}{2(1-cx)} = \frac{1}{2}$

474 **(c)**

Let $f(x) = (x-3)\log_e x$

Clearly, $f(x)$ is continuous on $[1, 3]$ and differentiable on $(1, 3)$

Also, $f(1) = 0 = f(3)$

Hence, by Rolle's theorem there exists at least one $c \in (1, 3)$ such that $f'(c) = 0$

$\Rightarrow \log_e c + \frac{(c-3)}{c} = 0 \Rightarrow c \log_e c = 3 - c$

$\Rightarrow x = c$ is a root of $x \log_e x = 3 - x$, where $c \in (1, 3)$

475 **(c)**

Given, $\frac{\delta r}{r} \times 100 = 2 \Rightarrow \delta r = \frac{2r}{100}$

Surface area, $S = 4\pi r^2$

$\delta S = 4\pi 2r.\delta r$

$8\pi r.\frac{2r}{100} = \frac{16\pi r^2}{100}$

Now percentage error in surface area

$\frac{\delta S}{s} \times 100 = \frac{16\pi r^2}{100} \times \frac{1}{4\pi r^2} \times 100 = 4\%$

476 **(a)**

$\because \ f(x) = x + \cos x$

On differentiating, we get $f'(x) = 1 - \sin x$

$f'(x) > 0$ for all values of x

($\because$ $\sin x$ is lying between-1 to +1)

$\therefore \ f(x)$ is always increasing

477 **(b)**

We have,

$y = ax^2 + bx + c$ ...(i)

$\Rightarrow \frac{dy}{dx} = 2ax + b$ ...(ii)

The curve (i) passes through (1, 2)

$\therefore 2 = a + b + c$ ...(iii)

The line $y = x$ touches the curve (i) at the origin

$\therefore \left(\frac{dy}{dx}\right)_{(0,0)} =$ (Slope of the line $y = x$)

$\Rightarrow b = 1$

Putting $b = 1$ in (iii), we get

$a + c = 1$

Also, the curve (i) passes through the origin

$\therefore c = 0$

Hence, $a = 1, b = 1$ and $c = 0$

478 **(b)**

We have,

$f(x) = \sin^4 x + \cos^4 x = \frac{3}{4} + \frac{1}{4}\cos 4x$

$\Rightarrow f'(x) = -\sin 4x$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow -\sin 4x > 0$

$\Rightarrow \sin 4x < 0 \Rightarrow \pi < 4x < 2\pi \Rightarrow \pi/4 < x < \pi/2$

479 **(b)**

The length of the normal at (x, y) to a curve is given by

$y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Here, $y = c\cosh\left(\frac{x}{c}\right) \Rightarrow \frac{dy}{dx} = \sin h\frac{x}{c}$

$\therefore$ Length of the normal

$= y\sqrt{1 + \sin h^2\frac{x}{c}} = y\cos h\left(\frac{x}{c}\right)$

$= \frac{y^2}{c}$ $\left[\because y = c\cos h\frac{x}{c}\right]$

Thus, length of the normal varies as the square of the ordinate

480 **(d)**

Let the equation of normal is

$Y - y = -\frac{dx}{dy}(X - x)$

It meets the x-axis at$G$. Therefore, coordinates of G are $\left(x + y\frac{dy}{dx}, 0\right)$

According to given condition,

$x + y\frac{dy}{dx} = 2x \quad \Rightarrow \quad y\,dy = x\,dx$

On integrating, we get

$\frac{y^2}{2} = \frac{x^2}{2} + c \quad \Rightarrow \quad x^2 - y^2 = 2c$

481 **(c)**

$P(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_nx^{2n}$

Where, $a_n > a_{n-1} > a_{n-2} ... > a_2 > a_1 > a_0 > 0$

$\Rightarrow \quad P'(x) = 2a_1x + 4a_2x^3 + \cdots + 2na_nx^{2n-1}$

$= 2x\{a_1 + 2a_2x^2 + \cdots + na_nx^{2n-2}\}$ ..... (i)

Where,

$(a_1 + 2a_2x^2 + 3a_3x^4 + \cdots + na_nx^{2n-2}) > 0$

For all $x \epsilon R$

Thus, $\begin{cases} P'(x) > 0, \text{when} x > 0 \\ P'(x) < 0, \text{when} x > 0 \end{cases}$

ie $P'(x)$ changes sign from (−ve) to (+ve) at $x = 0$

Hence, $P(x)$ attains minimum at $x = 0$

$\Rightarrow$ Only one minimum at $x = 0$.

483 **(b)**

Let radius vector is r

$\therefore \; r^2 = x^2 + y^2$

$\Rightarrow \; r^2 = \frac{a^2y^2}{y^2 - b^2} + y^2 \quad \left(\because \frac{a^2}{x^2} + \frac{b^2}{y^2} = 1\right)$

For minimum value of r

$\frac{d(r^2)}{dy} = 0 \;\Rightarrow\; \frac{-2yb^2a^2}{(y^2 - b^2)^2} + 2y = 0$

$\Rightarrow \quad y^2 = b(a + b)$

$\therefore \; x^2 = a(a + b)$

$\Rightarrow \quad r^2 = (a + b)^2$

$\Rightarrow \; r = a + b$

485 **(a)**

We have,

$f(x) = \cos x \sin 2x = 2\sin x - 2\sin^3 x$

Let $\sin x = t$. Then, for $x \in [-\pi, \pi]$, we have $t \in [-1, 1]$

Let $g(t) = 2t - 2t^3$. Then,

$\min f(x) = \min g(t)$ where $t \in [-1, 1]$

Now, $g'(t) = 2 - 6t^2 = 0 \Rightarrow t = \pm\frac{1}{\sqrt{3}}$

We have, $g''(t) = -12t$

Clearly $g''(t) > 0$ for $t = -1/\sqrt{3}$

Hence, $g(t)$ attains its minimum value at $t = -1/3$. The minimum value of $g(t)$ is given by

$g\left(-\frac{1}{\sqrt{3}}\right) = -\frac{2}{\sqrt{3}} + \frac{2}{3\sqrt{3}} = -\frac{4}{3\sqrt{3}} > -\frac{7}{9} > -\frac{9}{7}$

Hence, the minimum value of $f(x)$ is $-\frac{4}{3\sqrt{3}} > -\frac{7}{9} > -\frac{9}{7}$

486 **(c)**

Given, $f(x) = \int_{-10}^{x}(t^4 - 4)e^{-4t}\, dt$

On differentiating w.r.t. x, we get

$f'(x) = (x^4 - 4)e^{-4x}$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow \; x = \pm\sqrt{2}, \pm\sqrt{2}$

Again on differentiating w.r.t. x, we get

$f''(x) = -4(x^4 - 4)e^{-4x} + 4x^3\, e^{-4x}$

At $x = \sqrt{2}$ and $x = -\sqrt{2}$, the given function has two extreme values

488 **(b)**

$f(x) \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$= 1 - \frac{\sin^2 2x}{2} = 1 - \left(\frac{1 - \cos 4x}{4}\right)$

$= \frac{3}{4} + \frac{1}{4}\cos 4x$

For $f(x)$ to be increasing, $f'(x) > 0$

$\Rightarrow -\sin 4x > 0$

$\Rightarrow \sin 4x < 0$

$\Rightarrow \pi < 4x < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{4} < x < \frac{3\pi}{8}$

489 **(a)**

Let the required point be (x, y) on the curve

So, $d = \sqrt{(x-4)^2 + \left(y + \frac{1}{2}\right)^2}$ should be minimum

Let $D = (x - 4)^2 + \left(y + \frac{1}{2}\right)^2$

$\therefore \; D = (x - 4)^2 + \left(x^2 + \frac{1}{2}\right)^2$

$\Rightarrow \quad D = x^4 + 2x^2 - 8x$

$\therefore \quad D' = 4x^3 + 4x - 8$

Put $D' = 0$ for maxima or minima

$\therefore \; x^3 + x - 2 = 0 \quad\Rightarrow\quad x = 1$

Now, $D'' = 12x^2 + 4$

at $x = 1$, $D'' = 16 > 0$

$\therefore \; D$ is minimum when $x = 1$

Hence, the required point is (1, 1)

490 **(a)**

Given, $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$

At $x = 0$, $y = 1$

Let $y = u + v$ where $u = (1 + x)^y, v = \sin^{-1}(\sin^2 x)$

$\Rightarrow \quad \log u = y\log(1 + x)$

$\Rightarrow \quad \frac{du}{dx}\frac{1}{u} = \frac{y}{1 + x} + \log(1 + x)\frac{dy}{dx}$

and $\frac{dv}{dx} = \frac{1}{\sqrt{1 - \sin^2 x}} . 2\sin x \cos x$

$\therefore \; \frac{dy}{dx} = (1 + x)^y \left[\frac{y}{1 + x} + \log(1 + x)\frac{dy}{dx}\right] + \frac{\sin 2x}{\cos x}$

$\Rightarrow \quad \frac{dy}{dx}[1 - (1 + x)^y \log(x + 1)] = \frac{(1 + x)^y y}{1 + x} + \frac{\sin 2x}{\cos x}$

$\therefore \quad \left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1}{1+0} = 1$

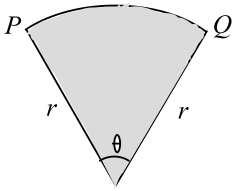
$\therefore$ Equation of tangent is

$(y-1) = 1(x) \quad \Rightarrow \quad x - y + 1 = 0$

491 **(c)**

Perimeter of sector$= 2r + r\theta$

$\Rightarrow \quad 60 = 2r + r\theta \quad$ [given]



$\Rightarrow \quad \theta = \frac{60-2r}{r}$

Area of sector, $A = \frac{\pi r^2 \theta}{360°} = \frac{\pi r^2 (60-2r)}{r360°}$

$= \frac{\pi r}{180°}(30 - r)$

$\Rightarrow \quad \frac{dA}{dr} = \frac{\pi}{180°}(30 - 2r)$

For maximum area, $\frac{dA}{dr} = 0 \quad \Rightarrow \quad r = 15$

Now, $\frac{d^2A}{dr^2} = \frac{\pi}{180°}\ (0-2) = -\frac{\pi}{90} < 0$

$\therefore$ It is maximum at $r = 15$ m

492 **(b)**

Let $\phi(x) = f(x) - 2\,g(x), x \in [0,1]$

Clearly, $\phi(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$ as $f(x)$ and $g(x)$ are differentiable on $[0,1]$

Also, $\phi(0) = f(0) - 2\,g(0) = 2 - 0 = 2$

and, $\phi(1) = f(1) - 2\,g(1) = 6 - 2 \times 2 = 2$

$\therefore \phi(0) = \phi(1)$

Thus, $\phi(x)$ satisfies all the three conditions of Rolle's theorem. Consequently, there exists a point $c \in (0,1)$ such that

$\phi'(c) = 0 \Rightarrow f'(c) - 2g'(c) = 0 \Rightarrow f'(c) = 2g'(c)$

493 **(b)**

Given, $f(x) = 2x^3 - 3x^2 - 12x + 4$

On differentiating w.r.t. x, we get

$f(x) = 6x^2 - 6x - 12$

For maxima or minima put $f'(x) = 0$

$\Rightarrow \quad x^2 - x - 2 = 0 \quad \Rightarrow \quad x = 2, -1$

Again differentiating, we get $f''(x) = 12x - 6$

$\Rightarrow \quad f''(2) = +\text{ve}, \qquad f''(-1) = -\text{ve}$

$\therefore$ Given function has one maximum and one minimum

494 **(c)**

We have,

$$f(x) = \frac{|x-1|}{x^2} = \begin{cases} \frac{1-x}{x^2}, & x < 1, x \neq 0 \\ \frac{x-1}{x^2}, & x \geq 1 \end{cases}$$

Clearly, $f(x)$ is not differentiable at $x = 1$. Also,

$$f'(x) = \begin{cases} \frac{1}{x^2} - \frac{2}{x^3}, & x < 1, x \neq 0 \\ \frac{2}{x^3} - \frac{1}{x^2}, & x > 1, \end{cases}$$

$$= \begin{cases} \frac{x-2}{x^3}, & x < 1, x \neq 0 \\ \frac{2-x}{x^3}, & x > 1 \end{cases}$$

Now,

$f'(x) < 0$

$$\Rightarrow \begin{cases} \frac{x-2}{x^3} < 0 \text{ given that } x < 1 \\ \frac{2-x}{x^3} < 0 \text{ given that } x > 1 \end{cases}$$

$\Rightarrow x < 1$ or $x > 2$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

495 **(b)**

Given curve is $y = e^{2x}$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 2e^{2x} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,1)} = 2e^0 = 2$

Equation of tangent at $(0, 1)$ with slope 2 is

$y - 1 = 2(x - 0) \quad \Rightarrow \quad y = 2x + 1$

This tangent meets x-axis

$\therefore \quad y = 0$

$\Rightarrow \quad 0 = 2x + 1 \quad \Rightarrow \quad x = -\frac{1}{2}$

$\therefore$ Coordinates of the point on x-axis is $\left(-\frac{1}{2}, 0\right)$

496 **(b)**

We have,

$f(x) = \cot^{-1}\{g(x)\}$

$\Rightarrow f'(x) = -\frac{1}{1 + \{g(x)\}^2}\,g'(x)$

$\Rightarrow f'(x) < 0$ for all $x \in (0, \pi)$

$\left[\begin{array}{c} \because g(x) \text{ is an increasing} \\ \text{function on } (0,\pi) \\ \therefore g'(x) > 0 \text{ for all } x \in (0,\pi) \end{array}\right]$

$\Rightarrow f(x)$ is decreasing on $(0, \pi)$

497 **(a)**

Let h and r be the height and radius of cylinder.

Given that $,\frac{dr}{dt} = 3m/s, \qquad \frac{dh}{dt} = -4m/s$

Let volume of cylinder, $V = \pi r^2 h$

$\Rightarrow \quad \frac{dV}{dt} = \pi\left[r^2 \frac{dh}{dt} + h.2r\frac{dr}{dt}\right]$

At $\quad r = 4m$

And $h = 6m$

$\therefore \frac{dV}{dt} = \pi[-64 + 144] = 80\pi cu\ m/s$

498 **(b)**

Let $f(x) = x^2$,

$\therefore \quad f(1) = f(-1) = 1$

Also, $f(x)$ is continuous and differentiable in the given interval

499 **(b)**

Let $f(x) = \frac{ab(a^2-b^2)\sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}$

Then,

$$f(x) = \frac{ab(a^2 - b^2)}{a^2 \tan x + b^2 \cot x}$$

$$\Rightarrow f(x) = \frac{ab(a^2 - b^2)}{\left(a\sqrt{\tan x} - b\sqrt{\cot x}\right)^2 + 2ab}$$

Clearly, $f(x)$ will be maximum, when $\left(a\sqrt{\tan x} - b\sqrt{\cot x}\right)^2$ is minimum. But, the minimum value of $a\sqrt{\tan x} - b\sqrt{\cot x}$ is zero. Therefore,

Maximum value of $f(x) = \frac{a^2-b^2}{2}$

500 **(d)**

Given $f(x)$satisfy Rolle's theorem in [3, 5].

$\therefore f(x)$is continuous in [3,5]

And $f(x)$is differentiable in]3,5[

And $f(a) = f(b) = 0 ie, f(3) = f(5) = 0$

Let $f(x) = (x - 3)(x - 5)$

$\therefore f(x) = x^2 - 8x + 15$

$$\therefore \int_3^5 (x^2 - 8x = 15)dx = \left[\frac{x^3}{3} - \frac{8x^2}{2} + 15x\right]_3^5$$

$$= \left(\frac{50}{3} - 18\right) = -\frac{4}{3}$$

501 **(a)**

Let x be the length of an edge of a cube and S be its surface area. Then,

$$S = 60x^2 \Rightarrow \frac{dS}{dx} = 12x$$

$$\therefore \Delta S = \frac{dS}{dx}\Delta x$$

$$\Rightarrow \Delta S = 12x\ \Delta x$$

$$\Rightarrow \frac{\Delta S}{S} \times 100 = \frac{12x\ \Delta x}{6x^2} \times 100 = 2\left(\frac{\Delta x}{x} \times 100\right)$$

$$= 2a$$

502 **(b)**

We have,

$f(x) = (a^2 - 7a + 12)\cos x + 2(a - 4)x + \log 2$

$\Rightarrow f'(x) = -(a - 4)(a - 3)\sin x + 2(a - 4)$

$\Rightarrow f'(x) = (a - 4)\{-(a - 3)\sin x + 2\}$

If $f(x)$ does not have any critical point, then $f'(x) = 0$ does not have any solution in R

$\therefore a - 4 \neq 0$ and $\sin x = \frac{2}{a-3}$ is not solvable in R

$\Rightarrow a \neq 4$ and $\left|\frac{2}{a-3}\right| > 1$

$\Rightarrow a \neq 4$ and $|a - 3| < 2$

$\Rightarrow a \neq 4$ and $1 < a < 5 \Rightarrow a \in (1, 4) \cup (4, 5)$

504 **(d)**

$\because \quad y = be^{-x/a} \quad$ ...(i)

$$\therefore \frac{dy}{dx} = -\frac{b}{a}\ e^{-x/a} = -\frac{b}{a} \quad \left(\text{slope of} \frac{x}{a} + \frac{y}{b} = 1\right)$$

$\Rightarrow \quad e^{-x/a} = 1 = e^0$

$\therefore \quad x = 0$

From Eq. (i), $y = b$

Hence, point of contact is $(0, b)$

505 **(c)**

Given curve is $xy = 4$

$$\Rightarrow \quad y = \frac{4}{x}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{4}{x^2}$$

$$\Rightarrow -\frac{4}{x^2} = -\frac{a}{b}$$

$$\Rightarrow \quad x^2 = \frac{4b}{a} > 0$$

Which is true when

$a > 0, b > 0\ \ or\ \ a < 0, b < 0$

506 **(b)**

We have,

$\sqrt{x} + \sqrt{y} = 3$

$$\Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)_{(4,1)}$$

$$= -2$$

$\therefore$ Length of the subtangent $= \left|\frac{y}{dy/dx}\right| = \frac{1}{2}$

507 **(d)**

Let $\ y = \frac{\log x}{x}$

$$\frac{dy}{dx} = \frac{x.\frac{1}{x} - \log x.1}{x^2} = \frac{1 - \log x}{x^2}$$

For maxima, put $\frac{dy}{dx} = 0$

$$\Rightarrow \quad \frac{1 - \log x}{x^2} = 0 \quad \Rightarrow \quad x = e$$

Now, $\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)2x}{(x^2)^2}$

At $\ x = e, \frac{d^2y}{dx^2} \leq 0,$ maxima

$\therefore$ The maximum value at $x = e$ is $y = \frac{1}{e}$

508 **(a)**

Let $f(x)=\frac{\log x}{x}$. Clearly, $f(x)$ is defined for $x>0$

Now,

$$f(x)=\frac{\log x}{x} \Rightarrow f'(x)=\frac{1-\log x}{x^2}$$

For maximum or minimum, we must have

$f'(x)=0 \Rightarrow \log x=1 \Rightarrow x=e$

$f'(x)>0$ for $x<e$ and $f'(x)<0$ for $x>e$

Thus, $f'(x)$ changes its sign from positive to negative in the neighbourhood of $x=e$

So, $x=e$ is the point of local maximum

Now, $f(e)=\frac{\log e}{e}=\frac{1}{e}$

509 **(b)**

Let x be the radius and V be the volume of the sphere. Then,

$$V=\frac{4}{3}\pi x^3 \Rightarrow \frac{dV}{dx}=4\pi x^2$$

We have,

$$\frac{\Delta x}{x}\times 100=k$$

$$\therefore \Delta V=\frac{dV}{dx}\Delta x$$

$$\Rightarrow \Delta V=4\pi x^2\Delta x$$

$$\Rightarrow \frac{\Delta V}{V}\times 100=\frac{4\pi x^2\,\Delta x}{\frac{4}{3}\pi x^3}\times 100=3\left(\frac{\Delta x}{x}\times 100\right)$$

$$=3k$$

510 **(d)**

Given curve is $y^3+3x^2=12y$

On differentiating w.r.t. y, we get

$$3y^2\frac{dy}{dx}+6x=12\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(3y^2-12)+6x=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{6x}{12-3y^2}$$

$$\Rightarrow \frac{dx}{dy}=\frac{12-3y^2}{6x}$$

Since, tangent is parallel to y-axis

$$\frac{dy}{dx}=0 \Rightarrow 12-3y^2=0$$

$$\Rightarrow y^2=4 \Rightarrow y=\pm 2$$

Then, $x=\pm\frac{4}{\sqrt{3}}$

At $y=-2, x$ cannot be real

$\therefore$ The required point is $\left(\pm\frac{4}{\sqrt{3}},2\right)$

511 **(b)**

Given, $y=a\log x+bx^2+x$

$$\Rightarrow \frac{dy}{dx}=\frac{a}{x}+2bx+1$$

At $x=-1$ and $x=2, y$ has its extrema.

$\therefore\ -a-2b+1=0 \Rightarrow a+2b=1$ ...(i)

And $\frac{a}{2}+4b+1=0 \Rightarrow a+8b=-2$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a=2, b=-\frac{1}{2}$

512 **(a)**

Let $P(x_1,y_1)$ be any point on the curve $x^ny=a^n$

Then,

$x_1^ny_1=a^n$ ...(i)

Now,

$x^ny=a^n$

$$\Rightarrow n\,x^{n-1}y+x^n\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{ny}{x}$$

$\therefore \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\frac{-ny_1}{x_1} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=-\frac{na^n}{x_1^{n+1}}$ [Using (i)]

The equation of the tangent at $P(x_1,y_1)$ is

$$y-y_1=-\frac{n\,a^n}{x_1^{n+1}}(x-x_1)$$

This meets the coordinates axes at $A\left(\frac{x_1^{n+1}y_1}{n\,a^n}+x_1,0\right)$ and $B\left(0,y_1+\frac{n\,a^n}{x_1^n}\right)$ respectively

$\therefore$ Area of $\Delta AOB=\frac{1}{2}(OA\times OB)$

$\Rightarrow$ Area of $\Delta AOB=\frac{1}{2}\left(\frac{x_1^{n+1}y_1}{n\,a^n}+x_1\right)\left(y_1+\frac{n\,a^n}{x_1^n}\right)$

$\Rightarrow$ Area of $\Delta AOB=\frac{1}{2}\left(\frac{x_1}{n}+x_1\right)\left(\frac{a^n}{x_1^n}+\frac{n\,a^n}{x_1^n}\right)$

[Using (i)

$\Rightarrow$ Area of $\Delta AOB=\frac{1}{2}\frac{(n+1)^2}{n}a^nx_1^{1-n}$

Clearly, the area will be constant if $1-n=0$ $i.e.\, n=1$

513 **(c)**

Given, $y=\frac{c^2}{x}$

$$\Rightarrow \frac{dy}{dx}=c^2\left(-\frac{1}{x^2}\right)$$

$\therefore$ Subnormal at any point $=y.\frac{dy}{dx}$

$$=y\times\left(-\frac{c^2}{x^2}\right)=\frac{-y^3}{c^2}$$

$\therefore$ Subnormal $\propto y^3$

514 **(a)**

Given that, $\frac{dv}{dt}=30$ ft^3/ min and $r=15$ ft

Volume of spherical balloon

$$V=\frac{4}{3}\pi r^2 \Rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

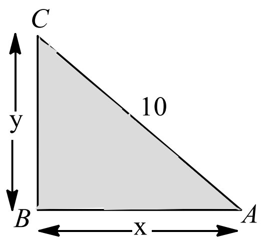
$$\Rightarrow 30=4\pi r^2\frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt}=\frac{30}{4\times\pi\times 15\times 15}=\frac{1}{30\pi}\text{ ft/min}$$

515 **(d)**

Let $AB = x\text{m}$, BC = ym and $AC = 10\text{m}$

$\therefore \quad x^2 + y^2 = 100 \quad ...(i)$



$\Rightarrow \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \quad 2x(3) - 2y(4) = 0$

$\left[Given = \frac{dx}{dt} = 3m/s, \frac{dy}{dt} = -4m/s\right]$

$\Rightarrow \quad x = \frac{4y}{3}$

On putting this value in Eq.(i),we get

$\frac{16}{9}y^2 + y^2 = 100 \Rightarrow y = 6m$

516 **(d)**

Let there be a value of k for which $x^3 - 3x + k = 0$ has two distinct roots between 0 and 1

Let a, b be two distinct roots of $x^3 - 3x + k = 0$ lying between 0 and 1 such that $a < b$

Let $f(x) = x^3 - 3x + k$. Then, $f(a) = f(b) = 0$

Since between any two roots of a polynomial $f(x)$ there exists at least one roots of its derivative $f'(x)$. Therefore, $f'(x) = 3x^2 - 3$ has at least one root between a and b. But, $f'(x) = 0$ has two roots equal to ± 1 which do not lie between a and b

Hence, $f(x) = 0$ has no real roots lying between 0 and 1 for any value of k

517 **(b)**

$\because \quad f'(x) = 2\left(\frac{1}{3}\right)\sin\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right) + \left(\frac{1}{3}\right)\cos\frac{x}{3} - \left(\frac{1}{3}\right)$

$= \left(\frac{1}{3}\right)\left[2\sin\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right) - 2\sin^2\left(\frac{x}{6}\right)\right]$

$= \left(\frac{2}{3}\right)\sin\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right) - 2\sin\left(\frac{x}{6}\right)$

Put $f'(x) = 0 \quad \Leftrightarrow \quad \sin\left(\frac{x}{6}\right) = 0$

$\Rightarrow \tan\left(\frac{x}{6}\right) = 1$

$\Rightarrow \frac{x}{6} = k\pi, k \in I \;\; or \;\; \frac{x}{6} = n\pi + \frac{\pi}{4}, \quad n \in I$

$x^2 - 10 < -19.5x$

$\Leftrightarrow \quad (x + 9.75)^2 < 105.0625$

$\Leftrightarrow \quad (x - 0.5)(x + 20) < 0$

$\Leftrightarrow \quad -20 < x < 0.5$

So, the critical points satisfying the last inequality will be $0, 6\pi, -\frac{9\pi}{2}$

518 **(d)**

Given curve is $f(x) = \int_0^x \frac{\sin t}{t}dt$

On differentiating w.r.t. x, we get

$f'(x) = \frac{\sin x}{x}$

For point of extreme, put $f'(x) = 0$

$\Rightarrow \quad \frac{\sin x}{x} = 0 \quad \Rightarrow \quad \sin x = 0$

$\Rightarrow \quad x = n\pi, n = 1, 2, 3, ...$

519 **(a)**

Given , $s = 22t - 12t^2$

$\therefore \quad \text{v} = \frac{ds}{dt} = 22 - 24t$

When the car stop, $v = 0, 22 - 24t = 0$

$\Rightarrow t = \frac{11}{12}$

$\therefore \quad s = 22\left(\frac{11}{12}\right) - 12\left(\frac{11}{12}\right)^2 = 10.08 \text{ ft}$

520 **(a)**

Let $\quad f(x) = e^{ax} + e^{-ax}, \; a < 0$

Now, $\quad f'(x) = ae^{ax} - ae^{-ax}$

For decreasing, $f'(x) < 0 \quad \Rightarrow \quad a(e^{ax} - e^{-ax}) < 0$

$\Rightarrow \quad a(e^{2ax} - 1) < 0$

$\Rightarrow \quad e^{2ax} > 1 \quad \Rightarrow \quad x < 0 \quad [\because \quad a < 0]$

523 **(d)**

We have,

$f(x) = \frac{x^2 - 1}{x^2 + 1}$

$\Rightarrow f(x) = \frac{x^2 + 1 - 2}{x^2 + 1}$

$\Rightarrow f(x) = 1 - \frac{2}{x^2 + 1} \Rightarrow f'(x) = \frac{4x}{(x^2 + 1)^2}$

Clearly, $f'(x) = 0$ for $x = 0$

Also, $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$

Therefore, $x = 0$ is a point of minimum with the local minimum $f(0) = -1$

524 **(d)**

$f(x) = x + \frac{1}{x}$

On differentiating w.r.t. x, we get

$f'(x) = 1 - \frac{1}{x^2}$

For maximum put $f'(x)$=0

$\therefore \quad x^2 - 1 = 0 \Rightarrow x = \pm 1$

Again differentiating, we get

$f'(x) = \frac{2}{x^3}$

Since, $\; x > 0$, so we can check only at $x = 1$

At y $x = 1,\ f''(x) > 0$
$\Rightarrow\ f(c)$ is minimum at $x = 1$
At $x = 1,\ f''(x) > 0$
$\Rightarrow\ f(x)$ is minimum at $x = 1$
$\therefore$ No maximum value can be found

525 **(b)**

Let $f(x) = \left(\frac{1}{x}\right)^{2x^2}$
Clearly, $f(x)$ is defined for all $x > 0$
Now,
$f(x) = e^{-2x^2 \log_e x}$
$$\Rightarrow f'(x) = \left(\frac{1}{2}\right)^{2x^2} \{-4x \log_e x - 2x\}$$
$$\Rightarrow f'(x) = -\left(\frac{1}{x}\right)^{2x^2} 2x\ (2 \log_e x + 1)$$
For local maximum, we must have
$f'(x) = 0 \Rightarrow 2x(\log_e x + 1) = 0 \Rightarrow x = e^{-1/2}\ [\because x > 0]$
It can be checked that $f''(e^{-1/2}) < 0$
Therefore, $f(x)$ attains maximum value at $x = e^{-1/2}$. The maximum value of $f(x)$ is given by
$f(e^{-1/2}) = (e^{1/2})^{2/e} = e^{1/e}$

527 **(b)**

Given, $y = x^{40} - x^{20}$
$$\therefore \quad \frac{dy}{dx} = 40x^{39} - 20x^{19}$$
Now, put $\frac{dy}{dx} = 0$ *ie*, $20x^{19}\ (2x^{20} - 1) = 0$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x^{20} = \frac{1}{2}$$
When $x = 0,\ y = 0$
And $x^{20} = \frac{1}{2}$
$$y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$
Also, $x = 1,\ y = 0$
Hence, absolute maximum value of y is 0

528 **(a)**

We have, $f(x) = x\sqrt{ax - x^2}$
Clearly, $f(x)$ exist for $0 < x < a$
Now,
$$f'(x) = \sqrt{ax - x^2} + \frac{x(a - 2x)}{2\sqrt{ax - x^2}} = \frac{2ax - 2x^2 + ax - 2x^2}{2\sqrt{ax - x^2}}$$
$$\Rightarrow f'(x) = \frac{3ax - 4x^2}{2\sqrt{ax - x^2}}$$
For $f(x)$ to be increasing, we must have
$$f'(x) > 0 \Rightarrow \frac{3ax - 4x^2}{2\sqrt{ax - x^2}} > 0 \Rightarrow 3ax - 4x^2 > 0$$
$$\Rightarrow 0 < x < \frac{3a}{4}$$

530 **(c)**

Let x be the length of a side of the square and r be the radius of the circle. Then,
$x = 2r$ [Given] and $\frac{dx}{dt} = \frac{dr}{dt}$
Let A_1 and A_2 be the areas of the square and circle respectively.
Then,
$A_1 = x^2$ and $A_2 = \pi r^2$
$\Rightarrow \frac{dA_1}{dt} = 2x\frac{dx}{dt}$ and $\frac{dA_2}{dt} = 2\pi r\frac{dr}{dt}$
$$\Rightarrow \frac{\frac{dA_1}{dt}}{\frac{dA_2}{dt}} = \frac{2x\frac{dx}{dt}}{2\pi r\frac{dr}{dt}} = \frac{2}{\pi} \quad \left[\because x = 2r \text{ and } \frac{dx}{dt} = \frac{dr}{dt}\right]$$

531 **(d)**

We have,
$$f(x) = \begin{cases} \tan^{-1} a - 3x^2, & 0 < x < 1 \\ -6x, & x \geq 1 \end{cases}$$
If $f(x)$ attains a maximum at $x = 1$, then $f'(1)$ must exist and should be zero. This means that $f(x)$ must be continuous and differentiable at $x = 1$
We observe that $f(x)$ will be continuous at $x = 1$, if $\tan^{-1} a = -3$
But, (LHD at $x = 1$) = (RHD at $x = 1$) $= -6 \neq 0$ for any value of a
Hence, there is no value of a for which $f(x)$ has a maximum at $x = 1$

532 **(b)**

The equations of given curves are
$y = a^x$ ...(i)
And $y = b^x$ ...(ii)
From Eq. (i) $m_1 = \frac{dy}{dx} = a^x \log a$
And from Eq. (ii) $m_2 = \frac{dy}{dx} = b^x \log b$
From Eqs. (i) and (ii), we get
$a^x = b^x \Rightarrow x = 0$
Let α be the angle at which the two curves intersect
$$\therefore \tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{a^x \log a - b^x \log b}{1 + a^x b^x (\log a)(\log b)} = \frac{\log\frac{a}{b}}{1 + (\log a)(\log b)}$$

533 **(a)**

Let $\angle C = \theta$. Then, $b = a\cos\theta$

$\therefore a + b = 8 \Rightarrow a(1+\cos\theta) = 8 \Rightarrow a = \dfrac{8}{1+\cos\theta}$

Let Δ be the area of ΔABC. Then,

$$\Delta = \frac{1}{2}ab\sin\theta$$

$$\Rightarrow \Delta = \frac{1}{2}\frac{8}{1+\cos\theta} \times \frac{8\cos\theta}{1+\cos\theta} \times \sin\theta$$

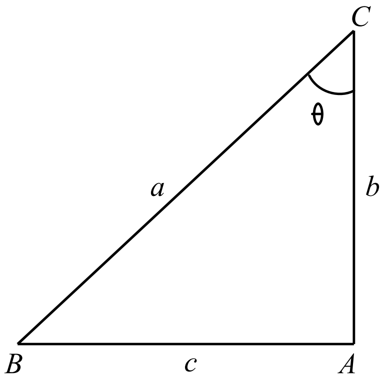
$$\Rightarrow \Delta = \frac{32\cos\theta\sin\theta}{(1+\cos\theta)^2} = \frac{16\sin 2\theta}{(1+\cos\theta)^2}$$

$$\Rightarrow \frac{d}{d\theta}(\Delta) = 16\left\{\frac{2\cos 2\theta\ (1+\cos\theta)^2 + 2\sin\theta(1+\cos\theta)\sin 2\theta}{(1+\cos\theta)^4}\right\}$$

$$\Rightarrow \frac{d}{d\theta}(\Delta) = 32\left\{\frac{\cos 2\theta(1+\cos\theta) + 2\sin^2\theta\cos\theta}{(1+\cos\theta)^3}\right\}$$

$$\Rightarrow \frac{d}{d\theta}(\Delta) = 32\left\{\frac{(2\cos^2\theta - 1)(1+\cos\theta) + 2\cos\theta - 2\cos^3\theta}{(1+\cos\theta)^3}\right\}$$

$$\Rightarrow \frac{d}{d\theta}(\Delta) = \frac{32(2\cos^2\theta + \cos\theta - 1)}{(1+\cos\theta)^3}$$



$$\Rightarrow \frac{d}{d\theta}(\Delta) = \frac{32(2\cos\theta - 1)(\cos\theta + 1)}{(1+\cos\theta)^3} = \frac{32(2\cos\theta - 1)}{(1+\cos\theta)^2}$$

For maximum or minimum, we must have

$$\frac{d}{d\theta}(\Delta) = 0 \Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Clearly, $\frac{d}{d\theta}(\Delta) > 0$ in the left neighbourhood of $\theta = \pi/3$ and $\frac{d}{d\theta}(\Delta) < 0$ in the right neighbourhood of $\theta = \pi/3$

So, Δ maximum when $\theta = \pi/3$

534 **(a)**

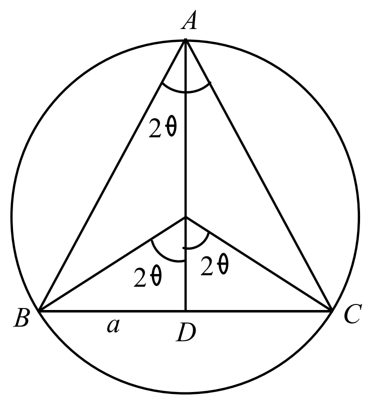
We have,

$AD = a + a\cos 2\theta$ and $BC = 2\,BD = 2\,a\sin 2\theta$.

Therefore, area Δ of the triangle ABC is given by

$$\Delta = \frac{1}{2}BC \times AD = \frac{2}{2}a^2(\sin 2\theta + \sin 2\theta\cos 2\theta)$$

$$\Rightarrow \Delta = a^2\sin 2\theta + \frac{1}{2}a^2\sin 4\theta$$



$\therefore \frac{d\Delta}{d\theta} = 2a^2 \cos 2\theta + 2a^2 \cos 4\theta$

For maximum or minimum, we must have

$\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta \Rightarrow 2\theta = \pi - 4\theta$

$\Rightarrow \theta = \frac{\pi}{6}$

For this value of θ, we find that $\frac{d^2\Delta}{d\theta^2} < 0$

Hence, Δ is maximum for $\theta = \pi/6$

537 **(b)**

The equation of the curve is

$y = x^2 - x$ ...(i)

The abscissae of the points of intersection of (i) and the line $y = 2$ are the roots of the equation

$2 = x^2 - x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1, 2$

$\Rightarrow x = 2$ [$\because$ Point of intersection is in first quadrant]

Putting $x = 2$ in (i), we get $y = 2$

Thus, the required point of intersection is $P(2, 2)$

From (i), we have

$\frac{dy}{dx} = 2x - 1 \Rightarrow \left(\frac{dy}{dx}\right)_P = 4 - 1 = 3$

538 **(c)**

Given, $f(x) = (x^2 + 3x)e^{-(1/2)x}$

$\therefore \quad f'(x) = (x^2 + 3x)\, e^{-(1/2)x}.\left(-\frac{1}{2}\right) + (2x+3)e^{-(1/2)x}$

$= -\frac{1}{2}e^{-(1/2)x}\{x^2 - x - 6\}$

Since, $f(x)$ satisfies the Rolle's theorem

$\therefore \quad f'(c) = 0 \quad \Rightarrow \quad -\frac{1}{2}e^{-(c/2)}(c^2 - c - 6) = 0$

$\Rightarrow \quad c = 3, -2$

But $\quad c = 3 \notin [-3, 0]$

$\therefore \ c = -2$

539 **(c)**

Given, $y^2 = \frac{a^4}{x^2}$

$\Rightarrow \quad 2y\frac{dy}{dx} = \frac{-2a^4}{x^3}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-a^4}{x^3 y}$

At $(-a, a), \frac{dy}{dx} = \frac{-a^4}{-a^3.a} = 1$

Now, length of subtangent to the given curve at $(-a, a)$ is

$\frac{y}{dy/dx} = \frac{a}{1} = a$

540 **(a)**

For y-axis, $x = 0$

$\therefore \quad y = 1 - e^0 = 1 - 1 = 0$

$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}\, e^{x/2} \Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = -\frac{1}{2}$

$\therefore$ Equation of tangent is

$y - 0 = -\frac{1}{2}(x - 0)$

$\Rightarrow \quad x + 2y = 0$

541 **(d)**

Given, $\frac{dV}{dt} = k$ [say]...(i)

$\because \quad V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\therefore \quad \frac{dr}{dt} = \frac{k}{4\pi r^2}$ [from Eq.(i)]

$\Rightarrow$ Rate of increasing radius is inversely proportional to its surface area

542 **(d)**

We have,

$$f(x) = \begin{vmatrix} x-1 & x+1 & 2x+1 \\ x+1 & x+3 & 2x+3 \\ 2x+1 & 2x-1 & 4x+1 \end{vmatrix}$$

$\Rightarrow f(x)$

$$= \begin{vmatrix} x-1 & x+1 & 2x+1 \\ 2 & 2 & 2 \\ 3 & -3 & -1 \end{vmatrix} \begin{bmatrix} \text{Applying } R_2 \to R_2 - R \\ R_3 \to R_3 - 2R_1 \end{bmatrix}$$

$\Rightarrow f(x)$

$$= \begin{vmatrix} x-1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -6 & -7 \end{vmatrix} \begin{bmatrix} \text{Applying } C_2 \to C_2 - C_1 \\ C_3 \to C_3 - 2C_1 \end{bmatrix}$$

$\Rightarrow f(x) = -12(x-1) - 2(-14+6) + 3(-12)$

$\Rightarrow f(x) = -12x - 8$

Clearly, $f'(x) \neq 0$ for any $x \in R$

So, $f(x)$ has no point of maximum or minimum

543 **(d)**

Let length of sector is l and radius of sector is$r$.

$\therefore \quad l = \frac{2\pi r\theta}{360°}$

Perimeter of sector $P = \frac{2\pi r\theta}{360°} + 2r$

$\Rightarrow \quad r = \frac{p}{\left(\frac{2\pi r\theta}{360°} + 2\right)}$

$\because \quad A = \frac{\pi r^2\theta}{360°} = \frac{\pi}{360°}\left[\frac{P^2}{\left(\frac{2\pi\theta}{360°} + 2\right)^2}\right]\theta$

$\Rightarrow \quad A = \frac{\pi P^2}{360°}\left[\frac{\theta}{\left(\frac{2\pi\theta}{360°} + 2\right)^2}\right]$

$$\frac{dA}{d\theta}=\frac{\pi P^2}{360°}\left[\frac{\left(\frac{2\pi\theta}{360°}+2\right)^2-\theta.2\left(\frac{2\pi\theta}{360°}+2\right)\frac{2\pi}{360°}}{\left(\frac{2\pi\theta}{360°}+2\right)^4}\right]$$

For maxima for minima, put $\frac{dA}{d\theta}=0$

$$\left(\frac{2\pi\theta}{360°}+2\right)-\frac{4\theta\pi}{360°}=0$$

$\Rightarrow\ \frac{2\pi\theta}{360°}=2\quad\Rightarrow\quad\theta=2\text{rad}$

Thus, area of sector will be maximum, if sectorial angle is of 2 rad.

544 **(c)**

Let $f(x)=x^3$ and $g(x)=6x^2+15x+5$

It is given that the rate of increase of $f(x)$ is less than that $g(x)$.

Therefore,

$$\frac{d}{dx}(f(x))<\frac{d}{dx}(g(x))$$

$\Rightarrow 3x^2<12x+15$

$\Rightarrow x^2-4x-5<0$

$\Rightarrow (x-5)(x+1)<0\Rightarrow -1<x<5\Rightarrow x\in(-1,5)$

545 **(a)**

If velocity acquired by the particle is proportional to the square root of the distance covered, then its acceleration is a constant

546 **(c)**

We have,

$x=a(\theta+\sin\theta), y=a(1-\cos\theta)$

$$\therefore\ \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{a(1+\cos\theta)}=\tan\frac{\theta}{2}$$

Since the tangent is inclined at an angle $\pi/4$ with x-axis

$$\therefore\ \tan\frac{\theta}{2}=\tan\frac{\pi}{4}\Rightarrow\theta=\frac{\pi}{2}$$

Putting, $\theta=\frac{\pi}{2}$, the point on the curve is $\left(a\left(\frac{\pi}{2}+1\right),a\right)$

547 **(d)**

Given, $f(x)=xe^x$

$\Rightarrow\quad f'(x)=e^x+xe^x$

$\Rightarrow\quad f''(x)=e^x+xe^x+e^x=2e^x+xe^x$

For maxima or minima, put $f'(x)=0$

$\Rightarrow\quad e^x(1+x)=0$

$\Rightarrow\quad x=-1$

At $x=-1,\ f''(x)>0$

$\therefore$ At $x=-1,\ f(x)$ is minimum.

548 **(c)**

Given circumference of a circle $S=2\pi R=56$

$$\Rightarrow\quad R=\frac{28}{\pi}$$

$\therefore\quad Error\ \delta S=2\pi\,\delta r=0.02$

$$\Rightarrow\quad\delta r=\frac{0.02}{2\pi}$$

Let area of circle , $A=\pi r^2$

$$\therefore\text{ Percentage error in }A=\frac{\delta A}{A}\times100$$

$$=2\times\frac{\delta r}{r}\times100$$

$$=2\times\frac{0.02\times\pi}{2\pi\times28}\times100=\frac{1}{14}$$

549 **(c)**

Given, $f(x)=\frac{\sin x}{e^x}$

Here, $f(0)=0,\ f(\pi)=0$

Also. $f(x)$Is continuous in $[0,\pi]$Since, every exponential function and trigonometric function is continuous in their domain and it is differentiable in the open interval.

Now, $f'(x)=\frac{e^x(\cos x-\sin x)}{e^x}$

Put $f'(x)=0$

$$\Rightarrow\cos x-\sin x=0\Longrightarrow x=\frac{\pi}{4}$$

$$\therefore f'\left(\frac{\pi}{4}\right)=0$$

550 **(d)**

Given curve is $xy=c^2$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{-c^2}{x^2}$$

Length of subnormal $=y\frac{dy}{dx}$

$$=\frac{y\times(-c)^2}{x^2}=\frac{-yc^2}{\left(\frac{c^2}{y}\right)^2}\quad\left[\text{from Eq. (i)}, x=\frac{c^2}{y}\right]$$

$$=\frac{-yc^2y^2}{c^4}=\frac{-y^3}{c^2}$$

$\therefore$ Subnormal varies as y^3

551 **(b)**

Given, $y=e^{(2x^2-2a+1)\sin^2 x}$

For minimum or maximum, put $\frac{dy}{dx}=0$

$$\therefore\quad e^{(2x^2-2x+1)\sin^2 x}[(4x-2)\sin^2 x+2(2x^2-2x+1)\sin x\cos x]=0$$

$$\Rightarrow\quad(4x-2)\sin^2 x+2(2x^2-2x+1)\sin x\cos x=0$$

$$\Rightarrow\quad 2\sin x\,[(2x-1)\sin x+(2x^2-2x+1)\cos x]=0$$

$\Rightarrow\quad\sin x=0$

$\therefore\quad y$ is minimum for $\sin x=0$

Thus, minimum value of

$y = e^{(2x^2-2x+1)(0)} = e^0 = 1$

Alternate

Let $f(x) = e^{(2x^2-2x+1)\sin^2 x}$

Since $\sin^2 x \geq 0$

And $2x^2 - 2x + 1 \geq \frac{4(2)(1)-2^2}{4(2)}$ $\left(\because \text{ minima } = \frac{4ac-b^2}{4a}\right)$

$\Rightarrow \quad 2x^2 - 2x + 1 \geq \frac{1}{2}$

$\therefore \quad (2x^2 - 2x + 1)\sin^2 x \geq 0$

$\therefore \quad f(x)_{\min} = e^0 = 1$

552 **(c)**

Given , $s = 48t - 16t^2$

$\Rightarrow \frac{ds}{dt} = 48 - 32t$

At the greatest height, v=$\frac{ds}{dt} = 0$

$\Rightarrow$ 48-32t=0

$\Rightarrow \quad t = \frac{3}{2}$

$\therefore \quad s = 48 \times \frac{3}{2} - 16\left(\frac{3}{2}\right)^2 = 36m$

$\therefore$ Total height = 64 + 36 = 100m

553 **(c)**

Let $f(x) = 2x^3 - 6x + 5$

On differentiating w.r.t. x, we get

$f'(x) = 6x^2 - 6$

Since, it is increasing function

$\Rightarrow \; 6x^2 - 6 > 0$

$\Rightarrow \; (x-1)(x+1) > 0$

$\Rightarrow \; x > 1$ or $x < -1$

554 **(c)**

$\frac{dy}{dx} = a^2 - 3x^2 = 0 \;\Leftrightarrow\; x = \pm\frac{a}{\sqrt{3}}$

Since, $\frac{d^2y}{dx^2} = -6x$, so y is minimum for $x = -\frac{a}{\sqrt{3}}$

Since, $x^2 + x + 2 > 0$ for all$x$, so for $\frac{x^2+x+2}{x^2+5x+6} \leq 0$, we must have $x^2 + 5x + 6 < 0$. If $x = -\frac{a}{\sqrt{3}}$, we have,

$\frac{a^2}{3} - \frac{5a}{\sqrt{3}} + 6 < 0$

$\Rightarrow \; a^2 - 5\sqrt{3}a + 18 < 0$

$\Rightarrow \; (a - 2\sqrt{3})(a - 3\sqrt{3}) < 0$

$\Rightarrow \; a \in (2\sqrt{3}, 3\sqrt{3})$.

555 **(b)**

We have,

$e^{2y} = 1 + 4x^2$

$\Rightarrow e^{2y} \cdot 2\frac{dy}{dx} = 8x$

$\Rightarrow (1 + 4x^2)\frac{dy}{dx} = 4x$

$\Rightarrow \frac{dy}{dx} = \frac{4x}{1+4x^2} \Rightarrow m = \frac{4x}{1+4x^2} \Rightarrow |m|$

$= \frac{4|x|}{1+4|x|^2}$

Now,

A.M. $\geq$ G.M.

$\Rightarrow \frac{1+4|x|^2}{2} \geq \sqrt{4|x|^2}$

$\Rightarrow 1 + 4|x|^2 \geq 4|x|$

$\Rightarrow 1 \geq \frac{4|x|}{1+4|x|^2} \Rightarrow 1 \geq |m| \Rightarrow |m| \leq 1$

556 **(c)**

Function $2^{(x^2-3)^3+27}$ is minimum when $(x^2 - 3)^3 + 27$ is minimum

Clearly, $27 + (x^2-3)^3 = x^2\left\{\left(x^2 - \frac{9}{2}\right)^2\right\} \geq 0$ for all x

Therefore, the minimum value of $(x^2-3)^3 + 27$ is zero for $x = 0$. Hence, the minimum value of the given function is $2^0 = 1$

557 **(a)**

The given curve is $(1 + x^2)y = 2 - x$ ...(i)

It meets x-axis, where $y = 0 \Rightarrow 0 = 2 - x \Rightarrow x = 2$

So, Eq. (i) meets x-axis at the point at the point (2, 0)

Also, from Eq. (i), $y = \frac{2-x}{1+x^2}$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{(1+x^2)(-1) - (2-x)(2x)}{(1+x^2)^2}$

$\Rightarrow \; \frac{dy}{dx} = \frac{x^2 - 4x - 1}{(1+x^2)^2}$

$\therefore$ Slope of tangent at (2, 0)$= \frac{2^2-4(2)-1}{(1+2^2)^2}$

$= \frac{4-8-1}{(1+4)^2} = -\frac{5}{25} = -\frac{1}{5}$

$\therefore$ Equation of tangent at (2, 0) with slope $-\frac{1}{5}$ is

$y - 0 = -\frac{1}{5}(x-2)$

$\Rightarrow \quad 5y = -x + 2$

$\Rightarrow \quad x + 5y = 2$