Session : 2025-26 **AS PER NEW NTA SYLLABUS**

Total Questions : 206

MATHS (QUESTION BANK)

5.CONTINUITY AND DIFFERENTIABILITY

Single Correct Answer Type

1. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,
   a) $\lim_{x\to 0} f(x)$ does not exist
   b) $f(x)$ is continuous at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$
   d) $f'(0) = 1$
2. The value of $f(0)$ so that $\frac{(-e^x+2^x)}{x}$ may be continuous at $x = 0$ is
   a) $\log\left(\frac{1}{2}\right)$ b) 0 c) 4 d) $-1 + \log 2$
3. Let $f(x)$ be an even function. Then $f'(x)$
   a) Is an even function b) Is an odd function c) May be even or odd d) None of these
4. If $f(x) = \begin{cases} [\cos \pi\ x], x < 1 \\ |x-2|,\ 2 > x \geq 1 \end{cases}$, then $f(x)$ is
   a) Discontinuous and non-differentiable at $x = -1$ and $x = 1$
   b) Continuous and differentiable at $x = 0$
   c) Discontinuous at $x = 1/2$
   d) Continuous but not differentiable at $x = 2$
5. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, x \neq -2 \\ 2, \quad x = -2 \end{cases}$, then $f(x)$ is
   a) Continuous at $x = -2$
   b) Not continuous $x = -2$
   c) Differentiable at $x = -2$
   d) Continuous but not derivable at $x = -2$
6. If $f(x) = |\log|x||$, then
   a) $f(x)$ is continuous and differentiable for all x in its domain
   b) $f(x)$ is continuous for all x in its domain but not differentiable at $x = \pm 1$
   c) $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
   d) None of the above
7. If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$ equals
   a) $2a - 4$ b) $4 - 2a$ c) $2a + 4$ d) None of these
8. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then
   a) $f(x)$ is continuous but not differentiable at $x = 0$ b) $f(x)$ is differentiable at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$ d) None of the above
9. If $f(x) = \begin{cases} ax^2 + b,\ b \neq 0, x \leq 1 \\ x^2 b + ax + c,\ \ x > 1 \end{cases}$, then, $f(x)$ is continuous and differentiable at $x = 1$, if
   a) $c = 0, a = 2b$ b) $a = b, c \in R$ c) $a = b\ , c = 0$ d) $a = b, c \neq 0$
10. For the function $f(x) = \begin{cases} |x-3|,\ x \geq 1 \\ \frac{x^2}{4} - \frac{3\,x}{2} + \frac{13}{4},\ x < 1 \end{cases}$ which one of the following is incorrect?

a) Continuous at $x = 1$ b) Derivable at $x = 1$ c) Continuous at $x = 3$ d) Derivable at $x = 3$

11. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x} & \text{, if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Then the value of a so that f is continuous at 0 is

a) 2 b) 1 c) -1 d) 0

12. $f(x) = x + |x|$ is continuous for

a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function

$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\dfrac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \dfrac{\pi}{6} \end{cases}$$

Is continuous at $x = 0$

a) $a = \log_e b, b = \dfrac{2}{3}$ b) $b = \log_e a, a = \dfrac{2}{3}$ c) $a = \log_e b, b = 2$ d) None of these

14. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots + \frac{x^2}{(1+x^2)^n} + \cdots$, then at $x = 0, f(x)$

a) Has no limit

b) Is discontinuous

c) Is continuous but not differentiable

d) Is differentiable

15. Let $f(x) = \begin{cases} 1, & \forall \quad x < 0 \\ 1 + \sin x, & \forall \quad 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

a) 1 b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is

a) Continuous at $x = 1$ b) Discontinuous at $x = 1$

c) Not defined at $x = 1$ d) None of the above

17. If $f(x + y + z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals

a) 12 b) 9 c) 16 d) 6

18. If $f(x) = |\log_e |x||$, then $f'(x)$ equals

a) $\dfrac{1}{|x|}, x \neq 0$

b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$

c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$

19. If the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) 0 c) $\dfrac{1}{2}$ d) -1

20. Function $f(x) = |x - 1| + |x - 2|, x \in R$ is

a) Differentiable everywhere in R

b) Except $x = 1$ and $x = 2$ differentiable everywhere in R

c) Not continuous at $x = 1$ and $x = 2$

d) Increasing in R

21. The set of points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-1, \infty)$ d) None of these

22. If $f(x) = x\sin\left(\frac{1}{x}\right), x \neq 0$, then the value of function at $x = 0$, so that the function is continuous at $x = 0$ is

a) 1 b) -1 c) 0 d) Indeterminate

23. The value of $f(0)$ so that the function $f(x) = \frac{2-(256-7\,x)^{1/8}}{(5x+32)^{1/5}-2}\,(x \neq 0)$ is continuous everywhere, is given by

a) -1 b) 1 c) 26 d) None of these

24. The derivative of $f(x) = |x|^3$ at $x = 0$ is

a) -1 b) 0 c) Does not exist d) None of these

25. If $f(x) = \begin{cases} \frac{(4^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, x \neq 0 \\ 9(\log 4)^3, x = 0 \end{cases}$ is continuous function at $x = 0$, then the value of a is equal to

a) 3 b) 1 c) 2 d) 0

26. $f(x) = |[x] + x|$ in $-1 < x \leq 2$ is

a) Continuous at $x = 0$
b) Discontinuous at $x = 1$
c) Not differentiable at $x = 2, 0$
d) All the above

27. Let$f(x) = [x^3 - x]$, where $[x]$the greatest integer function is. Then the number of points in the interval (1, 2), where function is discontinuous is

a) 4 b) 5 c) 6 d) 7

28. If $y = \cos^{-1}\cos(|x| - f(x))$, where

$f(x) = \begin{cases} 1, if\ x > 0 \\ -1, if\ x < 0 \\ 0, if\ x = 0 \end{cases}$. Then, $(dy/dx)\ x = \frac{5\pi}{4}$ is equal to

a) -1 b) 1
c) 0 d) Cannot be determined

29. Let $f(x + y) = f(x) + f(y)$ and $f(x) = x^2\ g(x)$ for all $x, y \in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(0)$ c) $g(0) + g'(x)$ d) 0

30. Let a function $f(x)$ be defined by $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \in R - Q \end{cases}$ Then, $f(x)$ is

a) Everywhere continuous
b) Nowhere continuous
c) Continuous only at some points
d) Discontinuous only at some points

31. The function $f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \cdots \text{ to } \infty, & x \neq -1 \\ 1, & x = -1 \end{cases}$ is

a) Continuous and derivable at $x = -1$
b) Neither continuous nor derivable at $x = -1$
c) Continuous but not derivable at $x = -1$
d) None of these

32. $f(x) = \begin{cases} 2a - x & \text{in } -a < x < a \\ 3x - 2a & \text{in } a \leq x \end{cases}$. Then, which of the following is true?

a) $f(x)$ is discontinuous at $x = a$ b) $f(x)$ is not differentiable at $x = a$
c) $f(x)$ is differentiable at $x \geq a$ d) $f(x)$ is continuous at all $x < a$

33. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2\,x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

a) $f(x)g(0)$ b) $2f(x)g(0)$ c) $2g(0)$ d) None of these

34. If $f(x) = [x\sin\pi\ x]$, then which of the following is incorrect?

a) $f(x)$ is continuous at $x = 0$
b) $f(x)$ is continuous in $(-1, 0)$
c) $f(x)$ is differentiable at $x = 1$

d) $f(x)$ is differentiable in $(-1, 1)$

35. If $f(x) = \begin{cases} 1, x < 0 \\ 1 + \sin x, 0 \le x \le \frac{\pi}{2} \end{cases}$ then derivative of $f(x)$ at $x = 0$

a) Is equal to 1 b) Is equal to 0 c) Is equal to -1 d) Does not exist

36. If the derivative of the function $f(x)$ is everywhere continuous and is given by $f(x) = \begin{cases} bx^2 + ax + 4; & x \ge -1 \\ ax^2 + b; & x < -1 \end{cases}$, then

a) $a = 2, b = -3$ b) $a = 3, b = 2$ c) $a = -2, b = -3$ d) $a = -3, b = -2$

37. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \ne 0 \\ 0, & x = 0 \end{cases}$, then

a) $f(x)$ is not continuous at $x = 0$

b) $f(x)$ is not continuous and differentiable at $x = 0$

c) $f(x)$ is not continuous at $x = 0$ but not differentiable at $x = 0$

d) None of these

38. If the function $f(x) = \begin{cases} Ax - B, & x \le 1 \\ 3x, & 1 < x < 2 \\ B\,x^2 - A, & x \ge 2 \end{cases}$ be continuous at $x = 1$ and discontinuous at $x = 2$, then

a) $A = 3 + B, B \ne 3$ b) $A = 3 + B, B = 3$ c) $A = 3 + B$ d) None of these

39. If $f(x) = \begin{cases} |x - 4|, \text{for } x \ge 1 \\ (x^3/2) - x^2 + 3x + (1/2), \text{for } x < 1 \end{cases}$, then

a) $f(x)$ is continuous at $x = 1$ and $x = 4$

b) $f(x)$ is differentiable at $x = 4$

c) $f(x)$ is continuous and differentiable at $x = 1$

d) $f(x)$ is not continuous at $x = 1$

40. The function $f(x) = a[x + 1] + b[x - 1]$, where $[x]$ is the greatest integer function, is continuous at $x = 1$, is

a) $a + b = 0$ b) $a = b$ c) $2a - b = 0$ d) None of these

41. Let $f(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \ge 0 \end{cases}$ and $\lambda \in R$, then at $x = 0$

a) f is discontinuous b) f is continuous only, if $\lambda = 0$

c) f is continuous only, whatever λ may be d) None of the above

42. If for a continuous function f, $f(0) = f(1) = 0, f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to

a) 1 b) 2 c) 0 d) None of these

43. If $f(x) = \begin{cases} ax^2 - b, |x| < 1 \\ \frac{1}{|x|}, & |x| \ge 1 \end{cases}$ is differentiable at $x = 1$, then

a) $a = \frac{1}{2}, b = -\frac{1}{2}$ b) $a = -\frac{1}{2}, b = -\frac{3}{2}$ c) $a = b = \frac{1}{2}$ d) $a = b = -\frac{1}{2}$

44. Let $f(x) = \frac{\sin 4\pi [x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then

a) $f(x)$ is not differentiable at some points

b) $f'(x)$ exists but is different from zero

c) $f'(x) = 0$ for all x

d) $f'(x) = 0$ but f is not a constant function

45. The value of k which makes $f(x) = \begin{cases} \sin(1/k), & x \ne 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$ is

a) 8 b) 1 c) -1 d) None of these

46. The function $f(x) = \max[(1 - x), (1 + x), 2], x \in (-\infty, \infty)$ is

a) Continuous at all points b) Differentiable at all points

c) Differentiable at all points except at $x = 1$ and $x = -1$ d) None of the above

47. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then,

a) $f(x)$ is bounded b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ c) $xf(x) \to 1$ as $x \to 0$ d) $f(x) = \ln x$

48. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x+y) = f(x)f(y)$ (ii) $f(x) = 1 + x\,g(x)\log a$, where $a > 1$ and $\lim_{x\to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $\log a$ b) $\log a^{f(x)}$ c) $\log(f(x))^a$ d) None of these

49. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

a) $\dfrac{1}{1+\left(g(x)\right)^3}$ b) $\dfrac{1}{1+\left(f(x)\right)^3}$ c) $1+\left(g(x)\right)^3$ d) $1+\left(f(x)\right)^3$

50. If $f(x) = |x^2 - 4x + 3|$, then

a) $f'(1) = -1$ and $f'(3) = 1$
b) $f'(1) = -1$ and $f'(3)$ does not exist
c) $f'(1) = -1$ does not exist and $f'(3) = 1$
d) Both $f'(1)$ and $f'(3)$ do not exist

51. The points of discontinuity of $\tan x$ are

a) $n\pi, n \in I$ b) $2n\pi, n \in I$ c) $(2n+1)\dfrac{\pi}{2}, n \in I$ d) None of these

52. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)

a) $0, \pm 1$ b) ± 1 c) 0 d) 1

53. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$. Then

a) $f(x)$ is continuous at $x = 0$ b) $f(|x|)$ is continuous at $x = 0$ c) $f(x)$ is discontinuous at $x = 0$ d) None of the above

54. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,

a) $f(x)$ is continuous on R^+
b) $f(x)$ is continuous on R
c) $f(x)$ is continuous on $R - Z$
d) None of these

55. The function $f(x) = \frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is

a) -1/2 b) ½ c) -1 d) 1

56. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?

a) f is differentiable at $x = 1$ but not at $x = 0$
b) f is neither differentiable at $x = 0$ nor at $x = 1$
c) f is differentiable at $x = 0$ and at $x = 1$
d) f is differentiable at $x = 0$ but not at $x = 1$

57. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$ b) $m = \dfrac{n\pi}{2} + 1$ c) $n = \dfrac{m\pi}{2}$ d) $m = n = \dfrac{\pi}{2}$

58. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

59. If $\lim_{x\to a^+} f(x) = l = \lim_{x\to a^-} g(x)$ and $\lim_{x\to a^-} f(x) = m \lim_{x\to a^+} g(x)$, then the function $f(x)\,g(x)$

a) Is not continuous at $x = a$
b) Has a limit when $x \to a$ and it is equal to lm

c) Is continuous at $x = a$
d) Has a limit when $x \to a$ but it is not equal to lm

60. Let $f(x)$ be a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) = 1 + x\,g\,(x)$ where $\lim_{x\to 0} g(x) = 1$. Then, $f'(x)$ is equal to
a) $g'(x)$ b) $g(x)$ c) $f(x)$ d) None of these

61. The set of points where the function $f(x) = x|x|$ is differentiable is
a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $[0, \infty)$

62. If $f(x+y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is
a) 30 b) 13 c) 10 d) 0

63. If $f(x) = |x-a|\phi(x)$, where $\phi(x)$ is continuous function, then
a) $f'(a^+) = \phi(a)$ b) $f'(a^-) = \phi(a)$ c) $f'(a^+) = f'(a^-)$ d) None of these

64. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
a) Continuous as well as differentiable for all x
b) Continuous for all x but not differentiable at $x = 0$
c) Neither differentiable nor continuous at $x = 0$
d) Discontinuous everywhere

65. If $f(x) = \begin{cases} 3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$, then
a) Both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$
b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$
c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$
d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$

66. If $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists finitely, then
a) $\lim_{x\to c} f(x) = f(c)$
b) $\lim_{x\to c} f'(x) = f'(c)$
c) $\lim_{x\to c} f(x)$ does not exist
d) $\lim_{x\to c} f(x)$ may or may not exist

67. The number of points at which the function $f(x) = |x-0.5| + |x-1| + \tan x$ does not have a derivative in the interval $(0, 2)$, is
a) 1 b) 2 c) 3 d) 4

68. If $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to
a) -2 b) 2 c) 1 d) -1

69. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x\to c} f(x)$ equals
a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these

70. If $f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are
a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$

71. Let $f(x) = (x + |x|)|x|$. The, for all x
a) f and f' are continuous
b) f is differentiable for some x
c) f' is not continuous
d) f'' is continuous

72. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$

73. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

a) 6 b) 5 c) 4 d) 3

74. If $f: R \to R$ is defined by

$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1,-2\} \\ -1, & \text{if } x = -2 \\ 0, & \text{if } x = -1 \end{cases}$, then f is continuous on the set

a) R b) $R - \{-2\}$ c) $R - \{-1\}$ d) $R - (-1,-2)$

75. Let $f(x) = \frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$ for $x \neq 0$ and $f(0) = 12$. If f is continuous at $x = 0$, then the value of a is equal to

a) 1 b) -1 c) 2 d) 3

76. If a function $f(x)$ is given by $f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots \infty$ then at $x = 0, f(x)$

a) Has no limit
b) Is not continuous
c) Is continuous but not differentiable
d) Is differentiable

77. If $f(x)$ is continuous function and $g(x)$ be discontinuous, then

a) $f(x) + g(x)$ must be continuous
b) $f(x) + g(x)$ must be discontinuous
c) $f(x) + g(x)$ for all x
d) None of these

78. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$, then $f'(x)$ equals

a) $f(x)$ b) $-f(x)$ c) $2f(x)$ d) None of these

79. Consider $f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

a) $f(x)$ is discontinuous everywhere
b) $f(x)$ is continuous everywhere
c) $f'(x)$ exists in $(-1, 1)$
d) $f'(x)$ exists in $(-2, 2)$

80. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then

$\lim_{x\to 0}\frac{\int_0^x f(u)du}{x}$ is

a) 0 b) 2 c) $f(2)$ d) None of these

81. Let $f(x+y) = f(x)f(y)$ for all $x, y \in R$. Suppose that $f(3) = 3$ and $f'(0) = 11$ then, $f'(3)$ is equal to

a) 22 b) 44 c) 28 d) None of these

82. If $f(x) = \begin{cases} x-5, & \text{for } x \leq 1 \\ 4x^2-9, & \text{for } 1 < x < 2 \\ 3x+4, & \text{for } x \geq 2 \end{cases}$, then $f'(2^+)$ is equal to

a) 0 b) 2 c) 3 d) 4

83. $f(x) = \sin|x|$. Then $f(x)$ is not differentiable at

a) $x = 0$ only b) All x c) Multiples of π d) Multiples of $\frac{\pi}{2}$

84. If $f(x) = \sum_{n=0}^{\infty}\frac{x^n}{n!}(\log_e a)^n, a > 0, a \neq 0$, then at $x = 0, f(x)$ is

a) Everywhere continuous but not differentiable
b) Everywhere differentiable
c) Nowhere continuous
d) None of these

85. The function $f(x) = [x]\cos\left[\frac{2x-1}{2}\right]\pi$ where [.] denotes the greatest integer function, is discontinuous at

a) All x
b) No x
c) All integer points
d) x which is not an integer

86. The function $f(x) = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2, 3, \ldots \\ 0, & x = 0 \end{cases}$

a) Is discontinuous at finitely many points
b) Is continuous everywhere
c) Is discontinuous only at $x = \pm\frac{1}{n}, n \in Z - \{0\}$ and $x = 0$
d) None of these

87. Let f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2,\ x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

a) 1
b) 2
c) 0
d) -1

88. Let $f(x) = [2x^3 - 5]$, [] denotes the greatest integer function. Then number of points (1, 2) where the function is discontinuous, is

a) 0
b) 13
c) 10
d) 3

89. ln[1, 3] the function $[x^2 + 1]$, $[x]$ denoting the greatest integer function, is continuous

a) For all x
b) For all x except at four points
c) For all except at seven points
d) For all except at eight-points

90. If $f(x) = |\log_{10} x|$, then at $x = 1$

a) $f(x)$ is continuous and $f'(1^+) = \log_{10} e, f'(1^-) = -\log_{10} e$
b) $f(x)$ is continuous and $f'(1^+) = \log_{10} e, f'(1^-) = \log_{10} e$
c) $f(x)$ is continuous and $f'(1^-) = \log_{10} e, f'(1^+) = -\log_{10} e$
d) None of these

91. The function $f(x) = |\cos x|$ is

a) Everywhere continuous and differentiable
b) Everywhere continuous and but not differentiable at $(2n+1)\pi/2, n \in Z$
c) Neither continuous nor differentiable at $(2n+1)\pi/2, n \in Z$
d) None of these

92. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$

Then, $f(x)$ is continuous at $x = 4$ when

a) $a = 0, b = 0$
b) $a = 1, b = 1$
c) $a = -1, b = 1$
d) $a = 1, b = -1$

93. If $f(x) = \begin{cases} \frac{2^x - 1}{\sqrt{1+x} - 1}, -1 \leq x < \infty,\ x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous everywhere, then k is equal to

a) $\frac{1}{2}\log_e 2$
b) $\log_e 4$
c) $\log_e 8$
d) $\log_e 2$

94. The function $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0,\ x = 0 \end{cases}$ is continuous and differentiable at $x = 0$, if

a) $n \in (0, 1]$
b) $n \in [1, \infty)$
c) $n \in (1, \infty)$
d) $n \in (-\infty, 0)$

95. The function $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, x \neq 0 \\ 0,\ x = 0 \end{cases}$

a) Is continuous at $x = 0$
b) Is not continuous at $x = 0$

c) Is not continuous at $x = 0$, but can be made continuous $x = 0$
d) None of these

96. A function $f(x) = \begin{cases} 1 + x, & x \le 2 \\ 5 - x, & x > 2 \end{cases}$ is
a) Not continuous at $x = 2$
b) Differenti8able at $x = 2$
c) Continuous but not differentiable at $= 2$
d) None of the above

97. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to
a) 4
b) 1
c) 1/2
d) 8

98. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in Z \\ x^2, & x \in R - Z \end{cases}$ Then, which one of the following is incorrect?
a) $\lim_{x \to 1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$
b) $\lim_{x \to 1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$
c) gof is continuous for all x
d) fog is continuous for all x

99. If $f(x) = \begin{cases} x, & \text{for} \quad 0 < x < 1 \\ 2 - x, & \text{for} \quad 1 \le x < 2 \\ x - (1/2)x^2, & \text{for } x = 2 \end{cases}$.Then, $f'(1)$ is equal to
a) -1
b) 1
c) 0
d) None of these

100. The function $f(x) = |x| + \frac{|x|}{x}$ is
a) Discontinuous at origin because $|x|$ is discontinuous there
b) Continuous at origin
c) Discontinuous at origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there
d) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there

101. $f(x) = |x - 3|$ is ... at $x = 3$
a) Continuous and not differentiable
b) Continuous and differentiable
c) Discontinuous and not differentiable
d) Discontinuous and differentiable

102. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is
a) Continuous
b) Discontinuous
c) Differentiable
d) Non-zero

103. The following functions are differentiable on $(-1, 2)$
a) $\int_x^{2x} (\log t)^2 dt$
b) $\int_x^{2x} \frac{\sin t}{t} dt$
c) $\int_x^{2x} \frac{1 - t + t^2}{1 + t + t^2} dt$
d) None of these

104. Let$f(x) = \frac{1-\tan x}{4x-\pi}, x \ne \frac{\pi}{4}, \ x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is
a) 1
b) 1/2
c) -1/2
d) -1

105. If $f(x) = \begin{cases} \frac{1-\cos x}{x}, x \ne 0 \\ k, \quad x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
a) 0
b) $\frac{1}{2}$
c) $\frac{1}{4}$
d) $-\frac{1}{2}$

106. Let $f(x) = |x| + |x - 1|$, then
a) $f(x)$ is continuous at $x = 0$, as well as at $x = 1$
b) $f(x)$ is continuous at $x = 0$, but not at $x = 1$
c) $f(x)$ is continuous at $x = 1$, but not at $x = 0$
d) None of these

107. The function $f(x)$ is defined as$f(x) = \frac{2x-\sin^{-1} x}{2x+\tan^{-1} x}$, if $x \ne 0$. The value of f to be assigned at $x = 0$ so that the function is continuous there, is
a) $-\frac{1}{3}$
b) 1
c) $\frac{2}{3}$
d) $\frac{1}{3}$

108. Let $f(x)$ be an odd function. Then $f'(x)$

a) Is an even function b) Is an odd function c) May be even or odd d) None of these

109. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) -13 d) 1/3

110. If $f: R \to R$ given by

$f(x) = \begin{cases} 2\cos x, \text{if } x \leq -\frac{\pi}{2} \\ a + \sin x + b, \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 + \cos^2 x, \text{ if } x \geq \frac{\pi}{2} \end{cases}$ is a continuous

Function on R, then (a, b) is equal to

a) (1/2, 1/2) b) (0, -1) c) (0, 2) d) (1, 0)

111. If $f(x+y) = f(x)f(y)$ for all $x, y \in R, f(5) = 2, f'(0) = 3$. Then $f'(5)$ equals

a) 6 b) 3 c) 5 d) None of these

112. Let $f(x)$ be a function satisfying $f(x+y) = f(x) + f(y)$ and $f(x) = x\, g\,(x)$ for all $x, y \in R$, where $g(x)$ is continuous. Then,

a) $f'(x) = g'(x)$ b) $f'(x) = g(x)$ c) $f'(x) = g(0)$ d) None of these

113. If $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2\,x-4}}$, then $f(x)$ is differentiable on

a) $(-\infty, \infty)$ b) $[2, \infty) - \{4\}$ c) $[2, \infty)$ d) None of these

114. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

a) f and f' are continuous at $x = 0$

b) f is derivable at $x = 0$ and f' is continuous at $x = 0$

c) f is derivable at $x = 0$ and f' is not continuous at $x = 0$

d) f' is derivable at $x = 0$

115. If a function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x}{\sqrt{x^2}}, x \neq 0 \\ 0, & x = 0 \end{cases}$ then

a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

b) $f(x)$ is continuous as well as differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d) None of these

116. If $f(x) = [\sqrt{2}\sin x]$, where $[x]$ represents the greatest integer function, then

a) $f(x)$ is periodic

b) Maximum value of $f(x)$ is 1 in the interval $[-2\,\pi, 2\,\pi]$

c) $f(x)$ is discontinuous at $x = \frac{n\,\pi}{2} + \frac{\pi}{4}, n \in Z$

d) $f(x)$ is differentiable at $x = n\,\pi, n \in Z$

117. $\lim_{x \to 0}[(1+3x)^{1/x}] = k$, then for continuity at $x = 0, k$ is

a) 3 b) -3 c) e^3 d) e^{-3}

118. Let $f(x) = \begin{cases} \int_0^x \{5 + |1-t|\}dt, \text{if } x > 2 \\ 5x + 1, \text{if } x \leq 2 \end{cases}$

a) $f(x)$ is continuous at $x = 2$

b) $f(x)$ is continuous but not differentiable at $x = 2$

c) $f(x)$ is everywhere differentiable

d) The right derivative of $f(x)$ at $x = 2$ does not exist

119. Let $f(x) = \begin{cases} \frac{1}{|x|} \text{ for } |x| \geq 1 \\ ax^2 + b \text{ for } |x| < 1 \end{cases}$

If $f(x)$ is continuous and differentiable at any point, then

a) $a = \frac{1}{2}, b = -\frac{3}{2}$ b) $a = -\frac{1}{2}, b = \frac{3}{2}$ c) $a = 1, b = -1$ d) None of these

120. If function $f(x) = \begin{cases} x, \text{if } x \text{ is rational} \\ 1 - x, \text{ if } x \text{ is irrational} \end{cases}$, then the number of points at which $f(x)$ is continuous, is

a) ∞ b) 1 c) 0 d) None of these

121. The function $f(x) = e^{-|x|}$ is

a) Continuous everywhere but not differentiable at $x = 0$ b) Continuous and differentiable everywhere

c) Not continuous at $x = 0$ d) None of the above

122. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

Becomes continuous for all x, is given by

a) $a^{3/2}$ b) $a^{1/2}$ c) $-a^{1/2}$ d) $-a^{3/2}$

123. The value of k for which the function

$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, x \neq 0 \\ k \quad x = 0 \end{cases}$ is continuous at $a = 0$, is

a) $k = 0$ b) $k = 1$ c) $k = -1$ d) None of these

124. The number of points at which the function $f(x) = (|x - 1| + |x - 2| + \cos x)$ where $x \in [0, 4]$ is not continuous, is

a) 1 b) 2 c) 3 d) 0

125. If $f(x) = \begin{cases} x \sin\frac{1}{x}, \ x \neq 0 \\ k, \quad x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) -1 c) 0 d) 2

126. Let $f(x)$ be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x), h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to

a) 22 b) 11 c) 0 d) None of these

127. if $f(x) = |x|^3$, then $f'(0)$ equals

a) 0 b) 1/2 c) -1 d) -1/2

128. Let function $f(x) = \sin^{-1}(\cos x)$, is

a) Discontinuous at $x = 0$

b) Continuous at $x = 0$

c) Differentiable at $x = 0$

d) None of these

129. Let $f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, x \neq 1, 2 \\ 6, \ x = 10 \\ 12, \quad x = 2 \end{cases}$ Then, $f(x)$ is continuous on the set

a) R b) $R - \{1\}$ c) $R - \{2\}$ d) $R - \{1, 2\}$

130. The set of points, where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

a) $(-\infty, -1) \cup (-1, \infty)$ b) $(-\infty, \infty)$ c) $(0, \infty)$ d) $(-\infty, 0) \cup (0, \infty)$

131. Given $f(0) = 0$ and $f(x) = \frac{1}{(1-e^{-1/x})}$ for$x \neq 0$. Then only one of the follo0wing statements on $f(x)$ is true. That id $f(x)$, is

a) Continuous at $x = 0$

b) Not continuous at $x = 0$

c) Both continuous and differentiable at $x = 0$
d) Not defined at $x = 0$

132. Let f and g be differentiable functions satisfying $g'(a) = 2, g(a) = b$ and $fog = I$ (identify function). Then, $f'(b)$ is equal to
a) $1/2$ b) 2 c) $2/3$ d) None of these

133. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, if $f(x)$ is continuous at $x = 0$, then k is equal to
a) $\frac{\pi}{5}$ b) $\frac{5}{\pi}$ c) 1 d) 0

134. The number of discontinuities of the greatest integer function $f(x) = [x], x \in \left(-\frac{7}{2}, 100\right)$ is equal to
a) 104 b) 100 c) 102 d) 103

135. For the function$f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}, x = 0$, which of the following is correct?
a) $\lim_{x\to 0} f(x)$ does not exist
b) $\lim_{x\to 0} f(x) = 1$
c) $\lim_{x\to 0} f(x)$ exists but $f(x)$ is not continuous at $x = 0$
d) $f(x)$ is continuous at $x = 0$

136. If $f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \cdots$ to ∞ then at $x = 0, f(x)$ is
a) Continuous but not differentiable
b) Differentiable
c) Continuous
d) None of these

137. If $f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$ then the set of points of discontinuity of $g(x) = fof(x)$, is
a) $\{1, 2\}$ b) $\{0, 1, 2\}$ c) $\{0, 1\}$ d) None of these

138. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$, then $g'(f(c))$ equals
a) $f'(c)$ b) $\frac{1}{f'(c)}$ c) $f(c)$ d) None of these

139. If $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at $x = 0$, then
a) $p < 0$ b) $0 < p < 1$ c) $p = 1$ d) $p > 1$

140. At $x = 0$, the function $f(x) = |x|$ is
a) Continuous but not differentiable
b) Discontinuous and differentiable
c) Discontinuous and not differentiable
d) Continuous and differentiable

141. If $f(x) = \begin{cases} (x-2)^2 \sin\left(\frac{1}{x-2}\right) - |x-1|, & x \neq 2 \\ -1, & x = 2 \end{cases}$ then the set of points where $f(x)$ is differentiable, is
a) R b) $R - \{1, 2\}$ c) $R - \{1\}$ d) $R - \{2\}$

142. The value of f at $x = 0$ so that function $f(x) = \frac{2^x - 2^{-x}}{x}, x \neq 0$ is continuous at $x = 0$, is
a) 0 b) $\log 2$ c) 4 d) $\log 4$

143. If $f(x) = |\log_e x|$, then
a) $f'(1^+) = 1, f'(1^-) = -1$
b) $f'(1^-) = -1, f'(1^+) = 0$
c) $f'(1) = 1, f'(1^-) = 0$
d) $f'(1) = -1, f'(1^+) = -1$

144. Let $f(x)$ be a function such that $f(x + y) = f(x) + f(y)$ and $f(x) = \sin x\, g(x)$ for all $x, y \in R$. If $g(x)$ is a continuous function such that $g(0) = k$, then $f'(x)$ is equal to
a) k b) kx c) $kg(x)$ d) None of these

145. The function $f(x) = |x| + |x-1|$, is
a) Continuous at $x = 1$, but not differentiable
b) Both continuous and differentiable at $x = 1$
c) Not continuous at $x = 1$
d) None of these

146. The set of points of differentiability of the function $f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{x}, \text{for } x \neq 0 \\ 0, \text{for } x = 0 \end{cases}$ is
a) R b) $[0, \infty]$ c) $(-\infty, 0)$ d) $R - \{0\}$

147. Given that $f(x)$ is a differentiable function of x and that $f(x).f(y) = f(x) + f(y) + f(xy) - 2$ and that $f(2) = 5$. Then, $f(3)$ is equal to
a) 10 b) 24 c) 15 d) None of these

148. If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$,
a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous
b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous
c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
d) $\tan[f(x)]$ s continuous but $\frac{1}{f(x)}$ is not

149. If $f(x) = (x+1)^{\cot x}$ be continuous at $= 0$, then $f(0)$ is equal to
a) 0 b) $-e$ c) e d) None of these

150. Let $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \frac{\pi}{4}}, & x \neq \frac{\pi}{4} \\ a, & x = \frac{\pi}{4} \end{cases}$ the value of a so that $f(x)$ is continuous at $x = \frac{\pi}{4}$ is
a) 2 b) 4 c) 3 d) 1

151. If $f(x) = \int_{-1}^{x} |t|\, dt, x \geq -1$, then
a) f and f' are continuous for $x + 1 > 0$
b) f is continuous but f' is not so for $x + 1 > 0$
c) f and f' are continuous at $x = 0$
d) f is continuous at $x = 0$ but f' is not so

152. The set of points of discontinuity of the function
$f(x) = \lim_{n \to \infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}, n \in Z$ is
a) $\{1\}$ b) $\{-1\}$ c) $\{-1, 1\}$ d) None of these

153. The number of points of discontinuity of the function
$f(x) = \frac{1}{\log|x|}$, is
a) 4 b) 3 c) 2 d) 1

154. $f(x) = \begin{cases} \frac{\sin 3x}{\sin x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous, if k is
a) 3 b) 0 c) -3 d) -1

155. For the function $f(x) = \frac{\log_e(1+x) + \log_e(1-x)}{x}$ to be continuous at $= 0$, the value of $f(0)$ is
a) -1 b) 0 c) -2 d) 2

156. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases}$
Then, $f(x)$ is continuous at $x = 4$, when
a) $a = 0, b = 0$ b) $a = 1, b = 1$ c) $a = -1, b = 1$ d) $a = 1, b = -1$

157. If $f(x)\begin{cases}\frac{[x]-1}{x-1}, x \neq 1\\ 0, \quad x = 1\end{cases}$ then at $x = 1, f(x)$ is

a) Continuous and differentiable

b) Differentiable but not continuous

c) Continuous but not differentiable

d) Neither continuous nor differentiable

158. If $f(x) = \begin{cases}\frac{1-\sqrt{2}\sin x}{\pi-4x}, \text{if } x \neq \frac{\pi}{4}\\ a, \quad \text{if } x = \frac{\pi}{4}\end{cases}$ is continuous at $\frac{\pi}{4}$, then a is equal to

a) 4 b) 2 c) 1 d) 1/4

159. If the function $f: R \to R$ given by $f(x) = \begin{cases} x + a, \text{if } x \leq 1\\ 3 - x^2, \text{ if } x > 1\end{cases}$ is continuous at $x = 1$, thyen a is equal to

a) 4 b) 3 c) 2 d) 1

160. If $f: R \to R$ is defined by

$f(x) = \begin{cases}\frac{\cos 3x - \cos x}{x^2}, \quad \text{for } x \neq 0\\ \lambda, \quad \text{for } x = 0\end{cases}$ and if f is continuous at $x = 0$, then λ is equal to

a) -2 b) -4 c) -6 d) -8

161. For the function $f(x) = \begin{cases}\frac{x^3-a^3}{x-a}, x \neq a\\ b, \quad x = a\end{cases}$, if $f(x)$ is continuous at $x = a$, then b is equal to

a) a^2 b) $2a^2$ c) $3a^2$ d) $4a^2$

162. If $y = f(x) = \frac{1}{u^2+u-1}$ where $u = \frac{1}{x-1}$, then the function is discontinuous at $x =$

a) 1 b) 1/2 c) 2 d) -2

163. If $f(x) = \text{Min}\{\tan x, \cot x\}$, then

a) $f(x)$ is not differentiable at $x = 0, \pi/4, 5\pi/4$

b) $f(x)$ is continuous at $x = 0, \pi/2, 3\pi/2$

c) $\int_0^{\pi/2} f(x)dx = \ln\sqrt{2}$

d) $f(x)$ is periodic with period $\frac{\pi}{2}$

164. If $f(x) = \{|x| - |x-1|\}^2$, then $f'(x)$ equals

a) 0 for all x

b) $2\{|x| - |x-1|\}$

c) $\begin{cases}0 \text{ for } x < 0 \text{ and for } x > 1\\ 4(2x-1) \text{for } 0 < x < 1\end{cases}$

d) $\begin{cases}0 \text{ for } x < 0\\ 4(2x-1) \text{for } x > 0\end{cases}$

165. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then $f'(x_0)$ is equal to

a) $\phi'(x_0)$ b) $\phi(x_0)$ c) $x_0\phi(x_0)$ d) None of these

166. The function defined by

$f(x) = \begin{cases}\left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} \quad x \neq 2\\ k, \quad x = 2\end{cases}$ is continuous from right at the point $x = 2$, then k is equal to

a) 0 b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) None of these

167. If $f(x) = \begin{cases}\frac{1-\sin x}{(\pi-2x)^2} \cdot \frac{\log \sin x}{(\log 1+\pi^2-4\pi x+x^2)}, x \neq \frac{\pi}{2}\\ k, \ x = \frac{\pi}{2}\end{cases}$ is continuous at $x = \pi/2$, then $k =$

a) $-\frac{1}{16}$ b) $-\frac{1}{32}$ c) $-\frac{1}{64}$ d) $-\frac{1}{28}$

168. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) -2 c) 2 d) $\frac{1}{2}$

169. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (-\infty, 0)$ d) $n = 0$

170. The function $f(x) = \begin{cases} |x-3|, & \text{if } x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & \text{if } x < 1 \end{cases}$ is

a) Continuous and differentiable at $x = 3$
b) Continuous at $x = 3$, but not differentiable at $x = 3$
c) continuous and differentiable everywhere
d) continuous at $x = 1$, but not differentiable at $x = 1$

171. Let $f(x) = |x|$ and $g(x) = |x^3|$, then

a) $f(x)$ and $g(x)$ Both are continuous at $x = 0$
b) $f(x)$ and $g(x)$ Both are differentiable at $x = 0$
c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$
d) $f(x)$ and $g(x)$ Both are not differentiable at $x = 0$

172. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, x < 0 \\ c, \ x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx\sqrt{x}}, x > 0 \end{cases}$ is continuous at $x = 0$, then

a) $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$
b) $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$
c) $a = -\frac{3}{2}, b \in R - \{0\}, c = \frac{1}{2}$
d) None of these

173. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, x \neq 0 \\ k, \ x = 0 \end{cases}$ is continuous at $x = 0$, then k equals

a) $16\sqrt{2}\log 2 \log 3$ b) $16\sqrt{2}\ln 6$ c) $16\sqrt{2}\ln 2 \ln 3$ d) None of these

174. Let [ ] denotes the greatest integer function and $f(x) = [\tan^2 x]$. Then,

a) $\lim_{x \to 0} f(x)$ does not exist b) $f(x)$ is continuous at $x = 0$
c) $f(x)$ is not differentiable at $x = 0$ d) $f(x) = 1$

175. Let a function $f: R \to R$, where R is the set of real numbers satisfying the equation $f(x+y) = f(x) + f(y), \forall\, x, y$ if $f(x)$ is continuous at $x = 0$, then

a) $f(x)$ is discontinuous, $\forall\, x \in R$ b) $f(x)$ is continuous, $\forall\, x \in R$
c) $f(x)$ is continuous for $x \in \{1, 2, 3, 4\}$ d) None of the above

176. Let $f(x) = \begin{cases} \sin x, & \text{for } x \geq 0 \\ 1 - \cos x, & \text{for } x \leq 0 \end{cases}$ and $g(x) = e^x$. Then, $(gof)'(0)$ is

a) 1 b) -1 c) 0 d) None of these

177. The function $f(x) \begin{cases} (x+1)^{2-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, x \neq 0 \\ 0, \qquad x = 0 \end{cases}$ is

a) Continuous everywhere
b) Discontinuous at only one point
c) Discontinuous at exactly two points
d) None of these

178. If $f(x) = \begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x}, x \neq 0 \\ k, \ x = 0 \end{cases}$ and $f(x)$ is continuous at $x = 0$, then the value of k is

a) $a - b$ b) $a + b$ c) $\log a + \log b$ d) None of these

179. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^{1/3}-3}{9-3(243+5\,x)^{1/5}} (x \neq 0)$ is continuous is given by

a) $\frac{2}{3}$ b) 6 c) 2 d) 4

180. The function $f: R/\{0\} \to R$ given by

$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$

Can be made continuous at $x = 0$ by defining $f(0)$ as function

a) 2 b) -1 c) 0 d) 1

181. Which one of the following is not true always?

a) If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$

b) If $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$

c) If $f(x)$ and $g(x)$ are differentiable at $x = a$, then $f(x) + g(x)$ is also differentiable at $x = a$

d) If a function $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x)$ exists

182. The value of the derivative of $|x - 1| + |x - 3|$ at $x = 2$ is

a) 2 b) 1 c) 0 d) -2

183. On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x+1)\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, \ x \neq 0 \\ 0, \quad x = 0 \end{cases}$

a) Is continuous for all $x \in I - \{0\}$

b) Assumes all intermediate values from $f(-2)$ to $f(2)$

c) Has a maximum value equal to $3/e$

d) All the above

184. Function $f(x) = \begin{cases} x - 1, x < 2 \\ 2x - 3, x \geq 2 \end{cases}$ is a continuous function

a) For $x = 2$ only b) For all real values of x such that $x \neq 2$

c) For all real values of x d) For all integer values of x only

185. The function $f(x) = \begin{cases} \frac{\tan x}{x}, \ x \neq 0 \\ 1, \ x = 0 \end{cases}$, is

a) Continuous but not differentiable at $x = 0$ b) Discontinuous at $x = 0$

c) Continuous and differentiable at $x = 0$ d) Not defined at $x = 0$

186. At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, \ 1 < x < \infty \\ x - 1, \ -\infty < x \leq 1 \end{cases}$

a) Continuous and differentiable

b) Continuous and not differentiable

c) Discontinuous and differentiable

d) Discontinuous and not differentiable

187. If $f(x)$ defined by $f(x) = \begin{cases} \frac{|x^2-x|}{x^2-x}, x \neq 0, 1 \\ 1, \ x = 0 \\ -1, \ x = 1 \end{cases}$ then $f(x)$ is continuous for all

a) x

b) x except at $x = 0$

c) x except at $x = 1$

d) x except at $x = 0$ and $x = 1$

188. The value of derivative of $|x - 1| + |x - 3|$ at $x = 2$, is

a) -2 b) 0 c) 2 d) Not defined

189. If $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ 1 + \sin x & \text{for } 0 \le x \le \pi/2 \end{cases}$, then at $x = 0$, the derivative $f'(x)$ is

a) 1 b) 0 c) Infinite d) Does not exist

190. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2, m$ and n are integers, $m \neq 0, n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x \to 1^+} g(x) = p$, then

a) $n = 1, m = 1$ b) $n = 1, m = -1$ c) $n = 2, m = 2$ d) $n > 2, m = n$

191. The function $f(x) = \frac{2x^2+7}{x^3+3x^2-x-3}$ is discontinuous for

a) $x = 1$ only b) $x = 1$ and $x = -1$ only

c) $x = 1, x = -1, x = -3$ only d) $x = 1, x = -1, x = -3$ and other values of x

192. If for a function $f(x), f(2) = 3, f'(2) = 4$, then $\lim_{x \to 2}[f(x)]$, where $[\cdot]$ denotes the greatest integer function, is

a) 2 b) 3 c) 4 d) Non-existent

193. A function $f(x)$ is defined as fallows for real x,

$f(x) = \begin{cases} 1 - x^2, \text{for } x < 1 \\ 0, \quad \text{for } x = 1 \\ 1 + x^2, \text{for } x > 1 \end{cases}$ Then,

a) $f(x)$, is not continuous at $x = 1$

b) $f(x)$ is continuous but not differentiable at $x = 1$

c) $f(x)$ is both continuous and differentiable at $x = 1$

d) None of the above

194. Let $f: R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then, which of the following is true?

a) $f(x) \ge 1$ for all $x \in R$ b) $f(x)$ is not differentiable at $x = 1$

c) $f(x)$ is differentiable everywhere d) $f(x)$ is not differentiable at $x = 0$

195. If $f(x) = \begin{cases} mx + 1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous t $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = m\frac{\pi}{2}$ d) $m = n = \frac{\pi}{2}$

196. If $f(x) = \frac{\log_e(1+x^2 \tan x)}{\sin x^3}, x \neq 0$, is to be continuous at $x = 0$, then $f(0)$ must be defined as

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

197. Let $f(x) = \begin{cases} x^P \sin\frac{1}{x}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ then $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $0 < p \le 1$ b) $1 \le p < \infty$ c) $-\infty < p < 0$ d) $p = 0$

198. The function f defined by

$f(x) = \begin{cases} \frac{\sin x^2}{x}, x \neq 0 \\ 0, \ x = 0 \end{cases}$ is

a) Continuous and derivable at $x = 0$

b) Neither continuous nor derivable at $x = 0$

c) Continuous but not derivable at $x = 0$

d) None of these

199. A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$, there exists, $\delta > 0$ such that

a) $|f(x) - f(a)| < \in \Rightarrow |x - a| < \delta$ b) $|f(x) - f(a)| > \in \Rightarrow |x - a| > \delta$

c) $|x - a| > \delta |f(x) - f(a)| > \in$ d) $|x - a| < \delta |f(x) - f(a)| < \in$

200. The function $f(x) = x - |x - x^2|, -1 \le x \le 1$ is continuous on the interval

a) $[-1, 1]$ b) $(-1, 1)$ c) $[-1, 0) \cup (0, 1]$ d) $(-1, 0) \cup (0, 1)$

201. if $f(x) = a|\sin x| + b\, e^{|x|} + c\, |x|^3$ and if $f(x)$ is differentiable at $x = 0$, then

a) $a = b = c = 0$ b) $a = 0, b = 0; c \in R$ c) $b = c = 0, a \in R$ d) $c = 0, a = 0, b \in R$

202. Let $f(x)$ be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u+v) = f(u) + kuv - 2\,v^2$ for all $u, v \in R$ (k is a fixed constant). Then,

a) $f'(x) = 8x$ b) $f(x) = 8x$ c) $f'(x) = x$ d) None of these

203. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on

a) $[-1, 1]$ b) $R - \{-1, 1\}$ c) $R - (-1, 1)$ d) None of these

204. Define f on R into itself by

$$f(x) = \begin{cases} x \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}, \text{ then}$$

a) f is continuous at 0 but not differentiable at 0 b) f is both continuous and differentiable at 0

c) f is differentiable but not continuous at 0 d) None of the above

205. The set of points where the function $f(x) = |x-1|e^x$ is differentiable, is

a) R b) $R - \{1\}$ c) $R - \{-1\}$ d) $R - \{0\}$

206. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$, where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0} G(x) = b$. Then $f'(x)$ is equal to

a) $1 + ab$ b) ab c) a/b d) None of these

Session : 2025-26 **AS PER NEW NTA SYLLABUS**

Total Questions : 206

MATHS (QUESTION BANK)

5.CONTINUITY AND DIFFERENTIABILITY

: ANSWER KEY :

1)	b	2)	d	3)	b	4)	c	109)	b	110)	a	111)	a	112)	c
5)	b	6)	b	7)	b	8)	c	113)	b	114)	c	115)	c	116)	c
9)	a	10)	d	11)	d	12)	a	117)	c	118)	b	119)	b	120)	c
13)	a	14)	b	15)	d	16)	a	121)	a	122)	c	123)	b	124)	d
17)	a	18)	b	19)	c	20)	b	125)	c	126)	b	127)	a	128)	b
21)	b	22)	c	23)	d	24)	b	129)	d	130)	b	131)	b	132)	a
25)	a	26)	d	27)	c	28)	b	133)	a	134)	d	135)	a	136)	d
29)	d	30)	b	31)	b	32)	b	137)	a	138)	b	139)	d	140)	a
33)	b	34)	c	35)	d	36)	c	141)	c	142)	d	143)	a	144)	a
37)	b	38)	a	39)	a	40)	a	145)	a	146)	d	147)	a	148)	b
41)	c	42)	b	43)	b	44)	c	149)	c	150)	b	151)	a	152)	c
45)	d	46)	c	47)	d	48)	b	153)	b	154)	a	155)	b	156)	d
49)	c	50)	d	51)	c	52)	a	157)	d	158)	d	159)	d	160)	b
53)	c	54)	b	55)	c	56)	d	161)	c	162)	a	163)	a	164)	c
57)	c	58)	d	59)	b	60)	c	165)	b	166)	b	167)	c	168)	c
61)	a	62)	a	63)	a	64)	b	169)	a	170)	b	171)	a	172)	c
65)	d	66)	a	67)	c	68)	d	173)	c	174)	b	175)	b	176)	c
69)	d	70)	a	71)	a	72)	b	177)	b	178)	b	179)	c	180)	d
73)	b	74)	c	75)	d	76)	b	181)	b	182)	c	183)	d	184)	c
77)	b	78)	c	79)	b	80)	b	185)	c	186)	b	187)	d	188)	b
81)	d	82)	c	83)	a	84)	b	189)	d	190)	c	191)	c	192)	c
85)	c	86)	c	87)	c	88)	b	193)	a	194)	c	195)	c	196)	a
89)	c	90)	a	91)	b	92)	d	197)	a	198)	a	199)	a	200)	a
93)	b	94)	c	95)	b	96)	c	201)	b	202)	a	203)	b	204)	a
97)	d	98)	d	99)	d	100)	d	205)	b	206)	d				
101)	a	102)	b	103)	c	104)	c								
105)	a	106)	a	107)	d	108)	a								

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5.CONTINUITY AND DIFFERENTIABILITY

: HINTS AND SOLUTIONS :

1 **(b)**

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \le \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, $f(x)$ is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) = 0$ for all $x \in (-\pi/4, \pi/4)$

2 **(d)**

Since, $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 **(b)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 **(c)**

We have,

$$f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x-2|, 1 \le x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2-x, & 1 \le x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \le 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \le x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$

5 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$= \lim_{h\to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h\to 0} \frac{-h}{\tan^{-1} h} = -1$$

and,

$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h)$$

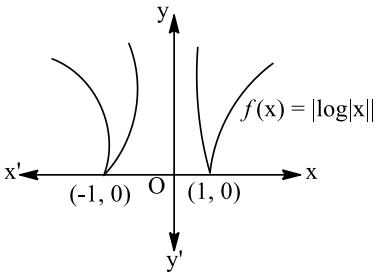
$$= \lim_{h\to 0} \frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}$$

$$\Rightarrow \lim_{x\to -2^+} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\therefore \lim_{x\to -2^-} f(x) \ne \lim_{\to -2^+} f(x)$$

So, $f(x)$ is neither continuous nor differentiable at $x = -2$

6 **(b)**



From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

7 **(b)**

We have,

$$\lim_{x\to a} \frac{xf(a) - a\,f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{x\,f(a) - a\,f(a) - a\big(f(x) - f(a)\big)}{x-a}$$

$$\Rightarrow \lim_{x\to a}\frac{x\,f(a)-a\,f(x)}{x-a}$$
$$=\lim_{x\to a}\frac{f(a)(x-a)}{x-a}$$
$$-\,a\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$
$$\Rightarrow \lim_{x\to a}\frac{x\,f(a)-a\,f(x)}{x-a}=f(a)-a\,f'(a)=4-2a$$

8 **(c)**

Given, $f(x)=x\left(\sqrt{x}+\sqrt{x+1}\right)$. At $x=0$ LHL of $\sqrt{x}$ is not defined, therefore it is not continuous at $x=0$

Hence, it is not differentiable at $x=0$

9 **(a)**

Here, $f'(x)=\begin{cases}2ax, & b\neq 0, x\leq 1\\ 2bx+a, & x>1\end{cases}$

Since, $f(X)$ is continuous at $x=1$

$\therefore \lim_{h\to 0} f(x)=\lim_{h\to 1^+} f(x)$

$\Rightarrow \quad a+b=b+a+c \Rightarrow c=0$

Also, $f(x)$ is differentiable at $x=1$

$\therefore$ (LHD at $x=1$)=(RHD at $x=1$)

$\Rightarrow \quad 2a=2b(1)+a \quad \Rightarrow \quad a=2b$

10 **(d)**

We have,

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\left\{\frac{x^2}{4}-\frac{3x}{4}+\frac{13}{4}\right\}=\frac{1}{4}-\frac{3}{2}+\frac{13}{4}$$
$$=2$$

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}|x-3|=2$

and, $f(1)=|1-3|=2$

$\therefore \lim_{x\to 1^-} f(x)=f(1)=\lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x=1$

We have,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3}|x-3|=0,\ \lim_{x\to 3^+} f(x)$$
$$=\lim_{x\to 3}|x-3|=0$$

and, $f(3)=0$

$\therefore \lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3)$

So, $f(x)$ is continuous at $x=3$

Now,

(LHD at $x=1$)

$$=\left\{\frac{d}{dx}\left(\frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}\right)\right\}_{x=1}=\left\{\frac{x}{2}-\frac{3}{2}\right\}_{x=1}=\frac{1}{2}-\frac{3}{2}$$
$$=-1$$

(RHD at $x=1$) $=\left\{\frac{d}{dx}\left(-(x-3)\right)\right\}_{x=1}=-1$

$\therefore$ (LHD at $x=1$) $=$ (RHD at $x=1$)

So, $f(x)$ is differentiable at $x=1$

11 **(d)**

$$f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0,\\ a, & \text{if } x=0\end{cases}$$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{2\sin x-\sin 2x}{2x\cos x}$ $\left(\frac{0}{0}\text{ form}\right)$

$$=\lim_{x\to 0}\frac{2\cos x-2\cos 2x}{2\,(\cos x-x\sin x)}$$
$$=\lim_{x\to 0}\frac{2-2}{2(1-0)}=0$$

Since, $f(x)$ is continuous at $x=0$

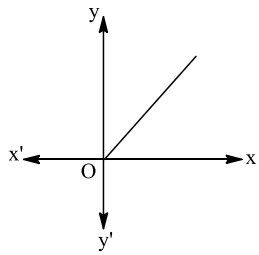
$\therefore \quad f(0)=\lim_{x\to 0} f(x)$

$\Rightarrow a=0$

12 **(a)**

Given, $f(x)=x+|x|$

$\therefore \quad f(x)=\begin{cases}2x, & x\geq 0\\ 0, & x<0\end{cases}$



It is clear from the graph of $f(x)$ is continuous for every value of x

Alternate

Since, x and $|x|$ is continuous for every value of x, so their sum is also continous for every value of x

13 **(a)**

Since $f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to 0^-} f(x)=f(0)=\lim_{x\to 0^+} f(x)$

$\Rightarrow \lim_{x\to 0}\{1+|\sin x|\}^{\frac{a}{|\sin x|}}=b=\lim_{x\to 0} e^{\frac{\tan 2x}{\tan 3x}}$

$\Rightarrow e^a=b=e^{2/3}\Rightarrow a=\frac{2}{3}$ and $a=\log_e b$

14 **(b)**

We have,

$$f(x)=\begin{cases}x^2+\dfrac{(x^2/1+x^2)}{1-(1/1+x^2)}=x^2+1, x\neq 0\\ 0, \quad x=0\end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=1\neq f(0)$

So, $f(x)$ is discontinuous at $x=0$

15 **(d)**

LHD$=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$$=\lim_{h\to 0}\frac{1-1}{-h}=0$$

RHD$=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$$=\lim_{h\to 0}\frac{1+\sin(0+h)-1}{h}=\lim_{h\to 0}\frac{\sin h}{h}=1$$

$\Rightarrow$ LHD$\neq$RHD

16 **(a)**

Given, $f(x) = x - |x - x^2|$

At $x = 1$, $f(1) = 1 - |1 - 1| = 1$

$\lim_{x\to 1^{-1}} f(x) = \lim_{h\to 0}[(1-h) - |(1-h) - (1-h)^2|]$

$= \lim_{h\to 0}[(1-h) - |h - h^2|] = 1$

$\lim_{x\to 1^{+}} f(x) = \lim_{h\to 0}[(1+h) - |(1+h) - (1+h)^2|]$

$= \lim_{h\to 0}[1 + h - |-h^2 - h|] = 1$

$\because \lim_{x\to 1^{-1}} f(x) = \lim_{x\to 1^{+}} = f(1)$

17 **(a)**

We have,

$f(x+y+z) = f(x)f(y)f(z)$ for all x, y, z ...(i)

$\Rightarrow f(0) = f(0)f(0)f(0)$ [Putting $x = y = z = 0$]

$\Rightarrow f(0)\{1 - f(0)^2\} = 0$

$\Rightarrow f(0) = 1$ [$\because f(0) = 0 \Rightarrow f(x) = 0$ for all x]

Putting $z = 0$ and $y = 2$ in (i), we get

$f(x+2) = f(x)f(2)f(0)$

$\Rightarrow f(x+2) = 4f(x)$ for all x

$\Rightarrow f'(2) = 4f'(0)$ [Putting $x = 0$]

$\Rightarrow f'(2) = 4 \times 3 = 12$

18 **(b)**

For $x > 1$, we have

$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$

For $x < -1$, we have

$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$

For $0 < x < 1$, we have

$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \frac{-1}{x}$

For $-1 < x < 0$, we have

$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\frac{1}{x}$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

19 **(c)**

Since, $\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{1 - \cos x}{x^2} = k$

$\Rightarrow \lim_{x\to 0} \frac{-(-\sin x)}{2x} = k$ [using L 'Hospital's rule]

$\Rightarrow \frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x} = k \quad \Rightarrow \quad k = \frac{1}{2}$

20 **(b)**

Given, $f(X) = |x - 1| + |x - 2|$

$= \begin{cases} x - 1 + x - 2, & x \geq 2 \\ x - 1 + 2 - x, & 1 \leq x < 2 \\ 1 - x + 2 - x, & x < 1 \end{cases}$

$= \begin{cases} 2x - 3, & x \geq 2 \\ 1, & 1 \leq x < 2 \\ 3 - 2x, & x < 1 \end{cases}$

$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$

Hence, except $x = 1$ and $x = 2$, $f(x)$ is differentiable everywhere in R

21 **(b)**

Clearly, $f(x)$ is differentiable for all non-zero values of x. For $x \neq 0$, we have

$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1 - e^{-x^2}}}$

Now,

$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{f(0-h) - f(0)}{x - 0}$

$\Rightarrow (\text{LHD at } x = 0) = \lim_{h\to 0} \frac{\sqrt{1 - e^{-h^2}}}{-h}$

$= \lim_{h\to 0} -\frac{\sqrt{1 - e^{-h^2}}}{h}$

$\Rightarrow (\text{LHD at } x = 0) = -\lim_{h\to 0} \sqrt{\frac{e^{h^2} - 1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$

and, $(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} =$

$\lim_{h\to 0} \frac{\sqrt{1 - e^{-h^2}} - 0}{h}$

$\Rightarrow (\text{RHD at } x = 0) = \lim_{h\to 0} \sqrt{\frac{e^{h^2} - 1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$

So, $f(x)$ is not differentiable at $x = 0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

22 **(c)**

Since $f(x)$ is continuous at $x = 0$

$\therefore f(0) = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$

23 **(d)**

For $f(x)$ to be continuous everywhere, we must have,

$f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow f(0) = \lim_{x\to 0} \frac{2 - (256 - 7x)^{1/8}}{(5x + 32)^{1/5} - 2} \quad \left[\text{Form } \frac{0}{0}\right]$

$\Rightarrow f(0) = \lim_{x\to 0} \frac{\frac{7}{8}(256 - 7x)^{-\frac{7}{8}}}{(5x + 32)^{-4/5}} = \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$

24 **(b)**

We have,

$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

$\therefore$ (LHD at $x = 0$) $= \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} -\frac{x^3}{x}$
$= 0$

and,

$\therefore$ (RHD at $x = 0$) $= \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^3}{x}$
$= 0$

Clearly, (LHD at $x = 0$) = (RHD at $x = 0$)

Hence, $f(x)$ is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

25 **(a)**

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1 + \frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 . 1 . a \lim_{x \to 0} \left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \ldots}\right)$$

$= 3a\,(\log 4)^3$

$\because \quad \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad 3a\,(\log 4)^3 = 9(\log 4)^3$

$\Rightarrow \quad a = 3$

26 **(d)**

We have,

$f(x) = |[x]x|$ for $-1 < x \leq 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \leq x < 1 \\ x, & 1 \leq x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

27 **(c)**

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore \quad g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, $f(x)$ is discontinuous at six points

28 **(b)**

Given, $y = \cos^{-1}\cos(x - 1), \quad x > 0$

$\Rightarrow \quad y = x - 1, \qquad 0 \leq x - 1 \leq \pi$

$\therefore \quad y = x - 1, \qquad 1 \leq x \leq \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$$\Rightarrow \frac{dy}{dx} = 1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{x = \frac{5\pi}{4}} = 1$$

29 **(d)**

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h} \qquad [\because f(x + y) = f(x) + f(y)]$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0 \times g(0) = 0 \qquad \left[\begin{array}{l} \because g \text{ is continuous} \\ \therefore \lim_{h \to 0} g(h) = g(0) \end{array}\right]$$

30 **(b)**

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

31 **(b)**

For $x \neq -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \cdots \infty$

$$\Rightarrow f(x) = (1 + x)^{-2} = \frac{1}{(1 + x)^2}$$

Thus, we have

$$f(x) = \begin{cases} \frac{1}{(1 + x)^2}, & x \neq -1 \\ 1, & x = -1 \end{cases}$$

We have, $\lim_{x \to -1^-} f(x) \to \infty$ and $\lim_{x \to -1^-} f(x) \to \infty$

So, $f(x)$ is not continuous at $x = -1$

Consequently, it is not differentiable there at

32 **(b)**

At $x = a$,

LHL= $\lim_{x \to a^-} f(x) = \lim_{x \to a} 2a - x = a$

And RHL= $\lim_{x \to a^+} f(x) = \lim_{x \to a} 3x - 2a = a$

And $f(a) = 3(a) - 2a = a$

$\therefore$ LHL=RHL= $f(a)$

Hence, it is continuous at $x = a$

Again, at $x = a$

LHD= $\lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

$$= \lim_{h \to 0} \frac{2a - (a - h) - a}{-h} = -1$$

and RHD= $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$= \lim_{h \to 0} \frac{3(a + h) - 2a - a}{h} = 3$$

$\therefore$ LHD$\neq$RHD

Hence, it is not differentiable at $x = a$

33 **(b)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$
$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{f(h)-1}{h}$$
$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{1+(\sin 2\,h)g(h)-1}{h}$$
$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{\sin 2\,h}{h}\times\lim_{h\to 0}g(h)$$
$$= 2f(x)g(0)$$

34 **(c)**

If $-1 \le x \le 1$, then $0 \le x\sin\pi x \le 1/2$

$\therefore f(x) = [x\sin\pi x] = 0$, for $-1 \le x \le 1$

If $1 < x < 1+h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin\pi x < 0 \Rightarrow -1 < x\sin\pi x < 0$

$\therefore f(x) = [x\sin\pi x] = -1$ in the right neighbourhood of $x = 1$

Thus, $f(x)$ is constant and equal to zero in $[-1,1]$ and so $f(x)$ is differentiable and hence continuous on $(-1,1)$

At $x = 1$, $f(x)$ is discontinuous because

$\Rightarrow \lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = -1$

Hence, $f(x)$ is not differentiable at $x = 1$

35 **(d)**

We have,

$$(\text{LHD at } x=0) = \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$$
$$(\text{RHD at } x=0) = \left\{\frac{d}{dx}(1+\sin x)\right\}_{x=0} = \cos 0 = 1$$

Hence, $f'(x)$ at $x = 0$ does not exist

36 **(c)**

Here, $f'(x) = \begin{cases} 2bx + a, & x \ge -1 \\ 2a, & x < -1 \end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$

$\Rightarrow -2b + a = -2a$

$\Rightarrow 3a = 2b$

$\Rightarrow a = 2, \quad b = 3$

or $a = -2, \ b = -3$

37 **(b)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{\log\cos x}{\log(1+x^2)}$$
$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{\log(1-1+\cos x)}{\log(1+x^2)}\cdot\frac{1-\cos x}{1-\cos x}$$
$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{\log\{1-(1-\cos x)\}}{1-\cos x}\cdot\frac{1-\cos x}{\log(1+x^2)}$$
$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = -\lim_{x\to 0}\log\frac{[1-(1-\cos x)]}{-(1-\cos x)}\times\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\times\frac{x^2}{\log(1+x^2)}$$
$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = -\frac{1}{2}$$

Hence, $f(x)$ is differentiable and hence continuous at $x = 0$

38 **(a)**

Since $f(x)$ is continuous at $x = 1$. Therefore,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow A - B = 3 \Rightarrow A = 3 + B$ ...(i)

If $f(x)$ is continuous at $x = 2$, then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \Rightarrow 6 = 4\,B - A$ ...(ii)

Solving (i) and (ii) we get $B = 3$

As $f(x)$ is not continuous at $x = 2$. Therefore, $B \ne 3$

Hence, $A = 3 + B$ and $B \ne 3$

39 **(a)**

We have,

$$f(x) = \begin{cases} x-4, & x \ge 4 \\ -(x-4), & 1 \le x < 4 \\ (x^3/2) - x^2 + 3x + (1/2), & x < 1 \end{cases}$$

Clearly, $f(x)$ is continuous for all x but it is not differentiable at $x = 1$ and $x = 4$

40 **(a)**

It is given that $f(x)$ is continuous at $x = 1$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x\to 1^-} a[x+1] + b[x-1]$

$= \lim_{x\to 1^+} a[x+1] + b[x-1]$

$\Rightarrow a - b = 2a + 0\times b$

$\Rightarrow a + b = 0$

41 **(c)**

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \lambda[x] = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore$ f is continuous only whatever λ may be

42 **(b)**

We have,

$y(x) = f(e^x)\, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x) \cdot e^x \cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)}f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

43 **(b)**

Since $f(x)$ is differentiable at $x = 1$. Therefore,

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{a(1-h)^2 - b - 1}{-h} = \lim_{h\to 0} \frac{\frac{1}{|1+h|} - 1}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{(a - b - 1) - 2\,ah + ah^2}{-h} = \lim_{h\to 0} \frac{-h}{h(1+h)}$$

$$\Rightarrow \lim_{h\to 0} \frac{-(a - b - 1) - 2\,ah - ah^2}{h} = -1$$

$\Rightarrow -(a - b - 1) = 0$ and so $\lim_{h\to 0} \frac{2ah - ah^2}{h} = -1$

$\Rightarrow a - b - 1 = 0$ and $2a = -1 \Rightarrow a = -\frac{1}{2}, b = -\frac{3}{2}$

44 **(c)**

We have,

$f(x) = \frac{\sin 4\,\pi[x]}{1+[x]^2} = 0$ for all x $[\because$ $4\pi[x]$is a multiple of $\pi]$

$\Rightarrow f'(x) = 0$ for all x

45 **(d)**

We have,

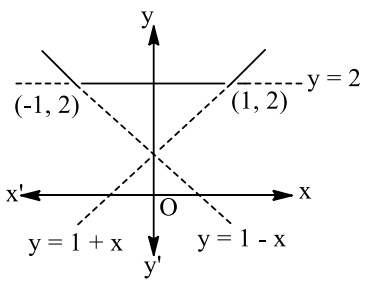
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to 0} f(x) =$ An oscillating number which oscillates between -1 and 1

Hence, $\lim_{x\to 0} f(x)$ does not exist

Consequently, $f(x)$ cannot be continuous at $x = 0$ for any value of k

46 **(c)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except $\{-1, 1\}$ due to sharp edge

47 **(d)**

We have,

$\log\left(\frac{x}{y}\right) = \log x - \log y$ and $\log(e) = 1$

$\therefore f(x) = \log x$

Clearly, $f(x)$ is unbounded because $f(x) \to -\infty$ as $x \to 0$ and $f(x) \to +\infty$ as $x \to \infty$

We have,

$$f\left(\frac{1}{x}\right) = \log\left(\frac{1}{x}\right) = -\log x$$

As $x \to 0, f\left(\frac{1}{x}\right) \to \infty$

Also,

$$\lim_{x\to 0} xf(x) = \lim_{x\to 0} x\log x = \lim_{x\to 0} \frac{\log x}{1/x}$$

$$\Rightarrow \lim_{x\to 0} xf(x) = \lim_{x\to 0} \frac{1/x}{-1/x^2} = -\lim_{x\to 0} x = 0$$

49 **(c)**

Since $g(x)$ is the inverse of $f(x)$. Therefore,

$fog(x) = x$, for all x

$\Rightarrow \frac{d}{dx}\{fog\,(x)\} = 1$, for all x

$\Rightarrow f'\big(g(x)\big)\, g'(x) = 1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3} \times g'(x) = 1$ for all x $\left[\because f'(x) = \frac{1}{1+x^3}\right]$

$\Rightarrow g'(x) = 1 + \{g(x)\}^3$, for all x

50 **(d)**

We have,

$f(x) = |x^2 - 4x + 3|$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x^2 - 4x + 3 \geq 0 \\ -(x^2 - 4x + 3), & \text{if } x^2 - 4x + 3 < 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x \leq 1 \text{ or } x \geq 3 \\ -x^2 + 4x - 3, & \text{if } 1 < x < 3 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous

Now,

$$(\text{LHD at } x = 1) = \left(\frac{d}{dx}(x^2 - 4x + 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{LHD at } x = 1) = (2x - 4)_{\text{at } x=1} = -2$

and,

$$(\text{RHD at } x = 1) = \left(\frac{d}{dx}(-x^2 + 4x - 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{RHD at } x = 1) = (-2x + 4)_{\text{at } x=1} = 2$

Clearly, $(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$

So, $f(x)$ is not differentiable at $x = 1$

Similarly, it can be checked that $f(x)$ is not differentiable at $x = 3$ also

ALITER We have,

$f(x) = |x^2 - 4x + 3| = |x - 1|\,|x - 3|$

Since, $|x - 1|$ and $|x - 3|$ are not differentiable at 1 and 3 respectively

Therefore, $f(x)$ is not differentiable at $x = 1$ and $x = 3$

51 **(c)**

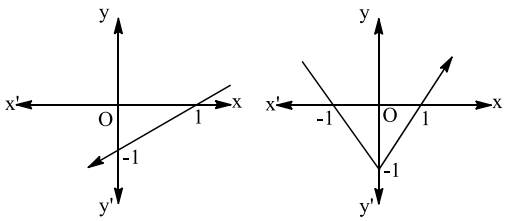
The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.

$ie,\quad \tan x = \tan\infty$

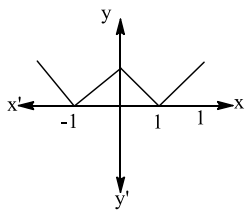
$\Rightarrow \quad x = (2n+1)\dfrac{\pi}{2}, \qquad n \in I$

52 **(a)**

Using graphical transformation



(i) y = x - 1 (ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \big||x| - 1\big|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

53 **(c)**

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} 2(0-h) = 0$

And $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} 2(0+h) + 1 = 1$

$\because \quad \lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$

$\therefore \quad f(x)$ is discontinuous at $x = 0$

54 **(b)**

Draw a rough sketch of $y = f(x)$ and observe its properties

55 **(c)**

$\lim\limits_{x\to\pi} \dfrac{(1+\cos x) - \sin x}{(1+\cos x) + \sin x}$

$= \lim\limits_{x\to\pi} \dfrac{2\cos^2 x/2 - 2(\sin x/2)\cos\ x/2}{2\cos^2 x/2 + 2(\sin x/2)\cos\ x/2}$

$= \lim\limits_{x\to\pi} \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{2}\right) = -1$

Since, $f(x)$ is continuous at $x = \pi$

$\therefore \quad f(\pi) = \lim\limits_{x\to\pi} f(x) = -1$

56 **(d)**

$f'(1^-) = \lim\limits_{h\to 0} \dfrac{f(1-h) - f(1)}{-h}$

$= \lim\limits_{h\to 0} \dfrac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right) - 0}{-h}$

$= -\lim\limits_{h\to 0} \sin\dfrac{1}{h}$

And $f'(1^+) = \lim\limits_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h}$

$= \lim\limits_{h\to 0} \sin\dfrac{1}{h}$

$\therefore \quad f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$

Again, now

$f'(0^+) = \lim\limits_{h\to 0} \dfrac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{h}$

$= \lim\limits_{h\to 0} \dfrac{\left[-\left\{(h-1)\cos\left(\frac{1}{h-1}\right) \times \left(\frac{1}{(h-1)^2}\right)\right\} + \sin\left(\frac{1}{h-1}\right)\right]}{1}$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

And $f'(0^-) = \lim\limits_{h\to 0} \frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right) - \sin 1}{-h}$

$= \lim\limits_{h\to 0} \dfrac{(-h-1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right) - \sin\left(\frac{1}{-h-1}\right)}{-1}$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

$\Rightarrow \quad f'(0^-) = f'(0^+)$

$\therefore \quad f$ is differentiable at $x = 0$

57 **(c)**

As $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\therefore \lim\limits_{x\to\frac{\pi}{2}^-} f(x) = \lim\limits_{x\to\frac{\pi}{2}^+} f(x)$

$\Rightarrow m\dfrac{\pi}{2} + 1 = \sin\dfrac{\pi}{2} + n \Rightarrow m\dfrac{\pi}{2} + 1 = 1 + n \Rightarrow n = \dfrac{m\,\pi}{2}$

58 **(d)**

Since, $\frac{f(6)-f(1)}{6-1} \geq 2 \quad \left[\because \quad f'(x) = \frac{y_2-y_1}{x_2-x_1}\right]$

$\Rightarrow \quad f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

$\Rightarrow \quad f(6) \geq 8$

59 **(b)**

We have,

$\lim\limits_{x\to a^-} f(x)\ g(x) = \lim\limits_{x\to a^-} f(x)\,.\lim\limits_{x\to a^-} g(x) = m \times l = ml$

and,

$\lim\limits_{x\to a^+} f(x)\ g(x) = \lim\limits_{x\to a^+} f(x) \lim\limits_{x\to a^+} g(x) = lm$

$\therefore \lim\limits_{x\to a^-} f(x)\ g(x) = \lim\limits_{x\to a^+} f(x)\ g(x) = lm$

Hence, $\lim_{x\to a} f(x)\ g(x)$ exists and is equal to lm

60 **(c)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{f(h)-1}{h} \qquad [\because f(x+y) = f(x)f(y)]$$

$$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0}\frac{1+h\,g(h)-1}{h}\right\} \qquad [\because f(x) = 1+x\,g(x)]$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0} g(h) = f(x)\cdot 1 = f(x)$$

61 **(a)**

We have, $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Clearly, $f(x)$ is differentiable for all $x > 0$ and for all$x < 0$. So, we check the differentiable at $x = 0$

Now, (RHD at $x = 0$)

$$\left(\frac{d}{dx}(x)^2\right)_{x=0} = (2x)_{x=0} = 0$$

And (LHD at $= 0$)

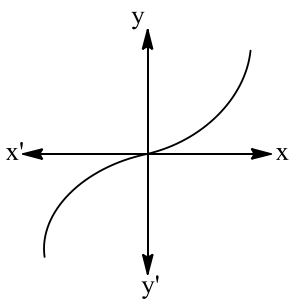
$$\left(\frac{d}{dx}(-x)^2\right)_{x=0} = (-2x)_{x=0} = 0$$

$\therefore$ (LHD at $x = 0$)=(RHD at $x = 0$)

So, $f(x)$ is differentiable for all x ie, the set of all points where $f(x)$ is differentiable is $(-\infty, \infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.



62 **(a)**

Since, $f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = 10$

$$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = 10$$

$$\Rightarrow f(0)\left(\lim_{h\to 0}\frac{f(h)-1}{h}\right) = 10 \quad ...(i)$$

$[\because\ f(0+h) = f(0)f(h), \text{given}]$

Now, $f(0) = f(0)f(0)$

$\Rightarrow f(0) = 1$

$\therefore$ From Eq. (i)

$$\lim_{h\to 0}\frac{f(h)-1}{h} = 10 \quad ...(ii)$$

Now, $f'(6) = \lim_{h\to 0}\frac{f(6+h)-f(6)}{h}$

$= \lim_{x\to 0}\left(\frac{f(h)-1}{h}\right)f(6)$ [from Eq. (ii)]

$= 10 \times 3 = 30$

63 **(a)**

We have,

$$f'(a^+) = \lim_{x\to a^+}\frac{f(x)-f(0)}{x-a}$$

$$\Rightarrow f'(a^+) = \lim_{x\to a^+}\frac{|x-a|\phi(x)}{x-a}$$

$$\Rightarrow f'(a^+) = \lim_{x\to a}\frac{(x-a)}{(x-a)}\phi(x) \quad [\because x > a\ \therefore |x-a| = x-a]$$

$$\Rightarrow f'(a^+) = \lim_{x\to a}\phi(x)$$

$\Rightarrow f'(a^+) = \phi(a)$ $[\because \phi(x)$ is continuous at $x = a]$

and,

$$f'(a^-) = \lim_{x\to a^-}\frac{f(x)-f(0)}{x-a}$$

$$\Rightarrow f'(a^-) = \lim_{x\to a^-}\frac{|x-a|\phi(x)}{x-a}$$

$$\Rightarrow f'(a^-) = \lim_{x\to a}\frac{(x-a)\phi(x)}{(x-a)} \quad [\because x < a\ \therefore |x-a| = -(x-a)]$$

$$\Rightarrow f'(a^-) = -\lim_{x\to a}\phi(x)$$

$\Rightarrow f'(a^-) = -\phi(a)$ $[\because$ $\phi(x)$ is continuous at $x = a]$

64 **(b)**

LHL= $\lim_{h\to 0}(0-h)_e^{-\left(\frac{1}{|-h|}+\frac{1}{(-h)}\right)} = \lim_{h\to 0}(-h) = 0$

RHL= $\lim_{h\to 0}(0+h)_e^{-\left(\frac{1}{|h|}+\frac{1}{(h)}\right)} = \lim_{h\to 0}\frac{h}{e^{2/h}} = 0$

LHL=RHL= $f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x = 0$

$$Lf'(0) = \lim_{h\to 0}\frac{(-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}}{(-h)-0} = 1$$

$$Rf'(0) = \lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)-0}}{h-0}$$

$$= \lim_{h\to 0}\frac{1}{e^{2/h}} = 0$$

$\Rightarrow\ Rf'(0)Lf'(0)$

Therefore, $f(x)$ is not differentiable at $x = 0$

65 **(d)**

We have,

$$f(x) = \begin{cases} 3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$$

Clearly, f is continuous but not differentiable at $x = 0$

Now,
$f(|x|) = 2|x| + 1$ for all x
Clearly, $f(|x|)$ is everywhere continuous but not differentiable at $x = 0$

67 **(c)**
We have,
$f(x) = |x - 0.5| + |x - 1| + \tan x, 0 < x < 2$

$$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \le x < 1 \\ 2x - 1.5 + \tan x, & 1 \le x < 2 \end{cases}$$

It is evident from the above definition that
$Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$
Also, the function is not continuous at $= \pi/2$. So, it cannot be differentiable thereat

68 **(d)**
Given, $f(x) = \begin{cases} \log_{(1-3x)}(1 + 3x), \text{for } x \neq 0 \\ k, \quad \text{for } x = 0 \end{cases}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\log(1 + 3x)}{\log(1 - 3x)}$$

$$= -\lim_{x\to 0} \frac{\log(1 + 3x)}{3x} \cdot \frac{(-3x)}{\log(1 - 3x)}$$

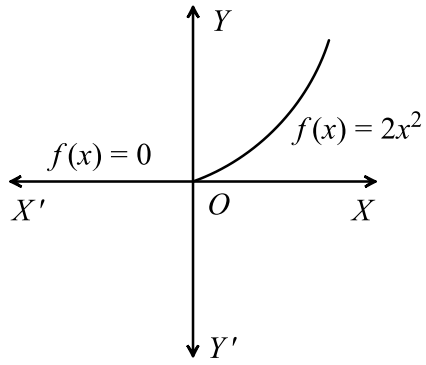
$= -1$
And $f(0) = k$
$\because f(x)$ is continuous at $x = 0$
$\therefore \quad k = -1$

69 **(d)**
Since $f(x)$ is differentiable at $x = c$. Therefore, it is continuous at $x = c$
Hence, $\lim_{x\to c} f(x) = f(c)$

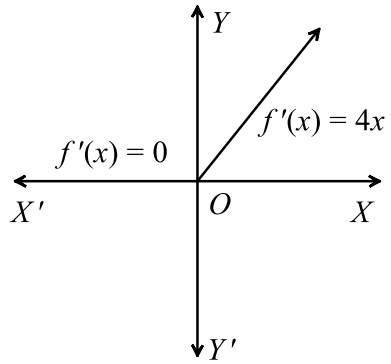
70 **(a)**
Given, $f(x) = ae^{|x|} + b\,|x|^2$
We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$ is differentiable at $x = 0$
$\therefore \quad f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$

71 **(a)**
We have,

$$f(x) = \begin{cases} (x - x)(-x) = 0, x < 0 \\ (x + x)x = 2x^2, x \ge 0 \end{cases}$$



(i)



(ii)

As is evident from the graph of $f(x)$ that it is continuous and differentiable for all x
Also, we have

$$f''(x) = \begin{cases} 0, x < 0 \\ 4x, x \ge 0 \end{cases}$$

Clearly, $f''(x)$ is continuous for all x but it is not differentiable at $x = 0$

72 **(b)**

Given, $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$

$$f(x) = \begin{cases} \dfrac{1}{2x - 5}, & x \neq 1 \\ -\dfrac{1}{3}, & x = 1 \end{cases}$$

$$f'(1) = \lim_{h\to 0} \frac{f(1 + h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2(1+h)-5} - \left(-\frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2h-3} + \frac{1}{3}}{h} = \lim_{h\to 0} \frac{3 + 2h - 3}{3h(2h - 3)} = -\frac{2}{9}$$

$$Lf'(1) = \lim_{h\to 0} \frac{f(1 - h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2(1-h)-5} - \left(-\frac{1}{3}\right)}{-h}$$

$$= \lim_{h\to 0} -\frac{2}{3(2h + 3)} = -\frac{2}{9}$$

$\therefore\ f'(1) = -\frac{2}{9}$

73 **(b)**

$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0}\frac{f(1+h)}{h} - \lim_{h\to 0}\frac{f(1)}{h}$

Given, $\lim_{h\to 0}\frac{f(1+h)}{h} = 5$

So, $\lim_{h\to 0}\frac{f(1)}{h}$ must be finite as $f'(1)$ exist and

$\lim_{h\to 0}\frac{f(1)}{h}$ can be finite only, if $f(1) = 0$ and

$\lim_{h\to 0}\frac{f(1)}{h} = 0$

So, $f'(1) = \lim_{h\to 0}\frac{f(1+h)}{h} = 5$

74 **(c)**

Since, $f(x)$ is continuous for every value of R except {-1, -2}. Now, we have to check that points

At $x = -2$

LHL= $\lim_{h\to 0}\frac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$

$= \lim_{h\to 0}\frac{-h}{h^2+h} = -1$

RHL= $\lim_{h\to 0}\frac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$

$= \lim_{h\to 0}\frac{h}{h^2-h} = -1$

$\Rightarrow$ LHL=RHL=$f(-2)$

$\therefore$ It is continuous at $x = -2$

Now, check for $x = -1$

LHL= $\lim_{h\to 0}\frac{(-1-h)+2}{(-1-h)^2+3(-1-h)+2}$

$= \lim_{h\to 0}\frac{1-h}{h^2-h} = \infty$

RHL= $\lim_{h\to 0}\frac{(-1+h)+2}{(-1+h)^2+3(-1+h)+2}$

$= \lim_{h\to 0}\frac{1+h}{h^2+h} = \infty$

$\Rightarrow$ LHL=RHL$\neq f(-1)$

$\therefore$ It is not continuous at $x = -1$

The required function is continuous in $R - \{-1\}$

75 **(d)**

$f(0) = \lim_{x\to 0}\frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$

$\Rightarrow \quad \lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^2 . \frac{\frac{x}{a}.a}{\sin\frac{x}{a}} . \frac{\frac{x}{4}.4}{\log\left(1+\frac{x}{4}\right)} = 12$

$\Rightarrow \quad 1^2.a.4 = 12$

$\Rightarrow \quad a = 3$

76 **(b)**

We have,

$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots\infty$

$\Rightarrow f(x) = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{x}{((r-1)x+1)\,(rx+1)}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty}\sum_{r=1}^{n}\left\{\frac{1}{(r-1)x+1} - \frac{1}{rx+1}\right\}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty}\left\{1 - \frac{2}{nx+1}\right\} = 1$, for $x \neq 0$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$

77 **(b)**

If possible, let $f(x) + g(x)$ be continuous. Then, $\{f(x) + g(x)\} - f(x)$ must be continuous

$\Rightarrow g(x)$ must be continuous

This is a contradiction to the given fact that $g(x)$ is discontinuous

Hence, $f(x) + g(x)$ must be discontinuous

78 **(c)**

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\therefore f(0) = f(0)f(0)$

$\Rightarrow f(0)\{f(0) - 1\} = 0$

$\Rightarrow f(0) = 1 \qquad [\because f(0) \neq 1]$

Now,

$f'(0) = 0$

$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = 2$

$\Rightarrow \lim_{h\to 0}\frac{f(h)-1}{h} = 2 \quad [\because f(0) = 1] \quad$ ....(i)

$\therefore f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h} \quad [\because f(x+y) = f(x)f(y)]$

$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0}\frac{f(h)-1}{h}\right\} = 2f(x) \quad$ [Using (i)]

79 **(b)**

We have,

$f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} \frac{x^2}{2} = x, & x > 0 \\ 0, & x = 0 \\ \frac{x^2}{-x} = -x, & x < 0 \end{cases}$$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} -x = 0, \lim_{x\to 0^+} f(x) = \lim_{x\to 0} x = 0$ and $f(0) = 0$

So, $f(x)$ is continuous at $x = 0$. Also, $f(x)$ is continuous for all other values of x

Hence, $f(x)$ is everywhere continuous

Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$

Therefore, $f(x)$ is not differentiable at $x = 0$

80 **(b)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) \Rightarrow f(0) = 2$ ...(i)

Now, using L' Hospital's rule, we have

$$\lim_{x\to 0} \frac{\int_0^x f(u)\ du}{x} = \lim_{x\to 0} \frac{f(x)}{1}$$

$= f(0)$ [$\because f(x)$ is continuous at $x = 0$]

$\Rightarrow \lim_{x\to 0} \frac{\int_0^x f(u)\ du}{x} = 2$ [Using (i)]

82 **(c)**

$$f'(2^+) = \lim_{x\to 2^+} \left(\frac{f(x) - f(2)}{x - 2}\right)$$

$$= \lim_{x\to 2^+} \frac{3x + 4 - (6 + 4)}{x - 2} = \lim_{x\to 2} \frac{3x - 6}{x - 2} = 3$$

83 **(a)**

Here, $f(x) = \begin{cases} \sin x, x > 0 \\ 0,\ x = 0 \\ -\sin x,\ x < 0 \end{cases}$

RHD= $\lim_{h\to 0} \frac{\sin|0+h| - \sin(0)}{h}$

$= \lim_{h\to 0} \frac{\sin h}{h} = 1$

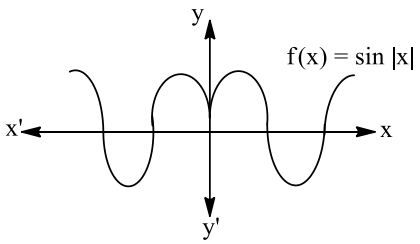
LHD= $\lim_{h\to 0} \frac{\sin|(0-h)| - \sin(0)}{-h}$

$= \frac{-\sin h}{h} = -1$

$\therefore$ LHD$\neq$RHD at $x = 0$

$\therefore$ $f(x)$ is not derivable at $x = 0$

Alternate



It is clear from the graph that $f(x)$ is not differentiable at $x = 0$

84 **(b)**

We have,

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} (\log_e a)^n$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(x \log_e a)^n}{n!} = \sum_{n=0}^{\infty} \frac{(\log_e a^x)^n}{n!}$$

$\Rightarrow f(x) = e^{\log_e a^x} = a^x$, which is everywhere continuous and differentiable

85 **(c)**

$$f(x) = [x] \cos\left[\frac{2x - 1}{2}\right]\pi$$

Since, $[x]$ is always discontinuous at all integer value, hence $f(x)$ is discontinuous for all integer value

86 **(c)**

The function f is clearly continuous for $|x| > 1$

We observe that

$$\lim_{x\to -1^+} f(x) = 1, \lim_{x\to -1^-} f(x) = \frac{1}{4}$$

Also, $\lim_{x\to \frac{1}{n}^+} f(x) = \frac{1}{n^2}$ and, $\lim_{x\to \frac{1}{n}^-} f(x) = \frac{1}{(n+1)^2}$

Thus, f is discontinuous for $x = \pm\frac{1}{n}, n = 1, 2, 3, \ldots$

87 **(c)**

Since, $|f(x) - f(y)| \leq (x - y)^2$

$\Rightarrow \lim_{x\to y} \frac{|f(x) - f(y)|}{|x - y|} \leq \lim_{x\to y} |x - y|$

$\Rightarrow |f'(y)| \leq 0$

$\Rightarrow f'(y) = 0$

$\Rightarrow f(y) =$constant

$\Rightarrow f(y) = 0 \Rightarrow f(1) = 0$ [$\because f(0) = 0$, given]

88 **(b)**

Since $\phi(x) = 2x^3 - 5$ is an increasing function on $(1, 2)$ such that $\phi(1) = -3$ and $\phi(2) = 11$

Clearly, between -3 and 11 there are thirteen points where $f(x) = [2x^3 - 5]$ is discontinuous

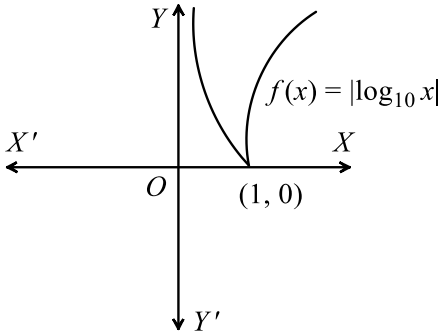
89 **(c)**

Clearly, $[x^2 + 1]$ is discontinuous at $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

Note that it is right continuous at $x = 1$ but not left continuous at $x = 3$

90 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$$f''(1^+) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log_{10}(1+h) - 0}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$

$$f''(1^-) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{h}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{\log_{10}(1-h)}{h} = \lim_{h\to 0} \frac{\log_e(1-h)}{h\log_e 10}$$

$$= -\log_{10} e$$

91 **(b)**

It can be easily seen from the graph of $f(x) = |\cos x|$ that it is everywhere continuous but not differentiable at odd multiples of $\pi/2$

92 **(d)**

We have,

$$\lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} \frac{4-h-4}{|4-h-4|} + a$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} -\frac{h}{h} + a = a - 1$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{4+h-4}{|4+h-4|} + b$$

$$= b + 1$$

and, $f(4) = a + b$

Since $f(x)$ is continuous at $x = 4$. Therefore,

$$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$$

$\Rightarrow a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1$

93 **(b)**

We have,

$$f(x) = \begin{cases} \dfrac{2^x - 1}{\sqrt{1+x} - 1}, -1 \le x < \infty, & x \ne 0 \\ k, & x = 0 \end{cases}$$

Since, $f(x)$ is continuous everywhere

$\therefore \quad \lim_{x\to 0^-} f(x) = f(0) \quad$ ...(i)

Now, $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} \frac{2^{(0-h)} - 1}{\sqrt{1+(0-h)} - 1}$

$$= \lim_{h\to 0} \frac{2^{-h} - 1}{\sqrt{1-h} - 1}$$

$$= \lim_{h\to 0} \frac{-2^{-h}\log_e 2}{\frac{-1}{2\sqrt{1-h}}} \text{ [by L' Hospital's rule]}$$

$$= 2\lim_{h\to 0} 2^{-h}\log_e 2\sqrt{1-h}$$

$$= 2\log_e 2$$

From Eq. (i),

$f(0) = 2\log_e 2 = \log_e 4$

95 **(b)**

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-h) = \lim_{h\to 0} \frac{e^{-1/h} - 1}{e^{-1/h} + 1} = -1$$

and,

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h) = \lim_{x\to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1} = \lim_{h\to 0} \frac{e^{-1/h}}{e^{-1/h}}$$

$$= 1$$

$\therefore \lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x)$

Hence, $f(x)$ is not continuous at $x = 0$

96 **(c)**

LHL= $\lim_{x\to 2^-} f(x) = \lim_{h\to 0} 1 + (2-h) = 3$

RHL= $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} 5 - (2+h) = 3, \quad f(2) = 3$

Hence, f is continuous at $x = 2$

Now, $Rf''(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$$= \lim_{h\to 0} \frac{5 - (2+h) - 3}{h} = -1$$

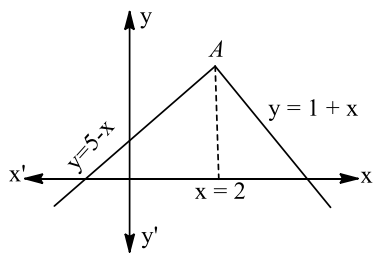
$$Lf''(2) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h\to 0} \frac{1 + (2-h) - 3}{-h} = 1$$

$\therefore \quad Rf''(2) \ne Lf''(2)$

$\therefore \quad f$ is not differentiable at $x = 2$

Alternate



It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x = 2$

97 **(d)**

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

Putting $x = 1, y = 0$, we get

$f(0) = f(1)f(0) \Rightarrow f(0)\big(1 - f(1)\big) = 0$

$\Rightarrow f(1) = 1 \qquad [\because f(0) \neq 0]$

Now,

$f'(1) = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(1)f(h) - f(1)}{h} = 2$

$\Rightarrow f(1) \lim_{h\to 0} \frac{f(h) - 1}{h} = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \quad$ [Using $f(1) = 1$] ...(i)

$\therefore f'(4) = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$

$\Rightarrow f'(4) = \lim_{h\to 0} \frac{f(4)f(h) - f(4)}{h}$

$\Rightarrow f'(4) = \left\{\lim_{h\to 0} \frac{f(h) - 1}{h}\right\} f(4)$

$\Rightarrow f'(4) = 2\, f(4) \qquad$ [From (i)]

$\Rightarrow f'(4) = 2 \times 4 = 8$

98 **(d)**

We have,

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = 1$ and $g(1) = 0$

So, $g(x)$ is not continuous at $x = 1$ but $\lim_{x\to 1} g(x)$ exists

We have,

$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}[1-h] = 0$

and,

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}[1+h] = 1$

So, $\lim_{x\to 1} f(x)$ does not exist and so $f(x)$ is not continuous at $x = 1$

We have, $gof(x) = g\big(f(x)\big) = g([x]) = 0$, for all $x \in R$

So, gof is continuous for all x

We have,

$fog(x) = f\big(g(x)\big)$

$\Rightarrow fog(x) = \begin{cases} f(0), & x \in Z \\ f(x^2), & x \in R - Z \end{cases}$

$\Rightarrow fog(x) = \begin{cases} 0, & x \in Z \\ [x^2], & x \in R - Z \end{cases}$

Which is clearly not continuous

99 **(d)**

At $x = 1$,

RHD= $\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{2 - (1+h) - (2-1)}{h} = -1$

LHD= $\lim_{h\to 0^-} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{(1-h) - (2-1)}{-h} = 1$

$\therefore$ LHD≠RHD

100 **(d)**

Given, $f(x) = |x| + \frac{|x|}{x}$

Let $f_1(x) = |x|,\ f_2(x) = \frac{|x|}{x}$

1. LHL= $\lim_{x\to 0^-} f_1(x) = \lim_{x\to 0^-} |x| = 0$

And RHL $\lim_{x\to 0^+} f_1(x) = \lim_{x\to 0^+} |x| = 0$

Here, LHL=RHL=$f(0)$, $f_1(x)$ is continuous

2. LHL= $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0-h|}{0-h} = -1$

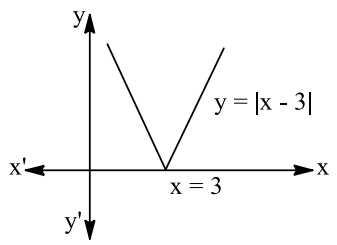
RHL= $\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0+h|}{h} = 1$

$\therefore$ LHL≠RHL, $f_2(x)$ is discontinuous

Hence, $f(x)$ is discontinuous at $x = 0$

101 **(a)**

From the graph it is clear that $f(x)$ is continuous everywhere but not differentiable at $x = 3$



102 **(b)**

Given, $f(x) = \begin{cases} \frac{2x-3}{2x-3}, & \text{if } x > \frac{3}{2} \\ \frac{-(2x-3)}{2x-3}, & \text{if } x < \frac{3}{2} \end{cases}$

$= \begin{cases} 1, & \text{if } x > \frac{3}{2} \\ -1, & \text{if } x < \frac{3}{2} \end{cases}$

Now, RHL= $\lim_{x\to \frac{3}{2}^+} f(x) = \lim_{x\to \frac{3}{2}^+} 1 = 1$

And LHL= $\lim_{x\to \frac{3}{2}^-} f(x) = \lim_{x\to \frac{3}{2}^-} (-1) = -1$

$\because$ RHL≠LHL

$\therefore$ $f(x)$ is discontinuous at $x = \frac{3}{2}$

103 **(c)**

Since the functions $(\log t)^2$ and $\frac{\sin t}{t}$ are not defined on $(-1, 2)$. Therefore, the functions in options (a) and (b) are not defined on $(-1, 2)$

The function $g(t) = \frac{1-t+t^2}{1+t+t^2}$ is continuous on $(-1, 2)$ and

$f(x) = \int_0^x \frac{1-t+t^2}{1+t+t^2} dt$ is the integral function of $g(t)$

Therefore, $f(x)$ is differentiable on $(-1,2)$ such that $f'(x)=g(x)$

104 **(c)**

Since, $f(x)=\frac{1-\tan x}{4x-\pi}$

Now, $\lim_{x\to\pi/4} f(x)=\lim_{x\to\pi/4}\left(\frac{1-\tan x}{4x-\pi}\right)$

$=\lim_{x\to\pi/4}\left(\frac{-\sec^2 x}{4}\right)=-\frac{1}{2}$

Since, $f(x)$ is continuous at $x=\frac{\pi}{4}$

$\therefore \lim_{x\to\pi/4} f(x)=f\left(\frac{\pi}{4}\right)=-\frac{1}{2}$

105 **(a)**

$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}.x=0$

Also, $f(0)=k$

For, $\lim_{x\to 0} f(x)=f(0) \Rightarrow k=0$

106 **(a)**

We have,

$f(x)=|x|+|x-1|$

$\Rightarrow f(x)=\begin{cases}-2x+1, & x<0\\ x-x+1, & 0\le x<1\\ x+x-1, & x\ge 1\end{cases}$

$\Rightarrow f(x)=\begin{cases}-2x+1, & x<0\\ 1, & 0\le x<1\\ 2x-1, & x\ge 1\end{cases}$

Clearly, $\lim_{x\to 0^-} f(x)=1=\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x=0,1$

107 **(d)**

$f(0)=\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

$=\lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}$

$=\frac{2-1}{2+1}=\frac{1}{3}$

109 **(b)**

$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5}-\left(\frac{1}{3}\right)}{h}$

$=\lim_{h\to 0}\frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h}=\lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right)=-\frac{2}{9}$

110 **(a)**

LHL$=\lim_{h\to 0} f\left(-\frac{\pi}{2}-h\right)=\lim_{h\to 0} 2\cos\left(-\frac{\pi}{2}-h\right)=0$

RHL$=\lim_{h\to 0} f\left(-\frac{\pi}{2}+h\right)=\lim_{h\to 0} 2\,a\sin\left(-\frac{\pi}{2}+h\right)+b$

$=-a+b$

Since, function is continuous.

$\therefore$ RHL=LHL $\Rightarrow a=b$

From the given options only (a) *ie*, $\left(\frac{1}{2},\frac{1}{2}\right)$ satisfies this condition

111 **(a)**

We have,

$f'(0)=3$

$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=3$

$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=3$ [Using: (RHD at $x=0$) $=3$]

$\Rightarrow \lim_{h\to 0}\frac{f(0)f(h)-f(0)}{h}$

$=3$ $\left[\because f(x+y)=f(x)f(y)\right]$ $\left[\therefore f(0+h)=f(0)f(h)\right]$

$\Rightarrow f(0)\left(\lim_{h\to 0}\frac{f(h)-1}{h}\right)=3$ ...(i)

Now, $f(x+y)=f(x)f(y)$ for all $x,y\in R$

$\Rightarrow f(0)=f(0)f(0)$

$\Rightarrow f(0)\{1-f(0)\}=0\Rightarrow f(0)=1$

Putting $f(0)=1$ in (i), we get

$\lim_{h\to 0}\frac{f(h)-1}{h}=3$ ...(ii)

Now,

$f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$

$\Rightarrow f'(5)=\lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$

$\Rightarrow f'(5)=\left\{\lim_{h\to 0}\frac{f(h)-1}{h}\right\}f(5)=3\times 2=6$

[Using (ii)]

112 **(c)**

We have,

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f(x)'=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$

$\Rightarrow f(x)'=\lim_{h\to 0}\frac{f(h)}{h}$

$\Rightarrow f(x)'=\lim_{h\to 0}\frac{h\,g\,(h)}{h}\lim_{h\to 0} g(h)=g(0)$ [$\because g$ is conti. at $x=0$]

113 **(b)**

The domain of $f(x)$ is $[2,\infty)$

We have,

$f(x)=\sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2}+2+2\sqrt{2x-4}}$

$$+\sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2}+2-2\sqrt{2x-4}}$$

$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2+4\sqrt{2x-4}+4}$$

$$+\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2-4\sqrt{2x-4}+4}$$

$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\left|\sqrt{2x-4}+2\right|+\frac{1}{\sqrt{2}}\left|\sqrt{2x-4}-2\right|$$

$$\Rightarrow f(x)=\begin{cases}\frac{1}{\sqrt{2}}\times 4, & \text{if } \sqrt{2x-4}<2\\ \sqrt{2}\cdot\sqrt{2x-4}, & \text{if } \sqrt{2x-4}\geq 2\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}2\sqrt{2}, & \text{if } x\in[2,4)\\ 2\sqrt{x-2}, & \text{if } x\in[4,\infty)\end{cases}$$

Hence, $f'(x)=\begin{cases}0 & \text{if } x\in[2,4)\\ \frac{1}{\sqrt{x-2}} & \text{if } x\in(4,\infty)\end{cases}$

114 **(c)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim_{x\to 0}x\sin\frac{1}{x}=0$$

So, $f(x)$ is differentiable at $x=0$ such that $f'(0)=0$

For $x\neq 0$, we have

$$f'(x)=2x\sin\left(\frac{1}{x}\right)+x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow f'(x)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$$

$$\Rightarrow \lim_{x\to 0}f'(x)=\lim_{x\to 0}2x\sin\frac{1}{x}-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$$

$$=0-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$$

Since $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist

$\therefore \lim_{x\to 0}f'(x)$ does not exist

Hence, $f'(x)$ is not continuous at $x=0$

115 **(c)**

We have,

$$f(x)=\begin{cases}\frac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\\ 0, & x=0\end{cases}$$

Clearly, $f(x)$ is not continuous at $x=0$

117 **(c)**

Given, $\lim_{x\to 0}\left[(1+3x)^{\frac{1}{x}}\right]=k$

$\therefore \quad e^3=k$

118 **(b)**

For $x>2$, we have

$$f(x)=\int_0^x\{5+|1-t|\}dt$$

$$\Rightarrow f(x)=\int_0^1(5+(1-t)dt+\int_1^x(5-(1-t)\,dt$$

$$\Rightarrow f(x)=\int_0^1(6-t)dt+\int_1^x(4+t)dt$$

$$\Rightarrow f(x)=\left[6t-\frac{t^2}{2}\right]_0^1+\left[4t+\frac{t^2}{2}\right]_1^x$$

$$\Rightarrow f(x)=1+4x+\frac{x^2}{2}$$

Thus, we have

$$f(x)=\begin{cases}5x+1, & \text{if } x\leq 2\\ \frac{x^2}{2}+4x+1, & \text{if } x>2\end{cases}$$

Clearly, $f(x)$ is everywhere continuous and differentiable except possibly at $x=2$

Now,

$\lim_{x\to 2^-}f(x)=\lim_{x\to 2}5x+1=11$

and,

$$\lim_{x\to 2^+}f(x)=\lim_{x\to 2}\left(\frac{x^2}{2}+4x+1\right)=11$$

$\therefore \lim_{x\to 2^-}f(x)=\lim_{x\to 2^+}f(x)$

So, $f(x)$ is continuous at $x=2$

Also, we have (LHD at $x=2$) $=\lim_{x\to 2^-}f'(x)=\lim_{x\to 2}5=5$

119 **(b)**

The given function is clearly continuous at all points except possibly at $x=\pm 1$

For $f(x)$ to be continuous at $x=1$, we must have

$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)=f(1)$

$$\Rightarrow \lim_{x\to 1}ax^2+b=\lim_{x\to 1}\frac{1}{|x|}$$

$\Rightarrow a+b=1 \qquad \ldots(\text{i})$

Clearly, $f(x)$ is differentiable for all x, except possibly at $x=\pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x=1$ only

For $f(x)$ to be differentiable at $x=1$, we must have

$$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow \lim_{x\to 1}\frac{ax^2+b-1}{x-1}=\lim_{x\to 1}\frac{\frac{1}{|x|}-1}{x-1}$$

$$\Rightarrow \lim_{x\to 1}\frac{ax^2-a}{x-1}=\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1} \qquad [\because a+b=1$$

$$\therefore b-1=-a]$$

$\Rightarrow \lim_{x\to 1} a(x+1) = \lim_{x\to 1} \frac{-1}{x}$

$\Rightarrow 2a = -1 \Rightarrow a = -1/2$

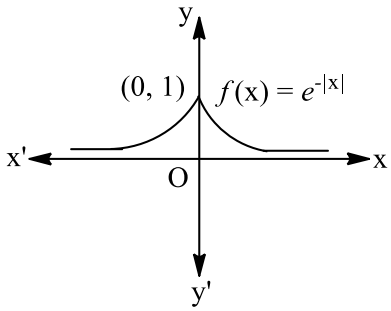
Putting $a = -1/2$ in (i), we get $b = 3/2$

120 **(c)**

At no point, function is continuous

121 **(a)**

It is clear from the figure that $f(x)$ is continuous everywhere and not differentiable at $x = 0$ due to sharp edge



122 **(c)**

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}} \times \frac{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$$

$\Rightarrow \lim_{x\to 0} f(x)$

$$= \lim_{x\to 0} \frac{-2ax(\sqrt{a+x} + \sqrt{a-x})}{2x\,(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$$

$$= \frac{-a(2\sqrt{a})}{(a+a)} = -\sqrt{a}$$

123 **(b)**

Given, $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, x \neq 0 \\ k \quad x = 0 \end{cases}$

LHL= $\lim_{x\to 0^-} f(x)$

$$= \lim_{h\to 0} \frac{1 - \cos 4(0-h)}{8(0-h)^2}$$

$$= \lim_{h\to 0} \frac{1 - \sin 4h}{8h^2}$$

$= \lim_{h\to 0} \frac{4\sin 4h}{16h} = 1$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) =$LHL $\Rightarrow \quad k = 1$

124 **(d)**

Given, $f(x) = |x-1| + |x-2| + \cos x$

Since, $|x-1|, |x-2|$ and $\cos x$ are continuous in $[0, 4]$

$\therefore \quad f(x)$ being sum of continuous functions is also continuous

125 **(c)**

If function $f(x)$ is continuous at $x = 0$, then

$f(0) = \lim_{x\to 0} f(x)$

$\therefore \quad f(0) = k = \lim_{x\to 0} x \sin\frac{1}{x}$

$\Rightarrow k = 0 \qquad \left[\because -1 \leq \sin\frac{1}{x} \leq 1\right]$

126 **(b)**

We have,

$h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$\Rightarrow h'(x) = 2f(x)2f'(x) + 2\,g(x)\,g'(x)$

Now,

$f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, we have

$f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2\,g(x)g'(x) + 2\,g\,(x)g'(x) = 0$, for all x

$\Rightarrow h(x) =$ Constant for all x

But, $h(5) = 11$. Hence, $h(x) = 11$ for all x

127 **(a)**

$$f(x) = |x|^3 = \begin{cases} 0, & x = 0 \\ x^3, & x > 0 \\ -x^3, & x < 0 \end{cases}$$

Now, $Rf'(0) = \lim_{h\to 0} \frac{h^3 - 0}{h} = 0$

And $Lf'(0) = \lim_{h\to 0} \frac{-h^3 - 0}{-h} = 0$

$\because \quad Rf'(0) = Lf'(0) = 0$

$\therefore \quad f'(0) = 0$

128 **(b)**

We have,

(LHL at $x = 0$) $= \lim_{n\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$\Rightarrow$ (LHL at $x = 0$) $= \lim_{n\to 0} \sin^{-1}(\cos\ (-h))$

$= \lim_{h\to 0} \sin^{-1}(\cosh h)$

$\Rightarrow$ (LHL at $x = 0$) $= \sin^{-1} 1 = \pi/2$

(RHL at $x = 0$) $= \lim_{x\to 0^+} f(x)$

$\Rightarrow$ (RHL at $x = 0$) $= \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} \sin^{-1}(\cos h)$

$\Rightarrow$ (RHL at $x = 0$) $= \sin^{-1}(1) = \pi/2$

and, $f(0) = \sin^{-1}(\cos 0) = \sin^{-1}(1) = \pi/2$

$\therefore$ (LHL at $x = 0$) = (RHL at $x = 0$) $= f(0)$

So, $f(x)$ is continuous at $x = 0$

Now,

$$f'(x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}} = \frac{\sin x}{|\sin x|}$$
$$= \begin{cases} \frac{-\sin x}{-\sin x} = 1, x < 0 \\ \frac{-\sin x}{\sin x} = -1, x > 0 \end{cases}$$
$\therefore$ (LHD at $x = 0$) $= 1$ and (RHD at $x = 0$) $= -1$

Hence, $f(x)$ is not differentiable at $x = 0$

129 **(d)**

For any $x \neq 1, 2$, we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore, $f(x)$ is continuous for all $x \neq 1, 2$

Continuity at $x = 1$:

We have,

$$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$$
$$\Rightarrow \lim_{x\to 1^-} f(x)$$
$$= \lim_{h\to 0} \frac{(1-h-2)(1-h+2)(1-h+1)(1-h-1)}{|(1-h-1)(1-h-2)|}$$
$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \frac{(3-h)(2-h)(-1-h)(-h)}{|(-h)(-1-h)|}$$
$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \frac{(3-h)(2-h)h(h+1)}{h(h+1)} = 6$$

and,

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$$
$$\lim_{x\to 1^+} f(x)$$
$$= \lim_{h\to 0} \frac{(1+h-2)(1+h+2)(1+h+1)(1+h-1)}{|(1+h-1)(1+h-2)|}$$
$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} \frac{(h-1)(3+h)(2+h)(h)}{|h(h-1)|}$$
$$\lim_{x\to 1^+} f(x) = -\lim_{h\to 0} \frac{(h-1)(3+h)(2+h)h}{h(1-h)} = -6$$
$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

So, $f(x)$ is not continuous at $x = 1$

Similarly, $f(x)$ is not continuous at $x = 2$

130 **(b)**

Let $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1+|x|}$

It is clear that $g(x) = x$ and $h(x) = 1 + |x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively

Thus, $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.Now, we have to check the differentiable at $x = 0$

$$\therefore \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{\frac{x}{1+|x|} - 0}{x} = \lim_{x\to 0} \frac{1}{1+|x|}$$
$$= 1$$

Hence, $f(x)$ is differntaible on $(-\infty, \infty)$

131 **(b)**

At $x = 0$,

LHL= $\lim_{h\to 0} \frac{1}{1-e^{-1/(0-h)}} = \lim_{h\to 0} \frac{1}{1-e^{1/h}} = 0$

RHL= $\lim_{h\to 0} \frac{1}{1-e^{-1/(0+h)}} = \lim_{h\to 0} \frac{1}{1-e^{-1/h}} = 1$

$\therefore$ FUnction is not continuous at $x = 0$

132 **(a)**

We have,

$fog = I$

$\Rightarrow fog(x) = x$ for all x

$\Rightarrow f'(g(x))g'(x) = 1$ for all x

$$\Rightarrow f'(g(a)) = \frac{1}{g'(a)} = \frac{1}{2} \Rightarrow f'(b)$$
$$= \frac{1}{2} \quad [\because f(a) = b]$$

133 **(a)**

Since, $\lim_{x\to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0} \frac{\sin \pi x}{5x} = k$$
$$\Rightarrow (1)\frac{\pi}{5} = k \quad \Rightarrow \quad k = \frac{\pi}{5} \qquad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

134 **(d)**

Given, $f(x) = [x], x \in (-3.5, 100)$

As we know greatest integer is discontinuous on integer values.

In given interval, the integer values are $(-3, -2, -1, 0, \dots, 99)$

$\therefore$ Total numbers of integers are 103.

135 **(a)**

LHL= $\lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = -1 \quad \left[\because \lim_{h\to 0} \frac{1}{e^{1/h}} = 0\right]$

RHL= $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$

$$= \lim_{h\to 0} \frac{1 - \frac{1}{e^{1/h}}}{1 + \frac{1}{e^{1/h}}} = 1$$

$\therefore$ LHL$\neq$RHL

So, limit does not exist at $x = 0$

136 **(d)**

We have,

$$f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)} + \cdots$$
$$\Rightarrow f(x) = \frac{x^4}{1 - \frac{1}{1+x^4}} = 1 + x^4, \text{if } x \neq 0$$

Clearly, $f(x) = 0$ at $x = 0$

Thus, we have

$$f(x) = \begin{cases} 1 + x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is neither continuous nor differentiable at $x = 0$

137 **(a)**

We have,

$$f(x) = \begin{cases} 1 + x, & 0 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$$

$\therefore g(x) = fof(x)$

$\Rightarrow f(x) = f\big(f(x)\big)$

$$\Rightarrow g(x) = \begin{cases} f(1 + x), & 0 \le x \le 2 \\ f(3 - x), & 2 < x \le 3 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} 1 + (1 + x), & 0 \le x \le 1 \\ 3 - (1 + x), & 1 < x \le 2 \\ 1 + (3 - x), & 2 < x \le 3 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} 2 + x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 4 - x, & 2 < x \le 3 \end{cases}$$

Clearly, $g(x)$ is continuous in $(0, 1) \cup (1, 2) \cup (2, 3)$ except possibly at $x = 0, 1, 2$ and 3

We observe that

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (2 + x) = 2 = g(0)$

and $\lim\limits_{x \to 3^-} g(x) = \lim\limits_{x \to 3^-} 4 - x = 1 = g(3)$

Therefore, $g(x)$ is right continuous at $x = 0$ and left continuous at $x = 3$

At $x = 1$, we have

$\lim\limits_{x \to 1^-} g(x) = \lim\limits_{x \to 1^-} 2 + x = 3$

and, $\lim\limits_{x \to 1^+} g(x) = \lim\limits_{x \to 1^+} 2 - x = 1$

$\therefore \lim\limits_{x \to 1^+} g(x) \ne \lim\limits_{x \to 1^-} g(x)$

So, $g(x)$ is not continuous at $x = 1$

At $x = 2$, we have

$\lim\limits_{x \to 2^-} g(x) = \lim\limits_{x \to 2^-} (2 - x) = 0$

and,

$\lim\limits_{x \to 2^+} g(x) = \lim\limits_{x \to 2^+} (4 - x) = 0$

$\therefore \lim\limits_{x \to 2^-} g(x) \ne \lim\limits_{x \to 2^+} g(x)$

So, $g(x)$ is not continuous at $x = 2$

Hence, the set of points of discontinuity of $g(x)$ is $\{1, 2\}$

138 **(b)**

Since $g(x)$ is the inverse of function $f(x)$

$\therefore gof(x) = I(x)$, for all x

Now, $gof(x) = I(x)$, for all x

$\Rightarrow gof(x) = x$, for all x

$\Rightarrow (gof)'(x) = 1$, for all x

$\Rightarrow g'\big(f(x)\big)f'(x) = 1$, for all x [Using Chain Rule]

$\Rightarrow g'\big(f(x)\big) = \frac{1}{f'(x)}$, for all x

$\Rightarrow g'\big(f(c)\big) = \frac{1}{f'(c)}$ [Putting $x = c$]

139 **(d)**

Given, $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), x \ne 0 \\ 0, \quad x = 0 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore it is continuous at $x = 0$

$\therefore \lim\limits_{x \to 0} f(x) = f(0) = 0$

$\Rightarrow \lim\limits_{x \to 0} x^p \cos\left(\dfrac{1}{x}\right) = 0 \quad \Rightarrow \quad p > 0$

As $f(x)$ is differentiable at $x = 0$

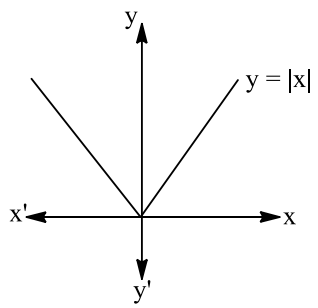
$\therefore \lim\limits_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim\limits_{x \to 0} \frac{x^p \cos\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim\limits_{x \to 0} x^{p-1} \cos\frac{1}{x} - 0$ exists finitely

$\Rightarrow \quad p - 1 > 0 \quad \Rightarrow \quad p > 1$

140 **(a)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x = 0$

141 **(c)**

We know that the function

$$\phi(x) = (x - a)^2 \sin\left(\frac{1}{x - a}\right)$$

Is continuous and differentiable at $x = a$ whereas the function $\Psi(x) = |x - a|$ is everywhere continuous but not differentiable at $x = a$

Therefore, $f(x)$ is not differentiable at $x = 1$

142 **(d)**

$$\lim_{x \to 0} \frac{2^x - 2^{-x}}{x} = \lim_{x \to 0} 2^x \log 2 + 2^{-x} \log 2$$

[by L' Hospital's rule]

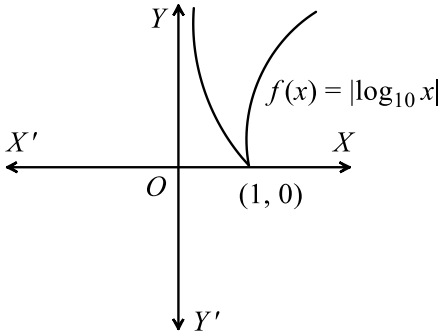
$= \log 4$

Since, the function is continuous at $x = 0$

$\therefore \quad f(0) = \lim\limits_{x \to 0} f(x) \quad \Rightarrow \quad f(0) = \log 4$

143 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$$f''(1^+) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log_{10}(1+h)-0}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$

$$f''(1^-) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{h}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{\log_{10}(1-h)}{h} = \lim_{h\to 0} \frac{\log_e(1-h)}{h\log_e 10}$$

$$= -\log_{10} e$$

144 **(a)**

We have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)+f(h)-f(x)}{h} \qquad [\because f(x+y) = f(x)+f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(h)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\sin h\ g(h)}{h} = \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} g(h)$$

$$= g(0) = k$$

145 **(a)**

We have,

$$f(x) = |x| + |x-1| = \begin{cases} -2x+1, & x<0 \\ 1, & 0 \le x < 1 \\ 2x-1, & 1 \le x \end{cases}$$

Clearly, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 1 = 1$, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x-1) = 1$

and, $f(1) = 2\times 1 - 1 = 1$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

So, $f(x)$ is continuous at $x = 1$

Now, $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{1-1}{-h} = 0$

and,

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{h\to 0} \frac{2(1+h)-1-1}{h} = 2$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$

146 **(d)**

The given function is differentiable at all points except possibly at $x = 0$

Now,

(RHD at $x = 0$)

$$= \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{h+1}-1}{h^{3/2}}$$

$$= \lim_{h\to 0} \frac{h}{h^{3/2}(\sqrt{h+1}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{h}(\sqrt{h+1}+1)}$$

$$\to \infty$$

So, the function is not differentiable at $x = 0$

Hence, the required set is $R - \{0\}$

147 **(a)**

We have,

$$f(x)\,f(y) = f(x) + f(y) + f(xy) - 2$$

$$\Rightarrow f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) + f(1) - 2$$

$$\Rightarrow f(x).f\left(\frac{1}{x}\right)$$

$$= f(x)$$

$$+ f\left(\frac{1}{x}\right) \quad \left[\begin{array}{c}\because f(1) = 2 \text{ (Putting } x = y = 1 \\ \text{in the given relation)}\end{array}\right]$$

$$\Rightarrow f(x) = x^n + 1$$

$$\Rightarrow f(2) = 2^n + 1$$

$$\Rightarrow 5 = 2^n + 1 \qquad [\because f(2) = 5 \text{ (given)}]$$

$$\Rightarrow n = 2$$

$$\therefore f(x) = x^2 + 1 \Rightarrow f(3) = 10$$

148 **(b)**

We have,

$f(x) = \frac{1}{2}x - 1$, for $0 \le x \le \pi$

$$\therefore \{f(x)\} = \begin{cases} -1, \text{for } 0 \le x < 2 \\ 0, \text{for } 2 \le x \le \pi \end{cases}$$

$$\Rightarrow \tan[f(x)] = \begin{cases} \tan(-1) = -\tan(1), 0 \le x < 2 \\ \tan 0 = 0, \qquad 2 \le x \le \pi \end{cases}$$

It is evident from the definition of $\tan[f(x)]$ that $\lim_{x\to 2^-} \tan[f(x)] = -\tan 1$ and, $\lim_{x\to 2^+} \tan[f(x)] = 0$

So, $\tan[f(x)]$ is not continuous at $x = 2$

Now,

$$f(x) = \frac{1}{2}x - 1 \Rightarrow f(x) = \frac{x-2}{2} \Rightarrow \frac{1}{f(x)} = \frac{2}{x-2}$$

Clearly, $f(x)$ is not continuous at $x = 2$

So, $\tan[f(x)]$ and $\tan\left[\frac{1}{f(x)}\right]$ both are discontinuous at $x = 2$

149 **(c)**

$\lim_{x\to 0}(1+x)^{\cot x} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x\cot x}$

$= \lim_{x\to 0} e^{x\cot x} = e$

Since $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) = \lim_{x\to 0} f(x) = e$

150 **(b)**

LHL= $\lim_{h\to 0} f\left(\frac{\pi}{4} - h\right)$

$$= \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4} - h\right) - \cot\left(\frac{\pi}{4} - h\right)}{\frac{\pi}{4} - h - \frac{\pi}{4}}$$

$$= \lim_{h\to 0} \frac{-\sec^2\left(\frac{\pi}{4} - h\right) - \operatorname{cosec}^2\left(\frac{\pi}{4} - h\right)}{-1} = 4$$

[by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$, then LHL= $f\left(\frac{\pi}{4}\right)$

$\therefore \quad a = 4$

151 **(a)**

If $-1 \le x < 0$, then

$$f(x) = \int_{-1}^{x} |t|\, dt = \int_{-1}^{x} -t\, dt = -\frac{1}{2}(x^2 - 1)$$

If $x \ge 0$, then

$$f(x) = \int_{-1}^{0} -t\, dt + \int_{-1}^{x} -t\, dt = \frac{1}{2}(x^2 + 1)$$

$$\therefore f(x) = \begin{cases} -\frac{1}{2}(x^2 - 2), & -1 \le x < 0 \\ \frac{1}{2}(x^2 + 1), & 0 \le x \end{cases}$$

It can be easily seen that $f(x)$ is continuous at $x = 0$

So, it is continuous for all $x > -1$

Also, $Rf'(0) = 0 = Lf'(0)$

So, $f(x)$ is differentiable at $x = 0$

$$\therefore f'(x) = \begin{cases} -x, & -1 < x = 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$$

Clearly, $f'(x)$ is continuous at $x = 0$

Consequently, it is continuous for all $x > -1$ i.e. for $x + 1 > 0$

Hence, f and f' are continuous for $x + 1 > 0$

152 **(c)**

We have,

$$f(x) = \lim_{n\to\infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}$$

$$\Rightarrow f(x) = \lim_{n\to\infty} \frac{1 - x^{2n}}{1 + x^{2n}}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1-0}{1+0} = 1, & \text{if } -1 < x < 1 \\ \frac{1-1}{1+1} = 0, & \text{if } x = \pm 1 \\ \frac{0-1}{0+1} = -1, & \text{if } |x| > 1 \end{cases}$$

Clearly, $f(x)$ is discontinuous at $x = \pm 1$

153 **(b)**

Clearly, $\log|x|$ is discontinuous at $x = 0$

$f(x) = \frac{1}{\log|x|}$ is not defined at $x = \pm 1$

Hence, $f(x)$ is discontinuous at $x = 0, 1, -1$

154 **(a)**

For continuity, $\lim_{x\to 0} f(x) = k$

$$\Rightarrow \lim_{x\to 0} \frac{\sin 3x}{\sin x} = k \Rightarrow \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{3x}{\sin 3x} = k$$

$\Rightarrow \quad 3 = k$

155 **(b)**

Since, the function $f(x)$ is continuous

$\therefore \quad f(0) =$RHL $f(x) =$LHL$f(x)$

Now, RHL $f(X) = \lim_{h\to 0} \frac{\log(1+0+h) + \log(1-0-h)}{0+h}$

$$= \lim_{h\to 0} \frac{\log(1+h) + \log(1-h)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+h} - \frac{1}{1-h}}{1} = 0$$

[by L 'Hospital's rule]

$\therefore \quad f(0) =$RHL $f(x) = 0$

156 **(d)**

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases} = \begin{cases} -1 + a, x < 4 \\ a + b \\ 1 + b, x > 4 \end{cases}$$

LHL= $\lim_{x\to 4^{\mp}} f(x) = a - 1$

RHL= $\lim_{x\to 4^{\mp}} f(x) = 1 + b$

Since, LHL=RHL= $f(4)$

$\Rightarrow \quad a - 1 = a + b = b + 1$

$a = 1$ and $b = -1$

157 **(d)**

We have,

$$f(x) = \begin{cases} \frac{-1}{x-1}, & 0 < x < 1 \\ \frac{1-1}{x-1} = 0, & 1 < x < 2 \\ 0, & x = 1 \end{cases}$$

Clearly, $\lim_{x\to 1^-} f(x) \to -\infty$ and $\lim_{x\to 1^+} f(x) = 0$

So, $f(x)$ is not continuous at $x = 1$ and hence it is not differentiable at $x = 1$

158 **(d)**

$$\lim_{x\to\frac{\pi}{4}} f(x) = \lim_{x\to\frac{\pi}{4}} \frac{1-\sqrt{2}\sin x}{\pi - 4x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\cos x}{4} = \frac{1}{4} \quad \text{[by L 'Hospital's rule]}$$

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \quad \Rightarrow \quad \frac{1}{4} = a$$

159 **(d)**

LHL= $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} 1 - h + a = 1 + a$

RHL= $\lim_{x\to 1^+} f(x) = \lim_{h\to 0} 3 - (1+h)^2 = 2$

For $f(x)$ to be continuous, LHL=RHL

$\Rightarrow 1 + a = 2 \quad \Rightarrow \quad a = 1$

160 **(b)**

LHL= $\lim_{h\to 0} \frac{\cos 3(0-h) - \cos(0-h)}{(0-h)^2}$

$$= \lim_{h\to 0} \frac{\cos 3h - \cos h}{h^2}$$

$$= \lim_{h\to 0} \frac{-3\sin 3h + \sin h}{2h}$$

$$= \lim_{h\to 0} \frac{-9\cos 3h + \cos h}{2} = \frac{-9+1}{2} = -4$$

$\because \lim_{x\to 0^-} f(x) = f(0) \quad \Rightarrow \quad \lambda = -4$

161 **(c)**

LHL= $\lim_{x\to a^-} \frac{x^3 - a^3}{x - a} = \lim_{h\to 0} \frac{(a-h)^3 - a^3}{a - h - a}$

$$= \lim_{h\to 0} \frac{(a-h-a)\{(a-h)^2 + a^3 + a(a-h)\}}{-h}$$

$$= 3a^2$$

Since, $f(x)$ is continuous at $x = a$

$\therefore \quad \text{LHL} = f(a)$

$\Rightarrow \quad 3a^2 = b$

163 **(a)**

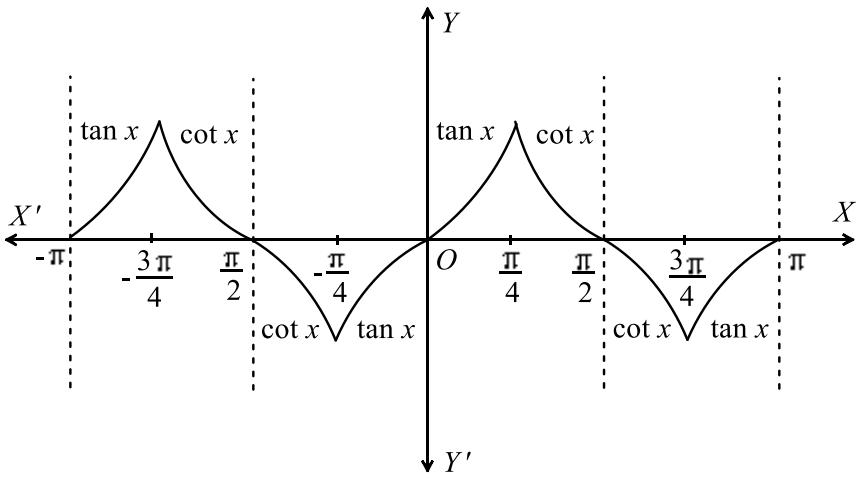
We have,

$$f(x) = \begin{cases} \tan x, & 0 \le x \le \pi/4 \\ \cot x, & -\pi/4 \le x \le \pi/2 \\ \tan x, & \pi/2 < x \le 3\pi/4 \\ \cot x, & 3\pi/4 \le x < \pi \end{cases}$$

Since $\tan x$ and $\cot x$ are periodic functions with period π. So, $f(x)$ is also periodic with period π

It is evident from the graph that $f(x)$ is not continuous at $x = \pi/2$. Since $f(x)$ is periodic with period π. So, it is not continuous at $x = 0, \pm\pi/2, \pm\pi, \neq 3\pi/2$

Also, $f(x)$ is not differentiable $x = \pi/4, 3\pi/4, 5\pi/4$ etc



164 **(c)**

We have,

$f(x) = \{|x| - |x-1\}^2$

$$\Rightarrow f(x) = \begin{cases} (-x + x - 1)^2, & \text{if } x < 0 \\ (x + x - 1)^2, & \text{if } 0 \le x < 1 \\ (x - x + 1)^2, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x < 0 \\ (2x-1)^2, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 0, & \text{if } x < 0 \text{ or if } x > 1 \\ 4(2x-1), & \text{if } 0 < x < 1 \end{cases}$$

165 **(b)**

We have,

$$f'(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow f'(x_0) = \lim_{x\to x_0} \frac{(x - x_0)\phi(x) - 0}{(x - x_0)}$$

$\Rightarrow f'(x_0) = \lim_{x\to x_0} \phi(x) = \phi(x_0)$ [

$\because \phi(x)$ is continuous at $x = x_0$]

166 **(b)**

Since, $\lim_{x\to 2^+} f(x) = f(2) = k$

$\Rightarrow \quad k = \lim_{h\to 0} f(2+h)$

$\Rightarrow \quad k = \lim_{h\to 0}\left[(2+h)^2 + e^{\frac{1}{2-(2+h)}}\right]^{-1}$

$\Rightarrow \quad \lim_{h\to 0} \left[4 + h^2 + 4h + e^{-1/h}\right]^{-1} = \frac{1}{4}$

167 **(c)**

For $f(x)$ to be continuous at $x = \pi/2$, we must have

$\lim_{x\to\pi/2} f(x) = f(\pi/2)$

$\Rightarrow \lim_{x\to\pi/2} \frac{1-\sin x}{(\pi-2x)^2} \cdot \frac{\log\sin x}{\log(1+\pi^2-4\pi x+4x^2)} = k$

$\Rightarrow \lim_{h\to 0} \frac{1-\cos h}{4h^2} \times \frac{\log\cos h}{\log(1+4h^2)} = k$

$\Rightarrow \lim_{h\to 0} \frac{1-\cos h}{4h^2} \times \frac{\log\{1+\cos h-1\}}{\cos h - 1}$

$\times \frac{4h^2}{\log(1+4h^2)} \times \frac{\cos h - 1}{4h^2} = k$

$\Rightarrow -\lim_{h\to 0}\left(\frac{1-\cos h}{4h^2}\right)^2 \frac{\log\big(1+(\cos h-1)\big)}{\cos h - 1}$

$\times \frac{4\,h^2}{\log(1+4h^2)} = k$

$\Rightarrow -\lim_{h\to 0}\left(\frac{\sin^2 h/2}{2\,h^2}\right)^2 \frac{\log\big(1+(\cos h-1)\big)}{\cos h - 1}$

$\times \frac{4h^2}{\log(1+4h^2)} = k$

$\Rightarrow -\frac{1}{64}\lim_{h\to 0}\left(\frac{\sin h/2}{h/2}\right)^4 \frac{\log\big(1+(\cos h-1)\big)}{\cos h - 1}$

$\times \frac{4h^2}{\log(1+4h^2)} = k$

$\Rightarrow -\frac{1}{64} = k$

168 **(c)**

LHL$= \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$

$= -\lim_{h\to 0} \frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)} = \frac{5}{2}$

Since, it is continuous at $x = 0$, therefore LHL$=$ $f(0)$

$\Rightarrow \frac{5}{2} = k + \frac{1}{2} \quad \Rightarrow \quad k = 2$

169 **(a)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n > 0$

$f(x)$ is differentiable at $x = 0$, if

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists finitely

$\Rightarrow \lim_{x\to 0} \frac{x^n \sin\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow n - 1 > 0 \Rightarrow n > 1$

If $n \le 1$, then $\lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ does not exist and hence $f(x)$ is not differentiable at $x = 0$

Hence $f(x)$ is continuous but not differentiable at $x = 0$ for $0 < n \le 1$ i.e. $n \in (0, 1]$

170 **(b)**

Clearly, $f(x)$ is not differentiable at $x = 3$

Now, $\lim_{h\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} |3-h-3|$

$= 0$

$\lim_{h\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} |3+h-3| = 0$

and $f(3) = |3-3| = 0$

$\therefore \quad f(x)$ is continuous at $x = 3$

171 **(a)**

It can easily be seen from the graphs of $f(x)$ and that both are continuous at $x = 0$

Also, $f(x)$ is not differentiable at $x = 0$ whereas $g(x)$ is differentiable at $x = 0$

172 **(c)**

We have,

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} \frac{-\sin(a+1)h - \sin h}{-h}$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0}\left\{\frac{\sin(a+1)h}{h} + \frac{\sin h}{h}\right\}$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = (a+1)+1$

$= a + 2$

and, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{b\,h^{3/2}}$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{h + bh^2 - h}{bh^{3/2}(\sqrt{h+bh^2} - \sqrt{h})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{1+bh}+1} = \frac{1}{2}$

Since, $f(x)$ is continuous at $x = 0$. Therefore,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow a+2=\frac{1}{2}=c \Rightarrow c=\frac{1}{2}, a=-\frac{3}{2}$ and $b \in R-\{0\}$

173 **(c)**

For $f(x)$ to be continuous at $x=0$, we must have

$\lim_{x\to 0} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0}\frac{(9^x-1)(4^x-1)}{\sqrt{2}-\sqrt{2\cos^2 x/2}}=k$

$\Rightarrow \lim_{x\to 0}\frac{(9^x-1)(4^x-1)}{\sqrt{2}.2\sin^2 x/4}=k$

$\Rightarrow \lim_{x\to 0}\frac{16\times\left(\frac{9^x-1}{x}\right)\left(\frac{4^x-1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2}=k$

$\Rightarrow \frac{16}{2\sqrt{2}}\log 9.\log 4=k=4\sqrt{2}\log 9.\log 4$

$=16\sqrt{2}\log 3\log 2$

174 **(b)**

Given, $f(x)=[\tan^2 x]$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}[\tan^2 x]=0$

And $f(0)=[\tan^2 0]=0$

Hence, $f(x)$ is continuous at $x=0$

175 **(b)**

Let, $f(x)=x$

Which is continuous at $x=0$

Also, $f(x+y)=f(x)+f(y)$

$\Rightarrow \quad f(0+0)=f(0)+f(0)$

$=0+0$

$\Rightarrow f(0)=0$

$f(1+0)=f(1)+f(0)$

$\Rightarrow \quad f(1)=1+0$

$\Rightarrow \quad f(1)=1$

As, it satisfies it.

Hence, $f(x)$ is continous for every values of x

176 **(c)**

Here, $gof=\begin{cases} e^{\sin x}, & x\geq 0 \\ e^{1-\cos x}, & x\leq 0\end{cases}$

$\therefore$ LHD$=\lim_{h\to 0}\frac{gof(0-h)-gof(h)}{-h}$

$=\lim_{h\to 0}\frac{e^{1-\cos h}-e^{1-\cos h}}{-h}=0$

RHD$=\lim_{h\to 0}\frac{gof(0+h)-gof(h)}{h}$

$=\lim_{h\to 0}\frac{e^{\sin h}-e^{\sin h}}{h}=0$

Since, RHD=LHD=0

$\therefore \quad (gof)'(0)=0$

177 **(b)**

We have,

$f(x)\begin{cases}(x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^2, & x<0\\ 0, & x=0\\ (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^{2-\frac{2}{x}}, & x>0\end{cases}$

Clearly, $f(x)$ is everywhere continuous except possibly at $x=0$

At $x=0$, we have

$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}(x+1)^2=1$

and, $\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(x+1)^{2-\frac{2}{x}}=\lim_{x\to 0}(x+1)^{-2/x}$

$\Rightarrow \lim_{x\to 0^+} f(x)=e^{\lim_{x\to 0}-\frac{2}{x}\log(1+x)}=e^{-2}$

Clearly, $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x=0$

178 **(b)**

Since $f(x)$ is continuous at $x=0$. Therefore,

$\lim_{x\to 0} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0} f(x)=k$

$\Rightarrow \lim_{x\to 0}\frac{\log(1+ax)-\log(1-bx)}{x}=k$

$\Rightarrow a\lim_{x\to 0}\frac{\log(1+ax)}{ax}-(-b)\lim_{x\to 0}\frac{\log(1-bx)}{-bx}=k$

$\Rightarrow a+b=k$

179 **(c)**

Since $f(x)$ is continuous at $x=0$

$\therefore f(0)=\lim_{x\to 0} f(x)$

$\Rightarrow f(0)=\lim_{x\to 0}\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}} \quad \left[\text{Form}\frac{0}{0}\right]$

$\Rightarrow f(0)=\lim_{x\to 0}\frac{\frac{1}{3}(27-2x)^{-\frac{2}{3}}(-2)}{-\frac{3}{5}(243+5x)^{-\frac{4}{5}}(5)}$

$=\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)\frac{3^4}{3^2}=2$

180 **(d)**

$\lim_{x\to 0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$=\lim_{x\to 0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$ [using L 'Hospital rule]

$=\lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}}=1$ [using L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=0$, then

$\lim_{x\to 0} f(x)=f(0) \quad \Rightarrow \quad 1=f(0)$

181 **(b)**

If a function $f(x)$ is continuous at $x=a$, then it may or may not be differentiable at $x=a$

$\therefore$ Option (b) is correct

182 **(c)**

Let $f(x)=|x-1|+|x-3|$

$=\begin{cases}x-1 \quad +x-3 & ,x\geq 3\\ x-1+3-x, & 1\leq x<3\\ 1-x \quad +3-x, & x\leq 1\end{cases}$

$$= \begin{cases} 2x-4, x \geq 3 \\ 2, 1 \leq x < 3 \\ 4-2x, x \leq 1 \end{cases}$$

At $x = 2$, function is

$f(x) = 2$

$\Rightarrow \quad f'(x) = 0$

183 **(d)**

We have,

$$f(x) = \begin{cases} (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1), & x < 0 \\ (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)e^{-2/x}, & x > 0 \end{cases}$$

Clearly, $f(x)$ is continuous for all $x \neq 0$

So, we will check its continuity at $x = 0$

We have,

$(\text{LHL at } x = 0) = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0}(x+1) = 1$

$(\text{RHL } at\ x = 0) = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0}(x+1)\ e^{-2/x}$

$$= \lim_{x \to 0} \frac{x+1}{e^{2/x}} = 0$$

$\therefore \lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+ f(x)}$

So, $f(x)$ is not continuous at $x = 0$

Also, $f(x)$ assumes all values from $f(-2)$ to $f(2)$ and $f(2) = 3/e$ is the maximum value of $f(x)$

184 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x = 2$

LHL= $\lim\limits_{x \to 2^-} x - 1$

$= \lim\limits_{h \to 0} 2 - h - 1 = 1$

RHL= $\lim\limits_{x \to 2^+} 2x - 3 = \lim\limits_{h \to 0} 2(2+h) - 3 = 1$

And $\quad f(2) = 2(2) - 3 = 1$

$\therefore$ LHL+RHL= $f(2)$

Hence, $f(x)$ is continuous for all real values of x

185 **(c)**

Continuity at $x = 0$

LHL= $\lim\limits_{x \to 0^-} \frac{\tan x}{x} = \lim\limits_{h \to 0} \frac{-tan\, h}{-h} = 1$

RHL= $\lim\limits_{x \to 0^+} \frac{\tan x}{x} = \lim\limits_{h \to 0} \frac{tan\, h}{h} = 1$

$\therefore$ LHL=RHL= $f(0) = 1$, it is continuous

Differentiability at $x = 0$

LHD= $\lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{-h} = \lim\limits_{h \to 0} \frac{\frac{\tan(-h)}{-h}-1}{-h}$

$$= \lim_{h \to 0} \frac{+\frac{h^2}{3} + \frac{2h^4}{15} + \cdots}{-h} = 0$$

RHD= $\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0} \frac{\frac{\tan h}{h}-1}{h}$

$$= \lim_{h \to 0} \frac{\frac{h^2}{3} + \frac{2h^4}{15} + \cdots}{-h} = 0$$

$\therefore$ LHD=RHD

Hence, it is differentiable.

186 **(b)**

We have,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1}(x-1) = 0$

and,

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1}(x^3 - 1) = 0$. Also, $f(1) = 1 - 1 = 0$

So, $f(x)$ is continuous at $x = 1$

Clearly, $\left(f'(1)\right) = 3$ and $Rf'(1) = 1$

Therefore, $f(x)$ is not differentiable at $x = 1$

187 **(d)**

We have,

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - x} = 1, & \text{if } x < 0 \text{ or } x > 1 \\ -\frac{(x^2 - x)}{x^2 - x} = -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } x = 0 \\ -1, & \text{if } x = 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, \text{if } x \leq 0 \text{ or } x > 1 \\ -1, \text{if } 0 < x \leq 1 \end{cases}$$

Now,

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} 1 = 1$ and, $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} -1 = -1$

Clearly, $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$

So, $f(x)$ is not continuous at $x = 0$. It can be easily seen that it is not continuous at $x = 1$

188 **(b)**

We have,

$f(x) = |x-1| + |x-3|$

$$\Rightarrow f(x) = \begin{cases} -(x-1) - (x-3), & x < 1 \\ (x-1) - (x-3), & 1 \leq x < 3 \\ (x-1) + (x-3), & x \geq 3 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -2x+4, & x < 1 \\ 2, & 1 \leq x < 3 \\ 2x-4, & x \geq 3 \end{cases}$$

Since, $f(x) = 2$ for $1 \leq x < 3$. Therefore $f'(x) = 0$ for all $x \in (1, 3)$

Hence, $f'(x) = 0$ at $x = 2$

189 **(d)**

We have,

$Lf'(0) = 0$ and $Rf'(0) = 0 + \cos 0° = 1$

$\therefore Lf'(0) \neq Rf'(0)$

Hence, $f'(x)$ does not exist at $x = 0$

190 **(c)**

Given, $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $\ 0 < x < 2, \quad m \neq 0, \ \ n$ are integers and $|x-1| = \begin{cases} x-1; & x \geq 1 \\ 1-x; & x < 1 \end{cases}$

The left hand derivative of $|x-1|$ at $x = 1$ is $p = -1$

Also, $\lim\limits_{x \to 1^+} g(x) = p = -1$

$\Rightarrow \lim_{h\to 0} \frac{(1+h-1)^n}{\log\cos^m(1+h-1)} = -1$

$\Rightarrow \lim_{h\to 0} \frac{h^n}{m\log\cos h} = -1$

$\Rightarrow \lim_{h\to 0} \frac{n.h^{n-1}}{m\frac{1}{\cos h}(-\sin h)} = -1$

[using L 'Hospital's rule]

$\Rightarrow \left(\frac{n}{m}\right)\lim_{h\to 0}\frac{h^{n-2}}{\left(\frac{\tan h}{h}\right)} = 1$

$\Rightarrow n = 2$ and $\frac{n}{m} = 1$

$\Rightarrow m = n = 2$

191 **(c)**

Given, $f(x) = \frac{2x^2+7}{(x^2-1)(x+3)}$

Since, at $x = 1, -1, -3$, $f(x) = \infty$

Hence, function is discontinuous

193 **(a)**

LHL= $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} [1-(1-h)^2] = 0$

RHL= $\lim_{x\to 1^+} f(x) = \lim_{h\to 0}\{1+(1+h)^2\} = 2$

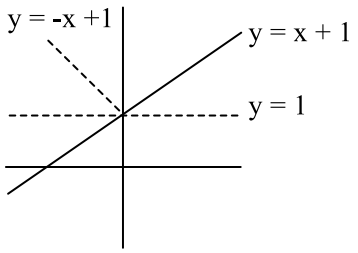
Also, $f(1) = 0$

$\Rightarrow$ RHL $\neq$ LHL $= f(1)$

Hence, $f(x)$ is not continuous at $x = 1$

194 **(c)**

It is clear from the graph that minimum $f(x)$ is



$f(x) = x + 1, \quad \forall x \in R$

Hence, it is a straight line, so it is differentiable everywhere

195 **(c)**

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\lim_{x\to\frac{\pi}{2}^{-1}} (mx+1) = \lim_{x\to\frac{\pi}{2}^{+}} (\sin x + n)$

$\Rightarrow m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n$

$\Rightarrow \frac{m\pi}{2} = n$

196 **(a)**

This function is continuous at $x = 0$, then

$\lim_{x\to 0}\frac{\log_e(1+x^2\tan x)}{\sin x^3} = f(0)$

$\Rightarrow \lim_{x\to 0}\frac{\log_e\left\{1+x^2\left(x+\frac{x^3}{3}+\ldots\right)\right\}}{x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\ldots} = f(0)$

$\Rightarrow \lim_{x\to 0}\frac{\log_e(1+x^3)}{x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\ldots} = f(0)$

[neglecting higher power of x in $x^2\tan x$]

$\Rightarrow \lim_{x\to 0}\frac{x^3-\frac{x^6}{2}+\frac{x^9}{3}-\cdots}{x^3+\frac{x^9}{3!}+\frac{x^{15}}{5!}-\ldots} = f(0)$

$\Rightarrow 1 = f(0)$

197 **(a)**

Given, $f(x)$ is continuous at $x = 0$

$\therefore$ Limit must exist

ie, $\lim_{x\to 0} x^p\sin\frac{1}{x} = (0)^p\sin\infty = 0$, when, $0 < p < \infty$ ...(i)

Now, RHD= $\lim_{h\to 0}\frac{h^p\sin\frac{1}{h}-0}{h} = \lim_{h\to 0} h^{p-1}\sin\frac{1}{h}$

LHD= $\lim_{h\to 0}\frac{(-h)^p\sin\left(-\frac{1}{h}\right)-0}{-h}$

$= \lim_{h\to 0}(-1)^p h^{p-1}\sin\frac{1}{h}$

Since, $f(x)$ is not differentiable at $x = 0$

$\therefore \quad p \le 1$ ...(ii)

From Eqs.(i) and (iii), $0 < p \le 1$

198 **(a)**

We have,

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{\sin x^2}{x} = \lim_{x\to 0}\left(\frac{\sin x^2}{x^2}\right)x = 1\times 0$

$= 0 = f(0)$

So, $f(x)$ is continuous at $x = 0$. $f(x)$ is also derivable at $x = 0$, because

$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{\sin x^2}{x} = \lim_{x\to 0}\frac{\sin x^2}{x^2} = 1$ exists finitely

199 **(a)**

A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$ there exist $\delta > 0$, such that

$|f(x) - f(a)| < \in \Rightarrow |x - a| < \delta$

200 **(a)**

We have,

$f(x) = x - |x - x^2|, \quad -1 \le x \le 1$

$\Rightarrow f(x) = \begin{cases} x + x - x^2, & -1 \le x < 0 \\ x - (x - x^2), & 0 \le x \le 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$

Clearly, $f(x)$ is continuous at $x = 0$

Also,

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} 2x - x^2 = -2 - 1 = -3$

$= f(-1)$

and,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1 = f(1)$

So, $f(x)$ is right continuous at $x=-1$ and left continuous at $x=1$

Hence, $f(x)$ is continuous on $[-1,1]$

201 **(b)**

Since $|\sin x|$ and $|e^{|x|}$ are not differentiable at $x=0$. Therefore, for $f(x)$ to be differentiable at $x=0$, we must have $a=0, b=0$ and c can be any real number

202 **(a)**

We have,

$f(u+v)=f(u)+kuv-2v^2$ for all $u,v\in R$ ...(i)

Putting $u=v=1$, we get

$f(2)=f(1)+k-2 \Rightarrow 8=2+k-2 \Rightarrow k=8$

Putting $u=x, v=h$ in (i), we get

$$\frac{f(x+h)-f(x)}{h}=kx-2h$$

$$\Rightarrow \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=kx \Rightarrow f'(x)$$

$$=8x \quad [\because k=8]$$

203 **(b)**

Given, $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow \quad f'(x)=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$=\frac{1+x^2}{\sqrt{(1+x^2)^2}}\times\frac{2(1-x^2)}{(1+x^2)^2}$$

$$=\frac{2}{1+x^2}\times\frac{1-x^2}{|1-x^2|}=\begin{cases}\frac{2}{1+x^2}, \text{if } |x|<1\\ -\frac{2}{1+x^2}, \quad \text{if } |x|>1\end{cases}$$

$\therefore \quad f'(x)$ does not exist for $|x|=i, ie, x=\pm 1$

Hence, $f(X)$ is differentiable on $R-\{-1,1\}$

204 **(a)**

LHL= $\lim_{x\to 0^-} f(x)=\lim_{h\to 0}-h\sin\left(\frac{1}{-h}\right)=0$

RHL= $\lim_{x\to 0^+} f(x)=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)=0$

$\therefore$ LHL=RHL= $f(0)$, it is continuous

LHD= $\lim_{x\to 0^-} f(x)=\lim_{h\to 0}\left[\frac{f(0-h)-f(0)}{-h}\right]$

$=\lim_{h\to 0}\left[\frac{-h\sin\frac{1}{h}-0}{-h}\right]$=does not exist

$\Rightarrow \quad f(x)$ is not differentiable at $x=0$

$\therefore \quad f(x)$ is continuous at $x=0$ but not differentiable at $x=0$

205 **(b)**

Since, $|x-1|$ is not differentiable at $x=1$

So, $f(x)=|x-1|e^x$ is not differentiable at $x=1$

Hence, the required set is $R-\{1\}$

206 **(d)**

We have,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h} \qquad [\because f(x+y)=f(x)f(y)]$$

$$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\frac{f(h)-1}{h}$$

$$\Rightarrow f'(x)=f(x)\cdot\lim_{h\to 0}\frac{1+h\,g\,(h)G(h)-1}{h}$$

$$\Rightarrow f'(x)=f(x)\cdot\lim_{h\to 0}g(h)\;G(h)$$

$$\Rightarrow f'(x)=f(x)\lim_{h\to 0}G(h)\lim_{h\to 0}g(h)=ab\,f(x)$$