

## MATHS ( QUESTION BANK )

### 4.DETERMINANTS

#### Single Correct Answer Type

- If  $\Delta_k = \begin{vmatrix} k & 1 & 5 \\ k^2 & 2n+1 & 2n+1 \\ k^3 & 3n^2 & 3n+1 \end{vmatrix}$ , then  $\sum_{k=1}^n \Delta_k$  is equal to

a)  $2 \sum_{k=1}^n k$                       b)  $2 \sum_{k=1}^n k^2$                       c)  $\frac{1}{2} \sum_{k=1}^n k^2$                       d) 0
- The solutions of the equation  $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ , are

a) 3, -1                      b) -3, 1                      c) 3, 1                      d) -3, -1
- The value of  $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$  is

a)  $441 \times 446 \times 4510$                       b) 0                      c) -1                      d) 1
- If  $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$ , then  $f(\sqrt[3]{3})$  is equal to

a) 1                      b) -4                      c) 4                      d) 2
- If  $a, b, c$  are respectively the  $p$ th,  $q$ th,  $r$ th terms of an AP, then  $\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix}$  is equal to

a) 1                      b) -1                      c) 0                      d)  $pqr$
- The minors of -4 and 9 and the cofactors of -4 and 9 in matrix  $\begin{bmatrix} -1 & -2 & 3 \\ -4 & -5 & -6 \\ -7 & 8 & 9 \end{bmatrix}$  are respectively

a) 42, 3, -42, 3                      b) -42, -3, 42, -3                      c) 42, 3, -42, -3                      d) 42, 3, 42, 3
- If  $\alpha, \beta, \gamma$  are the cube roots of unity, then the value of the determinant  $\begin{vmatrix} e^\alpha & e^{2\alpha} & (e^{3\alpha} - 1) \\ e^\beta & e^{2\beta} & (e^{3\beta} - 1) \\ e^\gamma & e^{2\gamma} & (e^{3\gamma} - 1) \end{vmatrix}$  is equal to

a) -2                      b) -1                      c) 0                      d) 1
- A root of the equation  $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ , is

a) 6                      b) 3                      c) 0                      d) None of these
- The value of  $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$  is

a) 7                      b) 10                      c) 13                      d) 17
- The value of the determinant  $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$  is independent of

a)  $n$                       b)  $a$                       c)  $x$                       d) None of these











69. If  $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ , then  $f(2x) - f(x)$  is equal to  
 a)  $ax$                                       b)  $ax(2a + 3x)$                                       c)  $ax(2 + 3x)$                                       d) None of these
70. If  $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$ , then the value of  $\lambda$  is  
 a)  $-1$                                       b)  $-2$                                       c)  $-3$                                       d)  $4$
71. If  $\omega$  is a complex cube root of unity, then  
 $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$  is equal to  
 a)  $-1$                                       b)  $1$                                       c)  $0$                                       d)  $\omega$
72. The value of  $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$ , when  $m$  is equal to  
 a)  $6$                                       b)  $5$                                       c)  $4$                                       d)  $1$
73. If  $\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 3 \\ 5 & -6 & x \end{vmatrix} = 29$ , then  $x$  is  
 a)  $1$                                       b)  $2$                                       c)  $3$                                       d)  $4$
74.  $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$   
 a)  $0$   
 b)  $12 \cos^2 x - 10 \sin^2 x$   
 c)  $12 \sin^2 x - 10 \cos^2 x - 2$   
 d)  $10 \sin 2x$
75. If  $A$  and  $B$  are square matrices of order 3 such that  $|A| = -1, |B| = 3$  then  $|3AB|$  is equal to  
 a)  $-9$                                       b)  $-81$                                       c)  $-27$                                       d)  $81$
76. If  $a, b, c$  are non-zero real numbers, then the system of equations  
 $(\alpha + a)x + \alpha y + \alpha z = 0$   
 $\alpha x + (\alpha + b)y + \alpha z = 0$   
 $\alpha x + \alpha y + (\alpha + c)z = 0$   
 has a non-trivial solution, if  
 a)  $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$   
 b)  $\alpha^{-1} = a + b + c$   
 c)  $\alpha + a + b + c = 1$   
 d) None of these
77. The determinant  $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix}$  vanishes, if  
 a)  $a, b, c$  are in AP                                      b)  $\alpha = \frac{1}{2}$                                       c)  $a, b, c$  are in GP                                      d) Both (b) or (c)
78. If  $-9$  is a root of the equation  $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ , then the other two roots are  
 a)  $2, 7$                                       b)  $-2, 7$                                       c)  $2, -7$                                       d)  $-2, -7$
79. If  $ab + bc + ca = 0$  and  $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ , then one of the value of  $x$  is  
 a)  $(a^2 + b^2 + c^2)^{1/2}$                                       b)  $\left[\frac{3}{2}(a^2 + b^2 + c^2)\right]^{1/2}$





102. In  $\Delta ABC$  if  $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$ , then  $\sin^2 A + \sin^2 B + \sin^2 C$  is equal to  
 a)  $\frac{4}{9}$                                       b)  $\frac{9}{4}$                                       c)  $3\sqrt{3}$                                       d) 1
103. The value of determinant  $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix}$  is equal to  
 a)  $a^3 + b^3 + c^3 - 3abc$     b)  $2abc(a+b+c)$     c) 0                                      d) None of these
104. If  $n = 3k$  and  $1, \omega, \omega^2$  are the cube roots of unity, then  $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$  has the value  
 a) 0                                      b)  $\omega$                                       c)  $\omega^2$                                       d) 1
105. If  $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$ , then  $x$  is equal to  
 a) 9                                      b) -9                                      c) 0                                      d) -1
106. the system of simultaneous equations  
 $kx + 2y - z = 1$   
 $(k - 1)y - 2z = 2$   
 $(k + 2)z = 3$   
 Have a unique solution if  $k$  equals  
 a) -2                                      b) -1                                      c) 0                                      d) 1
107. If  $\alpha, \beta$  are non-real numbers satisfying  $x^3 - 1 = 0$ , then the value of  $\begin{vmatrix} \lambda + 1 & \alpha & \beta \\ \alpha & \lambda + \beta & 1 \\ \beta & 1 & \lambda + \alpha \end{vmatrix}$  is equal to  
 a) 0                                      b)  $\lambda^3$                                       c)  $\lambda^3 + 1$                                       d)  $\lambda^3 - 1$
108. If  $x, y, z$  are different from zero and  $\Delta = \begin{vmatrix} a & b - y & c - z \\ a - x & b & c - z \\ a - x & b - y & c \end{vmatrix} = 0$ , then the value of expression  $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$  is  
 a) 0                                      b) -1                                      c) 1                                      d) 2
109. The value of the determinant  $\begin{vmatrix} 1 & \cos(\alpha - \beta) & \cos\alpha \\ \cos(\alpha - \beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$  is  
 a) 0                                      b) 1                                      c)  $\alpha^2 - \beta^2$                                       d)  $\alpha^2 + \beta^2$
110. If  $A, B, C$  are the angles of a triangle, then the determinant  $\Delta = \begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix}$  is equal to  
 a) 1                                      b) -1                                      c)  $\sin A + \sin B + \sin C$     d) None of these
111.  $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix}$  is equal to  
 a) 0                                      b)  $a + b + c$                                       c)  $(a + b + c)^2$                                       d)  $(a + b + c)^3$
112.  $A$  and  $B$  are two non-zero square matrices such that  $AB = O$ . Then,  
 a) Both  $A$  and  $B$  are singular  
 b) Either of them is singular  
 c) Neither matrix is singular  
 d) None of these
113. The system of linear equations



123. The value of  $\begin{vmatrix} \cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a) & \sin(x-a) & \sin x \\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$  is equal

- a) 1                                      b)  $\sin a \cos a$                                       c) 0                                      d)  $\sin x \cos x$

124. The roots of the equation

$$\begin{vmatrix} 3x^2 & x^2 + x \cos \theta + \cos^2 \theta & x^2 + x \sin \theta + \sin^2 \theta \\ x^2 + x \cos \theta + \cos^2 \theta & 3 \cos^2 \theta & 1 + \frac{\sin 2\theta}{2} \\ x^2 + x \sin \theta + \sin^2 \theta & 1 + \frac{\sin 2\theta}{2} & 3 \sin^2 \theta \end{vmatrix} = 0$$

- a)  $\sin \theta, \cos \theta$                                       b)  $\sin^2 \theta, \cos^2 \theta$                                       c)  $\sin \theta, \cos^2 \theta$                                       d)  $\sin^2 \theta, \cos \theta$

125. If  $A$  is a square matrix of order  $n$  such that its elements are polynomial in  $x$  and its  $r$ -rows become identical for  $x = k$ , then

- a)  $(x - k)^r$  is a factor of  $|A|$   
 b)  $(x - k)^{r-1}$  is a factor of  $|A|$   
 c)  $(x - k)^{r+1}$  is a factor  $|A|$   
 d)  $(x - k)^r$  is a factor of  $A$

126. If  $\begin{vmatrix} x^2 + x & 3x - 1 & -x + 3 \\ 2x + 1 & 2 + x^2 & x^3 - 3 \\ x - 3 & x^2 + 4 & 3x \end{vmatrix}$

$= a_0 + a_1x + a_2x^2 + \dots + a_7x^7,$

The value of  $a_0$  is

- a) 25                                      b) 24                                      c) 23                                      d) 21

127. If  $\begin{vmatrix} a & \cot \frac{A}{2} & \lambda \\ b & \cot \frac{B}{2} & \mu \\ c & \cot \frac{C}{2} & \gamma \end{vmatrix} = 0$  where,  $a, b, c, A, B$  and  $C$  are elements of a  $\Delta ABC$  with usual meaning. Then, the value

of  $a(\mu - \gamma) + b(\gamma - \lambda) + c(\lambda - \mu)$  is

- a) 0                                      b)  $abc$                                       c)  $ab + bc + ca$                                       d)  $2abc$

128. The value of the determinant  $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ , where  $a, b, c$  are the  $p^{th}, q^{th}$  and  $r^{th}$  terms of a H.P., is

- a)  $p + q + r$                                       b)  $(a + b + c)$                                       c) 1                                      d) None of these

129. If  $a, b, c$  are in AP, then the value of  $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$  is

- a)  $x - (a + b + c)$                                       b)  $9x^2 + a + b + c$                                       c)  $a + b + c$                                       d) 0

130. For the values of  $A, B, C$  and  $P, Q, R$  the value of

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$
 is

- a) 0                                      b)  $\cos A \cos B \cos C$                                       c)  $\sin A \sin B \sin C$                                       d)  $\cos P \cos Q \cos R$

131. If  $\Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin \frac{n\pi}{2} & \cos \frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$ , then the value of  $\frac{d^n}{dx^n} [\Delta(x)]$  at  $x = 0$  is

- a) -1                                      b) 0                                      c) 1                                      d) Dependent of  $a$

132. For positive numbers  $x, y$  and  $z$ , the numerical value of the determinant  $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$  is

- a) 0                                      b) 1                                      c)  $\log_e xyz$                                       d) None of these

133. The value of the determinant  $\begin{vmatrix} 15! & 16! & 17! \\ 16! & 17! & 18! \\ 17! & 18! & 19! \end{vmatrix}$  is equal to

134. a)  $15! + 16!$                       b)  $2(15!)(16!)(17!)$                       c)  $15! + 16! + 17!$                       d)  $16! + 17!$

If  $\Delta = \begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix}$ , then  $\Delta$  equals

- a)  $(y - 2z + 3x)^2$   
 b)  $(x - 2y + z)^2$   
 c)  $(x + y + z)^2$   
 d)  $x^2 + y^2 + z^2 - xy - yz - zx$

135. If the system of equations  $2x + 3y + 5 = 0, x + ky + 5 = 0, kx - 12y - 14 = 0$  be consistent, then value of  $k$  is

- a)  $-2, \frac{12}{5}$                       b)  $-1, \frac{1}{5}$                       c)  $-6, \frac{17}{5}$                       d)  $6, -\frac{12}{5}$

136. If  $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = k a^2 b^2 c^2$ , then  $k$  is equal to

- a) 3                      b) 2                      c) 4                      d) None of these

137. The repeated factor of the determinant

$\begin{vmatrix} y + z & x & y \\ z + x & z & x \\ x + y & y & z \end{vmatrix}$ , is

- a)  $z - x$                       b)  $x - y$                       c)  $y - z$                       d) None of these

138. The determinant  $\begin{vmatrix} 4 + x^2 & -6 & -2 \\ -6 & 9 + x^2 & 3 \\ -2 & 3 & 1 + x^2 \end{vmatrix}$  is not divisible by

- a)  $x$                       b)  $x^3$                       c)  $14 + x^2$                       d)  $x^5$

139. If  $a, b, c$  are different, then the value of  $x$  satisfying  $\begin{vmatrix} 0 & x^2 - a & x^3 - b \\ x^2 + a & 0 & x^2 + c \\ x^4 + a & x - c & 0 \end{vmatrix} = 0$  is

- a)  $a$                       b)  $b$                       c)  $c$                       d) 0

140. Determinant  $\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$  is equal to

- a)  $abc$                       b)  $4abc$                       c)  $4a^2b^2c^2$                       d)  $a^2b^2c^2$

141. If  $a \neq p, b \neq q, c \neq r$  and  $\begin{vmatrix} p & b & c \\ p + a & q + b & 2c \\ a & b & r \end{vmatrix} = 0$ , then

$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$  is equal to

- a) 0                      b) 1                      c) 2                      d) 3

142.  $\begin{vmatrix} a + b + 2c & a & b \\ c & 2a + b + c & b \\ c & a & a + 2b + c \end{vmatrix}$  is equal to

- a)  $(a + b + c)^2$                       b)  $2(a + b + c)^2$   
 c)  $(a + b + c)^3$                       d)  $2(a + b + c)^3$

143. If  $[ ]$  denotes the greatest integer less than or equal to the real number under consideration and  $-1 \leq x <$

$0; 0 \leq y < 1; 1 \leq z < 2$ , then the value of the determinant  $\begin{vmatrix} [x] + 1 & [y] & [z] \\ [x] & [y] + 1 & [z] \\ [x] & [y] & [z] + 1 \end{vmatrix}$  is

- a)  $[x]$                       b)  $[y]$                       c)  $[z]$                       d) None of these

144. The values of  $x$  for which the given matrix

$\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix}$  will be non-singular, are

a)  $-2 \leq x \leq 2$

c)  $x \geq 2$

b) For all  $x$  other than 2 and  $-2$

d)  $x \leq -2$

145. If all the elements in a square matrix  $A$  of order 3 are equal to 1 or  $-1$ , then  $|A|$ , is

a) An odd number

b) An even number

c) An imaginary number

d) A real number

146. Let  $a, b, c$  be such that  $(b + c) \neq 0$  and

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+ \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}^1 = 0$$

Then the value of  $n$  is

a) Zero

b) Any even integer

c) Any odd integer

d) Any integer

147. Determinant  $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}$  is equal to

a)  $abc$

b)  $\frac{1}{abc}$

c)  $ab + bc + ca$

d) 0

148. One root of the equation  $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$  is

a) 0

b) 1

c)  $-1$

d) 3

149. The value of  $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$  is

a)  $4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

b)  $3 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

c)  $2 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

d) None of these

150. The value of the determinant

$$\Delta = \begin{vmatrix} \frac{1-a_1^3 b_1^3}{1-a_1 b_1} & \frac{1-a_1^3 b_2^3}{1-a_1 b_2} & \frac{1-a_1^3 b_3^3}{1-a_1 b_3} \\ \frac{1-a_2^3 b_1^3}{1-a_2 b_1} & \frac{1-a_2^3 b_2^3}{1-a_2 b_2} & \frac{1-a_2^3 b_3^3}{1-a_2 b_3} \\ \frac{1-a_3^3 b_1^3}{1-a_3 b_1} & \frac{1-a_3^3 b_2^3}{1-a_3 b_2} & \frac{1-a_3^3 b_3^3}{1-a_3 b_3} \end{vmatrix}, \text{ is}$$

a) 0

b) Dependent only on  $a_1, a_2, a_3$

c) Dependent only on  $b_1, b_2, b_3$

d) Dependent on  $a_1, a_2, a_3, b_1, b_2, b_3$

151. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ , then the value of the determinant  $|A^{2009} - 5A^{2008}|$  is

a)  $-6$

b)  $-5$

c)  $-4$

d) 4

152. If  $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 3x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$ , then

$f(1) \cdot f(3) + f(3) \cdot f(5) + f(5) \cdot f(1)$  is equal to

a)  $f(1)$

b)  $f(3)$

c)  $f(1) + f(3)$

d)  $f(1) + f(5)$

153. The value of the determinant  $\begin{vmatrix} x & a & b+c \\ x & b & c+a \\ x & c & a+b \end{vmatrix} = 0$ , if

a)  $x = a$

b)  $x = b$

c)  $x = c$

d)  $x$  has any value

154. If the system of equations  $x + ky - z = 0$ ,  $3x - ky - z = 0$  and  $x - 3y + z = 0$  has non-zero solution then  $k$  is equal to

a)  $-1$

b) 0

c) 1

d) 2





- a)  $a + b + c$                       b) 0                      c)  $b^3$                       d)  $ab + bc$
175. Which one of the following is correct?  
If  $A$  non-singular matrix, then  
a)  $\det(A^{-1}) = \det(A)$       b)  $\det(A^{-1}) = \frac{1}{\det(A)}$       c)  $\det(A^{-1}) = 1$       d) None of these
176. If  $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$ , then  
a)  $a$  is one of the cube roots of unity                      b)  $b$  is one of the cube roots of unity  
c)  $\left(\frac{a}{b}\right)$  is one of the cube roots of unity                      d)  $\left(\frac{a}{b}\right)$  is one of the cube roots of  $-1$
177. If  $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ d & c & a \end{vmatrix}$ , then the value of  $k$ , is  
a) 1                      b) 2                      c) 3                      d) 4
178. If the value of the determinant  $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$  is positive, then  
a)  $abc > 1$                       b)  $abc > -8$                       c)  $abc < -8$                       d)  $abc > -2$
179. The value of the determinant  $\begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha + \beta) - \sin(\alpha + \beta) & 1 & 1 \end{vmatrix}$  is  
a) Independent of  $\alpha$                       b) Independent of  $\beta$   
c) Independent of  $\alpha$  and  $\beta$                       d) None of these
180. If  $B$  is a non-singular matrix and  $A$  is a square matrix such that  $B^{-1}AB$  exists, then  $\det(B^{-1}AB)$  is equal to  
a)  $\det(A^{-1})$                       b)  $\det(B^{-1})$                       c)  $\det(B)$                       d)  $\det(A)$
181. If matrix  $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{bmatrix}$  is singular, then  $\lambda$  is equal to  
a)  $-2$                       b)  $-1$                       c) 1                      d) 2
182. If  $x, y, z$  are in AP, then the value of the  $\det A$  is, where  
$$A = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$$
  
a) 0                      b) 1                      c) 2                      d) None of these
183. If  $\Delta_r = \begin{vmatrix} 1 & n & n \\ 2r & n^2 + n + 1 & n^2 + n \\ 2r - 1 & n^2 & n^2 + n + 1 \end{vmatrix}$  and  $\sum_{r=1}^n \Delta_r = 56$ , then  $n$  equals  
a) 4                      b) 6                      c) 7                      d) 8
184.  $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix}$  is equal to  
a) 0                      b)  $a^3 + b^3 + c^3 - 3abc$   
c)  $3abc$                       d)  $(a + b + c)^3$
185. If the matrix  $M_r$  is given by  $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$   $r = 1, 2, 3, \dots$ , then the value of  $\det(M_1) + \det(M_2) + \dots + \det(M_{2008})$  is  
a) 2007                      b) 2008                      c)  $(2008)^2$                       d)  $(2007)^2$
186. If  $\omega$  is the cube root of unity, then  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$  is equal to  
a) 1                      b) 0                      c)  $\omega$                       d)  $\omega^2$
187. If  $1, \omega, \omega^2$  are the cube roots of unity, then





212. a) 1                                      b)  $-1$                                       c) 0                                      d)  $-abc$   
 The value of the determinant  $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$ , where  $\omega$  is an imaginary cube root of unity, is

- a)  $(1 - \omega)^2$                                       b) 3                                      c)  $-3$                                       d) None of these

213. Let  $a, b, c$ , be positive and not all equal, the value of the

Determinant  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$  is

- a) Positive                                      b) Negative                                      c) Zero                                      d) None of these

214. If  $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$ , then the value of  $\lambda$ , is

- a)  $-1$                                       b)  $-2$                                       c)  $-3$                                       d) 4

215. If  $\begin{vmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{vmatrix} = 0$  and vectors  $(1, a, a^2)$ ,  $(1, b, b^2)$  and  $(1, c, c^2)$  are non-coplanar, then the product  $abc$  equals

- a) 2                                      b)  $-1$                                       c) 1                                      d) 0

216.  $\omega$  is an imaginary cube root of unity and

$\begin{vmatrix} x + \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 + x \\ 1 & x + \omega & \omega^2 \end{vmatrix} = 0$ , then one of the value of  $x$  is

- a) 1                                      b) 0                                      c)  $-1$                                       d) 2

217. if  $x, y, z$  are in A.P., then the value of the  $\det(A)$  is, where

$A = \begin{bmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix}$

- a) 0                                      b) 1                                      c) 2                                      d) None of these

218. If  $\alpha, \beta, \gamma \in R$ , then the determinant  $\Delta = \begin{vmatrix} (e^{i\alpha} + e^{-i\alpha})^2 & (e^{i\alpha} - e^{-i\alpha})^2 & 4 \\ (e^{i\beta} + e^{-i\beta})^2 & (e^{i\beta} - e^{-i\beta})^2 & 4 \\ (e^{i\gamma} + e^{-i\gamma})^2 & (e^{i\gamma} - e^{-i\gamma})^2 & 4 \end{vmatrix}$  is

- a) Independent of  $\alpha, \beta$  and  $\gamma$                                       b) Dependent of  $\alpha, \beta$  and  $\gamma$   
 c) Independent of  $\alpha, \beta$  only                                      d) Independent of  $\alpha, \beta$  only

219. If  $a > 0, b > 0, c > 0$  are respectively the  $p^{th}, q^{th}, r^{th}$  terms of a GP, then the value of the determinant

$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ , is

- a) 1                                      b) 0                                      c)  $-1$                                       d) None of these

220. The sum of the products of the elements of any row of a determinant  $A$  with the cofactors of the corresponding elements is equal to

- a) 1                                      b) 0                                      c)  $|A|$                                       d)  $\frac{1}{2}|A|$

221. If  $a, b, c, d, e$  and  $f$  are in GP, then the value of

$\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$

- a) Depends on  $x$  and  $y$                                       b) Depends on  $x$  and  $z$   
 c) Depends on  $y$  and  $z$                                       d) independents on  $x, y$  and  $z$





- a)  $a + b = 0$                       b)  $a + 2b = 0$                       c)  $2a + b = 0$                       d)  $a + 4b = 0$

246. If  $\omega$  is an imaginary cube root of unity, then the value of

$$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}, \text{ is}$$

- a)  $a^3 + b^3 + c^3$                       b)  $a^2b - b^2c$                       c) 0                      d)  $a^3 + b^3 + c^3 - 3abc$

247. The value of determinant  $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix}$  is

- a)  $a^2 + b^2 + c^2 - 3abc$                       b)  $3ab$                       c)  $3a + 5b$                       d) 0

248. The value of the determinant  $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$  is equal to

- a)  $6xyz$                       b)  $xyz$                       c)  $4xyz$                       d)  $xy + yz + zx$

249. If  $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$  and  $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ , then

- a)  $\Delta_1 = 3(\Delta_2)^2$                       b)  $\frac{d}{dx}(\Delta_1) = 3\Delta_2$                       c)  $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$                       d)  $\Delta_1 = 3(\Delta_2)^{3/2}$

250. For positive numbers  $x, y, z$  (other than unity) the numerical value of the determinant

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 3 & \log_y z \\ \log_z x & \log_z y & 5 \end{vmatrix}, \text{ is}$$

- a) 0                      b)  $\log x \log y \log z$                       c) 1                      d) 8

251. The value of  $\begin{vmatrix} 1990 & 1991 & 1992 \\ 1991 & 1992 & 1993 \\ 1992 & 1993 & 1994 \end{vmatrix}$  is

- a) 1992                      b) 1993                      c) 1994                      d) 0

252. If  $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ , then  $x$  is equal to

- a)  $0, 2a$                       b)  $a, 2a$                       c)  $0, 3a$                       d) None of these

253. The determinant  $\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix}$  is equal to zero, if

- a)  $b, c, d$  are in A.P.  
 b)  $b, c, d$  are in G.P.  
 c)  $b, c, d$  are in H.P.  
 d)  $\alpha$  is a root of  $ax^3 + bx^2 - cx - d = 0$

254. If  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$ , then the value of  $\begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & a_2b_3 - c_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$  is

- a) 5                      b) 25                      c) 125                      d) 0

255. The determinant  $\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$  is divisible by

- a)  $x^5$                       b)  $x^4$                       c)  $x^4 + 1$                       d)  $x^4 - 1$

256. If  $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$ , then  $\sum_{a=1}^n \Delta_a$  is equal to

- a) 0                      b) 1                      c)  $\left\{ \frac{n(n+1)}{2} \right\} \left\{ \frac{a(a+1)}{2} \right\}$                       d) None of these

257. Let the determinant of a  $3 \times 3$  matrix  $A$  be 6, then  $B$  is a matrix defined by  $B = 5A^2$ . Then, determinant of  $B$  is

- a) 180                                      b) 100                                      c) 80                                      d) None of These
258. The coefficient of  $x$  in  $f(x) = \begin{vmatrix} x & 1 + \sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}, -1 < x \leq 1$ , is
- a) 1                                      b) -2                                      c) -1                                      d) 0
259. The value of  $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$  is
- a) 1                                      b) 0                                      c)  $(a-b)(b-c)(c-a)$                                       d)  $(a+b)(b+c)(c+a)$
260. A factor of  $\Delta(x) = \begin{vmatrix} x^3+1 & 2x^4+3x^2 & 3x^5+4x \\ 2 & 5 & 7 \\ 3 & 14 & 19 \end{vmatrix}$  is
- a)  $x$                                       b)  $(x-1)^2$                                       c)  $(x+1)^2$                                       d) None of these
261. If  $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} b^2+c^2 & a^2+\lambda & a^2+\lambda \\ b^2+\lambda & c^2+a^2 & b^2+\lambda \\ c^2+\lambda & c^2+\lambda & a^2+b^2 \end{vmatrix}$  is an identity in  $\lambda$ , where  $p, q, r, s, t$  are constants, then the value of  $t$  is
- a) 1                                      b) 2                                      c) 0                                      d) None of these
262. The value of the determinant  $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$  is
- a)  $2(10!11!)$                                       b)  $2(10!13!)$                                       c)  $2(10!11!12!)$                                       d)  $2(11!12!13!)$
263. If  $A_i = \begin{bmatrix} a^i & b^i \\ b^i & a^i \end{bmatrix}$  and if  $|a| < 1, |b| < 1$ , then  $\sum_{i=1}^{\infty} \det(A_i)$  is equal to
- a)  $\frac{a^2}{(1-a)^2} - \frac{b^2}{(1-b)^2}$                                       b)  $\frac{a^2-b^2}{(1-a)^2(1-b)^2}$                                       c)  $\frac{a^2}{(1-a)^2} + \frac{b^2}{(1-b)^2}$                                       d)  $\frac{a^2}{(1+a)^2} - \frac{b^2}{(1+b)^2}$
264. If  $\begin{bmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix}$  is a singular matrix, then  $x$  is equal to
- a) 0                                      b) 1                                      c) -3                                      d) 3
265. The value of  $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$  is
- a)  $x(x-p)(x-q)$                                       b)  $(x-p)(x-q)(x+p+q)$   
c)  $(p-q)(x-q)(x-p)$                                       d)  $pq(x-p)(x-q)$
266. The roots of the equation  $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$  are
- a) -1, -2                                      b) -1, 2                                      c) 1, -2                                      d) 1, 2
267. If  $\begin{vmatrix} 6i-3i & 1 \\ 4 & 3i-1 \\ 40 & 3 & i \end{vmatrix} = x+iy$ , then
- a)  $x=3, y=1$                                       b)  $x=1, y=3$                                       c)  $x=0, y=3$                                       d)  $x=0, y=0$
268. The determinant  $\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\sin \alpha & \sin \alpha & \cos \beta \end{vmatrix}$  is independent of
- a)  $\alpha$                                       b)  $\beta$                                       c)  $\alpha$  and  $\beta$                                       d) Neither  $\alpha$  nor  $\beta$
269.  $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ , then  $a, b, c$  are
- a) In GP                                      b) In HP                                      c) Equal                                      d) In AP
270. If  $1 + \frac{1}{a} + \frac{1}{c} + \frac{1}{c} = 0$ , then  $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$  is equal to



## MATHS ( QUESTION BANK )

### : ANSWER KEY :

1)	b	2)	a	3)	b	4)	b	145)	b	146)	c	147)	d	148)	b
5)	c	6)	b	7)	c	8)	c	149)	a	150)	d	151)	a	152)	b
9)	b	10)	a	11)	b	12)	d	153)	d	154)	c	155)	a	156)	a
13)	d	14)	d	15)	b	16)	a	157)	b	158)	d	159)	b	160)	c
17)	c	18)	a	19)	b	20)	c	161)	a	162)	d	163)	c	164)	d
21)	c	22)	c	23)	b	24)	a	165)	d	166)	c	167)	a	168)	c
25)	b	26)	b	27)	c	28)	d	169)	c	170)	d	171)	b	172)	a
29)	d	30)	a	31)	c	32)	b	173)	c	174)	c	175)	b	176)	d
33)	d	34)	a	35)	c	36)	a	177)	b	178)	b	179)	a	180)	d
37)	a	38)	a	39)	b	40)	c	181)	d	182)	a	183)	c	184)	a
41)	d	42)	b	43)	b	44)	d	185)	c	186)	b	187)	a	188)	a
45)	d	46)	a	47)	b	48)	c	189)	c	190)	d	191)	c	192)	d
49)	d	50)	a	51)	c	52)	b	193)	a	194)	a	195)	c	196)	a
53)	c	54)	b	55)	c	56)	a	197)	d	198)	d	199)	c	200)	a
57)	d	58)	d	59)	d	60)	c	201)	a	202)	b	203)	a	204)	d
61)	b	62)	c	63)	d	64)	a	205)	c	206)	c	207)	c	208)	d
65)	c	66)	a	67)	b	68)	b	209)	b	210)	b	211)	c	212)	b
69)	b	70)	c	71)	c	72)	b	213)	b	214)	c	215)	b	216)	b
73)	b	74)	a	75)	b	76)	a	217)	a	218)	a	219)	b	220)	c
77)	d	78)	a	79)	a	80)	b	221)	d	222)	a	223)	d	224)	c
81)	c	82)	c	83)	b	84)	b	225)	b	226)	b	227)	c	228)	a
85)	a	86)	a	87)	b	88)	a	229)	b	230)	d	231)	d	232)	c
89)	d	90)	a	91)	c	92)	b	233)	c	234)	a	235)	c	236)	d
93)	c	94)	d	95)	a	96)	c	237)	c	238)	d	239)	d	240)	c
97)	b	98)	a	99)	b	100)	b	241)	d	242)	a	243)	b	244)	b
101)	a	102)	b	103)	a	104)	a	245)	b	246)	c	247)	d	248)	c
105)	b	106)	b	107)	b	108)	d	249)	b	250)	d	251)	d	252)	c
109)	a	110)	d	111)	d	112)	b	253)	b	254)	b	255)	b	256)	a
113)	a	114)	a	115)	b	116)	a	257)	d	258)	b	259)	c	260)	b
117)	d	118)	d	119)	d	120)	a	261)	d	262)	c	263)	b	264)	c
121)	c	122)	d	123)	c	124)	a	265)	b	266)	b	267)	d	268)	a
125)	a	126)	d	127)	a	128)	d	269)	d	270)	a	271)	a	272)	a
129)	d	130)	a	131)	b	132)	a	273)	b	274)	c	275)	a	276)	c
133)	b	134)	b	135)	c	136)	c	277)	b	278)	a				
137)	a	138)	d	139)	d	140)	c								
141)	c	142)	d	143)	c	144)	b								

## MATHS ( QUESTION BANK )

### : HINTS AND SOLUTIONS :

2 (a)

$$\text{Given } \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -3 & -3 & 0 \end{vmatrix} = 0 \quad [R_3 \rightarrow R_3 - R_2]$$

$$\Rightarrow -1(-6 + 15) - x[-3x + 6] = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x = 3, -1$$

3 (b)

$$\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = \begin{vmatrix} 441 & 1 & 1 \\ 445 & 1 & 1 \\ 449 & 1 & 1 \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_1$$

$$C_3 \rightarrow C_3 - C_2$$

$$= 0 \quad [\because \text{two columns are identical}]$$

4 (b)

$$\text{Given, } f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$$

$$= 1(\alpha^3 - 1) - \alpha(\alpha^2 - \alpha^2) + \alpha^2(\alpha - \alpha^4)$$

$$= \alpha^3 - 1 - 0 + \alpha^3 - \alpha^6$$

$$\Rightarrow f(\sqrt[3]{3}) = 3 - 1 - 0 + 3 - 3^2$$

$$= 6 - 10 = -4$$

5 (c)

Let the first term and common difference of an AP are  $A$  and  $D$  respectively.

$$\therefore a = A + (p - 1)D, b = A + (q - 1)D,$$

$$\text{and } c = A + (r - 1)D$$

$$\text{Now, } \begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix} = \begin{vmatrix} A + (p - 1)D & p & 1 \\ A + (q - 1)D & q & 1 \\ A + (r - 1)D & r & 1 \end{vmatrix}$$

$$\text{Applying } C_1 \rightarrow C_1 - DC_2 + DC_3$$

$$= \begin{vmatrix} A & p & 1 \\ A & q & 1 \\ A & r & 1 \end{vmatrix} = A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} = 0 \quad (\because \text{two columns are identical})$$

6 (b)

$$\text{Minor of } (-4) = \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = -42$$

$$\text{Minor of } 9 = \begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$$

$$\text{Cofactor of } (-4) = (-1)^{2+1} \cdot \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = 42$$

$$\text{and cofactor of } 9 = (-1)^{3+3} \cdot \begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$$

7 (c)

Given,  $\alpha, \beta$  and  $\gamma$  are the cube roots of unity, then assume

$$\alpha = 1, \beta = \omega \text{ and } \gamma = \omega^2.$$

$$\therefore \begin{vmatrix} e^\alpha & e^{2\alpha} & (e^{3\alpha} - 1) \\ e^\beta & e^{2\beta} & (e^{3\beta} - 1) \\ e^\gamma & e^{2\gamma} & (e^{3\gamma} - 1) \end{vmatrix}$$

$$= \begin{vmatrix} e^\alpha & e^{2\alpha} & e^{3\alpha} \\ e^\beta & e^{2\beta} & e^{3\beta} \\ e^\gamma & e^{2\gamma} & e^{3\gamma} \end{vmatrix} + \begin{vmatrix} e^\alpha & e^{2\alpha} & -1 \\ e^\beta & e^{2\beta} & -1 \\ e^\gamma & e^{2\gamma} & -1 \end{vmatrix}$$

$$= e^\alpha e^\beta e^\gamma \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix} - \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix} [e^\alpha e^\beta e^\gamma - 1] = 0$$

$$(\because e^\alpha e^\beta e^\gamma = e^{1+\omega+\omega^2} = e^0 = 1)$$

8 (c)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ , we obtain

$$-x \begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 3 \\ 1 & 3 & -6-x \end{vmatrix} = 0$$

$$\Rightarrow -x \begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

$$[\text{Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1]$$

$$\Rightarrow -x(9-x)(-9-x) = 0 \Rightarrow x = 0, 9, -9$$

9 (b)

$$\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$$

$$= (\log_3 512 \times \log_4 9 - \log_4 3 \log_3 8) \times (\log_2 3 \times \log_3 4 - \log_8 3 \times \log_3 4)$$

$$= \left( \frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4} \times \frac{\log 8}{\log 3} \right)$$

$$\times \left( \frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 3}{\log 8} \times \frac{\log 4}{\log 3} \right)$$

$$= \left( \frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} \times \frac{\log 2^3}{\log 2^2} \right) \times \left( \frac{\log 2^2}{\log 2} - \frac{\log 2^2}{\log 2^3} \right)$$

$$= \left( \frac{9 \times 2}{2} - \frac{3}{2} \right) \times \left( 2 - \frac{2}{3} \right) = \frac{15}{2} \times \frac{4}{3} = 10$$

10 (a)

$$\text{Let } \Delta = \begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

Since,  $\cos(nx) + \cos(n+2)x = 2 \cos(n+1)x \cos x$

and  $\sin(nx) + \sin(n+2)x = 2 \sin(n+1)x \cos x$

Applying  $C_1 \rightarrow C_1 - 2 \cos x \cdot C_2 + C_3$

$\therefore \Delta$

$$= \begin{vmatrix} a^2 - 2a \cos x + 1 & a & 1 \\ 0 & \cos(n+1)x & \cos(n+2)x \\ 0 & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

$$= (a^2 - 2a \cos x + 1)[\cos(n+1)x \sin(n+2)x - \cos(n+2)x \sin(n+1)x]$$

$$= (a^2 - 2a \cos x + 1) \sin x$$

$\therefore \Delta$  is independent of  $n$ .

11 (b)

$$\text{Given } \begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+4 & 6x+4 & 8x+4 \end{vmatrix} = 0$$

$$\Rightarrow 2 \begin{vmatrix} 0 & x & 2x \\ 2x & 4x+3 & 6x+3 \\ 2x+2 & 3x+2 & 4x+2 \end{vmatrix} = 0$$

[Using  $(R_1 \rightarrow 2R_1 - R_3)$ ]

$$\Rightarrow 2 \begin{vmatrix} 0 & x & 0 \\ 2x & 4x+3 & -2x-3 \\ 2x+2 & 3x+2 & -2x-2 \end{vmatrix} = 0$$

[Using  $(C_3 \rightarrow C_3 - 2C_2)$ ]

$$\Rightarrow -4x[2x^2 + 2x - (2x+3)(x+1)] = 0$$

$$\Rightarrow -4x[2x^2 + 2x - (2x^2 + 5x + 3)] = 0$$

$$\Rightarrow 4x(3x+3) = 0$$

$$\Rightarrow x+1 = 0 \quad [\because x \neq 0 \text{ given}]$$

13 (d)

$$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$$

(Using  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_2$ )

$$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$$

$$\Rightarrow ayz + bzx + cyx = 2xyz$$

$$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

14 (d)

We have,

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ 0 & b-q & r-c \end{vmatrix} = 0$$

[Applying  $R_3 \rightarrow R_3 - R_2$   
and  $R_2 \rightarrow R_2 - R_1$ ]

$$\Rightarrow \begin{vmatrix} p & b & c \\ p-a & q-b & r-c \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\Rightarrow \frac{p}{p-a} + \left( \frac{q}{q-b} - 1 \right) + \left( \frac{r}{r-c} - 1 \right) = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

15 (b)

We have,

$$\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3a+3b & a+b & a+2b \\ 3a+3b & a & a+b \\ 3a+3b & a+2b & a \end{vmatrix} \quad \text{Applying } C_1 \rightarrow$$

$C_1 + C_2 + C_3$

$$\Rightarrow \Delta = 3(a+b) \begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix}$$

$$\Rightarrow \Delta = 3(a+b) \begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \Delta = 3(a+b)(3b^2) = 9b^2(a+b)$$

16 (a)

Applying  $C_1 \rightarrow C_1 + C_2$ , we get

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 \theta & 4 \sin 4 \theta \\ 2 & 1 + \cos^2 \theta & 4 \sin 4 \theta \\ 1 & \cos^2 \theta & 1 + 4 \sin 4 \theta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 \theta & 4 \sin 4 \theta \\ 0 & 1 & 0 \\ 1 & \cos^2 \theta & 1 + 4 \sin 4 \theta \end{vmatrix} = 0$$

[ $R_2 \rightarrow R_2 - R_1$ ]

$$\Rightarrow (2 + 4 \sin 4 \theta) = 0$$

$$\Rightarrow \sin 4 \theta = -\frac{1}{2} = -\sin \frac{\pi}{6}$$

$$\Rightarrow 4 \theta = n\pi + (-1)^n \left( -\frac{\pi}{6} \right)$$

$\therefore$  The value of  $\theta$  between 0 and  $\frac{\pi}{2}$  will be  $\frac{7\pi}{24}$  and

$\frac{11\pi}{24}$

17 (c)

We have,  $a_i^2 + b_i^2 + c_i^2 = 1$

and  $a_i a_j + b_i b_j + c_i c_j = 0$  for  $(i = 1, 2, 3)$

$$\begin{aligned} \therefore \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 &= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\ &= \begin{vmatrix} a_1^2 + b_1^2 + c_1^2 & a_1a_2 + b_1b_2 + c_1c_2 & a_1a_3 + b_1b_3 + c_1c_3 \\ a_2a_1 + b_2b_1 + c_2c_1 & a_2^2 + b_2^2 + c_2^2 & a_2a_3 + b_2b_3 + c_2c_3 \\ a_3a_1 + b_3b_1 + c_3c_1 & a_3a_2 + b_3b_2 + c_3c_2 & a_3^2 + b_3^2 + c_3^2 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \end{aligned}$$

18 (a)

We have,

$$\alpha = 2, \beta = 2\omega \text{ and } \gamma = 2\omega^2 \Rightarrow \alpha + \beta + \gamma = 0$$

Now,

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} \alpha + \beta + \gamma & \beta & \gamma \\ \alpha + \beta + \gamma & \gamma & \alpha \\ \alpha + \beta + \gamma & \alpha & \beta \end{vmatrix} \text{ Applying } C_1 \rightarrow C_1 + C_2 +$$

$$\begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0 \quad [\because \alpha + \beta + \gamma = 0]$$

19 (b)

Take  $a, b, c$  common from  $R_1, R_2, R_3$  respectively,

$$\therefore \Delta = abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} + 1 & \frac{1}{b} + 2 & \frac{1}{b} \\ \frac{1}{c} + 1 & \frac{1}{c} + 1 & \frac{1}{c} + 3 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Delta = abc \left( 3 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & 1 & 1 \\ 1 + \frac{1}{b} & 2 + \frac{1}{b} & \frac{1}{b} \\ 1 + \frac{1}{c} & 1 + \frac{1}{c} & 3 + \frac{1}{c} \end{vmatrix}$$

Now, applying  $C_3 \rightarrow C_3 - C_2$  and  $C_2 \rightarrow C_2 - C_1$  and on expanding, we get

$$\Delta = 2abc \left[ 3 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right] = 0$$

$$\because a \neq 0, b \neq 0, c \neq 0$$

$$\therefore a^{-1} + b^{-1} + c^{-1} = -3$$

20 (c)

On expanding the given determinant, we obtain

$$2x^3 + 2x(ac - ab - bc) = 0 \Rightarrow x = 0$$

23 (b)

Given,  $A$  is a square matrix and  $AA^T = I = A^T A$

$$\Rightarrow |AA^T| = |I| = |A^T A|$$

$$\Rightarrow |A||A^T| = |I| = |A^T||A|$$

$$\Rightarrow |A|^2 = 1 \quad [\because |A^T| = |A|]$$

$$\Rightarrow |A| = \pm 1$$

25 (b)

We have,

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = k(x+y+z)(x-z)^2$$

$$\text{LHS} = (x+y+z) \begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix} (R_1 \rightarrow R_1 + R_2 + R_3)$$

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ x & z & x \\ x & y & z \end{vmatrix}$$

$$= (x+y+z) \{1(z^2 - xy) - 1(xz - x^2) + 1(xy - xz)\}$$

$$= (x+y+z)(x^2 + z^2 - 2xz)$$

$$\Rightarrow (x+y+z)(x-z)^2 = k(x+y+z)(x-z)^2$$

(given)

$$\Rightarrow k = 1$$

26 (b)

$$\det(2A) = 2^4 \det(A) = 16 \det(A)$$

27 (c)

$$\because \det(M_r) = r^2 - (r-1)^2 = 2r - 1$$

$$\therefore \det(M_1) + \det(M_2) + \dots + \det(M^{2008})$$

$$= 1 + 3 + 5 + \dots + 4015$$

$$= \frac{2008}{2} [2 + (2008 - 1)2]$$

$$= 2008(2008) = (2008)^2$$

28 (d)

Let  $A$  and  $R$  be the first term and common ratio respectively.

$$\therefore l = AR^{p-1}$$

$$\Rightarrow \log l = \log A + (p-1) \log R$$

$$m = AR^{q-1}$$

$$\Rightarrow \log m = \log A + (q-1) \log R$$

$$\text{and } n = AR^{r-1}$$

$$\Rightarrow \log n = \log A + (r-1) \log R$$

$$\text{Now, } \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} =$$

$$\begin{vmatrix} \log A + (p-1) \log R & p & 1 \\ \log A + (q-1) \log R & q & 1 \\ \log A + (r-1) \log R & r & 1 \end{vmatrix}$$

On multiplying  $R_1, R_2$  and  $R_3$  by  $(q-r), (r-p)$  and  $(p-q)$  and adding  $R_1 + R_2 + R_3$ , we get

$$= (q-r+r-p+p-q) \log A + \{(q-r)(p-1) + (r-p)(q-1) + (p-q)(r-1)\} \log R$$

$$= 0$$

29 (d)

Since, the given matrix is singular.

$$\therefore \begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$$

$$\Rightarrow 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0$$

$$\Rightarrow -20b + 60 + 20b - 60 = 0$$

$$\Rightarrow 0(b) = 0$$

$\therefore$  The given matrix is singular for any value of  $b$

31 (c)

$$\text{Given, } \begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$$

$$= (y-z)(z-x)(x-y) \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

The degree of determinant

$$= n + (n+2) + (n+3) = 3n + 5$$

and the degree of RHS = 2

$$\therefore 3n + 5 = 2 \Rightarrow n = -1$$

32 (b)

$$\text{Since, } \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$$

Applying  $R_3 \rightarrow R_3 - (\alpha R_1 + R_2)$

$$\Rightarrow \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ 0 & 0 & -a\alpha^2 - 2b\alpha - c \end{vmatrix} = 0$$

$$\Rightarrow -(a\alpha^2 + 2b\alpha + c)(ac + b^2) = 0$$

$$\Rightarrow b^2 = ac$$

Hence,  $a, b,$  and  $c$  are in GP.

33 (d)

The system of equations

$$kx + y + z = 1$$

$$x + ky + z = k$$

$$x + y + kz = k^2$$

Is inconsistent, if

$$\Delta = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0 \text{ and one of the } \Delta_1, \Delta_2, \Delta_3 \text{ is non-}$$

zero, where

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix}, \Delta_2 = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & k^2 & k \end{vmatrix}, \Delta_3 =$$

$$= \begin{vmatrix} k & 1 & 1 \\ 1 & k & k \\ 1 & 1 & k^2 \end{vmatrix}$$

$$\text{We have, } \Delta = (k+2)(k-1)^2, \Delta_1 = -(k+1)(k-1)^2$$

$$\Delta_2 = -k(k-1)^2, \Delta_3 = (k+1)^2(k-1)^2$$

Clearly, for  $k = -2$ , we have

$$\Delta = 0 \text{ and } \Delta_1, \Delta_2, \Delta_3 \text{ are non-zero. Therefore, } k = -2$$

34 (a)

We have,

$$\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 3a & 4a+3b & 5a+4b+3c \\ 6a & 9a+6b & 11a+9b+6c \end{vmatrix}$$

$$\text{Applying } R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - 2R_2$$

$$= \begin{vmatrix} a & a & a+b+c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$$

$$= a[a^2 + ab - 2a^2 - ab]$$

$$= -a^3 = i \quad (\because a = i, \text{ given})$$

35 (c)

$$\text{LHS} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

The determinant can be written sum of  $2 \times 2 \times 2 = 8$  determinants of which 6 are reduces to zero because of their two rows are identical.

$$\therefore \text{LHS} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

36 (a)

$$\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & \alpha & x \end{vmatrix} = 0 \Rightarrow \alpha^3 - \beta^3 = 0$$

$$\Rightarrow \left(\frac{\alpha}{\beta}\right)^3 = 1 \Rightarrow \frac{\alpha}{\beta} \text{ is one of the cube roots of unity.}$$

37 (a)

Applying  $R_3 \rightarrow R_3 - \alpha R_1 - R_2$ , we get

$$\Delta = \begin{vmatrix} b & c & a\alpha + b \\ c & d & c\alpha + d \\ 0 & 0 & a\alpha^3 + b\alpha^2 + c\alpha + d \end{vmatrix}$$

$$\Rightarrow \Delta = (a\alpha^3 + b\alpha^2 + c\alpha + d)(bd - c^2)$$

$$\therefore \Delta = 0$$

$\Rightarrow$  either  $b, c, d$  are in G.P. or  $\alpha$  is a root of  $a\alpha^3 + b\alpha^2 + c\alpha + d = 0$

38 (a)

We have,

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

$$= \frac{1}{\cos^2 A} \begin{vmatrix} \cos C \cos A & \sin A & 0 \\ \sin B \cos A & 0 & -\sin A \\ 0 & \sin B & \cos C \end{vmatrix}$$

$$\left[ \begin{array}{l} \text{Applying } R_1 \rightarrow R_1 \cos A \\ R_2 \rightarrow R_2 \cos A \end{array} \right]$$

$$= \frac{1}{\cos A} \begin{vmatrix} \cos C & \sin A & 0 \\ \sin B & 0 & -\sin A \\ 0 & \sin B & \cos C \end{vmatrix}$$

$$= \frac{1}{\cos A} \{ \sin A \sin B \cos C - \sin A \sin B \cos C \}$$

$$= 0$$

39 (b)

Applying  $C_3 \rightarrow C_3 - C_1$ , we get

$$\Delta = \begin{vmatrix} 1 & a & a^2 - 1 \\ \cos(p-d)a & \cos pa & 0 \\ \sin(p-d)a & \sin pa & 0 \end{vmatrix}$$

$$= (\alpha^2 - 1) \{ -\cos pa \sin(p-d)a + \sin pa \cos(p-d)a \}$$

$$= (\alpha^2 - 1) \sin \{ -(p-d)a + pa \}$$

$$\Rightarrow \Delta = (\alpha^2 - 1) \sin da$$

Which is independent of  $p$ .

40 (c)

$$\text{Let } \Delta = \begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$$

Applying  $C_3 \rightarrow C_3 - \omega C_1$

$$= \begin{vmatrix} a & b\omega^2 & 0 \\ b\omega & c & 0 \\ c\omega^2 & a\omega & 0 \end{vmatrix} = 0$$

42 (b)

$$\text{We have, } \Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx + ab^2 +$$

$$a^2b$$

$$\Rightarrow \frac{d}{dx} \Delta_1 = 3(x^2 - ab) \text{ and } \Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$$

$$\therefore \frac{d}{dx} (\Delta_1) = 3(x^2 - ab) = 3\Delta_2$$

43 (b)

$$\text{Given } f(\theta) = \begin{vmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix}$$

$$= \cos^2 \theta (0 + \cos^2 \theta) - \cos \theta \sin \theta (0 - \sin \theta \cos \theta)$$

$$- \sin \theta (-\cos^2 \theta \sin \theta - \sin^3 \theta)$$

$$= \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta$$

$$= \cos^4 \theta + \sin^2 \theta \cos^2 \theta + \sin^2 \theta$$

$$= \cos^2 \theta (\cos^2 \theta + \sin^2 \theta) + \sin^2 \theta = 1$$

$$\therefore \text{For all, } \theta, f(\theta) = 1$$

44 (d)

Given that  $C = 2 \cos \theta$

$$\text{and } \Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C(C^2 - 1) - 1(C - 6)$$

$$\Delta = 2 \cos \theta (4 \cos^2 \theta - 1) - (2 \cos \theta - 6) \quad (\because C = 2 \cos \theta)$$

$$\Rightarrow \Delta = 8 \cos^3 \theta - 4 \cos \theta + 6$$

45 (d)

We have,

$$f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{f(x)}{x} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \tan x \\ x^2 & x^2 & x^2 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{f(x)}{x^2} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \tan x \\ x & x & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -1(1 - 2) = 1$$

46 (a)

$$\begin{vmatrix} [e] & [\pi] & [\pi^2 - 6] \\ [\pi] & [\pi^2 - 6] & [e] \\ [\pi^2 - 6] & [e] & [\pi] \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$$= 2(9 - 4) - 3(9 - 6) + 3(6 - 9)$$

$$= 10 - 9 - 9 = -8$$

47 (b)

$$AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 2 & 34 \end{bmatrix}$$

$$\Rightarrow |AB| = \begin{vmatrix} 3 & 11 \\ 2 & 34 \end{vmatrix} = 100$$

48 (c)

Given that,

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + 2R_3$

$$\Delta = \begin{vmatrix} 7 & 20 & 29 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$$

$$\Rightarrow \Delta = \Delta'$$

49 (d)

$$\begin{vmatrix} 2xy & x^2 & y^2 \\ x^2 & y^2 & 2xy \\ y^2 & 2xy & x^2 \end{vmatrix}$$

$$= 2xy(x^2y^2 - 4x^2y^2)$$

$$- x^2(x^4 - 2xy^3) + y^2(2x^3y - y^4)$$

$$= -6x^3y^3 - x^6 + 2x^3y^3 + 2x^3y^3 - y^6$$

$$= -(x^6 + y^6 + 2x^3y^3)$$

$$= -(x^3 + y^3)^2$$

50 (a)

We have,

$$\Delta = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$\therefore \Delta = 0 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$$

51 (c)

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$$

On multiplying  $R_1, R_2, R_3$  by  $a, b, c$  respectively and divide the whole by  $abc$

$$= \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ a^2b^2c & abc & c(a+b) \end{vmatrix}$$

On taking common  $abc$  from  $C_1$  and  $C_2$ , we get

$$= \frac{(abc)(abc)}{abc} \begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$$

Now,  $C_1 \rightarrow C_1 + C_3$

$$\begin{aligned}
 &= abc \begin{vmatrix} ab + bc + ca & 1 & ab + ac \\ ca + bc + ab & 1 & bc + ab \\ ab + bc + ca & 1 & ca + bc \end{vmatrix} \\
 &= (abc)(ab + bc + ca) \begin{vmatrix} 1 & 1 & ab + ac \\ 1 & 1 & bc + ab \\ 1 & 1 & ca + bc \end{vmatrix} \\
 &= 0 \quad [\because \text{two columns are identical}]
 \end{aligned}$$

52 (b)

$$\text{We have, } \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$$

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = \lambda$$

On expanding w.r.t.  $R_3$ , we get

$$ab + bc + ca + abc = \lambda \dots(i)$$

Given  $a^{-1} + b^{-1} + c^{-1} = 0$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\Rightarrow ab + bc + ca = 0$$

From Eq. (i),  $\lambda = abc$

53 (c)

We have,

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+4 & x+c \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow 2R_2]$$

$$= \frac{1}{2} \begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+2 & x+4 & x+c \end{vmatrix} \quad [\text{Applying } R_2 - (R_1 + R_3)]$$

$$= 0$$

54 (b)

$$\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & p-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0$$

( $C_1 \rightarrow C_1 + C_2 + C_3$ )

55 (c)

$$\begin{vmatrix} a-b+c & -a-b+c & 1 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \\ 2a & -2a & 0 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$$

[using  $R_1 \rightarrow R_1 + R_2 - R_3$ ]

$$= 2a(-3a + 3b + 6c - 6c) + 2a(3a + 3b + 6c - 6c)$$

$$= 12ab$$

56 (a)

Ratio of cofactor to its minor of the element  $-3$ , which is in the 3rd row and 2nd column =  $(-1)^{3+2} = -1$

57 (d)

We have,

$$\Delta = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta$$

$$= \begin{vmatrix} x+1+\omega+\omega^2 & x+\omega+\omega^2+1 & x+1+\omega+\omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

[Applying  $R_1 \rightarrow R_1 + R_2 + R_3$ ]

$$\Rightarrow \Delta = (x+1+\omega+\omega^2) \begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta = x \begin{vmatrix} 1 & 0 & 0 \\ \omega & x+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & x+\omega-\omega^2 \end{vmatrix}$$

$$\Rightarrow \Delta = x[(x+\omega^2-\omega)(x+\omega-\omega^2) - (1-\omega)(1-\omega^2)]$$

$$\therefore \Delta = 0 \Rightarrow x = 0$$

59 (d)

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$  to the given determinant and expanding it along first row, we get

$$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)$$

$$\times \begin{vmatrix} 1 & 1 \\ 1 + \sin B + \sin A & 1 + \sin C + \sin A \end{vmatrix} = 0$$

$$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)(\sin C - \sin B) = 0$$

$$\Rightarrow \sin B = \sin A \text{ or } \sin C = \sin A \text{ or } \sin C = \sin B$$

$$\Rightarrow A = B \text{ or } B = C \text{ or } C = A$$

$\Rightarrow \Delta ABC$  is isosceles

60 (c)

$$\text{We have, } D_r = \begin{vmatrix} 2^{r-1} & 3^{r-1} & 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\Rightarrow \sum_{r=1}^n D_r = \begin{vmatrix} \sum_{r=1}^n 2^{r-1} & \sum_{r=1}^n 3^{r-1} & \sum_{r=1}^n 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\Rightarrow \sum_{r=1}^n D_r = \begin{vmatrix} 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\sum_{r=1}^n D_r = 0 \quad (\because \text{two rows are same})$$

61 (b)

We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0 \quad \text{Applying } R_3 \rightarrow$$

$R_3 - R_2$

$$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin A) = 0$$

$\Rightarrow \sin A = \sin B$  or,  $\sin B = \sin C$  or,  $\sin C = \sin A$

$\Rightarrow \Delta ABC$  is isosceles

62 (c)

We have,

$$\det(A) = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix} = 2(1 + \sin^2 \theta)$$

Now,

$0 \leq \sin^2 \theta \leq 1$  for all  $\theta \in [0, 2\pi)$

$\Rightarrow 2 \leq 2 + 2\sin^2 \theta \leq 4$  for all  $\theta \in [0, 2\pi)$

$\Rightarrow \det(A) \in [2, 4]$

63 (d)

$$\text{Let } \Delta = \begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & \pi \\ 1 & 5 & \sqrt{5} \\ 1 & 5 & e \end{vmatrix} = 5 \begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{5} \\ 1 & 1 & e \end{vmatrix} \quad (\because \log_a a =$$

1)

$= 0$  ( $\because$  two columns are identical)

64 (a)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ , we get

$f(x)$

$$= \begin{vmatrix} 1 + a^2x + x + xb^2 + x + c^2x & (1 + b^2)x & (1 + c^2)x \\ x + a^2x + 1 + b^2x + x + c^2x & (1 + b^2)x & (1 + c^2)x \\ x + a^2x + x + b^2x + 1 + c^2x & (1 + b^2)x & (1 + c^2)x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1 + b^2)x & (1 + c^2)x \\ 1 & 1 + b^2x & (1 + c^2)x \\ 1 & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

$[\because a^2 + b^2 + c^2 + 2 = 0]$

Applying  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} 0 & 0 & x - 1 \\ 0 & 1 - x & x - 1 \\ 1 & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

$$= 1[0 - (x - 1)(1 - x)]$$

$$= (x - 1)^2$$

$\Rightarrow f(x)$  is a polynomial of degree 2

65 (c)

Since system of equations is consistent.

$$\therefore \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0$$

$$\Rightarrow c + bc - 6b + b + 2c + 3bc = 0$$

$$\Rightarrow 3c + 4bc - 5b = 0$$

$$\Rightarrow c = \frac{5}{3 + 4b}$$

$$\text{But } c < 1 \Rightarrow \frac{5b}{3 + 4b} < 1$$

$$\Rightarrow \frac{b - 3}{3 + 4b} < 0$$

$$\Rightarrow b \in \left(-\frac{3}{4}, 3\right)$$

66 (a)

Applying  $R_2 \rightarrow R_2 - R_3$ , we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix} = 4 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix} = 4 \times 0 = 0 \quad [\because \text{two rows are identical}]$$

67 (b)

We have,

$$AA^{-1} = I$$

$$\Rightarrow \det(AA^{-1}) = \det(I)$$

$$\Rightarrow \det(A) \det(A^{-1}) = 1$$

$$[\because \det(AB) = \det(A) \det(B)]$$

$$\text{and, } \det(I) = 1$$

$$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$$

68 (b)

We have,

$$\begin{vmatrix} 1 + ax & 1 + bx & 1 + cx \\ 1 + a_1x & 1 + b_1x & 1 + c_1x \\ 1 + a_2x & 1 + b_2x & 1 + c_2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 + ax & (b - a)x & (c - a)x \\ 1 + a_1x & (b_1 - a_1)x & (c_1 - a_1)x \\ 1 + a_2x & (b_2 - a_2)x & (c_2 - a_2)x \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$

$$= x^2 \begin{vmatrix} 1 + ax & b - a & c - a \\ 1 + a_1x & b_1 - a_1 & c_1 - a_1 \\ 1 + a_2x & b_2 - a_2 & c_2 - a_2 \end{vmatrix}$$

$$= x^2[(1 + ax)\{(b_1 - a_1)(c_2 - a_2)$$

$$- (b_2 - a_2)(c_1 - a_1)\}$$

$$- (1 + a_1x)\{(b - a)(c_2 - a_2)$$

$$- (c - a)(b_2 - a_2)\}$$

$$+ (1 + a_2x)\{(b - a)(c_1 - a_1)$$

$$- (c - a)(b_1 - a_1)\}]$$

$$= x^2(\lambda x + \mu), \text{ where } \lambda \text{ and } \mu \text{ are constants}$$

$$= \mu x^2 + \lambda x^3$$

$$\text{Hence, } A_0 = A_1 = 0$$

69 (b)

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - xR_2$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ 0 & 0 & a+x \end{vmatrix} = (a+x)(a^2+ax)$$

$$\Rightarrow f(x) = a(a+x)^2$$

$$\therefore f(2x) = a(a+2x)^2$$

$$\Rightarrow f(2x) - f(x) = ax(2a+3x)$$

70 (c)

$$\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$$

$$\Rightarrow -12(30+1) - 4\lambda = -360$$

$$\Rightarrow -372 + 360 = 4\lambda \Rightarrow \lambda = -\frac{12}{4} = -3$$

71 (c)

$$\text{Let } A = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$$

$$\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$$

$$[C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0 \quad [\because 1+\omega+\omega^2=0]$$

72 (b)

Applying  $C_1 \rightarrow C_1 + C_2$ , we get

$$\begin{vmatrix} {}^{10}C_4 + {}^{10}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 + {}^{11}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 + {}^{12}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} {}^{11}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{12}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{13}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

It means either two rows or two columns are identical.

$$\therefore {}^{11}C_5 = {}^{11}C_m, {}^{12}C_7 = {}^{12}C_{m+2}, {}^{13}C_9 = {}^{13}C_{m+4}$$

$$\Rightarrow m = 5$$

73 (b)

$$\text{Given, } \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 3 \\ 5 & -6 & x \end{vmatrix} = 29$$

$$\Rightarrow 1(0+18) - 1(2x-15) = 29$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

74 (a)

Applying  $C_1 \rightarrow C_1 + C_2$ , we get

$$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = \begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix} = 0$$

75 (b)

$$\text{Since, } |A| = -1, |B| = 3$$

$$\therefore |AB| = |A||B| = -3$$

$$\text{Now, } |3AB| = (3)^3(-3) = -81$$

77 (d)

Applying  $C_3 \rightarrow C_3 - \alpha C_1 + C_2$  to the given determinant, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha + 1 \end{vmatrix} = (1-2\alpha)(ac-b^2)$$

So, if the determinant is zero, we must have

$$(1-2\alpha)(ac-b^2) = 0$$

$$\Rightarrow 1-2\alpha = 0$$

$$\text{or } (ac-b^2) = 0$$

$$\Rightarrow \alpha = \frac{1}{2} \text{ or } ac = b^2$$

Which means  $a, b, c$  are in GP.

78 (a)

$$\text{We have, } \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \quad (R_1 \rightarrow R_1 + R_2 + R_3)$$

$$\Rightarrow (x+9)\{1(x^2-12) - 1(2x-14) + 1(12-7x)\} = 0$$

$$\Rightarrow (x+9)(x^2-9x+14) = 0$$

$$\Rightarrow (x+9)(x-2)(x-7) = 0$$

$\therefore$  The other two roots are 2 and 7.

79 (a)

$$\text{Let } A \equiv \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}$$

$$= (a+b+c-x) \begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}$$

$$\Rightarrow (a+b+c-x)[1\{(b-x)(c-x) - a^2\} - c(c-x-a) + b(a-b+x)] = 0$$

$$\Rightarrow (a+b+c-x)[bc - bx - cx + x^2 - a^2 - c^2 + xc + ac + ab - b^2 + bx] = 0$$

$$\Rightarrow (a+b+c-x)[x^2 - (a^2 + b^2 + c^2) + ab + bc + ca] = 0$$

$\therefore ab + bc + ca = 0$  (given)

$$\Rightarrow \text{either } x = a + b + c \text{ or } x = (a^2 + b^2 + c^2)^{1/2}$$

80 (b)

We have,

$$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0 \text{ [Applying } C_1 \rightarrow$$

$$C_1 + C_2 + C_3]$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0$$

$$\text{[Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1]$$

$$\Rightarrow (x+1)(x-2)^2 = 0$$

$$\Rightarrow x = -1, 2$$

81 (c)

$$\text{Let } A = \begin{vmatrix} 1 & 2 & 3 \\ 1^3 & 2^3 & 3^3 \\ 1^5 & 2^5 & 3^5 \end{vmatrix} = 1.2.3 \begin{vmatrix} 1 & 2 & 3 \\ 1^2 & 2^2 & 3^2 \\ 1^4 & 2^4 & 3^4 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 16 & 81 \end{vmatrix} = 6 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 5 \\ 1 & 15 & 65 \end{vmatrix}$$

$$[C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_2]$$

$$= 6.3.5 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 5 & 13 \end{vmatrix} = 90[1(13-5)] = 720 = 6!$$

82 (c)

$$\because |A^3| = |A|^3 = 125$$

$$\Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$$

$$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$$

84 (b)

Given, angles of a triangle are  $A, B$  and  $C$ . We know that  $A + B + C = \pi$ , therefore

$$A + B = \pi - C$$

$$\Rightarrow \cos(A + B) = \cos(\pi - C) = -\cos C$$

$$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$$

$$\Rightarrow \cos A \cos B + \cos C = \sin A \sin B \dots(i)$$

$$\text{Let } \Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$

$$= -(1 - \cos^2 A)$$

$$+ \cos C(\cos C$$

$$+ \cos A \cos B)$$

$$+ \cos B(\cos B + \cos A \cos C)$$

$$= -\sin^2 A + \cos C(\sin A \sin B) +$$

$$\cos B(\sin A \sin C) \text{ [from Eq.(i)]}$$

$$= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C)$$

$$= -\sin^2 A + \sin A \sin(B + C)$$

$$= -\sin^2 A + \sin^2 A = 0 \text{ } [\because \sin(B + C) =$$

$$\sin(\pi - A) = \sin A]$$

85 (a)

We have,

$$\Delta = \begin{vmatrix} a^2 + x & ab & ac \\ ab & b^3 + x & bc \\ ac & bc & c^2 + x \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc} \begin{vmatrix} a^3 + ax & a^2b & a^2c \\ ab^2 & b^3 + bx & b^2c \\ ac^2 & bc^2 & c^3 + cx \end{vmatrix}$$

$$\text{[Applying } C_1(a), \\ C_2(b), C_3(c)]$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2 + x & a^2 & a^2 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

$$\text{[Applying } R_1 \rightarrow R_1 + R_2 + R_3]$$

$$\Rightarrow \Delta = (a^2 + b^2 + c^2 + x) \{ (b^2x + c^2x + x^2) \\ - (b^2x) + (-c^2x) \}$$

$$\Rightarrow \Delta = x^2(a^2 + b^2 + c^2 + x)$$

$$\Rightarrow x^2 \text{ is a factor } \Delta$$

86 (a)

$$\text{Given that, } \begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & x+3 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0 \text{ } (C_1 \rightarrow C_1 - C_2, \\ C_2 \rightarrow C_2 - C_3)$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0 \text{ } (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow (-1)(-b+c+a-b) = 0$$

$$\Rightarrow 2b - a - c = 0$$

$$\Rightarrow a + c = 2b$$

$$\therefore a, b, c \text{ in AP.}$$

87 (b)

$$\text{Given, } A = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{vmatrix} \Rightarrow A = 1$$

$$\therefore A^3 - 4A^2 + 3A + I = (1)^3 - 4(1)^2 + 3(1) + I \\ = I$$

88 (a)

$$\text{Let } \Delta = \begin{vmatrix} 1 & x & y \\ 2 & \sin x + 2x & \sin y + 3y \\ 3 & \cos x + 3x & \cos y + 3y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & y \\ 0 & \sin x & \sin y \\ 0 & \cos x & \cos y \end{vmatrix} \text{ } (R_2 \rightarrow R_2 - 2R_1, \\ R_3 \rightarrow R_3 - 3R_1)$$

$$= \sin x \cos y - \cos x \sin y = \sin(x - y)$$

89 (d)

$$\text{We have, } \Delta = \frac{1}{abc} \begin{vmatrix} a^3 + ax & a^2b & a^2c \\ ab^2 & b^3 + bx & b^2c \\ c^2a & c^2b & c^2 + xc \end{vmatrix}$$

Taking  $a, b, c$  common in columns Ist, IInd and IIIrd, we get,

$$\Delta = \begin{vmatrix} a^2 + x & a^2 & a^2 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$= (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$

$$= (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 0 \\ b^2 & x & 0 \\ c^2 & 0 & x \end{vmatrix}$$

$$= x(x - b^2)(a^2 + b^2 + c^2 + x)$$

Hence, option (d) is correct.

90 (a)

$$\text{Given, } \begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$$

$$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3a^2b^2c^2 = 0$$

$$\Rightarrow (ab + bc + ca)(a^2b^2 + b^2c^2 + c^2a^2 - ab^2c - bc^2a - ca^2b) = 0$$

$$\Rightarrow ab + bc + ca = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

91 (c)

Given,  $f(x) =$

$$\begin{vmatrix} 1 & 2(x-1) & 3(x-1)(x-2) \\ x-1 & (x-1)(x-2) & (x-1)(x-2)(x-3) \\ x & x(x-1) & x(x-1)(x-2) \end{vmatrix}$$

$$= (x-1)(x-1)(x-2) \begin{vmatrix} 1 & 2 & 3 \\ x-1 & x-2 & x-3 \\ x & x & x \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$

$$= (x-1)^2(x-2) \begin{vmatrix} -1 & -1 & 3 \\ 1 & 1 & x-3 \\ 0 & 0 & x \end{vmatrix}$$

$$= (x-1)^2(x-2)x(-1+1) = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(49) = 0$$

92 (b)

$$\text{Given that, } \begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix}$$

$$= A_0 + A_1x + A_2x^2 + A_3x^3$$

On putting  $x = 0$  on both sides, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = A_0$$

$$\Rightarrow A_0 = 0$$

94 (d)

We have,

$$\begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \\ \cos \gamma & \sin \gamma & 0 \end{vmatrix} \begin{vmatrix} \cos \alpha & \sin \alpha \\ \cos \beta & \sin \beta \\ \cos \gamma & \sin \gamma \end{vmatrix} = 0$$

$$= \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$$

$$\therefore \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha + \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix} = 0$$

95 (a)

$$\text{Given, } \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x) = 0$$

$$\Rightarrow x^3 - 67x + 126 = 0$$

$$\Rightarrow (x + 9)(x^2 - 9x + 14) = 0$$

$$\Rightarrow (x + 9)(x - 2)(x - 7) = 0$$

$$\Rightarrow x = -9, 2, 7$$

Hence, the other two roots are 2, 7

96 (c)

From the sine rule, we have

$$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say}),$$

$$\Rightarrow \sin A = ak, \sin B = bk \text{ and } \sin C = ck$$

$$\therefore \begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos(B - C) \\ c \sin A & \cos(B - C) & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & abk & ack \\ abk & 1 & \cos(B - C) \\ ack & \cos(B - C) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos(B - C) \\ \sin C & \cos(B - C) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & \sin(A + C) & \sin(A + B) \\ \sin(A + C) & 1 & \cos(B - C) \\ \sin(A + B) & \cos(B - C) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix} \begin{vmatrix} \sin A & \cos A \\ \cos C & \sin C \\ \cos B & \sin B \end{vmatrix} = a^2 \times 0 = 0$$

97 (b)

$$\text{Given, } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C$  and  $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$$

Hence,  $D$  is divisible by both  $x$  and  $y$ .

98 (a)

Taking  $x$  common from  $R_2$  and  $x(x - 1)$  common from  $R_3$ , we get

$$f(x) = x^2(x - 1) \begin{vmatrix} 1 & x & (x + 1) \\ 2 & (x - 1) & (x + 1) \\ 3 & (x - 2) & (x + 1) \end{vmatrix}$$

$$\begin{aligned} \Rightarrow f(x) &= x^2(x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3 & x-2 & 1 \end{vmatrix} \\ &= x^2(x^2-1) \begin{vmatrix} 1 & x & 1 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{vmatrix} \begin{matrix} [R_2 \rightarrow R_2 - R_1] \\ [R_3 \rightarrow R_3 - R_1] \end{matrix} \\ \Rightarrow f(x) &= x^2(x^2-1)(-2+2) = 0 \\ \Rightarrow f(x) &= 0 \text{ for all } x \\ \therefore f(11) &= 0 \end{aligned}$$

99 (b)

Applying  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ , we get

$$\begin{aligned} \begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} &= 0 \\ \Rightarrow 1[-6(5x^2-20) + 15(2x-4)] &= 0 \\ \Rightarrow x^2 - x - 2 &= 0 \\ \Rightarrow (x-2)(x+1) &= 0 \\ \Rightarrow x &= -1, 2 \end{aligned}$$

100 (b)

We have,

$$\begin{aligned} \begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} &= 0 \\ \Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} &= 0 \quad \text{Applying } C_1 \rightarrow \end{aligned}$$

$$\begin{aligned} C_1 + C_2 + C_3 \\ \Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} &= 0 \\ \Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} &= 0 \end{aligned}$$

Applying  $R_2 \rightarrow R_2 - R_1,$

$R_3 \rightarrow R_3 - R_1$

$$\begin{aligned} \Rightarrow (3x-2)(3x-11)^2 &= 0 \\ \Rightarrow x &= 2/3, 11/3 \end{aligned}$$

101 (a)

We have,

$$\begin{aligned} \begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix} \\ = \begin{vmatrix} \alpha & x-\alpha & x-\alpha & x-\alpha \\ x-(x-\beta) & 0 & 0 & 0 \\ x & 0 & -(x-\gamma) & 0 \\ x & 0 & 0 & -(x-\delta) \end{vmatrix} \\ = \alpha \begin{vmatrix} -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} \\ - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} \end{aligned}$$

$$\begin{aligned} + x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} \\ - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \end{vmatrix} \\ = -\alpha(x-\beta)(x-\gamma)(x-\delta) - (x-\alpha)(x-\gamma)(x-\delta) \\ - x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma) \\ = -\alpha(x-\beta)(x-\gamma)(x-\delta) + x(x-\beta)(x-\gamma)(x-\delta) \\ - x(x-\beta)(x-\gamma)(x-\delta) - x(x-\alpha)(x-\gamma)(x-\delta) \\ - x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma) \\ = (x-\beta)(x-\gamma)(x-\delta)(x-\alpha) \\ - x[(x-\alpha)(x-\beta)(x-\gamma) \\ + (x-\beta)(x-\gamma)(x-\delta) \\ + (x-\gamma)(x-\delta)(x-\alpha) + (x-\alpha)(x-\beta)(x-\delta)] \\ = f(x) - xf'(x), \text{ where, } f(x) = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta) \end{aligned}$$

102 (b)

$$\begin{aligned} \text{Given } \begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} &= 0 \\ \Rightarrow c^2 - ab - a(c-a) + b(b-c) &= 0 \\ \Rightarrow a^2 + b^2 + c^2 - ab - bc - ca &= 0 \\ \Rightarrow \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] &= 0 \\ \Rightarrow a = b = c \end{aligned}$$

So,  $\Delta ABC$  is equilateral triangle.

$$\therefore \angle A = 60^\circ, \angle B = 60^\circ, \angle C = 60^\circ$$

$$\sin^2 A + \sin^2 B + \sin^2 C$$

$$\begin{aligned} &= \sin^2 60^\circ + \sin^2 60^\circ + \sin^2 60^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 3 \times \frac{3}{4} = \frac{9}{4} \end{aligned}$$

104 (a)

$$\text{Given that, } \Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{aligned} &= \begin{vmatrix} 1 + \omega^n + \omega^{2n} & \omega^n & \omega^{2n} \\ 1 + \omega^n + \omega^{2n} & 1 & \omega^n \\ 1 + \omega^n + \omega^{2n} & \omega^{2n} & 1 \end{vmatrix} \\ &= \begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix} \end{aligned}$$

$$(\because \text{If } n \text{ multiple of } 3, \text{ then } 1 + \omega^n + \omega^{2n} = 0)$$

$$= 0$$

105 (b)

$$\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+9 & x+9 & x+9 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 9+x & x+9 & 9+x \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix}$$

$$= \begin{vmatrix} 9+x & 9+x & 9+x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & x-3 \\ 6 & x-6 & -3 \end{vmatrix} = (9+x) \begin{vmatrix} 1 & 0 & 0 \\ x & 7-x & 2-x \\ 7 & -5 & x-7 \end{vmatrix}$$

$$= (9+x) \begin{vmatrix} 1 & 0 & 0 \\ 5 & x-5 & -1 \\ x & 4-x & 5-x \end{vmatrix}$$

$$= 0$$

$$\Rightarrow x+9=0 \Rightarrow x=-9$$

106 (b)

The given system of equations will have a unique solution, if

$$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0 \Rightarrow k(k-1)(k+2) \neq 0$$

$$\Rightarrow k \neq 0, 1, -2$$

108 (d)

$$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_2$

$$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$$

$$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$$

$$\Rightarrow ayz + bzx + cyx = 2xyz$$

$$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

109 (a)

Given,

$$\begin{vmatrix} 1 & \cos(\alpha - \beta) & \cos \alpha \\ \cos(\alpha - \beta) & 1 & \cos \beta \\ \cos \alpha & \cos \beta & 1 \end{vmatrix} \text{ is symmetric}$$

determinant.

$\therefore$  Its value is

$$1 + 2 \cos(\alpha - \beta) \cos \alpha \cos \beta$$

$$- \cos^2 \alpha - \cos^2 \beta - \cos^2(\alpha - \beta)$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta - \cos(\alpha - \beta)$$

$$[\cos(\alpha - \beta) - 2 \cos \alpha \cos \beta]$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta - \cos(\alpha - \beta)$$

$$[\cos(\alpha - \beta) - \cos(\alpha + \beta) - \cos(\alpha - \beta)]$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta + \cos(\alpha - \beta) \cos(\alpha + \beta)$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta$$

$$+ \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta (1 - \cos^2 \alpha) - \sin^2 \alpha \sin^2 \beta$$

$$= (1 - \cos^2 \alpha)(1 - \cos^2 \beta) - \sin^2 \alpha \sin^2 \beta$$

$$= \sin^2 \alpha \sin^2 \beta - \sin^2 \alpha \sin^2 \beta = 0$$

110 (d)

We have,

$$\Delta = \begin{vmatrix} 2 \sin A \cos A & \sin C & \sin B \\ \sin C & 2 \sin B \cos B & \sin A \\ \sin B & \sin A & 2 \sin C \cos C \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2ka \cos A & kc & kb \\ kc & 2kb \cos B & ka \\ kb & ka & 2kc \cos C \end{vmatrix} \text{ [Using:}$$

Sine rule]

$$\Rightarrow \Delta = k^3 \begin{vmatrix} 2a \cos A & c & b \\ c & 2b \cos B & a \\ b & a & 2c \cos C \end{vmatrix}$$

$$\Rightarrow \Delta$$

$$= k^3 \begin{vmatrix} a \cos A + a \cos A & a \cos B + b \cos A & c \cos A \\ a \cos B + b \cos A & b \cos B + b \cos B & b \cos C \\ c \cos A + a \cos C & b \cos C + c \cos B & c \cos C \end{vmatrix}$$

$$\Rightarrow \Delta = k^3 \begin{vmatrix} \cos A & a & 0 \\ \cos B & b & 0 \\ \cos C & c & 0 \end{vmatrix} \begin{vmatrix} a & \cos A & 0 \\ b & \cos B & 0 \\ c & \cos C & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = k^3 \times 0 \times 0 = 0$$

111 (d)

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$  and taking common  $(a + b + c)$  from  $R_1$ , we get

$$= (a + b + c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b - c - a & 0 \\ 2c & 0 & c - a - b \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$ ,

$$= (a + b + c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b - c - a & -2b \\ 2c & 0 & -a - b - c \end{vmatrix}$$

$$= (c + b + c)[(-b - c - a)(-a - b - c)]$$

$$= (a + b + c)^3$$

112 (b)

We know that

$$|AB| = |A||B|$$

$$\Rightarrow AB = 0$$

$$\Rightarrow |AB| = 0$$

$$\Rightarrow |A||B| = 0$$

$$\Rightarrow \text{either } |A| = 0 \text{ or } |B| = 0$$

113 (a)

The given system of equations will have a unique solution, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

114 (a)

$\therefore a_1, a_2, \dots, a_n$  are in GP.

$\Rightarrow a_n, a_n + 2, a_n + 4, \dots$  are also in GP.

Now,  $(a_{n+2})^2 = a_n \cdot a_{n+4}$

$\Rightarrow 2 \log(a_{n+2}) = \log a_n + \log a_{n+4}$

Similarly,  $2 \log(a_{n+8}) = \log a_{n+6} + \log a_{n+10}$

$$\text{Now, } \Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

Applying  $C_2 \rightarrow 2C_2 - C_1 - C_3$

$$\begin{vmatrix} \log a_n & 2 \log a_{n+2} - \log a_n - \log a_{n+4} & \log a_{n+4} \\ \log a_{n+6} & 2 \log a_{n+8} - \log a_{n+6} - \log a_{n+10} & \log a_{n+10} \\ \log a_{n+12} & 2 \log a_{n+14} - \log a_{n+12} - \log a_{n+16} & \log a_{n+16} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_n & 0 & \log a_{n+4} \\ \log a_{n+6} & 0 & \log a_{n+10} \\ \log a_{n+12} & 0 & \log a_{n+16} \end{vmatrix} = 0$$

116 (a)

We have,

$$\text{Coefficient of } x \text{ in } \begin{vmatrix} x & (1 + \sin x)^3 & \cos x \\ 1 & \log(1 + x) & 2 \\ x^2 & (1 + x)^2 & 0 \end{vmatrix}$$

= coefficient of  $x$  in

$$= \begin{vmatrix} x & \left(1 + x - \frac{x^3}{3!} + \dots\right)^3 & 1 - \frac{x^2}{2!} + \dots \\ 1 & x - \frac{x^2}{2} + \frac{x^3}{3} \dots & 2 \\ x^2 & 1 + 2x + x^2 & 0 \end{vmatrix}$$

$$= \text{Coefficient of } x \text{ in } \begin{vmatrix} x & 1 & 1 \\ 1 & x & 2 \\ x^2 & 1 & 0 \end{vmatrix}$$

$$= \text{Coefficient of } x \text{ in } [x(0 - 2) - (0 - 2x^2) + (1 - x^3)] = -2$$

119 (d)

On putting  $x = 0$  in the given equation, we get

$$g = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 9$$

On differentiating given equation and then put  $x = 0$ , we get

$$f = -5$$

120 (a)

$$\text{In } \Delta ABC, \text{ given } \begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$$

$$\Rightarrow 1(c^2 - ab) - a(c - a) + b(b - c) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$$

$$\Rightarrow (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) = 0$$

$$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

Here, sum of squares of three numbers can be zero, if and only, if  $a = b = c$ .

$\Rightarrow \Delta ABC$  is an equilateral triangle.

$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$

$\therefore \sin^2 A + \sin^2 B$

$+ \sin^2 C$

$= \sin^2 60^\circ + \sin^2 60^\circ + \sin^2 60^\circ$

$$= \left(\frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right) = \frac{9}{4}$$

122 (d)

$\Delta(-x)$

$$= \begin{vmatrix} f(-x) + f(x) & 0 & x^4 \\ 3 & f(-x) - f(x) & \cos x \\ x^4 & -2x & f(-x)f(x) \end{vmatrix}$$

$$= \begin{vmatrix} f(x) + f(-x) & 0 & x^4 \\ 3 & f(x) - f(-x) & \cos x \\ x^4 & -2x & f(x)f(-x) \end{vmatrix}$$

$= -\Delta(x)$

So,  $\Delta(x)$  is an odd function.

$\Rightarrow x^4 \Delta(x)$  is an odd function

$$\Rightarrow \int_{-2}^2 x^4 \Delta(x) dx = 0$$

123 (c)

$$\begin{vmatrix} \cos(x - a) & \cos(x + a) & \cos x \\ \sin(x + a) & \sin(x - a) & \sin x \\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= \begin{vmatrix} \cos(x - a) + \cos(x - a) & \cos(x + a) & \cos x \\ \sin(x + a) + \sin(x - a) & \sin(x - a) & \sin x \\ \cos a (\tan x + \cot x) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= \begin{vmatrix} 2 \cos x \cos a & \cos(x + a) & \cos x \\ 2 \sin x \cos a & \sin(x - a) & \sin x \\ \cos a \left(\frac{\tan^2 x + 1}{\tan x}\right) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= 2 \cos a \begin{vmatrix} \cos x & \cos(x + a) & \cos x \\ \sin x & \sin(x - a) & \sin x \\ \operatorname{cosec} 2x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix} = 0$$

[ $\because$  two columns are identical]

125 (a)

Since  $(x - k)$  will be common from each row which vanish by putting  $x = k$ . Therefore,

$(x - k)^r$  will be a factor of  $|A|$

126 (d)

Putting  $x = 0$  in the given determinant equation we get

$$a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= 1(0 - 9) + 3(4 + 6)$$

$$= 30 - 9 = 21$$

127 (a)

$$\text{Given, } \begin{vmatrix} a & \cot \frac{A}{2} & \lambda \\ b & \cot \frac{B}{2} & \mu \\ c & \cot \frac{C}{2} & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & \frac{s(s-a)}{\Delta} & \lambda \\ b & \frac{s(s-b)}{\Delta} & \mu \\ c & \frac{s(s-c)}{\Delta} & \gamma \end{vmatrix} = 0$$

$$\left[ \because \cot \frac{A}{2} = \frac{s(s-a)}{\sqrt{(s-a)(s-b)(s-c)}} = \frac{s(s-a)}{\Delta} \right]$$

$$\Rightarrow \frac{1}{r} \begin{vmatrix} a & s-a & \lambda \\ b & s-b & \mu \\ c & s-c & \gamma \end{vmatrix} = 0 \text{ where } r = \frac{\Delta}{s}$$

Applying  $C_2 \rightarrow C_2 + C_1$

$$\Rightarrow \frac{1}{r} \begin{vmatrix} a & s & \lambda \\ b & s & \mu \\ c & s & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \frac{\Delta}{r^2} \begin{vmatrix} a & 1 & \lambda \\ b & 1 & \mu \\ c & 1 & \gamma \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \frac{\Delta}{r^2} \begin{vmatrix} a-b & 0 & \lambda-\mu \\ b-c & 0 & \mu-\gamma \\ c & 1 & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \frac{\Delta}{r^2} [(b-c)(\lambda-\mu) - (\mu-\gamma)(a-b)] = 0$$

$$\Rightarrow b(\lambda-\mu) - c(\lambda-\mu) - a(\mu-\gamma) + b(\mu-\gamma) = 0$$

$$\Rightarrow -a(\mu-\gamma) + b(\lambda-\mu + \mu-\gamma) - c(\lambda-\mu) = 0$$

$$\Rightarrow -a(\mu-\gamma) + b(\lambda-\gamma) - c(\lambda-\mu) = 0$$

$$\Rightarrow a(\mu-\gamma) + b(\gamma-\lambda) + c(\lambda-\mu) = 0$$

129 (d)

$$\text{Let } \Delta = \begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1$ , we get

$$\Delta = \begin{vmatrix} x+2 & 1 & x+a \\ x+4 & 1 & x+b \\ x+6 & 1 & x+c \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \Delta = \begin{vmatrix} x+2 & 1 & x+a \\ 2 & 0 & b-a \\ 4 & 0 & c-a \end{vmatrix}$$

$$= -1(2c - 2a - 4b + 4a)$$

$$\Rightarrow \Delta = 2(2b - c - a) \dots (i)$$

Since,  $a, b, c$  are in AP.

$$\therefore b = \frac{a+c}{2}$$

$$\therefore \Delta = 2(a+c - c - a)$$

$$= 0 \text{ [from Eq. (i)]}$$

130 (a)

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$= \begin{vmatrix} \cos A \cos P + \sin A \sin P & \cos A \cos Q + \sin A \sin Q & \cos A \cos R + \sin A \sin R \\ \cos B \cos P + \sin B \sin P & \cos B \cos Q + \sin B \sin Q & \cos B \cos R + \sin B \sin R \\ \cos C \cos P + \sin C \sin P & \cos C \cos Q + \sin C \sin Q & \cos C \cos R + \sin C \sin R \end{vmatrix}$$

The determinants can be rewritten as 8

determinants and the value of each of these 8 determinants is zero.

$$\text{ie, } \cos P \cos Q \cos R \begin{vmatrix} \cos A & \cos A & \cos A \\ \cos B & \cos B & \cos B \\ \cos C & \cos C & \cos C \end{vmatrix} = 0$$

Similarly, other determinants can be shown zero.

131 (b)

$$\text{We have, } \Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin \frac{n\pi}{2} & \cos \frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$$

$$\frac{d^n}{dx^n} [\Delta(x)] = \begin{vmatrix} \frac{d^n}{dx^n} x^n & \frac{d^n}{dx^n} \sin x & \frac{d^n}{dx^n} \cos x \\ n! & \sin \left(\frac{n\pi}{2}\right) & \cos \left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

( $\because$  Differentiation of  $R_2$  and  $R_3$  are zero)

$$\begin{vmatrix} n! & \sin \left(x + \frac{n\pi}{2}\right) & \cos \left(x + \frac{n\pi}{2}\right) \\ n! & \sin \left(\frac{n\pi}{2}\right) & \cos \left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$$\Rightarrow [\Delta^n(x)]_{x=0}$$

$$= \begin{vmatrix} n! & \sin \left(0 + \frac{n\pi}{2}\right) & \cos \left(0 + \frac{n\pi}{2}\right) \\ n! & \sin \left(\frac{n\pi}{2}\right) & \cos \left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$$= \begin{vmatrix} n! & \sin \left(\frac{n\pi}{2}\right) & \cos \left(\frac{n\pi}{2}\right) \\ n! & \sin \left(\frac{n\pi}{2}\right) & \cos \left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$$= 0 \text{ } (\because R_1 \text{ and } R_2 \text{ are identical})$$

132 (a)

$$\text{Let, } \Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= 1(1 - \log_z y \log_y z)$$

$$- \log_x y (\log_y x - \log_y z \log_z x) + \log_x z (\log_y x \log_z y - \log_z x)$$

$$= (1 - \log_z z) - \log_x y (\log_y x - \log_y z \log_z x) + \log_x z (\log_y x \log_z y - \log_z x)$$

$$= (1 - 1) - (1 - \log_x y \log_y x) + (\log_x z \log_z x - 1) = 0 \text{ (Since, } \log_x y \log_y x = 1)$$

$$= 0 - (1 - 1) + (1 - 1) = 0$$

133 (b)

Given determinant is

$$\Delta = \begin{vmatrix} 15! & 16! & 17! \\ 16! & 17! & 18! \\ 17! & 18! & 19! \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_2$

$$\Delta = \begin{vmatrix} 15! & 15 \times 15! & 16 \times 16! \\ 16! & 16 \times 16! & 17 \times 17! \\ 17! & 17 \times 17! & 18 \times 18! \end{vmatrix}$$

$$= (15!)(16!)(17!) \begin{vmatrix} 1 & 15 & 16 \times 16 \\ 1 & 16 & 17 \times 17 \\ 1 & 17 & 18 \times 18 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$

$$= (15!)(16!)(17!) \begin{vmatrix} 0 & -1 & -33 \\ 0 & -1 & -35 \\ 1 & 17 & 18 \times 18 \end{vmatrix}$$

$$= 2 \times (15!)(16!)(17!)$$

134 (b)

We have,

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{vmatrix} \begin{matrix} \text{Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1, R_4 \rightarrow R_4 - R_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 9 \\ 3 & 9 & 19 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 3 & 10 \end{vmatrix} \begin{matrix} \text{Applying } R_2 \rightarrow R_2 - 2R_1, \\ R_3 \rightarrow R_3 - 3R_1 \end{matrix}$$

$$= (10 - 9) = 1$$

135 (c)

The homogenous linear system of equations is consistent *ie*, possesses trivial solution, if  $\Delta \equiv$

$$\begin{vmatrix} 2 & 3 & 5 \\ 1 & k & 5 \\ k & -12 & -14 \end{vmatrix} \neq 0$$

$$\Rightarrow 2(-14k + 60) - 3(-14 - 5k) + 5(-12 - k^2) \neq 0$$

$$\Rightarrow 5k^2 + 13k - 102 \neq 0$$

$$\Rightarrow (5k - 17)(k + 6) \neq 0$$

$$\Rightarrow k \neq -6, \frac{17}{5}$$

136 (c)

We have,

$$\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} a(b^2 + c^2) & a^2b & a^2c \\ ab^2 & b(c^2 + a^2) & b^2c \\ c^2a & c^2b & c(a^2 + b^2) \end{vmatrix}$$

[Applying  $R_1 \rightarrow R_1(a), R_2 \leftrightarrow R_2(b), R_3 \leftrightarrow R_3(c)$ ]

$$= \frac{1}{abc} abc \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} \text{Applying } R_1 \rightarrow R_1 - (R_2 + R_3)$$

$$= 4a^2b^2c^2$$

$$\therefore ka^2b^2c^2 = 4a^2b^2c^2 \Rightarrow k = 4$$

137 (a)

We have,

$$\begin{vmatrix} y + z & x & y \\ z + x & z & x \\ x + y & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 2(x + y + z) & x + y + z & x + y + z \\ z + x & z & x \\ x + y & y & z \end{vmatrix} \begin{matrix} \text{App} \\ R_1 \rightarrow R_1 \cdot \end{matrix}$$

$$= (x + y + z) \begin{vmatrix} 2 & 1 & 1 \\ z + x & z & x \\ x + y & y & z \end{vmatrix}$$

$$= (x + y + z) \begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x - z & y & z \end{vmatrix} \text{Applying } C_1$$

$$\rightarrow C_1 - C_2 - C_3$$

Hence, the repeating factor is  $(z - x)$

138 (d)

$$\begin{vmatrix} 4 + x^2 & -6 & -2 \\ -6 & 9 + x^2 & 3 \\ -2 & 3 & 1 + x^2 \end{vmatrix}$$

$$= (4 + x^2)[(1 + x^2)(9 + x^2) - 9]$$

$$+ 6[-6(1 + x^2) + 6] - 2[-18 + 2(9 + x^2)]$$

$$= (4 + x^2)(10x^2 + x^4) - 36x^2 - 4x^2$$

$$= 40x^2 + 4x^4 + 10x^4 + x^6 - 40x^2$$

$$= x^4(x^2 + 14)$$

Which is not divisible by  $x^5$ .

139 (d)

Since, for  $x = 0$ , the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence,  $x = 0$  is the solution of the given equation.

140 (c)

$$\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1 \rightarrow R_1 - (R_2 + R_3)]$$

$$= -2 \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix} \begin{matrix} (R_2 \rightarrow R_2 - R_1) \\ (R_3 \rightarrow R_3 - R_1) \end{matrix}$$

$$= -2[-c^2(b^2a^2 - 0) + b^2(0 - a^2c^2)]$$

$$= -2[-2a^2b^2c^2] = 4a^2b^2c^2$$

141 (c)

$$\text{We have, } \begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & b & c \\ p & b & c \\ a & b & r \end{vmatrix} + \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow 0 + \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow p(qr - bc) - b(ar - ac) - c(ab - aq) = 0$$

$$\Rightarrow -pqr + pbc + bar + acq = 0$$

On simplifying, we get

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

142 (d)

$$\text{Let } \Delta = \begin{vmatrix} a+b+2c & a & b \\ c & 2a+b+c & b \\ c & a & a+2b+c \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & 2a+b+c & b \\ 2(a+b+c) & a & a+2b+c \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & 2a+b+c & b \\ 1 & a & a+2b+c \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 0 & -(a+b+c) & 0 \\ 0 & (a+b+c) & -(a+b+c) \\ 1 & a & a+2b+c \end{vmatrix}$$

$(\begin{smallmatrix} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - R_3 \end{smallmatrix})$

$$= 2(a+b+c)^3 \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a & a+2b+c \end{vmatrix}$$

$$= 2(a+b+c)^3$$

143 (c)

Since,  $-1 \leq x < 0$

$$\therefore [x] = -1$$

Also,  $0 \leq y < 1 \Rightarrow [y] = 0$

and  $1 \leq z < 2 \Rightarrow [z] = 1$

$\therefore$  Given determinant becomes

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

144 (b)

For singular matrix,

$$\begin{vmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{vmatrix} = 0$$

Applying  $C_2 \rightarrow C_2 + C_1, C_3 \rightarrow C_3 + C_1$

$$\Rightarrow \begin{vmatrix} -x & 0 & 2-x \\ 2 & 2+x & 2-x \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2 & 2+x & 1 \\ x & x-2 & 0 \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2+x & 2+x & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x)(2+x) \begin{vmatrix} -x & 0 & 1 \\ 1 & 1 & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x)(2+x)(x-2-x) = 0$$

$$\Rightarrow x = 2, -2$$

$\therefore$  Given matrix is non -

singular for all  $x$  other than 2 and -2.

146 (c)

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+ (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}^1$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+ (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+ (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$= (1 + (-1)^{n+2}) \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

This is equal to zero only, if  $n+2$  is odd i.e.,  $n$  is an odd integer.

147 (d)

$$\text{Given that, } \begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = \frac{1}{abc} \begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$$

$$= \frac{abc}{abc} \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$$= 0$$

( $\because$  columns  $C_1$  and  $C_3$  are same)

148 (b)

$$\text{Given that, } \begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & -6 & -1 \\ 5 & -5x & -5 \\ -3 & 2x & x+2 \end{vmatrix} = 0 \quad (R_2 \rightarrow R_2 - R_3)$$

$$\Rightarrow 5 \begin{vmatrix} x & -6 & -1 \\ 1 & -x & -1 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow x(-x^2 - 2x + 2x) - 1(-6x - 12 + 2x) - 3(6 - x) = 0$$

$$\Rightarrow -x^3 + 7x - 6 = 0$$

$$\Rightarrow x^3 - 7x + 6 = 0$$

$$\Rightarrow (x - 1)(x - 2)(x + 3) = 0$$

$$\Rightarrow x = 1, 2, -3$$

$\therefore$  Option (b) is correct.

149 (a)

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - (R_1 - 2R_2)$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

151 (a)

$$\text{Given, } A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow |A| = 5 - 6 = -1$$

$$\therefore |A^{2009} - 5A^{2008}| = |A^{2008}||A - 5I|$$

$$= (-1)^{2008} \left| \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \right|$$

$$= \begin{vmatrix} -4 & 2 \\ 3 & 0 \end{vmatrix} = -6$$

152 (b)

$$f(1) = \begin{vmatrix} -2 & -16 & -78 \\ -4 & -48 & -496 \\ 1 & 2 & 3 \end{vmatrix} = 2928$$

$$f(3) = \begin{vmatrix} 0 & 0 & 0 \\ -2 & -32 & -392 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\text{and } f(5) = \begin{vmatrix} 2 & 32 & 294 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\therefore f(1) \cdot f(3) + f(3) \cdot f(5) + f(5) \cdot f(1)$$

$$= f(1) \cdot 0 + 0 + f(1) \cdot 0 = 0 = f(3) \text{ or } f(5)$$

153 (d)

$$\Delta = (x + a + b + c) \begin{vmatrix} 1 & a & b + c \\ 1 & b & c + a \\ 1 & c & a + b \end{vmatrix} \quad [C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= (x + a + b + c)(a + b + c) \begin{vmatrix} 1 & 1 & b + c \\ 1 & 1 & c + b \\ 1 & 1 & a + b \end{vmatrix}$$

$$= 0 \quad [C_2 \rightarrow C_2 + C_3]$$

Hence,  $x$  may have any value.

154 (c)

$$\text{It has a non-zero solution, if } \begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1(-k - 3) - k(3 + 1) - 1(-9 + k) = 0$$

$$\Rightarrow -6k + 6 = 0$$

$$\Rightarrow k = 1$$

155 (a)

$$\text{Given, } \begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow 1 + xyz = 0$$

$$\Rightarrow xyz = -1$$

156 (a)

$$\begin{vmatrix} [e] & [\pi] & [\pi^2 - 6] \\ [\pi] & [\pi^2 - 6] & [e] \\ [\pi^2 - 6] & [e] & [\pi] \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$$= 2(9 - 4) - 3(9 - 6) + 3(6 - 9)$$

$$= 10 - 9 - 9$$

$$= -8$$

157 (b)

We have,

$$\Delta = \begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ ax + b & bx + c & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ 0 & 0 & -(ax^2 + 2bx + c) \end{vmatrix}$$

$$[\text{Applying } R_3 \rightarrow R_3 - x \begin{vmatrix} a & b \\ b & c \end{vmatrix} \\ R_1 - R_2]$$

$$\Rightarrow \Delta = (b^2 - ac)(ac^2 + 2bx + c)$$

$$\therefore \Delta = 0$$

$$\Rightarrow b^2 = ac \text{ or, } ax^2 + 2bx + c = 0$$

$$\Rightarrow a, b, c \text{ are in G.P. or, } x \text{ is a root of the equation } ax^2 + 2bx + c = 0$$

158 (d)

All statements are false.

159 (b)

Applying  $C_3 \rightarrow C_3 - C_1$ , we get

$$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 - 1 \\ \cos(p-d)a & \cos pa & 0 \\ \sin(p-d)a & \sin pa & 0 \end{vmatrix}$$

$$= (\alpha^2 - 1)\{\sin pa \cos(p-d)a - \cos pa \sin(p-d)a\}$$

$$= (\alpha^2 - 1)\sin\{-(p-d)a + pa\}$$

$$\Rightarrow \Delta = (\alpha^2 - 1)\sin da$$

Which is independent of  $p$ .

160 (c)

$$\text{Given, } \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$  and taking common  $(3a-x)$  from  $C_1$ , we get

$$(3a-x) \begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

$$\Rightarrow (3a-x) \begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$$= 0 \begin{bmatrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{bmatrix}$$

$$\Rightarrow (3a-x)(4x^2) = 0$$

$$\Rightarrow x = 3a, 0$$

161 (a)

Since, the given equations are consistent.

$$\therefore \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ a & 2 & -b \end{vmatrix} = 0$$

$$\Rightarrow 2(-b+4) - 3(-3b+2a) + 1(6-a) = 0$$

$$\Rightarrow -2b+8+9b-6a+6-a=0$$

$$\Rightarrow 7b-7a=-14$$

$$\Rightarrow a-b=2$$

162 (d)

Given,

$$\begin{vmatrix} 1 & \cos x & 1 - \cos x \\ 1 + \sin x & \cos x & 1 + \sin x - \cos x \\ \sin x & \sin x & 1 \end{vmatrix}$$

Applying  $C_3 \rightarrow C_3 + C_2 - C_1$

$$= \begin{vmatrix} 1 & \cos x & 0 \\ 1 + \sin x & \cos x & 0 \\ \sin x & \sin x & 1 \end{vmatrix}$$

$$= \cos x - \cos x(1 + \sin x)$$

$$= -\cos x \sin x$$

$$= -\frac{1}{2}\sin 2x$$

$$\therefore \int_0^{\pi/2} \Delta x \, dx = -\frac{1}{2} \int_0^{\pi/2} \sin 2x \, dx$$

$$= -\frac{1}{2} \left[ -\frac{\cos 2x}{2} \right]_0^{\pi/2} = -\frac{1}{2}$$

163 (c)

For the non-trivial solution, we must have

$$\begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1-a & 0 & a \\ b-1 & 1-b & b \\ 0 & c-1 & 1 \end{vmatrix} = 0$$

[Applying  $C_1 \rightarrow C_1 - C_2$ ;  
 $C_2 \rightarrow C_2 - C_3$ ]

$$\Rightarrow (1-a)[(1-b) - b(c-1)] + a(b-1)(c-1) = 0$$

$$\Rightarrow \frac{1}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 0$$

$$\Rightarrow \left( \frac{1}{c-1} + 1 \right) + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

$$\Rightarrow \frac{c}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

$$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1$$

164 (d)

Given system equations are

$$3x - 2y + z = 0$$

$$\lambda x - 14y + 15z = 0 \text{ and } x + 2y - 3z = 0$$

The system of equations has infinitely many (non-trivial) solutions, if  $\Delta = 0$ .

$$\Rightarrow \Delta = \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 3(42 - 30) - \lambda(6 - 2) + 1(-30 + 14) = 0$$

$$\Rightarrow 36 - 4\lambda - 16 = 0$$

$$\Rightarrow \lambda = 5$$

166 (c)

$$\text{Since, } \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\Rightarrow \sin x(\sin^2 x - \cos^2 x)$$

$$- \cos x(\cos x \sin x - \cos^2 x)$$

$$+ \cos x(\cos^2 x - \sin x \cos x) = 0$$

$$\Rightarrow \sin x(\sin^2 x - \cos^2 x)$$

$$- 2\cos^2 x(\sin x - \cos x) = 0$$

$$\Rightarrow (\sin x - \cos x)[\sin x(\sin x + \cos x) - 2\cos^2 x] = 0$$

$$\Rightarrow (\sin x - \cos x)[(\sin^2 x - \cos^2 x)$$

$$+ (\sin x \cos x - \cos^2 x)] = 0$$

$$\Rightarrow (\sin x - \cos x)^2 [\sin x + \cos x + \cos x] = 0$$

$$\Rightarrow (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$$

$$\Rightarrow \text{Either } (\sin x - \cos x)^2 = 0$$

$$\text{or } \sin x + 2\cos x = 0$$

$$\Rightarrow \text{Either } \tan x = 1 \text{ or } \tan x = -2$$

$$\Rightarrow \text{Either } x = \frac{\pi}{4} \text{ or } \tan x = -2$$

$$\text{As } x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \tan x \in [-1, 1]$$

$$\text{Hence, real solution is only } x = \frac{\pi}{4}$$

167 (a)

Applying  $R_1 \rightarrow R_1 + R_3 - 2R_2$ , we get

$$\Delta = \begin{vmatrix} 0 & 0 & 0 & x+z-zy \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix}$$

$$= -(x+z-2y) \begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ x & y & z \end{vmatrix} \quad [\text{Expanding along } R_1]$$

$$= -(x+z-2y) \begin{vmatrix} 0 & -1 & 6 \\ 0 & -1 & 7 \\ x-2y+z & y-z & z \end{vmatrix}$$

$$[\text{Applying } C_1 \rightarrow C_1 + C_3]$$

$$[-2C_2 \text{ and } C_2 \rightarrow C_2 - C_3]$$

$$= -(x+z-2y)^2 \begin{vmatrix} -1 & 6 \\ -1 & 7 \end{vmatrix} = (x-2y+z)^2$$

169 (c)

We have,  $a = 1 + 2 + 4 + 8 + \dots$  upto  $n$  terms

$$= 1 \left( \frac{2^n - 1}{2 - 1} \right) = 2^n - 1$$

$$b = 1 + 3 + 9 + \dots \text{ upto } n \text{ terms} = \frac{3^n - 1}{2}$$

$$\text{and } c = 1 + 5 + 25 + \dots \text{ upto } n \text{ terms} = \frac{5^n - 1}{4}$$

$$\therefore \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} = 2 \begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2^n & 3^n & 5^n \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix} \quad [R_1 \rightarrow R_1 + R_2]$$

$$= 2 \times 0 = 0 \quad [\because \text{two rows are identical}]$$

170 (d)

$$\text{Let } \Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix} = c(c^2 - 1) - 1(c - 6)$$

$$= 8 \cos^3 \theta - 4 \cos \theta + 6$$

171 (b)

We have,

$$\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 5 \\ 4 & 5 & 7 \\ 9 & 7 & 9 \end{vmatrix} \quad \text{Applying } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_2$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 2 \\ 4 & 5 & 2 \\ 9 & 7 & 2 \end{vmatrix} \quad \text{Applying } C_3 \rightarrow C_3 - C_2$$

$$\Rightarrow \Delta = 2 \begin{vmatrix} 1 & 3 & 1 \\ 4 & 5 & 1 \\ 9 & 7 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 2 \begin{vmatrix} 1 & 3 & 1 \\ 3 & 2 & 0 \\ 8 & 4 & 0 \end{vmatrix} \quad \text{Applying } R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \Delta = 2 \times -4 = -8$$

172 (a)

We have,

$$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$$

$$= \begin{vmatrix} x & a & b \\ a-x & x-a & 0 \\ a-x & b-a & x-b \end{vmatrix}$$

$$[\text{Applying } R_2 \rightarrow R_2 - R_1]$$

$$[\text{Applying } R_3 \rightarrow R_3 - R_1]$$

$$= (x-a) \begin{vmatrix} x & a & b \\ -1 & 1 & 0 \\ a-x & b-a & x-b \end{vmatrix}$$

$$= (x-a) \begin{vmatrix} x+a+b & a & b \\ 0 & 1 & 0 \\ 0 & b-a & x-b \end{vmatrix}$$

$$[\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= (x-a)(x+a+b)(x-b) \quad [\text{Expanding along } C_1]$$

173 (c)

We have,

$$\Delta = \begin{vmatrix} [x] + 1 & [y] & [z] \\ [x] & [y] + 1 & [z] \\ [x] & [y] & [z] + 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [y] & [z] + 1 \end{vmatrix}$$

$$[\text{Applying } R_1 \rightarrow R_1 - R_3]$$

$$[\text{Applying } R_2 \rightarrow R_2 - R_3]$$

$$\Rightarrow \Delta = [z] + 1 + [y] + [x] = [x] + [y] + [z] + 1$$

Since maximum values of  $[x]$ ,  $[y]$  and  $[z]$  are 1, 0 and 2 respectively

$\therefore$  Maximum value of  $\Delta = 2 + 1 + 0 + 1 = 4$

174 (c)

We have,

$$\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a-6 & 0 & 0 \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0 \quad \text{Applying } R_1 \rightarrow R_1 - 2R_2$$

$$\Rightarrow (a-6)(b^2 - ac) = 0 \Rightarrow b^2 = ac \Rightarrow b^3 = abc$$

176 (d)

$$\text{We have, } \Delta \equiv \begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$$

$$\Rightarrow \Delta \equiv a(a^2 - 0) - b(0 - b^2) = a^3 + b^3$$

$$\Rightarrow a^3 + b^3 = 0 \Rightarrow \left(\frac{a}{b}\right)^3 = -1$$

$\therefore \left(\frac{a}{b}\right)$  is one of the cube roots of  $-1$ .

177 (b)

We have,

$$\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying  $C_1 \leftarrow C_1 + (C_2 + C_3)$  on LHS, we have

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\Rightarrow 2 \begin{vmatrix} a+b+c & c+a & a+b \\ a+b+c & b+c & c+a \\ a+b+c & a+c & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$  on LHS, we have

$$\Rightarrow 2 \begin{vmatrix} a+b+c & -b & -c \\ a+b+c & -a & -b \\ a+b+c & -c & -a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$  on LHS, we have

$$\Rightarrow \begin{vmatrix} a & -b & -c \\ c & -a & -b \\ b & -c & -a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\Rightarrow 2 \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\therefore k = 2$$

178 (b)

$$\text{Let } \Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a+b+c) + 2$$

$$\because \Delta > 0 \Rightarrow abc + 2 > a + b + c$$

$$\Rightarrow abc + 2 > 3(abc)^{1/3}$$

$$\left[ \because \text{AM} > \text{GM} \Rightarrow \frac{a+b+c}{3} > (abc)^{1/3} \right]$$

$$\Rightarrow x^3 + 2 > 3x, \text{ where } x = (abc)^{1/3}$$

$$\Rightarrow x^3 - 3x + 2 > 0 \Rightarrow (x-1)^2(x+2) > 0$$

$$\Rightarrow x + 2 > 0 \Rightarrow x > -2 \Rightarrow (abc)^{1/3} > -2$$

$$\Rightarrow abc > -8$$

179 (a)

Applying  $R_3 \rightarrow R_3 - R_1(\cos \beta) + R_2(\sin \beta)$

$$\begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ 0 & 0 & 1 + \sin \beta - \cos \beta \end{vmatrix}$$

$$= (1 + \sin \beta - \cos \beta)(\cos^2 \alpha + \sin^2 \alpha)$$

$$= 1 + \sin \beta - \cos \beta, \text{ which is independent of } \alpha$$

180 (d)

$$\text{Given, } A = B^{-1}AB$$

$$\Rightarrow BA = AB$$

$$\therefore \det(B^{-1}AB) = \det(B^{-1}BA) = \det(A)$$

181 (d)

Given, matrix is singular.

$$\text{Therefore, } \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{vmatrix} = 0$$

$$\Rightarrow +1(0-6) + \lambda(3) = 0$$

$$\Rightarrow -6 + 3\lambda = 0$$

$$\Rightarrow \lambda = 2$$

182 (a)

We have,

$$|A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 10 & 12 & 14 & 2y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow 2R_2]$$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 0 & 0 & 0 & 0 \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow R_2 - (R_1 + R_3)]$$

$$\Rightarrow |A| = 0 \quad [\because 2y = x + z]$$

183 (c)

Putting  $r = 1, 2, 3, \dots, n$  and using the formula

$$\sum 1 = n \text{ and } \sum r = \frac{(n+1)n}{2}$$

$$\sum (2r-1) = 1 + 3 + 5 + \dots = n^2$$

$$\therefore \sum_{r=1}^n \Delta_r = \begin{vmatrix} n & n & n \\ n(n+1) & n^2 + n + 1 & n^2 + n \\ n^2 & n^2 & n^2 + n + 1 \end{vmatrix} = 56$$

Applying  $C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & n \\ 0 & 1 & n^2 + n \\ -n-1 & -n-1 & n^2 + n + 1 \end{vmatrix}$$

$$\Rightarrow n(n+1) = 56$$

$$\Rightarrow n^2 + n - 56 = 0$$

$$\Rightarrow (n+8)(n-7) = 0$$

$$\Rightarrow n = 7 \quad (n \neq -8)$$

184 (a)

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} =$$

$$\begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2 - ab \end{vmatrix} \quad \begin{matrix} (R_1 \rightarrow R_1 - R_2) \\ (R_2 \rightarrow R_2 - R_3) \end{matrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2 - ab \end{vmatrix} = 0 \quad (\because$$

rows  $R_1$  and  $R_2$  are identical)

185 (c)

$$\because \det(M_r) = r^2 - (r-1)^2 = 2r - 1$$

$$\therefore \det(M_1) + \det(M_2) + \dots + \det(M_{2008})$$

$$= 1 + 3 + 5 + \dots + 4015$$

$$= \frac{2008}{2} [2 + (2008-1)2]$$

$$= 2008(2008) = (2008)^2$$

186 (b)

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ 1 + \omega + \omega^2 & \omega^2 & 1 \\ 1 + \omega + \omega^2 & 1 & \omega \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0)$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

$$= 0$$

187 (a)

$$\text{Given, } \Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

$$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$$

$$= 1(1 - 1) - 0 + \omega^{2n}(\omega^n - \omega^n) \quad [\because \omega^3 = 1]$$

$$= 0$$

188 (a)

$$\text{Given, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 0 & 0 \\ a & a-b & a-c \\ a^3 & a^3-b^3 & a^3-c^3 \end{vmatrix}$$

$[C_2 \rightarrow C_1 - C_2, C_3 \rightarrow C_1 - C_3]$

$$= (a-b)(a$$

$$-c) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & a^2+ab+b^2 & a^2+ac+c^2 \end{vmatrix}$$

$$= (a-b)(a-c)(c^2+ac-ab-b^2)$$

$$= (a-b)(b-c)(c-a)(a+b+c)$$

189 (c)

We have,

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2(b+c) & 2(c+a) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + 2R_2 + R_3$

$$\Rightarrow \Delta = 2 \begin{vmatrix} b+c & c+a & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = 2 \begin{vmatrix} b+c & c+a & a+b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1,$

$R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \Delta = 2\{(b+c)(0-a^2) - (c+a)(0-ab) + (a+b)(ac-0)\}$$

$$\Rightarrow \Delta = 2\{-a^2(b+c) + ab(c+a) + ac(a+b)\}$$

$$\Rightarrow \Delta = 2(-a^2b - a^2c + abc + a^2b + a^2c + abc)$$

$$\Rightarrow \Delta = 4abc$$

190 (d)

$$\begin{vmatrix} \log_5 729 & \log_3 5 \\ \log_5 27 & \log_9 25 \end{vmatrix} = \begin{vmatrix} \log_3 3^6 & \log_3 5 \\ \log_5 3^3 & \log_{3^2} 5^2 \end{vmatrix}$$

$$= \begin{vmatrix} 6 \log_5 3 & \log_3 5 \\ 3 \log_5 3 & \frac{2}{2} \log_3 5 \end{vmatrix}$$

$$= 6 \log_5 3 \log_3 5 - 3 \log_5 3 \log_3 5$$

$$= 6 - 3 = 3$$

$$\text{And } \begin{vmatrix} \log_3 5 & \log_{27} 5 \\ \log_5 9 & \log_5 9 \end{vmatrix} = \begin{vmatrix} \log_3 5 & \log_{3^3} 5 \\ \log_5 3^2 & \log_5 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} \log_3 5 & \frac{1}{3} \log_3 5 \\ 2 \log_5 3 & 2 \log_5 3 \end{vmatrix}$$

$$= 2 \log_5 3 \log_3 5 - \frac{2}{3} \log_5 3 \log_3 5$$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

$$\text{Now, } \begin{vmatrix} \log_5 729 & \log_3 5 \\ \log_5 27 & \log_9 25 \end{vmatrix} \begin{vmatrix} \log_3 5 & \log_{27} 5 \\ \log_5 9 & \log_5 9 \end{vmatrix} =$$

$$3 \cdot \frac{4}{3} = 4$$

Take option (d),

$$\log_3 5 \cdot \log_5 81 = \log_3 81 = \log_3 3^4 = 4$$

191 (c)

$$\text{Given, } \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow (a+b+c-x) \begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow (a+b+c-x) \begin{vmatrix} 1 & 0 & 0 \\ c & b-x-c & a-c \\ b & a-b & c-x-b \end{vmatrix} = 0$$

$$\Rightarrow (a+b+c-x)[1(b-x-c)(c-x-b) - (a-c)(a-b)] = 0$$

$$\Rightarrow (a+b+c-x)[bc - xb - b^2 - xc + x^2 + bx - c^2 + cx + bc$$

$$- (a^2 - ab - ac + bc)] = 0$$

$$\Rightarrow (a+b+c-x)[x^2 - a^2 - b^2 - c^2 + ab + bc + ca] = 0$$

$$\Rightarrow x = a+b+c \text{ or } x^2 = a^2 + b^2 + c^2 + ab + bc + ca$$

$$\Rightarrow x = 0 \text{ or } x^2 = a^2 + b^2 + c^2 + \frac{1}{2}(a^2 + b^2 + c^2)$$

$$\Rightarrow x = 0 \text{ or } x = \pm \sqrt{\frac{3}{2}(a^2 + b^2 + c^2)}$$

192 (d)

We have,

$$\Delta = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = abc \times 0$$

193 (a)

Applying  $R_1 \rightarrow R_1 + R_3$ , we get

$$\begin{vmatrix} 1-i & \omega^2 + \omega & \omega^2 - 1 \\ 1-i & -1 & \omega^2 - 1 \\ -i & -1 + \omega - i & -1 \end{vmatrix} = 0$$

$\therefore \omega^2 + \omega = -1$ , so  $R_1$  and  $R_2$  become identical]

194 (a)

$$\sum_{n=1}^N U_n = \begin{vmatrix} \sum n & 1 & 5 \\ \sum n^2 & 2N+1 & 2N+1 \\ \sum n^3 & 3N^2 & 3N \end{vmatrix}$$

$$= \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N \end{vmatrix}$$

$$= \frac{N(N+1)}{12} \begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$$

Applying  $C_3 \rightarrow C_3 + C_2$

$$= \frac{N(N+1)}{12} \begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix}$$

$= 0$  ( $\therefore$  two columns are identical)

195 (c)

$$\begin{vmatrix} 215 & 342 & 511 \\ 6 & 7 & 8 \\ 36 & 49 & 54 \end{vmatrix}$$

$$= 215(378 - 392) - 342(324 - 288) + 511(294 - 252)$$

$$= -3010 - 12312 + 21462 = 6140$$

Which is exactly divisible by 20

196 (a)

$$\det(A^{-1} \text{adj } A) = \det(A^{-1}) \det(\text{adj } A)$$

$$= (\det A)^{-1} (\det A)^{3-1} = \det A$$

197 (d)

$$A = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$$

$$= 1(1 + \sin^2 \theta) - \sin \theta(-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1)$$

$$= 2(1 + \sin^2 \theta)$$

Since, the maximum and minimum value of  $\sin^2 \theta$  is 1 and 0.

$$\therefore |A| \in [2, 4]$$

198 (d)

Since, the first column consists of sum of two terms, second column consists of sum of three terms and third column consists of sum four terms.

$$\therefore n = 2 \times 3 \times 4 = 24$$

199 (c)

Given,  $a_1, a_2, a_3, \dots \in \text{GP}$

$\Rightarrow \log a_1, \log a_2, \dots \in \text{AP}$

$\Rightarrow \log a_n, \log a_{n+1}, \log a_{n+2}, \dots \in \text{AP}$

$$\Rightarrow \log a_{n+1} = \frac{\log a_n + \log a_{n+2}}{2} \quad \dots(i)$$

$$\text{Similarly, } \log a_{n+4} = \frac{\log a_{n+3} + \log a_{n+5}}{2} \quad \dots(ii)$$

$$\text{and } \log a_{n+7} = \frac{\log a_{n+6} + \log a_{n+8}}{2} \quad \dots(iii)$$

$$\text{Given, } \Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - \frac{C_1 + C_3}{2}$

$$\Delta = \begin{vmatrix} \log a_n & 0 & \log a_{n+2} \\ \log a_{n+3} & 0 & \log a_{n+5} \\ \log a_{n+6} & 0 & \log a_{n+8} \end{vmatrix} = 0$$

200 (a)

$$\begin{vmatrix} 1 & \omega & -\omega^2/2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\frac{1}{2} \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= -\frac{1}{2} \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} \quad (C_1 \rightarrow C_1 + C_2 + C_3)$$

$$= -\frac{1}{2} \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0)$$

$$= 0$$

201 (a)

$$\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix}$$

$$= \begin{vmatrix} \log e & 2\log e & 3\log e \\ 2\log e & 3\log e & 4\log e \\ 3\log e & 4\log e & 5\log e \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

(Using  $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$ )

$$= 0 [\because \text{two columns are identical}]$$

202 (b)

$$\begin{vmatrix} \sqrt{13} + \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} + \sqrt{26} & 5 & \sqrt{10} \\ 3 + \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$$

$$= \begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{16} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

$$= \sqrt{13} \cdot \sqrt{5} \cdot \sqrt{5} \begin{vmatrix} 1 & 2 & 3 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$+ \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{5} \begin{vmatrix} 1 & 2 & 3 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 0 + 5\sqrt{3} \begin{vmatrix} -1 & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix} = 5\sqrt{3}(\sqrt{6} - 5)$$

204 (d)

We can write  $\Delta = \Delta_1 + y_1\Delta_2$ , where

$$\Delta_1 = \begin{vmatrix} 1 & 1 + x_1y_2 & 1 + x_1y_3 \\ 1 & 1 + x_2y_2 & 1 + x_2y_3 \\ 1 & 1 + x_3y_2 & 1 + x_3y_3 \end{vmatrix}$$

$$\text{and } \Delta_2 = \begin{vmatrix} x_1 & 1 + x_1y_2 & 1 + x_1y_3 \\ x_2 & 1 + x_2y_2 & 1 + x_2y_3 \\ x_3 & 1 + x_3y_2 & 1 + x_3y_3 \end{vmatrix}$$

In  $\Delta_1$ , use  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$  so that,

$$\Delta_1 = \begin{vmatrix} 1 & x_1y_2 & x_1y_3 \\ 1 & x_2y_2 & x_2y_3 \\ 1 & x_3y_2 & x_3y_3 \end{vmatrix} = 0 \quad (\because C_2 \text{ and } C_3 \text{ are}$$

proportional)

In  $\Delta_2$ ,  $C_2 \rightarrow C_2 - y_2C_1$  and  $C_3 \rightarrow C_3 - y_3C_1$  to get

$$\Delta_2 = \begin{vmatrix} x_1 & 1 & 1 \\ x_2 & 1 & 1 \\ x_3 & 1 & 1 \end{vmatrix} = 0 \quad (\because C_2 \text{ and } C_3 \text{ are identical})$$

$\therefore \Delta = 0$

206 (c)

$$\text{Let } \Delta = \begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$$

$$= (b - a)(b - a) \begin{vmatrix} b & b - c & c \\ a & a - b & b \\ c & c - a & a \end{vmatrix}$$

$$= (a - b)^2 \begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} \quad (C_2 \rightarrow C_2 + C_3)$$

$= 0$  ( $\because$  two columns are same)

208 (d)

$$\begin{vmatrix} x + 1 & \omega & \omega^2 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix}$$

$$= \begin{vmatrix} x + 1 + \omega + \omega^2 & \omega & \omega^2 \\ x + 1 + \omega + \omega^2 & x + \omega^2 & 1 \\ x + 1 + \omega + \omega^2 & 1 & x + \omega \end{vmatrix} \quad C_1 \rightarrow C_1 + C_2 + C_3$$

$$= x \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x + \omega^2 & 1 \\ 1 & 1 & x + \omega \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0)$$

$$= x[1\{(x + \omega^2)(x + \omega) - 1\} + \omega\{1 - (x + \omega)\} + \omega^2\{1 - (x + \omega^2)\}]$$

$$= x[(x^2 + \omega x + \omega^2 x + \omega^3 - 1 + \omega - \omega x - \omega^2 + \omega^2 - \omega^2 x - \omega^4)]$$

$$= x^3 \quad (\because \omega^3 = 1)$$

210 (b)

$$\text{Given, } f(x) = \begin{vmatrix} x & 1 + \sin x & \cos x \\ 1 & \log(1 + x) & 2 \\ x^2 & 1 + x^2 & 0 \end{vmatrix}$$

$$= x\{-2(1 + x^2)\} - (1 + \sin x)(-2x^2) + \cos x\{1 + x^2 - x^2 \log(1 + x)\}$$

$$= -2x - 2x^3 + 2x^2 + 2x^2 \sin x + \cos x\{1 + x^2 - x^2 \log(1 + x)\}$$

$\therefore$  Coefficient of  $x$  in  $f(x) = -2$ .

211 (c)

Clearly, the degree of the given determinant is 3.

So, there cannot be more than 3 linear factors.

Thus, the other factor is a numerical constant. Let it be  $\lambda$ . Then,

$$\begin{vmatrix} -2a & a + b & a + c \\ b + a & -2b & b + c \\ c + a & c + b & -2c \end{vmatrix} = \lambda(a + b)(b + c)(c + a)$$

Putting  $a = 0, b = 1$  and  $c = 1$  on both sides, we get

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -2 \end{vmatrix} = \lambda \times 1 \times 2 \times 1 \Rightarrow 2\lambda \Rightarrow \lambda = 4$$

212 (b)

We have,

$$\begin{vmatrix} 1 & \omega^2 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$$

$$= 2 - (\omega^2 - \omega) = 2 - (-1) = 3$$

213 (b)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$  and taking common  $(a + b + c)$  from  $C_1$ , we get

$$(a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$ , we get

$$(a + b + c) \begin{vmatrix} 1 & b & c \\ 0 & c - b & a - c \\ 0 & a - b & b - c \end{vmatrix}$$

$$= (a + b + c)\{-(c - b)^2 - (a - b)(a - c)\}$$

$$= -(a + b + c)\{a^2 + b^2 + c^2 - ab - bc - ca\}$$

$$= -\frac{1}{2}(a + b + c)\{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac\}$$

$$= -\frac{1}{2}(a + b + c)\{a - b\}^2 + (b - c)^2 + (c - a)^2\}$$

Which is always negative.

214 (c)

In a  $\Delta ABC$ , we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow b \sin A = a \sin B \quad c \sin A = a \sin C$$

$$\therefore \begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos A \\ c \sin A & \cos A & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & a \sin B & a \sin C \\ a \sin B & 1 & \cos A \\ a \sin C & \cos A & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos A \\ \sin C & \cos A & 1 \end{vmatrix} \quad \text{Taking a common fr } R_1 \text{ and } C_1 \text{ both}$$

$$\begin{aligned} &= a^2 \{ (1 - \cos^2 A) \\ &\quad - \sin B (\sin B - \cos A \sin C) \\ &\quad + \sin C (\sin B \cos A - \sin C) \} \\ &= a^2 \{ \sin^2 A - \sin^2 B \\ &\quad + 2 \sin B \sin C \cos A - \sin^2 C \} \\ &= a^2 \{ \sin(A+B) \sin(A-B) - \sin^2 C \\ &\quad + 2 \cos A \sin B \sin C \} \\ &= a^2 [ \sin C \{ \sin(A-B) - \sin C \} \\ &\quad + 2 \cos A \sin B \sin C ] \\ &= a^2 [ \sin C \{ \sin(A-B) - \sin(A+B) \} \\ &\quad + 2 \cos A \sin B \sin C ] \\ &= a^2 [ \sin C \times -2 \cos A \sin B + 2 \cos A \sin B \sin C ] \\ &= 0 \end{aligned}$$

215 (b)

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\left[ \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0 \right]$$

$$\Rightarrow 1+abc = 0$$

$$\Rightarrow abc = -1$$

216 (b)

$$\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} x & \omega & 1 \\ x & \omega^2 & 1+x \\ x & x+\omega & \omega^2 \end{vmatrix} = 0 \quad (\because 1+\omega+\omega^2=0)$$

$\Rightarrow x=0$  is one of the values of  $x$  which satisfy the above determinant equation.

217 (a)

We have,

$$|A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 0 & 0 & 0 & x-2y+z \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \quad \begin{array}{l} \text{Applying } R_1 \rightarrow R_1 \\ \quad -2R_2 + R_3 \end{array}$$

$$\Rightarrow |A| = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \quad \begin{array}{l} [\because x, y, z \text{ are in A.P.}] \\ [\because x-2y+z=0] \end{array}$$

$$\Rightarrow |A| = 0$$

218 (a)

$$\text{Given, } \Delta = \begin{vmatrix} (e^{i\alpha} + e^{-i\alpha})^2 & (e^{i\alpha} - e^{-i\alpha})^2 & 4 \\ (e^{i\beta} + e^{-i\beta})^2 & (e^{i\beta} - e^{-i\beta})^2 & 4 \\ (e^{i\gamma} + e^{-i\gamma})^2 & (e^{i\gamma} - e^{-i\gamma})^2 & 4 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_2$

$$= \begin{vmatrix} 4 & (e^{i\alpha} - e^{-i\alpha})^2 & 4 \\ 4 & (e^{i\beta} - e^{-i\beta})^2 & 4 \\ 4 & (e^{i\gamma} - e^{-i\gamma})^2 & 4 \end{vmatrix}$$

$$= 0 \quad (\because \text{two columns are same})$$

Hence, it is independent of  $\alpha, \beta$  and  $\gamma$ .

219 (b)

Let  $A$  be the first term and  $R$  be the common ratio of the GP. Then,

$$a = A R^{p-1} \Rightarrow \log a = \log A + (p-1) \log R$$

$$b = A R^{q-1} \Rightarrow \log b = \log A + (q-1) \log R$$

$$c = A R^{r-1} \Rightarrow \log c = \log A + (r-1) \log R$$

Now,

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (p-1) \log R & p & 1 \\ (q-1) \log R & q & 1 \\ (r-1) \log R & r & 1 \end{vmatrix}$$

$$= \log R = \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 -$$

$(\log A) C_3]$

$$= \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0 \quad [\text{Applying } C_1 \rightarrow C_1 - C_2 +$$

$C_3]$

220 (c)

We know that the sum of the products of the elements of a row with the cofactors of the corresponding elements is always equal to the value of the determinant .ie,  $|A|$ .

221 (d)

$\therefore a, b, c, d, e$  and  $f$  are in GP.

$\therefore a = a, b = ar, c = ar^2, d = ar^3, e = ar^4$  and  $f = ar^5$

$$\begin{aligned} \therefore \begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix} &= \begin{vmatrix} a^2 & a^2 r^6 & x \\ a^2 r^2 & a^2 r^8 & y \\ a^2 r^4 & a^2 r^{10} & z \end{vmatrix} \\ &= a^4 r^6 \begin{vmatrix} 1 & 1 & x \\ r^2 & r^2 & y \\ r^4 & r^4 & z \end{vmatrix} = 0 \end{aligned}$$

Thus, the given determinant is independent of  $x, y$  and  $z$ .

222 (a)

$$\begin{aligned} &\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} \\ &= 1(1 - \log_y z \log_z y) \\ &\quad - \log_x y (\log_y x - \log_z x \log_y z) \\ &\quad + \log_x z (\log_z y \log_y x - \log_z x) \\ &= (1 - \log_y z) - \log_x y (\log_y x - \log_z x) \\ &\quad + \log_x z (\log_z y - \log_z x) \\ &= (1 - 1) - 0 + 0 = 0 \end{aligned}$$

223 (d)

$$\begin{aligned} &\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -x & 0 \\ 1 & 0 & y \end{vmatrix} \begin{matrix} [C_2 \rightarrow C_2 - C_1] \\ [C_3 \rightarrow C_3 - C_1] \end{matrix} \\ &= -xy \end{aligned}$$

224 (c)

$$\begin{aligned} &\begin{vmatrix} x & y & z \\ -x & y & z \\ x & -y & z \end{vmatrix} = \begin{vmatrix} x & y & z \\ -x & y & z \\ 0 & 0 & 2z \end{vmatrix} \quad [R_3 \rightarrow R_3 + R_2] \\ &= 2z(xy + xy) = 4xyz \end{aligned}$$

On comparing with  $kxyz$ , we get  $k = 4$

225 (b)

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$  and taking common  $(2x + 10)$  from  $R_1$ , we get

$$\begin{aligned} &(2x + 10) \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0 \\ &\Rightarrow (2x + 10) \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2x - 2 & 0 \\ 7 & -1 & 2x - 7 \end{vmatrix} = 0 \\ &[C_3 \rightarrow C_3 - C_1 \text{ and } C_2 \rightarrow C_2 - C_1] \\ &\Rightarrow (2x + 10)(2x - 2)(2x - 7) = 0 \\ &\Rightarrow x = -5, 1, \frac{7}{2} \end{aligned}$$

Hence, other roots are 1 and  $\frac{7}{2}$  or 1 and 3.5

226 (b)

Let  $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y$  and  $\frac{z^2}{c^2} = Z$

Then the given system of equations becomes

$$X + Y - Z = 1, X - Y + Z = 1, -X + Y + Z = 1$$

$$\text{The coefficient matrix is } A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Clearly,  $|A| \neq 0$ . So, the given system of equations has a unique solution

227 (c)

Applying  $R_1 \rightarrow R_1 + R_2$ , we get

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 \theta & 1 + \cos^2 \theta & \cos^2 \theta \\ 4 \sin 4 \theta & 4 \sin 4 \theta & 1 + 4 \sin 4 \theta \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 - 2C_3, C_2 \rightarrow C_2 - 2C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -\cos^2 \theta & 1 - \cos^2 \theta & \cos^2 \theta \\ -2 - 4 \sin 4 \theta & -2 - 4 \sin 4 \theta & 1 + 4 \sin 4 \theta \end{vmatrix} = 0$$

$$\Rightarrow [\cos^2 \theta (2 + 4 \sin 4 \theta) + (1 - \cos^2 \theta)(2 + 4 \sin 4 \theta)] = 0$$

$$\Rightarrow [2 \cos^2 \theta + 4 \cos^2 \theta \sin 4 \theta + 2 + 4 \sin 4 \theta - 2 \cos^2 \theta - 4 \cos^2 \theta \sin 4 \theta] = 0$$

$$\Rightarrow 2 + 4 \sin 4 \theta = 0$$

$$\Rightarrow \sin 4 \theta = -\frac{1}{2}$$

228 (a)

$$\text{Given determinant, } \Delta \equiv \begin{vmatrix} a & a^2 & a^3 + 1 \\ b & b^2 & b^3 + 1 \\ c & c^2 & c^3 + 1 \end{vmatrix} = 0$$

On splitting the determinant into two determinants, we get

$$\Delta \equiv abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + abc)[1(bc^2 - cb^2) - a(c^2 - b^2) + a^2(c - b)] = 0$$

$$\Rightarrow (1 + abc)[(a - b)(b - c)(c - a)] = 0$$

Since  $a, b, c$  are different, the second factor cannot be zero.

Hence,  $1 + abc = 0$

229 (b)

We have,

$$\begin{vmatrix} a & a^2 - bc & 1 \\ b & b^2 - ca & 1 \\ c & c^2 - ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & -bc & 1 \\ b & -ca & 1 \\ c & -ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$+ \frac{1}{abc} \begin{vmatrix} a^2 & -abc & a \\ b^2 & -abc & b \\ c^2 & -abc & c \end{vmatrix} \text{ Applying } R_1 \rightarrow R_1(a) \\ R_2 \rightarrow R_2(b), R_3 \rightarrow R_3(c) \text{ in the IInd determinant}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

230 (d)

Given that,  $x^a y^b = e^m, x^c y^d = e^n$

$$\text{and } \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Rightarrow a \log x + b \log y = m$$

$$\Rightarrow c \log x + d \log y = n$$

By Cramer's rule

$$\log x = \frac{\Delta_1}{\Delta_3} \text{ and } \log y = \frac{\Delta_2}{\Delta_3}$$

$$\Rightarrow x = e^{\Delta_1/\Delta_3} \text{ and } y = e^{\Delta_2/\Delta_3}$$

231 (d)

Clearly,  $x = 0$  satisfies the given equation

232 (c)

$$\text{Let } \Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$$

$$= 10! 11! 12! \begin{vmatrix} 1 & 11 & 11 \times 12 \\ 1 & 12 & 12 \times 13 \\ 1 & 13 & 13 \times 14 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$= 10! 11! 12! \begin{vmatrix} 1 & 11 & 11 \times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$$

$$= (10! 11! 12!)(50 - 48)$$

$$= 2 \cdot (10! 11! 12!)$$

233 (c)

$$\text{We have, } \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} \sin x + 2 \cos x & \cos x & \cos x \\ \sin x + 2 \cos x & \sin x & \cos x \\ \sin x + 2 \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\Rightarrow (2 \cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (2 \cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$$\Rightarrow (2 \cos x + \sin x)(\sin x - \cos x)^2 = 0$$

$\therefore \tan x = -2, 1$  But  $\tan x \neq -2$ , because it does not lie in the interval  $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ .

$$\therefore \tan x = 1$$

$$\text{So, } x = \frac{\pi}{4}$$

234 (a)

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_2$

$$= \begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^x - b^{-x})^2 & 1 \\ 4 & (c^x - c^{-x})^2 & 1 \end{vmatrix} =$$

$$4 \begin{vmatrix} 1 & (a^x - a^{-x})^2 & 1 \\ 1 & (b^x - b^{-x})^2 & 1 \\ 1 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 0 \text{ (}\therefore \text{two columns are}$$

identical)

235 (c)

Given matrix is non-singular, then

$$\begin{vmatrix} \lambda & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & \lambda \end{vmatrix} \neq 0$$

$$\Rightarrow \lambda(2\lambda - 0) \neq 0$$

$$\Rightarrow \lambda \neq 0$$

236 (d)

$$\text{Let } \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - (R_1 - 2R_2)$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore k = 4$$

237 (c)

$$\text{Let } f(x) = a_0 x^2 + a_1 x + a_2$$

$$\text{and } g(x) = b_2 x^2 + b_1 x + b_2$$

$$\text{Also, } h(x) = c_0 x^2 + c_1 x + c_2$$

$$\text{Then, } \Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 x + a_1 & 2b_0 x + b_1 & 2c_0 x + c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$$= x \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$$= 0 + 2 \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ a_0 & b_0 & c_0 \end{vmatrix}$$

$$= 2[(b_1 c_0 - b_0 c_1)f(x) - (a_1 c_0 - a_0 c_1)g(x) + (a_1 b_0 - a_0 b_1)h(x)]$$

Hence, degree of  $\Delta(x) \leq 2$

238 (d)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ , we get

$$\begin{vmatrix} 2(x+y+z) & y+z & z+x \\ x+y+z & y & z \\ 0 & y-z & z-x \end{vmatrix}$$

$$= (x + y + z) \begin{vmatrix} 2 & y - z & z + x \\ 1 & y & z \\ 0 & y - z & z - x \end{vmatrix}$$

Applying  $R_2 \rightarrow 2R_2 - R_1$

$$= (x + y + z) \begin{vmatrix} 2 & y + z & z + x \\ 0 & y - z & z - x \\ 0 & y - z & z - x \end{vmatrix}$$

$$= 0 \quad [\because \text{two rows are identical}]$$

239 (d)

$$\text{Given, } f(x) = \begin{vmatrix} 1 + a & 1 + ax & 1 + ax^2 \\ 1 + b & 1 + bx & 1 + bx^2 \\ 1 + b & 1 + cx & 1 + cx^2 \end{vmatrix}$$

$$\Rightarrow f(x) = \begin{vmatrix} 1 + a & a(x - 1) & ax(x - 1) \\ 1 + b & b(x - 1) & bx(x - 1) \\ 1 + b & c(x - 1) & cx(x - 1) \end{vmatrix}$$

$$= (x - 1)x(x - 1) \begin{vmatrix} 1 + a & a & a \\ 1 + b & b & b \\ 1 + c & c & c \end{vmatrix} = 0$$

( $\because$  two columns are same)

240 (c)

We have,

$$ax^4 + bx^3 + cx^2 + 50x + d$$

$$= \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix}$$

On differentiating with respect to  $x$ , we get

$$4ax^3 + 3bx^2 + 2cx + 50$$

$$= \begin{vmatrix} 3x^2 - 28x & -1 & 3 \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

Now, put  $x = 0$ , we get

$$50 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

$$\Rightarrow 50 = 25\lambda$$

$$\Rightarrow \lambda = 2$$

241 (d)

$$\text{We have, } \begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} =$$

$$Ax - 12$$

On putting  $x = 1$  on both sides, we get

$$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = A - 12$$

Applying  $C_1 \rightarrow C_1 - C_2$

$$\Rightarrow \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 5 & 1 & 1 \end{vmatrix} = A - 12$$

$$\Rightarrow -2(1) + (-1)(-14) = A - 12$$

$$\Rightarrow A = 24$$

242 (a)

$$\text{We have, } \begin{vmatrix} x + \alpha & \beta & \gamma \\ \gamma & x + \beta & \alpha \\ \alpha & \beta & x + \gamma \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x + \alpha + \beta + \gamma & \beta & \gamma \\ x + \alpha + \beta + \gamma & x + \beta & \alpha \\ x + \alpha + \beta + \gamma & \beta & x + \gamma \end{vmatrix} = 0$$

$$\Rightarrow (x + \alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta & \gamma \\ 1 & x + \beta & \alpha \\ 1 & \beta & x + \gamma \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (x + \alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha - \gamma \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x + \alpha + \beta + \gamma)(x^2 - 0) = 0$$

$$\Rightarrow x = 0 \text{ or } x = -(\alpha + \beta + \gamma)$$

243 (b)

We have,

$$\Delta = \begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix}$$

$$\Rightarrow \Delta$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & a & abc \\ 1 & b & abc \\ 1 & c & abc \end{vmatrix} \quad \begin{array}{l} \text{Applying } R_1 \rightarrow R_1(a), \\ R_2 \rightarrow R_2(b) \text{ and } R_3 \rightarrow R_3(c) \end{array}$$

$$\Rightarrow \Delta = \frac{abc}{abc} \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} \quad [\text{Taking } abc \text{ common from } C_3]$$

$$\Rightarrow \Delta = \frac{abc}{abc} \times 0 = 0$$

244 (b)

We have,  $|A| \neq 0$ . Therefore,  $A^{-1}$  exists

Now,  $AB = AC$

$$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$$

$$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow B = C$$

246 (c)

Applying  $C_3 \rightarrow C_3 - \omega C_1$ , we get

$$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix} = \begin{vmatrix} a & b\omega^2 & 0 \\ b\omega & c & 0 \\ c\omega^2 & a\omega & 0 \end{vmatrix} = 0$$

247 (d)

$$\begin{vmatrix} a + b & a + 2b & a + 3b \\ a + 2b & a + 3b & a + 4b \\ a + 4b & a + 5b & a + 6b \end{vmatrix}$$

$$= \begin{vmatrix} a + b & a + 2b & a + 3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix} \quad \begin{array}{l} (R_2 \rightarrow R_2 - R_1) \\ (R_3 \rightarrow R_3 - R_2) \end{array}$$

$$= 0 \quad (\because R_2 \text{ and } R_3 \text{ are proportional})$$

248 (c)

Applying  $R_1 \rightarrow R_1 - (R_2 + R_3)$ , we get

$$\begin{vmatrix} 0 & -2z & -2y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$$= 2z(xy + y^2 - yz) - 2y(yz - z^2 - xz)$$

$$= 2xyz + 2y^2z - 2yz^2 - 2y^2z + 2yz^2 + 2xyz$$

$$= 4xyz$$

249 (b)

We have,

$$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{d}{dx}(\Delta_1) = \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2$$

251 (d)

Applying  $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_2$ , we get

$$\begin{vmatrix} 1990 & 1 & 1 \\ 1991 & 1 & 1 \\ 1992 & 1 & 1 \end{vmatrix} = 0$$

253 (b)

We have,

$$\Delta = \begin{vmatrix} 1 + a_1b_1 + a_1^2b_1^2 & 1 + a_1b_2 + a_1^2b_2^2 & 1 + a_1b_3 \\ 1 + a_2b_1 + a_2^2b_1^2 & 1 + a_2b_2 + a_2^2b_2^2 & 1 + a_2b_3 \\ 1 + a_3b_1 + a_3^2b_1^2 & 1 + a_3b_2 + a_3^2b_2^2 & 1 + a_2b_3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$$

$$\Rightarrow \Delta = (a_1 - a_2)(a_2 - a_3)(a_3 - a_1)(b_1 - b_2)(b_2 - b_3)(b_3 - b_1)$$

254 (b)

$$\text{Let } A \equiv \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5 \dots (i)$$

$$\therefore \begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & c_2b_3 - c_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$$

$$|\text{adj } A| = (5)^{3-1} \text{ [from Eq. (i)]}$$

$$= 5^2 = 25 \quad (\because |\text{adj } A| = |A|^{n-1})$$

255 (b)

Let  $a \neq 0$ . Then,

$$\Delta = \frac{1}{a} \begin{vmatrix} a^3 + ax^2 & ab & ac \\ a^2b & b^2 + x^2 & bc \\ a^2c & bc & c^2 + x^2 \end{vmatrix} \text{ [Applying}$$

$$C_1 \rightarrow aC_1]$$

$\Rightarrow \Delta$

$$= \frac{1}{a} \begin{vmatrix} a(a^2 + b^2 + c^2 + x^2) & ab & ac \\ b(a^2 + b^2 + c^2 + x^2) & b^2 + x^2 & bc \\ c(a^2 + b^2 + c^2 + x^2) & bc & c^2 + x^2 \end{vmatrix}$$

$$\text{[Applying } C_1 \rightarrow C_1 + bC_2 + cC_3]$$

$$\Rightarrow \Delta = \frac{1}{a} (a^2 + b^2 + c^2$$

$$+ x^2) \begin{vmatrix} a & ab & ac \\ b & b^2 + x^2 & bc \\ c & bc & c^2 + x^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{a} (a^2 + b^2 + c^2 + x^2) \begin{vmatrix} a & 0 & 0 \\ b & x^2 & 0 \\ c & 0 & x^2 \end{vmatrix},$$

$$\text{[Applying } C_2 \rightarrow C_2 - bC_1, C_3 \rightarrow C_3 - cC_1]$$

$$\Rightarrow \Delta = (a^2 + b^2 + c^2 + x^2)x^4$$

Clearly,  $\Delta$  is divisible by  $x^4$

If  $a = 0$ , then also it can be easily seen that  $\Delta$  is divisible by  $x^4$

256 (a)

We have,

$$\Delta_a = \begin{vmatrix} a-1 & 2 & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 2n^2-3n \end{vmatrix}$$

$$\therefore \sum_{a=1}^n \Delta_a = \begin{vmatrix} \sum_{a=1}^n (a-1) & n & 6 \\ \sum_{a=1}^n (a-1)^2 & 2n^2 & 4n-2 \\ \sum_{a=1}^n (a-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$$

$$\Rightarrow \sum_{a=1}^n \Delta_a$$

$$= \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4n-2 \\ \left(\frac{n(n-1)}{2}\right)^2 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$\Rightarrow \sum_{a=1}^n \Delta_a$$

$$= \frac{n(n-1)}{12} \begin{vmatrix} 6 & n & 6 \\ 4n-2 & 2n^2 & 4n-2 \\ 3n^2-3n & 3n^3 & 3n^2-3n \end{vmatrix} = 0$$

257 (d)

$$B = 5A^2$$

$$\Rightarrow \det(B) = \det(5A^2) = 5^3[\det(A)]^2$$

$$= 125(6)^2 = 4500 \text{ [given } \det A = 6]$$

258 (b)

$$\text{Given, } f(x) = \begin{vmatrix} x & 1 + \sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$$

$$= x\{-2(1+x^2)\} - (1+\sin x)(-2x^2)$$

$$+ \cos x\{1+x^2-x^2\log(1+x)\}$$

$$= -2x - 2x^3 + 2x^2 + 2x^2 \sin x$$

$$+ \cos x\{1+x^2-x^2\log(1+x)\}$$

∴ Coefficient of  $x$  in  $f(x) = -2$

259 (c)

$$\begin{aligned} \Delta &= \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & a(b-c) \\ b+c & (a-b) & (b-c) \end{vmatrix} \begin{matrix} [C_2 \rightarrow C_2 - C_1] \\ [C_3 \rightarrow C_3 - C_2] \end{matrix} \\ &= (a-b)(b-c) \begin{vmatrix} 1 & 0 & 0 \\ bc & c & a \\ b+c & 1 & 1 \end{vmatrix} \\ &= (a-b)(b-c)(c-a) \end{aligned}$$

260 (b)

Since,  $\Delta(1) = 0$  and  $\Delta'(1) = 0$  so,  $(x-1)^2$  is a factor of  $\Delta(x)$

261 (d)

On putting  $\lambda = 0$ , we get

$$t = \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} = 4a^2b^2c^2$$

Clearly, it depends on  $a, b, c$ .

262 (c)

$$\begin{aligned} \text{Let } \Delta &= \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} \\ &= (10!)(11!)(12!) \begin{vmatrix} 1 & 11 & 11 \times 12 \\ 1 & 12 & 12 \times 13 \\ 1 & 13 & 13 \times 14 \end{vmatrix} \\ &= (10!)(11!)(12!) \begin{vmatrix} 1 & 11 & 11 \times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix} \\ &= 2(10!)(11!)(12!) \end{aligned}$$

263 (b)

$$\therefore \det(A_1) = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2$$

$$\det(A_2) = \begin{vmatrix} a^2 & b^2 \\ b^2 & a^2 \end{vmatrix} = a^4 - b^4$$

$$\begin{aligned} \therefore \sum_{i=1}^{\infty} \det(A_i) &= \det(A_1) + \det(A_2) + \dots \\ &= a^2 - b^2 + a^4 - b^4 + \dots \\ &= \frac{a^2}{1-a^2} - \frac{b^2}{1-b^2} = \frac{a^2 - b^2}{(1-a^2)(1-b^2)} \end{aligned}$$

264 (c)

Since,  $A$  is a singular matrix

$$\therefore |A| = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow 1(6-28) - 2(-24-14) + x[16+2] &= 0 \\ \Rightarrow -22 + 76 + 18x = 0 &\Rightarrow x = -3 \end{aligned}$$

265 (b)

$$\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = \begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ x+p+q & q & x \end{vmatrix}$$

$$\begin{aligned} &= (x+p+q) \begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix} \\ &= (x+p+q) \begin{vmatrix} 1 & p & q \\ 0 & x-p & 0 \\ 0 & q-p & x-q \end{vmatrix} \\ &= (x+p+q) \begin{vmatrix} x-p & 0 \\ q-p & x-q \end{vmatrix} \\ &= (x-p)(x-q)(x+p+q) \end{aligned}$$

266 (b)

$$\text{We have, } \begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow \begin{vmatrix} 0 & 6 & 15 \\ 0 & -2-2x & 5(1-x^2) \\ 1 & 2x & 5x^2 \end{vmatrix} &= 0 \\ &= 0 \begin{matrix} (R_1 \rightarrow R_1 - R_2) \\ \text{and } R_2 \rightarrow R_2 - R_3 \end{matrix} \end{aligned}$$

$$\Rightarrow 3 \cdot 2 \cdot 5 \begin{vmatrix} 0 & 1 & 1 \\ 0 & -(1+x) & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$$

(Taking common, 3 from  $R_1$ , 2 from  $C_2$ , 5 from  $C_3$ )

$$\Rightarrow (1+x) \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1-x \\ 1 & x & x^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+x)(2-x) = 0$$

$$\Rightarrow x+1=0 \text{ or } x-2=0 \Rightarrow x = -1, 2$$

267 (d)

$$x + iy = -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$$

$$\Rightarrow x = 0, \quad y = 0$$

268 (a)

We have,

$$\Delta = \begin{vmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) & \cos 2\beta \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\sin \alpha & \sin \alpha & \cos \beta \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & \cos 2\beta + 1 \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\cos \alpha & \sin \alpha & \cos \beta \end{vmatrix}$$

$$\left[ \text{Applying } R_1 \rightarrow R_1 + R_2 \right]$$

$$\left[ \begin{matrix} \sin \beta + R_3 \cos \beta \end{matrix} \right]$$

$$\Rightarrow \Delta = (\cos 2\beta + 1)(\sin^2 \alpha + \cos^2 \alpha)$$

$$= \cos 2\beta + 1,$$

Which is independent of  $\alpha$

269 (d)

$$\text{Given } \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow +R_1 + R_3 - 2R_2$ , we get

$$\begin{vmatrix} 0 & 0 & a+c-2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$$\Rightarrow (a+c-2b)[x^2 + 6x + 8 - (x^2 + 6x + 9)] = 0$$

$$\Rightarrow (a + c - 2b)(-1) = 0$$

$$\Rightarrow 2b = a + c$$

$\Rightarrow a, b, c$  are in AP

270 (a)

We have,

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$$= abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix} \quad \begin{array}{l} \text{Applying } R_1 \rightarrow R_1 \\ R_2 \rightarrow R_2 \left(\frac{1}{b}\right), R_3 \rightarrow R_3 \left(\frac{1}{c}\right) \end{array}$$

$$= abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} \frac{1}{b} & 1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b}\right)$$

$$+ \frac{1}{c} \begin{vmatrix} \frac{1}{b} & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix} \quad \left[ \begin{array}{l} \text{Applying } C_2 \rightarrow C_2 - C_1 \\ C_3 \rightarrow C_3 - C_1 \end{array} \right]$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

271 (a)

On putting  $x = 0$ , we observe that the determinant becomes zero.

$$\therefore \Delta = \begin{vmatrix} 0-a-b & \\ a & 0-c \\ b & c & 0 \end{vmatrix}$$

$$= a(bc) - b(ac) = 0$$

Hence,  $x = 0$  is a root of the given equation

272 (a)

$$\sum_{r=0}^n D_r = \begin{vmatrix} \sum_{r=0}^n 1 & \frac{n(n+1)}{2} \\ \sum_{r=0}^n r & 4 & n^2 \\ \sum_{r=0}^n 2^{r-1} & 5 & 2^n - 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & 1 & \frac{n(n+1)}{2} \\ n^2 & 4 & n^2 \\ 2^n - 1 & 5 & 2^n - 1 \end{vmatrix} = 0$$

[ $\because$  two columns are identical]

273 (b)

$$\text{Given, } f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$$

$$= 1(\alpha^3 - 1) - \alpha(\alpha^2 - \alpha^2) + \alpha^2(\alpha - \alpha^4)$$

$$= \alpha^3 - 1 - 0 + \alpha^3 - \alpha^6$$

$$\Rightarrow f(\sqrt[3]{3}) = 3 - 1 - 0 + 3 - 3^2 = 6 - 10 = -4$$

274 (c)

We have,

$$\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix}$$

This can be written as

$$\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \begin{vmatrix} b_1 & a_1 & 0 \\ b_2 & a_2 & 0 \\ b_3 & a_3 & 0 \end{vmatrix} = 0$$

275 (a)

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$= 3 \begin{vmatrix} 2 & 1 \\ 2 & 6 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ 3 & 6 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$

$$= 3(12 - 2) - 2(6 - 3) + 4(2 - 6)$$

$$= 30 - 6 - 16 = 8$$

276 (c)

We have,

$$\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3x - (a+b+c) & x-b & x-c \\ 3x - (a+b+c) & x-c & x-a \\ 3x - (a+b+c) & x-a & x-b \end{vmatrix} = 0$$

[Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \{3x - (a+b+c)\} \begin{vmatrix} 1 & x-b & x-c \\ 1 & x-c & x-a \\ 1 & x-a & x-b \end{vmatrix} = 0$$

$$\Rightarrow \{3x - (a+b+c)\} \begin{vmatrix} 1 & x-b & x-c \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} = 0$$

$$\Rightarrow \{3x - (a+b+c)\}(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$\Rightarrow x = \frac{1}{3}(a+b+c) \quad [$$

$$\because a^3 + b^3 + c^3 - ab - bc - ca \neq 0]$$

277 (b)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ , we obtain

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$$

$$= \begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ c+p+q & q & x \end{vmatrix}$$

$$= (x+p+q) \begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$$

$$= (x + p + q) \begin{vmatrix} 1 & p & q \\ 0 & x - p & 0 \\ 0 & q - p & x - q \end{vmatrix}$$

$$\left[ \begin{array}{l} \text{Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1 \end{array} \right]$$

$$= (x + p + q)(x - p)(x - q) \quad [\text{Expanding along } C_1]$$

278 (a)

Let

$$\begin{vmatrix} 1 & 1 & 1 \\ 2x & (x-1) & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$$

$f(x) =$

$$= (x - 1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x - 1 & x \\ 3x & x - 2 & x \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$

$$= (x - 1) \begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x & -2 & x \end{vmatrix}$$

$$= (x - 1)[-2x + 2x] = 0$$

$$\therefore f(x) = 0$$

$$\Rightarrow f(50) = 0$$