MATHS (QUESTION BANK)

7.INTEGRALS

Single Correct Answer Type

1. The value of $\int_0^{\pi/2} \frac{1}{9\cos x + 12\sin x} dx$, is
a) $\frac{1}{15}\log_{10} 6$ b) $\frac{1}{15}\log_e 6$ c) $\log\left(\frac{6}{15}\right)$ d) $\log\left(\frac{15}{6}\right)$
2. The value of $\int_{-2}^{0}[x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)]\, dx$ is
a) 0 b) 3 c) 4 d) 1
3. $\int \frac{e^{2x} - 2e^x}{e^{2x}+1} dx$ is equal to
a) $\log(e^{2x}+1) - \tan^{-1}(e^x) + C$
b) $\frac{1}{2}\log(e^{2x}+1) - \tan^{-1}(e^x) + C$
c) $\frac{1}{2}\log(e^{2x}+1) - 2\tan^{-1}(e^x) + C$
d) None of these
4. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is
a) 1 b) 0 c) -1 d) None of these
5. $\int \sqrt{1+\sin\left(\frac{x}{4}\right)}\, dx$ is equal to
a) $8\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right) + c$ b) $8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$
c) $8\left(\cos\frac{x}{8} - \sin\frac{x}{8}\right) + c$ d) $\frac{1}{8}\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$
6. The value of the integral $\int_a^b \frac{\sqrt{x}\, dx}{\sqrt{x}+\sqrt{a+b-x.}}$ is
a) π b) $\frac{1}{2}(b-a)$ c) $\frac{\pi}{2}$ d) $b-a$
7. The value of the integral $\int_0^3 \frac{dx}{\sqrt{x+1}+\sqrt{5x+1}}$, is
a) $\frac{11}{15}$ b) $\frac{14}{15}$ c) $\frac{2}{5}$ d) None of these
8. The value of $\int_0^{12a} \frac{f(x)}{f(x)+f(12a-x)} dx$ is
a) a b) $2a$ c) $3a$ d) $6a$
9. The value of
$\int_{-2}^{2}\left[p\log\left(\frac{1+x}{1-x}\right) + q\log\left(\frac{1-x}{1+x}\right)^{-2} + r\right] dx$ depends on
a) The value of p b) The value of q
c) The value of r d) The value of p and q
10. The antiderivative of $\frac{3^x}{\sqrt{1-9^x}}$ with respect to x is

a) $(\log_3 e)\sin^{-1}(3^x) + C$
b) $\sin^{-1}(3^x) + C$
c) $(\log_3 e)\cos^{-1}(3^x)$
d) None of these

11. If $\int \frac{dx}{x\log x} = f(x) +$ constant, then $f(x)$ is equal to

a) $1/\log x$ b) $\log x$ c) $\log\log x$ d) $x/\log x$

12. The solution of the equation $\int_{\log 2}^{x} \frac{1}{e^x - 1} dx = \log\frac{3}{2}$ is given by $x =$

a) e^2 b) $1/e$ c) $\log 4$ d) None of these

13. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{10}\int_0^1 (r-1+x)dx$ is equal to

a) $\int_0^{10} f(x)dx$ b) $\int_0^1 f(x)dx$ c) $10\int_0^1 f(x)dx$ d) $9\int_0^1 f(x)dx$

14. The value of the integral $\int_{-1}^{1}[x^2 + \{x\}]\,dx$, where $[\cdot]$ and $\{\cdot\}$ denote respectively the greatest integer function and fractional part function, is equal to

a) $\frac{5+\sqrt{5}}{2}$ b) $\frac{5-\sqrt{5}}{2}$ c) $-\frac{5+\sqrt{5}}{2}$ d) None of these

15. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = (fofo\ldots of)(x)$. $g(x) = \underbrace{(fofo....of)}_{n\text{ times}}(x)$. Then, $\int x^{n-2}\,g(x)dx$ equals

a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + c$
b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + c$
c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + c$
d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + c$

16. Let $I_n = \int_0^{\pi/2}\sin^n x\,dx, n \in N$. Then

a) $I_n : I_{n-2} = n : (n-1)$ b) $I_n > I_{n-2}$ c) $n(I_{n-2} - I_n) = I_{n-2}$ d) None of these

17. The value of $\int_0^3 x\sqrt{1+x}\,dx$, is

a) $\frac{9}{2}$ b) $\frac{27}{4}$ c) $\frac{126}{15}$ d) None of these

18. $\int_0^1 \log\left\{\sin\left(\frac{\pi x}{2}\right)\right\}\,dx$ is equal to

a) $-\frac{\pi}{2}\log 2$ b) $-\log 2$ c) $-\frac{2}{\pi}\log 2$ d) $\frac{\pi}{2}\log 2$

19. $\int \frac{dx}{x(x^7+1)}$ is equal to

a) $\log\left(\frac{x^7}{x^7+1}\right) + c$ b) $\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + c$ c) $\log\left(\frac{x^7+1}{x^7}\right) + c$ d) $\frac{1}{7}\log\left(\frac{x^7+1}{x^7}\right) + c$

20. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to

a) $\frac{1}{8}(x^2-1) + c$ b) $\frac{x^2}{4} + c$ c) $\frac{x}{2} + c$ d) $\frac{x^2}{2} + c$

21. The value of $\int \frac{x^2+1}{x^2-1}\,dx$ is

a) $\log\left(\frac{x-1}{x+1}\right) + c$ b) $\log\left(\frac{x+1}{x-1}\right) + c$
c) $x + \log\left(\frac{x-1}{x+1}\right) + c$ d) $\log\,(x^2-1) + c$

22. $\int e^{\tan^{-1}x}\left|\frac{1+x+x^2}{1+x^2}\right|dx$ is equal to

a) $xe^{\tan^{-1}x}+c$ b) $x^2e^{\tan^{-1}x}+c$ c) $\frac{1}{x}e^{\tan^{-1}x}+c$ d) None of these

23. $\int\frac{dx}{x+\sqrt{x}}$ is equal to

a) $\frac{1}{2}\log(1+\sqrt{x})+c$ b) $2\log(1+\sqrt{x})+c$ c) $\frac{1}{4}\log(1+\sqrt{x})+c$ d) $3\log(1+\sqrt{x})+c$

24. $\int\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$ is equal to

a) $2\operatorname{cosec}\alpha\sqrt{\cos\alpha+\sin\alpha\tan x}+C$

b) $-2\operatorname{cosec}\alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$

c) $\operatorname{cosec}\alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$

d) None of these

25. $\int_0^{\pi/2}\log\sin x\,dx$ is equal to

a) $-\left(\frac{\pi}{2}\right)\log 2$ b) $\pi\log\frac{1}{2}$ c) $-\pi\log\frac{1}{2}$ d) $\frac{\pi}{2}\log 2$

26. $\int_0^1\frac{1}{x+\sqrt{x}}\,dx$ is equal to

a) $\log 3$ b) $\log 1$ c) $\log 4$ d) $\log 2$

27. The tangent of the curve $y=f(x)$ at the point with abscissa $x=1$ form an angle of $\pi/6$ and at the point $x=2$ an angle of $\pi/3$ and at the point $x=3$ an angle of $\pi/4$. If $f''(x)$ is continuous, then the value of $\int_1^3 f''(x)f'(x)dx+\int_2^3 f''(x)dx$, is

a) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ b) $\frac{3\sqrt{3}-1}{2}$ c) $\frac{4-3\sqrt{3}}{3}$ d) None of these

28. If $g(x)=\frac{f(x)-f(-x)}{2}$ defined over $[-3,3]$ and $f(x)=2x^2-4x+1$, then $\int_{-3}^{3}g(x)dx$ is equal to

a) 0 b) 4 c) -4 d) 8

29. $\int\frac{x+\sin x}{1+\cos x}dx$ is equal to

a) $x\tan\frac{x}{2}+C$ b) $\cot\frac{x}{2}+C$ c) $\log(1+\cos x)+C$ d) $\log(x+\sin x)+C$

30. The value of the integral $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

a) $\frac{3}{2}$ b) 2 c) 1 d) $\frac{1}{2}$

31. $\int\frac{dx}{(a^2+x^2)^{3/2}}$ is equal to

a) $\frac{x}{(a^2+x^2)^{1/2}}+c$ b) $\frac{x}{a^2(a^2+x^2)^{1/2}}+c$ c) $\frac{1}{a^2(a^2+x^2)^{1/2}}+c$ d) None of these

32. The value of the integral $\int_0^{\pi/2}(\sin^{100}x-\cos^{100}x)dx$ is

a) $\frac{1}{100}$ b) $\frac{100!}{(100)^{100}}$ c) $\frac{\pi}{100}$ d) 0

33. $\int e^x[f(x)+f'(x)]dx$ is equal to

a) $e^xf(x)+c$ b) e^x+c c) $e^xf'(x)+c$ d) None of these

34. $\int\frac{dx}{1-\cos x-\sin x}$ is equal to

a) $\log\left|1+\cot\frac{x}{2}\right|+c$ b) $\log\left|1-\tan\frac{x}{2}\right|+c$ c) $\log\left|1-\cot\frac{x}{2}\right|+c$ d) $\log\left|1+\tan\frac{x}{2}\right|+c$

35. The value of $\alpha\in[0,2\pi]$ which does not satisfy the equation $\int_{\pi/2}^{\alpha}\sin x\,dx=\sin 2\alpha$, is

a) π b) $\frac{3\pi}{2}$ c) $\frac{7\pi}{6}$ d) $\frac{11\pi}{6}$

36. If $\int_{-1}^{4} f(x)\,dx = 4$ and $\int_{2}^{4}(3 - f(x))dx = 7$, then the value of $\int_{2}^{-1} f(x)dx$, is

a) 2 b) -3 c) -5 d) None of these

37. If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ and $g(x) = e^{\sin^{-1} x}$, then $\int f(x)g(x)dx$ is equal to

a) $e^{\sin^{-1} x}(\sin^{-1} x - 1) + c$ b) $e^{\sin^{-1} x} + c$

c) $e^{(\sin^{-1} x)^2} + c$ d) $e^{2\sin^{-1} x} + c$

38. The value of $\int_{-1}^{3}(|x-2| + [x])\,dx$ is ($[x]$ stands for greatest integer less than or equal to x)

a) 7 b) 5 c) 4 d) 3

39. Let $g(x) = \int_0^x f(t)dt$, where $\frac{1}{2} \le f(t) \le 1, t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in (1,2]$, then

a) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ b) $0 \le g(2) < 2$ c) $\frac{3}{2} < g(2) \le \frac{5}{2}$ d) $2 < g(2) < 4$

40. Let $\int \frac{x^2}{\sqrt{1-x}}dx = p\sqrt{(1-x)}(3x^2 + 4x + 8)$, then value of p is

a) $\frac{-2}{15}$ b) $\frac{2}{15}$ c) $\frac{4}{15}$ d) None of these

41. $\int_0^1 \{x\}d\{x\}$ is equal to (where $\{\cdot\}$ denotes fractional part of x)

a) 0 b) 1 c) $\frac{1}{2}$ d) None of these

42. If $I = \int \frac{x^5}{\sqrt{1+x^3}}\,dx$, then I is equal to

a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}} + \frac{2}{3}(1+x^3)^{\frac{3}{2}} + c$ b) $\log\left|\sqrt{x} + \sqrt{1+x^3}\right| + c$

c) $\log\left|\sqrt{x} - \sqrt{1-x^3}\right| + c$ d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + c$

43. The value of the integral $\int_0^{100\pi} \sqrt{1-\cos 2x}\,\,dx$, is

a) $100\sqrt{2}$ b) $200\sqrt{2}$ c) 0 d) $400\sqrt{2}$

44. The value of $I = \int_{-\pi/2}^{\pi/2} |\sin x|\,dx\; is$

a) 0 b) 2 c) -2 d) $-2 < I < 2$

45. The value of $\int_{-2}^{3}|1-x^2|dx$ is

a) $\frac{28}{3}$ b) $\frac{14}{3}$ c) $\frac{7}{3}$ d) $\frac{1}{3}$

46. $\int_{-2}^{2}(x - |x|)dx$ is equal to

a) 0 b) 2 c) 4 d) -4

47. The value of $\int_0^1 \log[x]\,dx$ is

a) 0 b) 1 c) log 1 d) None of these

48. $\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$ is equal to

a) $\pi/3$ b) $\pi/2$ c) $1/2$ d) $\pi/4$

49. $\int \frac{(\sin\theta + \cos\theta)}{\sqrt{\sin 2\theta}}d\theta$ is equal to

a) $\log|\cos\theta - \sin\theta + \sqrt{\sin 2\theta}| + c$ b) $\log|\sin\theta - \cos\theta + \sqrt{\sin 2\theta}| + c$

c) $\sin^{-1}(\sin\theta - \cos\theta) + c$ d) $\sin^{-1}(\sin\theta + \cos\theta) + c$

50. $\int \frac{\cos x + x\sin x}{x(x+\cos x)}dx$ is equal to

a) $\log(x(x+\cos x)) + C$ b) $\log\left(\frac{x}{x+\cos x}\right) + C$ c) $\log\left(\frac{x+\cos x}{x}\right)$ d) None of these

51. If $\int \frac{dx}{x^2+2x+2} = f(x) + c$, then $f(x)$ is equal to

a) $\tan^{-1}(x+1)$ b) $2\tan^{-1}(x+1)$ c) $-\tan^{-1}(x+1)$ d) $3\tan^{-1}(x+1)$

52. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x} dx$ is equal to

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

53. $\int \operatorname{cosec}^4 x\, dx$ is equal to

a) $\cot x + \frac{\cot^3 x}{3} + c$ b) $\tan x + \frac{\tan^3 x}{3} + c$ c) $-\cot x - \frac{\cot^3 x}{3} + c$ d) $-\tan x - \frac{\tan^3 x}{3} + c$

54. $\int_0^{\pi/2} \frac{\sin^{2/3} x}{\sin^{2/3} x + \cos^{2/3} x} dx$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) π

55. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ is equal to

a) 1 b) -1 c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

56. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ is

a) 2π b) $\frac{\pi}{a}$ c) $\frac{\pi}{2}$ d) $a\pi$

57. The value of $\int_0^{\pi} x \sin^3 x\, dx$ is

a) $\frac{4\pi}{3}$ b) $\frac{2\pi}{3}$ c) 0 d) None of these

58. To find the numerical value of $\int_{-2}^{2} (px^3 + qx + s) dx$, it is necessary to know the values of the constants

a) p b) q c) s d) p and s

59. The value of $\int e^{\tan^{-1} x} . \frac{(1+x+x^2)}{1+x^2} dx$ is

a) $\tan^{-1} x + c$ b) $e^{\tan^{-1} x} + 2x + c$ c) $e^{\tan^{-1} x} + c$ d) $xe^{\tan^{-1} x} + c$

60. $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}} d\theta$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{5}$

61. $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\dots\left(1+\frac{n}{n}\right)\right\}^{1/n}$ is equal to

a) $e/4$ b) $4/e$ c) $2/e$ d) None of these

62. $\int_2^k (2x+1) dx = 6$, then k is equal to

a) 4 b) -2 c) -3 d) 3

63. If $I = \int_0^1 \cos\left\{2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$, then

a) $I > \frac{1}{2}$ b) $I = -\frac{1}{2}$ c) $0 < I < \frac{1}{2}$ d) $I = \frac{1}{2}$

64. $\int (\sin x - \cos x)^4 (\sin x + \cos x) dx$ is equal to

a) $\frac{\sin x - \cos x}{5} + c$ b) $\frac{(\sin x - \cos x)^5}{5} + c$ c) $\frac{(\sin x - \cos x)^4}{4} + c$ d) $\frac{(\sin x + \cos x)^5}{5} + c$

65. Primitive of $\frac{1}{4\sqrt{x}+x}$ is equal to

a) $2\log|1+4\sqrt{x}| + c$ b) $\frac{1}{2}\log|4-\sqrt{x}| + c$ c) $2\log|4+\sqrt{x}| + c$ d) $\frac{1}{2}\log|4+\sqrt{x}| + c$

66. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is

a) 0 b) 1 c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

67. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

a) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

b) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

c) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

d) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

68. The value of the integral $\int_{-\pi/2}^{\pi/2}\log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\,\theta, a>0$, is

a) 0 b) 1 c) 2 d) None of these

69. The value of the integral $\int_1^3\sqrt{3+x^3}\,dx$ lies in the interval

a) $(1,3)$ b) $(2,30)$ c) $(4,2\sqrt{300})$ d) None of these

70. The value of $\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)\,dx$ is equal to

a) 0 b) 2 c) π d) None of these

71. $\int_{-1}^{1}[x\sin\pi x]\,dx$ is equal to

a) 2 b) -2 c) 1 d) 0

72. The value of $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is

a) $\frac{3}{2}$ b) $\frac{5}{2}$ c) 3 d) 5

73. If $f(x)=\begin{vmatrix}\sec x & \cos x & \sec^2 x+\operatorname{cosec} x\cot x\\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x\\ 1 & \cos^2 x & \cos^2 x\end{vmatrix}$, then $\int_0^{\pi/2}f(x)\,dx$ equals

a) 0 b) 1 c) $-\left(\frac{\pi}{4}+\frac{8}{15}\right)$ d) -1

74. The value of $\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$, is

a) $\sin x-6\tan^{-1}(\sin x)+C$

b) $\sin x-2(\sin x)^{-1}+C$

c) $\sin x-2(\sin x)^{-1}-6\tan^{-1}(\sin x)+C$

d) $\sin x-2(\sin x)^{-1}+5\tan^{-1}(\sin x)+C$

75. $\int_0^1 x^{3/2}\sqrt{1-x}\;dx$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{9}$ c) $\frac{\pi}{12}$ d) $\frac{\pi}{16}$

76. The value of $I=\int_0^1 x\left|x-\frac{1}{2}\right|dx$ is

a) 1/3 b) 1/4 c) 1/8 d) None of these

77. The value of $\int_{-\pi/2}^{\pi/2}\sin\{\log(x+\sqrt{x^2+1})\}\,dx$, is

a) 1 b) -1 c) 0 d) None of these

78. $\int\frac{x^3-1}{x^3+x}dx$ is equal to

a) $x-\log x+\log(x^2+1)-\tan^{-1}x+C$

b) $x-\log x+\frac{1}{2}\log(x^2+1)-\tan^{-1}x+C$

c) $x+\log x+\frac{1}{2}\log(x^2+1)+\tan^{-1}x+C$

d) None of these

79. But for all arbitrary constants,

$$\int\sqrt{\frac{1+\sin\theta-\sin^2\theta-\sin^3\theta}{2\sin\theta-1}}\,d\theta \text{ is equal to}$$

a) $\frac{1}{2}\sqrt{\sin\theta-\cos 2\theta}+\frac{3}{4\sqrt{2}}\log_e\left|(4\sin\theta+1)+2\sqrt{2}\sqrt{\sin\theta-\cos 2\theta}\right|$

b) $\frac{1}{2}\sqrt{\sin\theta+\cos 2\theta}+\frac{3}{4\sqrt{2}}\log_e\left|(4\sin\theta-1)+2\sqrt{2}\sqrt{\sin\theta+\cos 2\theta}\right|$

c) $\frac{1}{2\sqrt{2}}\sqrt{\sin\theta-\cos 2\theta}+\frac{3}{4}\log_e\left|(4\sin\theta+1)-\sqrt{\sin\theta-\cos 2\theta}\right|$

d) $\frac{1}{2}\sqrt{\sin\theta+\cos 2\theta}+\frac{3}{4\sqrt{2}}\log_e\left|4\sin\theta+1-\sqrt{\sin\theta-\cos 2\theta}\right|$

80. $\int_{-1}^{0}\max\{x-[x],-x-[-x]\}dx$ is equal to

a) 3 b) $\frac{3}{2}$ c) $\frac{3}{4}$ d) 2

81. $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx$ is equal to

a) $\ln 3$ b) $2\ln 3$ c) $\frac{1}{2}\ln 3$ d) None of these

82. $\int_{0}^{2}\frac{2x-2}{2x-x^2}\,dx$ is equal to

a) 0 b) 2 c) 3 d) 4

83. $\int_0^{\pi}\frac{x\,dx}{1+\sin x}$ is equal to

a) $-\pi$ b) $\frac{\pi}{2}$ c) π d) None of these

84. If $I_n=\int_0^{\pi/2}\frac{\sin^2 nx}{\sin^2 x}dx$, then which one of the following is not true?

a) $I_n=\frac{n\pi}{2}$

b) $I_n=2\int_0^{\pi/2}\frac{\sin x\cos 2nx}{\sin x}dx$

c) $I_1, I_2, I_3, \dots I_n$ .... Is a A.P.

d) $\sin(I_{15})=0$

85. If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is

a) An odd function b) An even function c) Neither even nor odd d) 0

86. If $f(x)=a\,e^{2x}+b\,e^{x}+c\,x$ satisfies the conditions $f(0)=-1, f'(\log 2)=31, \int_0^{\log 4}(f(x)-c\,x)\;dx=\frac{39}{2}$, then

a) $a=5, b=-6, c=-7$

b) $a=5, b=6, c=7$

c) $a=-5, b=6, c=-7$

d) None of these

87. The value of $\int_0^{\infty}\frac{dx}{(x^2+4)(x^2+9)}$ is

a) $\frac{\pi}{60}$ b) $\frac{\pi}{20}$ c) $\frac{\pi}{40}$ d) $\frac{\pi}{80}$

88. $I_{m,n}=\int_0^1 x^m(\ln x)^n dx=$

a) $\frac{n}{n+1}I_{m,n-1}$ b) $\frac{-m}{n+1}I_{m,n-1}$ c) $\frac{-n}{m+1}I_{m,n-1}$ d) $\frac{m}{n+1}I_{m,n-1}$

89. $\int e^{3\log x}\;(x^4+1)^{-1}dx$ is equal to

a) $e^{3\log x}+c$ b) $\frac{1}{4}\log(x^4+1)+c$ c) $\log(x^4+1)+c$ d) $\frac{1}{2}\log(x^4+1)+c$

90. $\int_0^{\infty}\frac{x\,dx}{(1+x)(1+x^2)}$ is equal to

a) $\frac{\pi}{2}$ b) 0 c) 1 d) $\frac{\pi}{4}$

91. If $f(x)=\tan x-\tan^3 x+\tan^5 x-\dots\infty$ with $0<x<\frac{\pi}{4}$, then $\int_0^{\pi/4}f(x)dx$ is equal to

a) 1 b) 0 c) $\frac{1}{4}$ d) $\frac{1}{2}$

92. The value of $\int_0^1 \frac{x^4+1}{x^2+1}\, dx$ is

a) $\frac{1}{6}(3-4\pi)$ b) $\frac{1}{6}(3\pi+4)$ c) $\frac{1}{6}(3+4\pi)$ d) $\frac{1}{6}(3\pi-4)$

93. The value of $I = \int_0^{\pi/2} \frac{1}{1+\cot x} dx$, is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) π

94. $\frac{d}{dx}\int_{f(x)}^{g(x)} h(t)\ dt$ is equal to

a) $g'(x)h[g(x)]$ b) $h[g(x)] - h[f(x)]$

c) $g'(x)h[g(x)] - f'(x)h[f(x)]$ d) None of the above

95. If $\int_0^\infty e^{-ax}\, dx = \frac{1}{a}$, then $\int_0^\infty x^n\, e^{-ax} dx$, is

a) $\frac{(-1)^n n!}{a^{n+1}}$ b) $\frac{(-1)^n (n-1)!}{a^n}$ c) $\frac{n!}{a^{n+1}}$ d) None of these

96. The integral $\int_{-1}^{1} \frac{|x+2|}{x+2} dx$ is equal to

a) 1 b) 2 c) 0 d) -1

97. The value of the integral $\int \frac{dx}{x(1+\log x)^2}$ is equal to

a) $\frac{-1}{1+x}+c$ b) $\frac{-1}{1+\log x}+c$ c) $\frac{1}{1+\log x}+c$ d) $\frac{1}{1+x}+c$

98. The value of the integral $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\, dx$, is

a) $\frac{\pi}{4}(\beta-\alpha)^2$ b) $\frac{\pi}{2}(\beta-\alpha)^2$ c) $\frac{\pi}{8}(\beta-\alpha)^2$ d) None of these

99. $\int_0^{\pi/2} x \sin x\ dx$ is equal to

a) 0 b) 1 c) -1 d) 2

100. The value of $\int \frac{d(\sin x)}{\sqrt{1-\sin^2 x}}$ is equal to

a) $x+c$ b) $3x+c$ c) x^2+c d) None of these

101. $\int \sin^{-1} x\, dx$ is equal to

a) $\cos^{-1} x + c$ b) $x \sin^{-1} x + \sqrt{1-x^2} + c$

c) $\frac{1}{\sqrt{1-x^2}} + c$ d) $x \sin^{-1} x - \sqrt{1-x^2} + c$

102. Let m be any integer. Then, the integral $\int_0^\pi \frac{\sin 2m\, x}{\sin x} dx$ equals

a) 0 b) π c) 1 d) None of these

103. $I_1 = \int_0^{\pi/2} \sin x\, dx, I_2 = \int_0^{\pi/2} \sin^3 x\, dx$ then

a) $I_1 > I_2$ b) $I_1 < I_2$ c) $I_1 = I_2$ d) $I_2 = 0$

104. $f(x) = \min\{x+2, 1, 2-x\}$, then $\int_{-2}^{2} f(x)dx$ is equal to

a) 1 b) 2 c) 3 d) 0

105. $\int_0^3 |x^3+x^2+3x|\, dx$ is equal to

a) $\frac{171}{2}$ b) $\frac{171}{4}$ c) $\frac{170}{4}$ d) $\frac{170}{3}$

106. $\int_{-1/7}^{20/7} e^{5\{x\}}\, dx$ is equal to (where $\{\cdot\}$ denotes fractional part of x)

a) $\frac{3}{5}(e^5)$ b) $\frac{3}{5}(e^5-1)$ c) $\frac{3}{5}$ d) None of these

107. $\int_{\pi/3}^{\pi/2} \text{cosec}^2 x\, dx$ is equal to

a) -1 b) 1 c) 0 d) $\frac{1}{2}$

108. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where C_0, C_1, C_2 are all real, then the quadratic equation $C_2x^2+C_1x+C_0=0$ has
a) At least one root in $(0, 1)$
b) One root in $(1, 2)$ and the other in $(3, 4)$
c) One root in $(-1, 1)$ and the other in $(-5, -2)$
d) Both roots imaginary

109. $\int \frac{dx}{2\sqrt{x}(1+x)}$ is equal to
a) $\frac{1}{2}\tan^{-1}(\sqrt{x})+c$ b) $\tan^{-1}(\sqrt{x})+c$ c) $2\tan^{-1}(\sqrt{x})+c$ d) None of these

110. If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period T, then $\phi(x)=\int_a^x f(t)\,dt$ is
a) A periodic function with period $T/2$
b) A periodic function with period T
c) Not a periodic function
d) A periodic function with period $T/4$

111. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ is equal to
a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{12}$ d) $\frac{\pi}{2}$

112. $\int_{-1/7}^{20/7} \sin(x-[x])dx$ (where $[\cdot]$ denotes greatest integer) is
a) 0 b) $3(\cos 1-1)$ c) $3(1-\cos 1)$ d) None of these

113. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2f(x)=0$ for all non-zero x, then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx$ is equal to
a) 0 b) 1 c) 2 d) 3

114. $\int_{-\pi/2}^{\pi/2} \sin^4 x\cos^6 x\,dx$ is equal to
a) $\frac{3\pi}{64}$ b) $\frac{3\pi}{572}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

115. The value of the integral $\int_0^1 \cot^{-1}(1-x+x^2)dx$, is
a) $\pi-\log 2$ b) $\frac{\pi}{2}-\log 2$ c) $\pi+\log 2$ d) $\frac{\pi}{2}+\log 2$

116. $\int_{-100}^{100} f(x)\,dx$ is equal to
a) $\int_{-100}^{100} f(x^2)\,dx$ b) $\int_{-100}^{100} f(-x^2)\,dx$ c) $\int_{-100}^{100} f\left(\frac{1}{x}\right)dx$ d) $\int_{-100}^{100} f(-x)\,dx$

117. $\int \frac{\cos x-1}{\sin x+1}e^x\,dx$ is equal to
a) $\frac{e^x\cos x}{1+\sin x}+c$ b) $c-\frac{e^x\sin x}{1+\sin x}$ c) $c-\frac{e^x}{1+\sin x}$ d) $c-\frac{e^x\cos x}{1+\sin x}$

118. If $\int(\log x)^2\,dx=x[f(x)]^2+Ax[f(x)-1]+c$, then
a) $f(x)=\log x, A=2$ b) $f(x)=\log x, A=-2$
c) $f(x)=-\log x, A=2$ d) $f(x)=-\log x, A=-2$

119. Let $f(x)$ be an integrable function defined on $[a, b], b>a>0$. If $I_1=\int_{\pi/6}^{\pi/3} f(\tan\theta+\cot\theta)\sec^2\theta\ d\theta$ and, $I_2=\int_{\pi/6}^{\pi/3} f(\tan\theta+\cot\theta)\operatorname{cosec}^2\theta\ d\theta$, then $\frac{I_1}{I_2}=$
a) A positive integer b) A negative integer c) An irrational number d) None of these

120. The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x-\cos^3 x}\,dx$ is
a) 0 b) 2/3 c) 4/3 d) None of these

121. The value of the integral $\int \frac{1+x^2}{1+x^4}dx$ is equal to

a) $\tan^{-1} x^2 + C$

b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$

c) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+C$

d) None of these

122. If $(-1,2)$ and $(2,4)$ are two points on the curve $y=f(x)$ and if $g(x)$ is the gradient of the curve at point (x,y), then the value of the integral $\int_{-1}^{2} g(x)\,dx$, is

a) 2 b) −2 c) 0 d) 1

123. $\int_0^{\pi/3} [\sqrt{3}\tan x]\,dx =$

a) $\frac{5\pi}{6}$ b) $\frac{5\pi}{6}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\frac{\pi}{2}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ d) None of these

124. If $I=\int\frac{1}{x^4\sqrt{a^2+x^2}}dx$, then I equals

a) $\frac{1}{a^4}\left\{\frac{1}{x}\sqrt{a^2+x^2}-\frac{1}{3x^3}\sqrt{a^2+x^2}\right\}+C$

b) $\frac{1}{a^4}\left\{\frac{1}{x}\sqrt{a^2+x^2}-\frac{1}{3x^3}(a^2+x^2)^{3/2}\right\}+C$

c) $\frac{1}{a^4}\left\{\frac{1}{x}\sqrt{a^2+x^2}-\frac{1}{2\sqrt{x}}(a^2+x^2)^{3/2}\right\}+C$

d) None of these

125. The value of the integral $\int_0^{\pi/2}\log|\tan x|dx$ is

a) $\pi\log 2$ b) 0 c) $-\pi\log 2$ d) None of these

126. The value of the integral $\int_{-1}^{1}\sin^{11}x\ dx$, is

a) $\frac{10}{11},\frac{8}{9},\frac{6}{7},\frac{4}{5},\frac{2}{3}$ b) $\frac{10}{11},\frac{8}{9},\frac{6}{7},\frac{4}{5},\frac{2}{3},\frac{\pi}{2}$ c) 1 d) 0

127. If $\int\cos^4 x\,dx = Ax+B\sin 2x+C\sin 4x+D$, then $\{A,B,C\}$ equals

a) $\left\{\frac{3}{8},\frac{1}{32},\frac{1}{4}\right\}$ b) $\left\{\frac{3}{8},\frac{1}{4},\frac{1}{32}\right\}$ c) $\left\{\frac{1}{32},\frac{1}{4},\frac{3}{8}\right\}$ d) $\left\{\frac{1}{4},\frac{3}{8},\frac{1}{32}\right\}$

128. If $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = f(x)+\text{constant}$, then $f(x)$ is equal to

a) $e^x\cot\left(\frac{x}{2}\right)+c$ b) $e^{-x}\cot\left(\frac{x}{2}\right)+c$ c) $-e^x\cot\left(\frac{x}{2}\right)+c$ d) $-e^{-x}\cot\left(\frac{x}{2}\right)+c$

129. The value of $\int_1^a[x]f'(x)dx, a>1$,where$[x]$denotes the greatest integer not exceeding x, is

a) $[a]f(a)-\{f(1)+f(2)+\ldots+f([a])\}$ b) $[a]f([a])-\{f(1)+f(2)+\ldots+f(a)\}$

c) $af([a])-\{f(1)+f(2)+\ldots+f(a)\}$ d) $af(a)-\{f(1)+f(2)+\ldots+f([a])\}$

130. If $f(x)=f(a+b-x)$ for all $x\in[a,b]$ and $\int_a^b x\,f(x)dx = k\int_a^b f(x)dx$, then the value of k is

a) $\frac{a+b}{2}$ b) $\frac{a-b}{2}$ c) $\frac{a^2+b^2}{2}$ d) $\frac{a^2-b^2}{2}$

131. $\int|x|dx$ is equal to

a) $\frac{1}{2}x^2+c$ b) $-\frac{x^2}{2}+c$ c) $x|x|+c$ d) $\frac{1}{2}x|x|+c$

132. The value of $\int_{-1}^{1}\sin^{-1}\left[x^2+\frac{1}{2}\right]dx+\int_{-1}^{1}\cos^{-1}\left[x^2-\frac{1}{2}\right]dx$, where $[\cdot]$ denotes the greatest integer function, is

a) π b) 2π c) 4π d) 0

133. $\int\frac{x^2-2}{x^3\sqrt{x^2-1}}dx$ is equal to

a) $\frac{x^2}{\sqrt{x^2-1}}+C$ b) $-\frac{x^2}{\sqrt{x^2-1}}+C$ c) $\frac{\sqrt{x^2-1}}{x^2}+C$ d) $-\frac{\sqrt{x^2-1}}{x^2}+C$

134. $\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$ is equal to

a) 1/3 b) 1 c) 2/3 d) None of these

135. $\int e^x\frac{x^2+1}{(x+1)^2}\,dx$ is equal to

a) $\frac{-e^x}{x+1}+c$ b) $\frac{e^x}{x+1}+c$ c) $e^x\frac{x-1}{x+1}+c$ d) $\frac{xe^x}{x+1}+c$

136. If $\int x\log(1+1/x)dx = f(x)\cdot\log(x+1)+g(x)\cdot x^2+Ax+C$, then

a) $f(x)=\frac{1}{2}x^2$ b) $g(x)=\log x$ c) $A=1$ d) None of these

137. Suppose that $f(x)\geq 0$ for all $x\in[0,1]$ and f is continuous in $[0,1]$ and $\int_0^1 f(x)dx=0$, then $\forall\, x\in[0,1]$, f is

a) Entirely increasing b) Entirely decreasing c) Constant d) None of these

138. If $f\left(\frac{3x-4}{3x+4}\right)=x+2$, then $\int f(x)\,dx$ is

a) $e^{x+2}\log\left|\frac{3x-4}{3x+4}\right|+c$ b) $-\frac{8}{3}\log|1-x|+\frac{2}{3}x+c$

c) $\frac{8}{3}\log|1-x|+\frac{x}{3}+c$ d) $e^{[(3x-4)/(3x+4)]}-\frac{x^2}{2}-2x+c$

139. The value of $\int e^x\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx$ is equal to

a) $e^x\left(\sqrt{1-x^2}\right)+c$ b) $e^x\frac{\sqrt{1+x^{2n}}}{1+x^{2n}}+c$ c) $\frac{e^x\sqrt{1-x^n}}{1-x^{2n}}+c$ d) $\frac{e^x\sqrt{1-x^{2n}}}{1-x^n}+c$

140. The anti-derivative of $f(x)=\log(\log x)+(\log x)^{-2}$ whose graph passes through (e,e), is

a) $x[\log(\log x)+(\log x)^{-1}]$ b) $x[-\log(\log x)+(\log x)^{-1}]+e$

c) $x[\log(\log x)-(\log x)^{-1}]+2e$ d) None of the above

141. If $\int_a^b\frac{x^n}{x^n+(16-x)^n}dx=6$, then

a) $a=4, b=12, n\in R$ b) $a=2, b=14, n\in R$ c) $a=-4, b=20, n\in R$ d) $a=2, b=8, n\in R$

142. $\int\tan^{-1}x\;dx=\ldots+c$

a) $\frac{1}{1+x^2}$ b) $x\tan^{-1}x+\frac{1}{2}\log|1+x^2|$

c) $x\tan^{-1}x+\frac{1}{2}\cdot\frac{\tan^{-1}x}{1+x^2}$ d) $x\tan^{-1}x-\frac{1}{2}\log|1+x^2|$

143. $\int x^{51}(\tan^{-1}x+\cot^{-1}x)dx$

a) $\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c$ b) $\frac{x^{52}}{52}(\tan^{-1}x-\cot^{-1}x)+c$

c) $\frac{\pi x^{52}}{104}+\frac{\pi}{2}+c$ d) $\frac{x^{52}}{52}+\frac{\pi}{2}+c$

144. If $f(x)=|x-1|$, then $\int_0^2 f(x)dx$ is

a) 1 b) 0 c) 2 d) -2

145. $\int\frac{3-x^2}{1-2x+x^2}.e^x dx=e^x f(x)+c\Rightarrow f(x)$

a) $\frac{1+x}{1-x}$ b) $\frac{1-x}{1+x}$ c) $\frac{1+x}{x-1}$ d) $\frac{x-1}{1+x}$

146. $\int_0^1\tan^{-1}\left(\frac{1}{x^2-x+1}\right)dx$ is equal to

a) $\log 2$ b) $-\log 2$ c) $\frac{\pi}{2}+\log 2$ d) $\frac{\pi}{2}-\log 2$

147. Let $f: R \to R$ be a differentiable function and $f(1) = 4$. Then, the value of $\lim_{x\to 1} \int_4^{f(x)} \frac{2t}{x-1}\, dt$ is

a) $8f'(1)$ b) $4f'(1)$ c) $2f'(1)$ d) $f'(1)$

148. $\int e^x(\log \sin x + \cot x)dx$ is equal to

a) $e^x \cot x + c$
b) $e^x \log \sin x + c$
c) $e^x \log \sin x + \tan x + c$
d) $e^x + \sin x + c$

149. The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x \cos^2 x\ dx$, is

a) 0 b) $\pi - \frac{\pi^3}{3}$ c) $2\pi - \pi^3$ d) $\frac{\pi}{2} - 2\pi^3$

150. $\int \frac{x}{(x^2-a^2)(x^2-b^2)} dx$ is equal to

a) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-a^2}{x^2-b^2}\right| + c$
b) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-b^2}{x^2-a^2}\right| + c$
c) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right| + c$
d) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-b^2}{x^2-a^2}\right| + c$

151. If $\int_0^{n\pi} f(\cos^2 x)dx = k\int_0^{\pi} f(\cos^2 x)dx$, then the value of k, is

a) 1 b) n c) $n/2$ d) None of these

152. $\int \sin^3 x . \cos^2 x\ dx$ is equal to

a) $\frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + c$
b) $\frac{\sin^5 x}{5} + \frac{\sin^3 x}{3} + c$
c) $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$
d) $\frac{\cos^5 x}{5} + \frac{\cos^3 x}{3} + c$

153. The value of α which satisfying $\int_{\pi/2}^{\alpha} \sin x\, dx = \sin 2\alpha$, $\alpha \in (0, 2\pi)$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{2}$ c) $\frac{7\pi}{6}$ d) All of these

154. $\int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+a^x} dx$ is equal to

a) $\frac{\pi+4}{4}$ b) $\frac{\pi-4}{4}$ c) $\frac{a\pi}{4}$ d) $\frac{a+\pi}{4}$

155. The value of $\int e^x \frac{1+n\,x^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}} dx$, is

a) $\frac{e^x\sqrt{1-x^n}}{1-x^n} + C$ b) $e^x\frac{\sqrt{1+x^{2n}}}{1-x^{2n}} + C$ c) $\frac{e^x\sqrt{1-x^{2n}}}{1-x^{2n}} + C$ d) $e^x\frac{\sqrt{1-x^{2n}}}{1-x^n} + C$

156. Which of the following is true?

a) $\int_0^1 e^x\ dx = e$ b) $\int_0^1 2^x\ dx = \log 2$ c) $\int_0^1 \sqrt{x}\ dx = \frac{2}{3}$ d) $\int_0^1 x\ dx = \frac{1}{3}$

157. $\int \sec^{8/9} x \text{cosec}^{10/9}\ x\, dx$ is equal to

a) $-(\cot x)^{1/9} + c$ b) $9(\tan x)^{1/9} + c$ c) $-9(\cot x)^{1/9} + c$ d) $-\frac{1}{9}(\cot x)^9 + c$

158. If $\int_1^a (a-4x)\, dx \geq 6 - 5a$, $a > 1$, then a equals

a) 1 b) 2 c) 3 d) 4

159. The value of $\int_0^2 \{3x\}dx$, where $\{\cdot\}$ denotes the fractional part function, is

a) 7 b) 5 c) 3 d) 1

160. Let $I_n = \int_0^{\pi/4} \tan^n x\, dx$, ($n > 1$ and $n \in N$), then

a) $I_n = I_{n-2}$ b) $I_n + I_{n-2} = \frac{1}{n-1}$ c) $I_n - I_{n-2} = \frac{1}{n-1}$ d) None of these

161. $\int [e^{a\log x} + e^{x\log a}]dx = \ldots + c$; $a, x > 1$

a) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a}$ b) $\frac{e^{a\log x}}{a\log x} + \frac{e^{x\log a}}{x\log a}$ c) $\frac{x^{a-1}}{a-1} + a^x.\log a$ d) $\frac{e^{a\log x}}{a/x} + \frac{e^{x\log a}}{x}$

162. $\left|\int_{10}^{19} \frac{\sin x}{1+x^8} dx\right|$ is less than

a) 10^{-10} b) 10^{-11} c) 10^{-7} d) 10^{-9}

163. If $a < \int_0^{2\pi} \frac{1}{10+3\cos x} dx < b$, then the ordered pair (a, b) is

a) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ b) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$ c) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ d) None of these

164. The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$, is

a) 2 b) 3/4 c) 0 d) 1

165. If $\int \frac{x\tan^{-1} x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2} f(x) + A\log\left(x + \sqrt{x^2+1}\right) + C$, then

a) $f(x) = \tan^{-1} x, A = -1$

b) $f(x) = \tan^{-1} x, A = 1$

c) $f(x) = 2\tan^{-1} x, A = -1$

d) $f(x) = 2\tan^{-1} x, A = 1$

166. The value of integral $\int_{-2}^{4} x[x]dx$, is

a) 41/2 b) 20 c) 21/2 d) None of these

167. $\int_0^{\pi/4}(\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{2\pi}^{\pi/4}(\cos x - \sin x)\, dx$ is equal to

a) $\sqrt{2} - 2$ b) $2\sqrt{2} - 2$ c) $3\sqrt{2} - 2$ d) $4\sqrt{2} - 2$

168. Let f be a real-valued function defined on the interval $(-1,1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$, for all $x \in (-1,1)$ and let f^{-1} be the inverse function of f. Then, $(f^{-1})'(2)$ is equal to

a) 1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{e}$

169. $\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = \ldots + c; f(x) \neq 0$

a) $\frac{1}{2}\sqrt{f(x)}$ b) $2\sqrt{f(x)}$ c) $\frac{1}{2} f(x)$ d) $2 f(x)$

170. $3a\int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to

a) $a - 1 + (a-1)^{-2}$ b) $a + a^{-2}$ c) $a - a^2$ d) $a^2 + \frac{1}{a^2}$

171. The value of $\int_1^e 10^{\log_e x}\, dx$ is equal to

a) $10\log_e(10\, e)$ b) $\frac{10\, e - 1}{\log_e 10\, e}$ c) $\frac{10\, e}{\log_e 10\, e}$ d) $(10\, e)\log_e(10\, e)$

172. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

a) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$ b) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

c) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$ d) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

173. $\int \frac{1}{x(x^4-1)} dx$ is equal to

a) $\frac{1}{4}\log\left|\frac{x^4}{x^4-1}\right| + C$ b) $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$ c) $\log\left|\frac{x^4-1}{x^4}\right| + C$ d) $\log\left|\frac{x^4}{x^4-1}\right| + C$

174. If $\mathrm{g}(x) = \int_0^x \cos^4 t\, dt$, then $\mathrm{g}(x+\pi)$ is equal to

a) $\mathrm{g}(x) + \mathrm{g}(\pi)$ b) $\mathrm{g}(x) - \mathrm{g}(\pi)$ c) $\mathrm{g}(x).\mathrm{g}(\pi)$ d) $\frac{\mathrm{g}(x)}{\mathrm{g}(\pi)}$

175. The integral $\int_0^{\pi/2} f(\sin 2x)\sin x\, dx$ is equal to

a) $\int_0^{\pi/2} f(\cos 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\sin x\,dx$

b) $\int_0^{\pi/2} f(\sin 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$

c) $\int_0^{\pi/2} f(\cos 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\cos x\,dx$

d) $\int_0^{\pi/2} f(\sin 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\cos x\,dx$

176. Let $f(x) = \int_1^x \sqrt{2-t^2}\,dt$. Then, real roots of the equation $x^2 - f'(x) = 0$ are

a) ± 1 b) $\pm\frac{1}{\sqrt{2}}$ c) $\pm\frac{1}{2}$ d) 0 and 1

177. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) zero d) 1

178. $\int_0^3 \frac{3x+1}{x^2+9}\,dx$ is equal to

a) $\log(2\sqrt{2}) + \frac{\pi}{12}$ b) $\log(2\sqrt{2}) + \frac{\pi}{2}$ c) $\log(2\sqrt{2}) + \frac{\pi}{6}$ d) $\log(2\sqrt{2}) + \frac{\pi}{3}$

179. If $\phi(x) = \cos x - \int_0^x (x-t)\,\phi(t)dt$, then $\phi''(x) + \phi(x)$ is equal to

a) $-\cos x$ b) 0 c) $\int_0^x (x-t)\phi(t)dt$ d) $-\int_0^{-x} (x-t)\phi(t)dt$

180. If $I_n = \int_0^\infty e^{-x}x^{n-1}\,dx$, then $\int_0^\infty e^{-\lambda x}\,x^{n-1}dx$ is equal to

a) $\lambda\, I_n$ b) $\frac{1}{\lambda}\,I_n$ c) $\frac{I_n}{\lambda^n}$ d) $\lambda^n I_n$

181. $\int \frac{x^{49}\tan^{-1}(x^{50})}{(1+x^{100})}dx = k[\tan^{-1}(x^{50})]^2 + c$, then k is equal to

a) $\frac{1}{50}$ b) $-\frac{1}{50}$ c) $\frac{1}{100}$ d) $-\frac{1}{100}$

182. If $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}dx, a > 0$, then I equals

a) π b) $\frac{\pi}{2}$ c) $a\,\pi$ d) $\frac{a\pi}{2}$

183. $\int \frac{x\tan^{-1}x}{\sqrt{(1+x^2)^{3/2}}}\,dx$ is equal to

a) $\frac{x-\tan^{-1}x}{1-x^2} + c$ b) $\frac{x+\tan^{-1}x}{\sqrt{1-x^2}} + c$ c) $\frac{x-\tan^{-1}x}{\sqrt{1+x^2}} + c$ d) $\frac{x+\sqrt{1-x^2}}{\sqrt{1+x^2}} + c$

184. If $f(x)$ is a quadratic polynomial in x such that $6\int_0^1 f(x)dx - \left\{f(0) + 4f\left(\frac{1}{2}\right)\right\} = kf(1)$, then $k =$

a) -1 b) 0 c) 1 d) 2

185. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then $\int_0^a f(x)\;g(x)dx$ is equal to

a) $\int_0^a g(x)dx$

b) $\int_0^a f(x)dx$

c) 0

d) None of these

186. $\int_{3\pi/2}^{5\pi/3} [2\cos x]\, dx =$

a) $\frac{5\pi}{3}$ b) $\frac{4\pi}{3}$ c) $\frac{2\pi}{3}$ d) None of these

187. $\int_0^{\pi/8} \cos^3 4\theta\, d\theta$ is equal to

a) 5/3 b) 5/4 c) 1/3 d) 1/6

188. If $\int_{[x]}^{[x]+1} f(t)dt = [x]$, then the value of $\int_{-2}^{4} f(x)dx$ is equal to

a) 1 b) 2 c) −2 d) 3

189. $\int |x| \log |x| dx$ equals $(x \neq 0)$

a) $\frac{x^2}{2}\log|x| - \frac{x^2}{4} + c$ b) $\frac{1}{2}x|x|\log x + \frac{1}{4}x|x| + c$

c) $-\frac{x^2}{2}\log|x| + \frac{x^2}{4} + c$ d) $\frac{1}{2}x|x|\log|x| - \frac{1}{4}x|x| + c$

190. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = fofofo \dots \cdot\ of\ (n$ times$)$ · Then, $\int x^{n-2}g(x)dx$ equals

a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + k$

b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + k$

c) $\frac{1}{n(n-1)}(1+nx^n)^{1+\frac{1}{n}} + k$

d) $\frac{1}{n-1}(1+nx^n)^{1+\frac{1}{n}} + k$

191. The value of the integral $\int_0^{2\pi} \frac{\sin 2\theta}{a-b\cos\theta} d\theta$ when $a > b > 0$, is

a) 1 b) π c) $\pi/2$ d) 0

192. Suppose f is such that $f(-x) = -f(x)$ for every real x and $\int_0^1 f(x)dx = 5$, then $\int_{-1}^0 f(t)dt$ is equal to

a) 10 b) 5 c) 0 d) −5

193. If $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then the value of $\int_0^a f(x)g(x)dx$ is

a) $\int_0^a f(x)dx$ b) $\int_0^a g(x)dx$ c) $\int_0^a [g(x) - f(x)dx$ d) $\int_0^a [g(x) + f(x)dx$

194. $\int_{-10}^{10} \log\left(\frac{a+x}{a-x}\right) dx$ is equal to

a) 0 b) $-2\log(a+10)$ c) $2\log\left(\frac{a+10}{a-10}\right)$ d) $2\log(a+10)$

195. The value of $\int_0^{\pi} |\sin^3\theta|\, d\theta$ is

a) 0 b) 3/8 c) 4/3 d) π

196. $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx$ is equal to

a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) 2 d) $\frac{1}{3}$

197. $\int_0^2 \sqrt{\frac{2+x}{2-x}}\, dx$ is equal to

a) $\pi + 2$ b) $\pi + \frac{3}{2}$ c) $\pi + 1$ d) π

198. If $\int f(x)dx = f(x) + c$, then $\int (f(x))^2 dx$ is

a) $(f(x))^2 + k$ b) $\frac{1}{3}(f(x))^3 + k$ c) $f(x) + k$ d) $\frac{1}{2}(f(x))^2 + k$

199. $\int \sqrt{\frac{x-1}{x+1}}\, dx$ is equal to

a) $2\sqrt{x^2+1}+\sin^{-1} x+c$
b) $\sqrt{x^2-1}-\sin^{-1} x+c$
c) $2\sqrt{x^2-1}+\sin^{-1} x+c$
d) $\sqrt{\frac{x^2-1}{2}+\sin^{-1} x+c}$

200. $\int \sin x\, d(\cos x)$ is equal to

a) $\frac{1}{4}\sin 2x+\frac{x}{2}+c$
b) $\frac{1}{4}\sin 2x-\frac{x}{2}+c$
c) $2\sin 2x+c$
d) $\sin x+\cos x+c$

201. Value of the integral $\int_{-\pi/2}^{\pi/2} \cos x\, dx$ is

a) 4
b) 2
c) 0
d) 1

202. If $f(x)=f(a+x)$and $\int_0^a f\,(x)dx=k$, then $\int_0^{na} f(x)dx$ is equal to

a) nk
b) $(n-1)k$
c) $(n+1)k$
d) 0

203. The value of $\int_0^{\pi} \frac{1}{5+3\cos x} dx$, is

a) π
b) $2\pi/3$
c) $\pi/4$
d) 2

204. If $I_1=\int_0^x e^{zx}\, e^{-z^2} dz$ and $I_2=\int_0^x e^{-z^2/4}\, dz$, then

a) $I_1=e^x I_2$
b) $I_1=e^{x^2} I_2$
c) $I_1=e^{x^2/2} I_2$
d) None of these

205. If $f(x)$ is an integrable function over every interval on the real line such that $f(t+x)=f(x)$ for every x and every x real t, then $\int_a^{a+t} f(x)\, dx$ is equal to

a) $\int_0^a f(x)\, dx$
b) $\int_0^t f(x)\, dx$
c) $\int_a^t f(x)\, dx$
d) None of these

206. If $f(\pi)=2$ and $\int_0^{\pi}[f(x)+f''(x)]\sin x\, dx=5$, then $f(0)$ is equal to, (it is given that $f(x)$ is continuous in $[0,\pi]$)

a) 7
b) 3
c) 5
d) 1

207. If $\int \frac{x+2}{2x^2+6x+5}\, dx = P\int \frac{4x+6}{2x^2+6x+5}\, dx+\frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) $\frac{1}{4}$
d) 2

208. $\int(1+x-x^{-1})e^{x+x^{-1}} dx$ is equal to

a) $(x+1)e^{x+x^{-1}}+c$
b) $(x-1)e^{x+x^{-1}}+c$
c) $-xe^{x+x^{-1}}+c$
d) $xe^{x+x^{-1}}+c$

209. If $I_1=\int_0^{3\pi} f(\cos^2 x)\, dx$ and $I_2=\int_0^{\pi} f\,(\cos^{2} x)dx$, then

a) $I_1=I_2$
b) $3I_1=I_2$
c) $I_1=3I_2$
d) $I_1=5I_2$

210. $\int_0^{\pi/2}(x\sqrt{\tan x}+\sqrt{\cot x})\, dx$ equals

a) $\frac{\pi}{2\sqrt{2}}$
b) $\frac{\pi^2}{2}$
c) $\frac{\pi^2}{2\sqrt{2}}$
d) $\frac{\pi^2}{2\sqrt{3}}$

211. $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x+b^2\sin^2 x}$ is equal to

a) $\frac{\pi}{2ab}$
b) $\frac{\pi}{ab}$
c) $\frac{\pi^2}{2ab}$
d) $\frac{\pi^2}{ab}$

212. If $f(x)=ae^{2x}+cx$, satisfies the conditions $f(0)=-1, f'(\log 2)=31, \int_0^{\log 4}(f(x)-cx)\, dx=\frac{39}{2}$, then

a) $a=5, b=6, c=3$
b) $a=5, b=-6, c=3$
c) $a=-5, b=6, c=3$
d) None of these

213. The value of the integral $\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$, is

a) $1/2$
b) $\sqrt{2}/2$
c) 1
d) $\sqrt{2}$

214. If $\int x\, f(x)dx=\frac{f(x)}{2}$, then $f(x)$ is equal to

a) e^x b) e^{-x} c) $\log x$ d) $\frac{e^{x^2}}{2}$

215. $\int_0^{\pi} \frac{1}{1+\sin x}\, dx$ is equal to

a) 1 b) 2 c) −1 d) −2

216. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then, $\int_{-1}^{1} f(x)dx$ is equal to

a) 1 b) 2 c) 0 d) $-\frac{1}{2}$

217. The value of $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ is

a) $-\frac{(x^4+1)^{1/4}}{x} + c$ b) $\frac{(x^4+1)^{1/4}}{x} + c$ c) Zero d) None of these

218. If $I_1 = \int_{1-k}^{k} x \sin\{x(1-x)\}\, dx$ and $I_2 = \int_{1-k}^{k} \sin\{x(1-x)\}\, dx$, then

a) $I_1 = 2I_2$ b) $2I_1 = I_2$ c) $I_1 = I_2$ d) None of these

219. If the primitive of $\frac{1}{f(x)}$ is equal to $\log\{f(x)\}^2 + c$, then $f(x)$ is

a) $x + d$ b) $\frac{x}{2} + d$ c) $\frac{x^2}{2} + d$ d) $x^2 + d$

220. $\int \frac{x^4+x^2+1}{x^2-x+1}\, dx$ is equal to

a) $\frac{x^3}{3} - \frac{x^2}{2} + x + c$ b) $\frac{x^3}{3} + \frac{x^2}{2} + x + c$ c) $\frac{x^3}{3} - \frac{x^2}{2} - x + c$ d) $\frac{x^3}{3} + \frac{x^2}{2} - x + c$

221. The value of $\int_{-1}^{1} \max\{2-x, 2, 1+x\}dx$ is

a) 4 b) 9/2 c) 2 d) None of these

222. $\int_0^1 \frac{x^2}{1+x^2}\, dx$ is equal to

a) $\frac{\pi}{4} - 1$ b) $1 - \frac{\pi}{2}$ c) $\frac{\pi}{2} - 1$ d) $1 - \frac{\pi}{4}$

223. $\int \frac{e^x}{(2+e^x)(e^x+1)}\, dx$ is equal to

a) $\log\left(\frac{e^x+1}{e^x+2}\right) + c$ b) $\log\left(\frac{e^x+2}{e^x+1}\right) + c$ c) $\left(\frac{e^x+1}{e^x+2}\right) + c$ d) $\left(\frac{e^x+2}{e^x+1}\right) + c$

224. If $f(x) = \begin{cases} 1-x, & 0 \le x \le 1 \\ 0, & 1 \le x \le 2 \\ (2-x)^2, & 2 \le x \le 3 \end{cases}$ and $\phi(x) = \int_0^x f(t)dt$. Then, for any $x \in [2,3]$, $\phi(x)$ equals

a) $\frac{(x-2)^3}{3}$ b) $\frac{1}{2} - \frac{(x-2)^3}{3}$ c) $\frac{1}{2} + \frac{(x-2)^3}{3}$ d) None of these

225. $\frac{d}{dx}(f(x)) = \phi(x)$ for $a \le x \le b$, $\int_a^b f(x)\,\phi(x)dx =$

a) $f(b) - f(a)$ b) $\phi(b) - \phi(a)$ c) $\frac{[f(b)]^2 - [f(a)]^2}{2}$ d) $\frac{[\phi(b)]^2 - [\phi(a)]^2}{2}$

226. The value of $\int_a^{a+(\pi/2)} (\sin^4 x + \cos^4 x)\, dx$, is

a) $\frac{3\pi}{8}$ b) $a\left(\frac{\pi}{2}\right)^2$ c) $\frac{3\pi a^2}{8}$ d) None of these

227. $\int_{-1/2}^{1/2} (\cos x)\left[\log\left(\frac{1-x}{1+x}\right)\right]\, dx$ is equal to

a) 0 b) 1 c) $e^{1/2}$ d) $2\, e^{1/2}$

228. $\int \frac{x^2-1}{x^4+x^2+1}\, dx$ is equal to

a) $\log(x^4+x^2+1) + c$ b) $\log\frac{x^2-x+1}{x^2+x+1} + c$ c) $\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1} + c$ d) $\frac{1}{2}\log\frac{x^2+x+1}{x^2-x+1} + c$

229. If $\int \frac{\sqrt{x}}{x+1} dx = A\sqrt{x} + B\tan^{-1}\sqrt{x} + c$, then

a) $A = 1, B = 1$ b) $A = 1, B = 2$ c) $A = 2, B = 2$ d) $A = 2, B = -2$

230. $\int xe^{x^2}dx$ is equal to

a) $-\frac{e^{x^2}}{2}+c$ b) $\frac{e^{x^2}}{2}+c$ c) $\frac{e^x}{2}+c$ d) $-\frac{e^x}{2}+c$

231. The value of $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) d\theta$ *is*

a) 0 b) 1 c) 2 d) None of these

232. If $\int \frac{dx}{(x+100)\sqrt{x+99}} = f(x)+c$, then $f(x)$ is

a) $2(x+100)^{1/2}$ b) $3(x+100)^{1/2}$ c) $2\tan^{-1}(\sqrt{x+99})$ d) $2\tan^{-1}(\sqrt{x+100})$

233. If $I=\int_0^{1/2}\frac{1}{\sqrt{1-x^{2n}}}dx$, then which one of the following is true?

a) $I\le\frac{\pi}{6}$ b) $I\ge\frac{1}{2}$ c) $I\ge 0$ d) All of these

234. $\int \frac{\operatorname{cosec} x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}\,dx$ is equal to

a) $\sin^2\left[1+\log\tan\frac{x}{2}\right]+c$ b) $\tan\left[1+\log\tan\frac{x}{2}\right]+c$

c) $\sec^2\left[1+\log\tan\frac{x}{2}\right]+c$ d) $-\tan\left[1+\log\tan\frac{x}{2}\right]+c$

235. If $I(m,n)=\int_0^1 t^m(1+t)^n\,dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n+1)$ is

a) $\frac{2^n}{m+1}-\frac{n}{m+1}(m+1,n-1)$

b) $\frac{n}{m+1}I(m+1,n-1)$

c) $\frac{2^n}{m+1}+\frac{n}{m+1}I(m+1,n-1)$

d) $\frac{m}{m+1}I(m+1,n-1)$

236. $\int \frac{dx}{x(\log x)(\log\log x)\ldots\underbrace{(\log\log\ldots x)}_{8\text{ times}}}$ is equal to

a) $\underbrace{(\log\log\ldots x)}_{8\text{ times}}+c$ b) $\underbrace{(\log\log\ldots x)}_{7\text{ times}}+c$ c) $\underbrace{(\log\log\ldots x)}_{9\text{ times}}+c$ d) None of these

237. If $I=\left|\int_2^5\frac{\sin x dx}{(1+x^2}\right|$, then

a) $I\ge\frac{1}{4}$ b) I lies in the interval $\left(\frac{1}{4},\frac{1}{5}\right)$

c) I lies in the interval $\left(\frac{1}{5},\frac{1}{6}\right)$ d) $I\le\frac{3}{10}$

238. $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\ldots+\frac{1}{3n}\right)$ is equal to

a) $\log 2$ b) $\log 3$ c) $\log 5$ d) 0

239. The difference between the greatest and least values of the function $\phi(x)=\int_0^x(t+1)dt$ on [2, 3], is

a) 3 b) 2 c) 7/2 d) 11/2

240. The value of the integral $\int_0^\infty\frac{x\log x}{(1+x^2)^2}dx$ is

a) 1 b) 0 c) 2 d) None of these

241. The value of $\int_0^1\frac{2^{2x+1}-5^{2x-1}}{10^x}dx$, is

a) $\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)}+\frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$

b) $-\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)}+\frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$

c) $\frac{3}{5}\left\{\frac{2}{\log_e\left(\frac{2}{5}\right)}-\frac{1}{2\log_e\left(\frac{5}{2}\right)}\right\}$

d) None of these

242. The integral $\int_0^{r\pi}\sin^{2n}x\,dx$ is equal to

a) $r\int_0^{\pi}\sin^{2n}x\,dx$ b) $2r\int_0^{\pi}\sin^{2n}x\,dx$ c) $r\int_0^{\pi/2}\sin^{2n}x\,dx$ d) None of these

243. $\int\{f(x)g''(x)-f''(x)g(x)\}dx$ is equal to

a) $\frac{f(x)}{g'(x)}+c$ b) $f'(x)g(x)-f(x)g'(x)+c$

c) $f(x)g'(x)-f'(x)g(x)+c$ d) $f(x)g'(x)+f'(x)g(x)+c$

244. $\int_0^{\lambda}\frac{y\,dy}{\sqrt{y+\lambda}}$ is equal to

a) $\frac{2}{3}(2-\sqrt{2})\lambda\sqrt{\lambda}$ b) $\frac{2}{3}(2+\sqrt{2})\lambda\sqrt{\lambda}$ c) $\frac{1}{3}(2-\sqrt{2})\lambda\sqrt{\lambda}$ d) $\frac{1}{3}(2+\sqrt{2})\lambda\sqrt{\lambda}$

245. Let f be a non-negative function defined on the interval $[0,1]$. If $\int_0^x\sqrt{1-(f'(t))^2}\,dt=\int_0^x f(t)\,dt, 0\le x\le 1$ and $f(0)=0$, then

a) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$

b) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$

c) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

d) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

246. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ is equal to

a) $e-\frac{e^2}{2}$ b) $\frac{e^2}{2}-e$ c) $\frac{e^2}{2}+e$ d) $\frac{e^2}{2}-2$

247. If a be a positive integer, the number of values of a satisfying $\int_0^{\pi/2}\left\{a^2\left(\frac{\cos 3x}{4}+\frac{3}{4}\cos x\right)+a\sin x-20\cos x\right\}dx\le-\frac{a^2}{3}$, is

a) Only one b) Two c) Three d) Four

248. $\int(x+1)^{3/2}[\sin^{-1}(\log x)+\cos^{-1}(\log x)]dx$ is equal to

a) $\frac{2\pi}{2}(x+1)^{5/2}+c$ b) $\frac{\pi}{5}(x+1)^{5/2}+c$ c) Does not exist d) None of these

249. If $\int_0^{\pi/3}\frac{\cos x}{3+4\sin x}dx=k\log\left(\frac{3+2\sqrt{3}}{3}\right)$, then k is

a) 1/2 b) 1/3 c) 1/4 d) 1/8

250. If $F(x)=\int_{x^2}^{x^3}\log t\;dt\;(x>0)$, then $F'(x)$ equals

a) $(9x^2-4x)\log x$ b) $(4x-9x^2)\log x$ c) $(9x^2+4x)\log x$ d) None of these

251. $\int\frac{\sin x\cos x}{\sqrt{1-\sin^4 x}}\,dx$ is equal to

a) $\frac{1}{2}\sin^{-1}(\sin^2 x)+c$ b) $\frac{1}{2}\cos^{-1}(\sin^2 x)+c$ c) $\tan^{-1}(\sin^2 x)+c$ d) $\tan^{-1}(2\sin^2 x)+c$

252. $\int\frac{1+x}{1+\sqrt[3]{x}}dx$ is equal to

a) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{4/3}+x+C$

b) $\frac{3}{5}x^{5/3}+\frac{3}{4}x^{4/3}+x+C$

c) $\frac{3}{5}x^{5/3}-\frac{3}{4}x^{4/3}+C$

d) None of these

253. The value of the integral $\int_0^1 x(1-x)^n dx$, is

a) $\frac{1}{n+1}+\frac{1}{n+2}$ b) $\frac{1}{(n+1)(n+2)}$ c) $\frac{1}{n+2}-\frac{1}{n+1}$ d) $2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

254. $\int_0^1 \frac{x\,dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}}$ is equal to

a) 0 b) 1 c) $\pi/4$ d) $\pi^2/2$

255. $\int_{-1}^{1}|1-x|dx$ is equal to

a) -2 b) 0 c) 2 d) 4

256. The value of $\int \frac{\sec x\,dx}{\sqrt{\sin(2x+\theta)+\sin\theta}}$ is

a) $\sqrt{(\tan x+\tan\theta)\sec\theta}+c$ b) $\sqrt{2(\tan x+\tan\theta)\sec\theta}+c$

c) $\sqrt{2(\sin x+\tan\theta)\sec\theta}+c$ d) None of the above

257. If an anti-derivate of $f(x)$ is e^x and that of $\mathrm{g}(x)$ is $\cos x$, then $\int f(x)\cos x\,dx+\int \mathrm{g}(x)\,e^x dx$ is equal to

a) $f(x)\mathrm{g}(x)+c$ b) $f(x)+\mathrm{g}(x)+c$ c) $e^x\cos x+c$ d) $f(x)-\mathrm{g}(x)+c$

258. If $P=\int_0^{3\pi} f(\cos^2 x)dx$ and $Q=\int_0^{\pi} f(\cos^2 x)dx$, then

a) $P-Q=0$ b) $P-2Q=0$ c) $P-3Q=0$ d) $P-5Q=0$

259. If $f(x)=\int_{x^2}^{x^4}\sin\sqrt{t}\,dt$, then $f'(x)$ equals

a) $\sin x^2-\sin x$ b) $4x^3\sin x^2-2x\sin x$ c) $x^4\sin x^2-x\sin x$ d) None of these

260. $\int_0^{50\pi} |\cos x|\,dx=$

a) 100 b) 50 c) 0 d) None of these

261. If $\int_{-1}^{4} f(x)dx=4$ and $\int_2^4\{3-f(x)\}dx=7$, then the value of $\int_{-1}^{2} f(x)\,dx$ is

a) -2 b) 3 c) 4 d) 5

262. $\int \frac{(x+1)^2}{x(x^2+1)}\,dx$ is equal to

a) $\log_e x+c$ b) $\log_e x+2\tan^{-1}x+c$ c) $\log_e \frac{1}{x^2+1}+c$ d) $\log_e\{x(x^2+1)\}+c$

263. $\lim_{x\to\infty}\frac{\pi}{n}\left\{\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{(n-1)\pi}{n}\right\}$ equals

a) 0 b) π c) 2 d) None of these

264. $f(x)$ is a continuous function for all real values of x and satisfies $\int_0^x f(t)dt=\int_x^1 t^2 f(t)dt+\frac{x^{16}}{8}+\frac{x^6}{3}+a$, then the value of $'a'$ is equal to

a) $-\frac{1}{24}$ b) $\frac{17}{168}$ c) $\frac{1}{7}$ d) None of these

265. If $f: R\to R$ and $\mathrm{g}: R\to R$ are one to one, real valued functions, then the value of the integral $\int_{-\pi}^{\pi}[f(x)+f(-x)][\mathrm{g}(x)-\mathrm{g}(-x)]dx$ is

a) 0 b) π c) 1 d) None of these

266. If $\int_0^{\pi} x\,f(\sin x)dx=A\int_0^{\pi/2} f(\sin x)dx$, then A is equal to

a) 0 b) π c) $\frac{\pi}{4}$ d) 2π

267. The number of solutions of $I(x)=\int_0^x [t]^2\,dt=2(x-1), \forall x>0$, where $[\cdot]$ denotes the greatest integer function, is

a) 2 b) 3 c) 4 d) 5

268. If n is an odd natural number, then

$$\int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^n}{1 - \sin\left(pq + \frac{\pi}{6}\right)} dx =$$

a) 4π b) $2\pi + \frac{1}{\sqrt{3}}$ c) $2\pi - \sqrt{3}$ d) $4\pi + \sqrt{3} - \frac{1}{\sqrt{3}}$

269. If $I_1 = \int_0^{\pi/4} \sin^2 x \, dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x \, dx$, then

a) $I_1 = I_2$ b) $I_1 < I_2$ c) $I_1 > I_2$ d) $I_2 = I_1 + \frac{\pi}{4}$

270. The set of values of $\alpha (\alpha > 0)$ for which the inequality $\int_{-\alpha}^{\alpha} e^x dx > \frac{3}{2}$ holds true, is

a) $(0, \infty)$ b) $(2, \infty)$ c) $(\log 2, \infty)$ d) None of these

271. If $\frac{d}{dx}\left[a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4-1}$, then $a - 2b$ is equal to

a) 1 b) -1 c) 0 d) 2

272. $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}} \, dx$ is equal to

a) $\frac{\pi}{2}(\alpha - \beta)$ b) $\frac{\pi}{2}(\beta - \alpha)$ c) $\pi(\alpha - \beta)$ d) $\pi(\beta - \alpha)$

273. $\int \frac{x \, dx}{x^2+4x+5}$ is equal to

a) $\frac{1}{2}\log(x^2 + 4x + 5) + 2\tan^{-1}(x) + c$ b) $\frac{1}{2}\log(x^2 + 4x + 5) - \tan^{-1}(x + 2) + c$

c) $\frac{1}{2}\log(x^2 + 4x + 5) + \tan^{-1}(x + 2) + c$ d) $\frac{1}{2}\log(x^2 + 4x + 5) - 2\tan^{-1}(x + 2) + c$

274. $\int \cos\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right] dx$ is equal to

a) $\frac{1}{2}x^2 + c$ b) $\frac{1}{2}\sin\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right] + c$

c) $-\frac{1}{2}x^2 + c$ d) $\frac{1}{2}x + c$

275. The value of $\int_0^a \sqrt{\frac{a-x}{x}} \, dx$ is

a) $\frac{a}{2}$ b) $\frac{a}{4}$ c) $\frac{\pi a}{2}$ d) $\frac{\pi a}{4}$

276. $$\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x \, (\sin x + \cos x) dx =$$

a) 2/15 b) 4/15 c) 2/5 d) 8/15

277. The value of $\int_{-1}^{1} \log\left(\frac{x-1}{x+1}\right) dx$ is

a) 1 b) 2 c) 0 d) 4

278. If for every integer n, $\int_n^{n+1} f(x) dx = n^2$, then the value of $\int_{-2}^{4} f(x) dx$ is

a) 16 b) 14 c) 19 d) None of these

279. The value of $\int_{-1}^{1} x|x| dx$, is

a) 2 b) 1 c) 0 d) None of these

280. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x) \, dx$ is

a) 3 b) 2/3 c) 1/3 d) 0

281. The value of the integral $\int_\alpha^\beta \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$ for $\beta > \alpha$, is

a) $\sin^{-1}(\alpha/\beta)$ b) $\pi/2$ c) $\sin^{-1}(\beta/2\alpha)$ d) π

282. $\int_{\pi/2}^{3\pi/2}[2\cos x]dx$ is equal to

a) $\frac{5\pi}{3}$ b) $-\frac{5\pi}{3}$ c) $-\pi$ d) -2π

283. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$ is equal to

a) $\log(\sin^{-1} x) + c$ b) $\frac{1}{2}\,(\sin^{-1} x)^2 + c$ c) $\log\left(\sqrt{1-x^2}\right) + c$ d) $\sin(\cos^{-1} x) + c$

284. $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$ is equal to

a) 0 b) 1 c) 2 d) 3

285. If f is a positive function and

$$I_1 = \int_{1-k}^{k} x\, f[x(1-x)]\, dx,$$

$$I_2 = \int_{1-k}^{k} f[x(1-x)]\, dx,$$

Where $2k-1>0$, then $\frac{I_1}{I_2}$ is

a) 2 b) k c) $\frac{1}{2}$ d) 1

286. $\int \frac{1}{(a^2+x^2)^{3/2}} dx$ is equal to

a) $\frac{x}{a^2\sqrt{a^2+x^2}} + C$ b) $\frac{x}{(a^2+x^2)^{3/2}} + C$ c) $\frac{1}{a^2\sqrt{a^2+x^2}} + C$ d) None of these

287. $\int \frac{\cos x - \sin x}{1+2\sin x \cos x}\, dx$ is equal to

a) $-\frac{1}{\cos x - \sin x} + c$ b) $\frac{\cos x + \sin x}{\cos x - \sin x} + c$ c) $-\frac{1}{\sin x + \cos x} + c$ d) $\frac{x}{\sin x + \cos x} + c$

288. The value of the integral $\int_0^{\pi/2} |\sin x - \cos x|\, dx$, is

a) 0 b) $2(\sqrt{2}-1)$ c) $2\sqrt{2}$ d) $2(\sqrt{2}+1)$

289. $\int_0^{\pi/4}(\cos x - \sin x) + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{2\pi}^{\pi/4}(\cos x - \sin x)\, dx$ is equal to

a) $\sqrt{2}-2$ b) $2\sqrt{2}-2$ c) $3\sqrt{2}-2$ d) $4\sqrt{2}-2$

290. The value of $\int_1^4 e^{\sqrt{x}}\, dx$, is

a) e^2 b) $2e^2$ c) $4e^2$ d) $3e^2$

291. Let $I_n = \int_0^{\pi/2} \cos^n x \cos n\, x\, dx$. Then, $I_n : I_{n+1}$ is equal to

a) 3 : 1 b) 2 : 3 c) 2 : 1 d) 3 : 4

292. If $\int \frac{2x^2+3}{(x^2-1)(x^2+4)} dx = a\log\left(\frac{x-1}{x+1}\right) + b\tan^{-1}\left(\frac{x}{2}\right) + c$, then the value of a and b are

a) $(1,-1)$ b) $(-1,1)$ c) $\left(\frac{1}{2},-\frac{1}{2}\right)$ d) $\left(\frac{1}{2},\frac{1}{2}\right)$

293. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x}\, dx$ is equal to

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

294. $\int_0^{\pi/2} \log\sin x\, dx$ is equal to

a) $-\pi\log 2$ b) $\pi\log 2$ c) $-\frac{\pi}{2}\log 2$ d) $\frac{\pi}{2}\log 2$

295. Let $\int e^x\left(f(x) - f'(x)\right)dx = \phi(x)$

Then, $\int e^x f(x) dx$ is equal to

a) $\phi(x) + e^x f(x)$ b) $\phi(x) - e^x f(x)$ c) $\frac{1}{2}[\phi(x) + e^x f(x)]$ d) $\frac{1}{2}[\phi(x) + e^x f'(x)]$

296. If $f(x)$ is an odd function and has a period T, then $\phi(x)=\int_0^x f(t)dt$ is

a) A periodic function with period $T/2$
b) A periodic function with period T
c) Not a periodic function
d) A periodic function with period $T/4$

297. If x is any arbitrary constant, then $\int 2^{2^{2^x}}.2^{2^x}.2^x\,dx$ is equal to

a) $\frac{\int 2^{2^x}dx}{(\log 2)^3}+c$
b) $\frac{\int 2^{2^{2^x}}dx}{(\log 2)^3}+c$
c) $\int 2^{2^{2^x}}(\log 2)^3dx+c$
d) None of these

298. The value of $\int\frac{2\,dx}{\sqrt{1-4x^2}}$ is

a) $\tan^{-1}(2x)+c$
b) $\cot^{-1}(2x)+c$
c) $\cos^{-1}(2x)+c$
d) $\sin^{-1}(2x)+c$

299. $\int\frac{d(\cos\theta)}{\sqrt{1-\cos^2\theta}}$ is equal to

a) $\cos^{-1}\theta+c$
b) $\theta+c$
c) $\sin^{-1}\theta+c$
d) $\sin^{-1}(\cos\theta)+c$

300. The value(s) of $\int_0^1\frac{x^4(1-x)^4}{1+x^2}\,dx$ is (are)

a) $\frac{22}{7}-\pi$
b) $\frac{2}{105}$
c) 0
d) $\frac{71}{15}-\frac{3\pi}{2}$

301. Let $f(x)=\int_0^x|x-2|dx$, $x\geq 0$. Then, $f'(x)$ is

a) Continuous and non differentiable at $x=2$
b) Discontinuous at $x=4$
c) Neither continuous nor differentiable at $x=2$
d) Non-differentiable at $x=4$

302. If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is

a) An odd function
b) An even function
c) Neither even nor odd
d) 0

303. $\int\frac{\sec x\,\mathrm{cosec}\,x}{2\cot x-\sec x\,\mathrm{cosec}\,x}\,dx$ is equal to

a) $\log|\sec x+\tan x|+c$
b) $\log|\sec x+\mathrm{cosec}\,x|+c$
c) $\frac{1}{2}\log|\sec 2x+\tan 2x|+c$
d) $\log|\sec 2x+\mathrm{cosec}\,2x|+c$

304. $\int_0^\pi\frac{\theta\sin\theta}{1+\cos^2\theta}d\theta$is equal to

a) $\frac{\pi^2}{2}$
b) $\frac{\pi^3}{3}$
c) π^2
d) $\frac{\pi^2}{4}$

305. The value of $\left[\int_0^{\sin^2\theta}\sin^{-1}\sqrt{\phi}\,d\phi+\int_0^{\cos^2\theta}\cos^{-1}\sqrt{\phi}\,d\phi\right]$ is equal to

a) π
b) $\pi/2$
c) $\pi/3$
d) $\pi/4$

306. $\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}$is equal to

a) π
b) $\frac{\pi}{2}$
c) $\frac{\pi}{4}$
d) $\frac{3\pi}{2}$

307. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}\,dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}\,dx$. Then, which one of the following is true?

a) $I>\frac{2}{3}$ and $J<2$
b) $I>\frac{2}{3}$ and $J>2$
c) $I<\frac{2}{3}$ and $J<2$
d) $I<\frac{2}{3}$ and $J>2$

308. If $f(x)=\begin{cases}|x|, & -1\leq x\leq 1\\ |x-2|, & 1<x\leq 3\end{cases}$, then $\int_{-1}^3 f(x)dx$ is equal to

a) 0
b) 1
c) 2
d) 4

309. $\int\frac{1}{x(x^n+1)}dx$ is equal to

a) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+C$ b) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right)$ c) $\log\left(\frac{x^n}{x^n+1}\right)+C$ d) None of these

310. $\int_0^{1/2}|\sin \pi x|\, dx$ is equal to

a) 0 b) π c) $-\pi$ d) $1/\pi$

311. The value of the integral $\int_0^{\pi} \frac{1}{a^2-2a\cos x+1} dx (a>1)$, is

a) $\frac{\pi}{1-a^2}$ b) $\frac{\pi}{a^2-1}$ c) $\frac{2\pi}{a^2-1}$ d) $\frac{2\pi}{1-a^2}$

312. If $I=\int_0^1 \sqrt{1+x^3} dx$ then

a) $I>2$ b) $I \neq \frac{\sqrt{5}}{2}$ c) $I>\frac{\sqrt{7}}{2}$ d) None of these

313. Assuming that f is everywhere continuous, $\frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right) dx$ is equal to

a) $\frac{1}{c}\int_a^b f(x)\, dx$ b) $\int_a^b f(x)\, dx$ c) $c\int_a^b f(x)\, dx$ d) $\int_{ac^2}^{bc^2} f(x)\, dx$

314. The value of the integral $\int e^x \left(\frac{1-x}{1+x}\right)^2 dx$ is

a) $e^x\left(\frac{1-x}{1+x^2}\right)+c$ b) $e^x\left(\frac{1+x}{1+x^2}\right)+c$ c) $\frac{e^x}{1+x^2}+c$ d) $e^x(1-x)+c$

315. Let $\frac{d}{dx}(F(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k)-f(1)$, then one of the possible values of k, is

a) 64 b) 15 c) 16 d) 63

316. The values of $'a'$ for which $\int_0^a (3x^2+4x-5)\, dx < a^3-2$ are

a) $\frac{1}{2}<a<2$ b) $\frac{1}{2}\le a\le 2$ c) $a\le\frac{1}{2}$ d) $a\ge 2$

317. The value of the integral $\int \frac{\log(x+1)-\log x}{x(x+1)} dx$ is

a) $\frac{1}{2}[\log(x+1)]^2+\frac{1}{2}(\log x)^2+\log(x+1)\log x+C$

b) $-[\{\log(x+1)\}^2-(\log x)^2]+\log(x+1)\cdot\log x+C$

c) $\frac{1}{2}[\log(1+1/x)]^2+C$

d) None of these

318. The value of $\int_0^{2\pi}[2\sin x]\, dx$, where $[\cdot]$ represents the greatest integral functions, is

a) $-\frac{5\pi}{3}$ b) $-\pi$ c) $\frac{5\pi}{3}$ d) -2π

319. $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right] dx$ is equal to

a) 0 b) π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

320. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x)+g(x)=x^2$. Then, the value of the integral $\int_0^1 f(x)\, g(x) dx$, is

a) $e-\frac{e^2}{2}-\frac{5}{2}$ b) $e+\frac{e^2}{2}-\frac{3}{2}$ c) $e-\frac{e^2}{2}-\frac{3}{2}$ d) $e+\frac{e^2}{2}+\frac{5}{2}$

321. The value of the integral $\int_0^{\pi}\log(1+\cos x)\, dx$ is

a) $\frac{\pi}{2}\log 2$ b) $-\pi\log 2$ c) $\pi\log 2$ d) None of these

322. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right) dx$ is equal to

a) $x\, e^{\tan^{-1}x}+C$ b) $x^2\, e^{\tan^{-1}x}+C$ c) $\frac{1}{x}\, e^{\tan^{-1}x}+C$ d) None of these

323. Let $I_1=\int_a^{\pi-a} xf(\sin x)\, dx, I_2=\int_a^{\pi-a} f(\sin x)\, dx$, then I_2 is equal to

a) $\frac{\pi}{2}I_1$ b) πI_1 c) $\frac{2}{\pi}I_1$ d) $2I_1$

324. $\int \cos^{-1}\left(\frac{1}{x}\right) dx$ equals

a) $x\sec^{-1}x + \cosh^{-1}x + c$
b) $x\sec^{-1}x - \cosh^{-1}x + c$
c) $x\sec^{-1}x - \sin^{-1}x + c$
d) None of these

325. The value of $\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{1}{2}$ d) $\frac{\pi}{4}$

326. For any natural number n, the value of the integral $\int_0^{\sqrt{n}}[x^2]dx$, is

a) $n\sqrt{n} + \sum_{r=1}^{n}\sqrt{r}$ b) $n\sqrt{n} - \sum_{r=1}^{n}\sqrt{r}$ c) $\sum_{r=1}^{n}\sqrt{r} - n\sqrt{n}$ d) None of these

327. $\int \frac{1}{x}(\log_{ex} e)dx$ is equal to

a) $\log_e(1-\log_e x) + c$
b) $\log_e(\log_e ex - 1) + c$
c) $\log_e(\log_e x - 1) + c$
d) $\log_e(1+\log_e x) + c$

328. Let f be integrable over $[0, a]$ for any real a. If we define

$$I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)\, d\theta$$

and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)\, d\theta$, then

a) $I_1 = I_2$ b) $I_1 = -I_2$ c) $I_1 = 2I_2$ d) $I_1 = -2I_2$

329. Consider the following statements:

1.$\int_{-\pi/2}^{\pi/2}\sqrt{\cos x - \cos^3 x}\ dx = \frac{3}{4}$

2.$\int_0^4(|x-1| + |x-3|)dx = 10$

Which of these is/are correct?

a) Only (1) b) Only (2) c) Both of these d) None of these

330. The value of the integral $\int_{-\pi/4}^{\pi/4}\sin^{-4}x\, dx$ is

a) $-\frac{8}{3}$ b) $\frac{3}{2}$ c) $\frac{8}{3}$ d) None of these

331. The value of $\lim_{x\to\infty}\frac{\left(\int_0^x e^x\, dx\right)^2}{\int_0^x e^{2x^3}\, dx}$, is

a) 1 b) 2 c) 3 d) 0

332. If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x, \forall\, x \in [0, \pi/2]$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

a) 3 b) $\sqrt{3}$ c) $\frac{1}{3}$ d) None of these

333. If a is fixed real number such that $f(a-x) + f(a+x) = 0$, then $\int_0^{2a} f(x)\, dx =$

a) $\frac{a}{2}$ b) 0 c) $-\frac{a}{2}$ d) $2a$

334. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is

a) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$

b) $\frac{-1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$

c) $\log\sin(x-a)\sin(x-b) + c$

d) $\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$

335. Let $\int \sqrt{\frac{5-x}{2+x}}dx$ equal

a) $\sqrt{x+2}\sqrt{5-x}+3\sin^{-1}\sqrt{\frac{x+2}{3}}+C$

b) $\sqrt{x+2}\sqrt{5-x}+7\sin^{-1}\sqrt{\frac{x+2}{7}}+C$

c) $\sqrt{x+2}\sqrt{5-x}+5\sin^{-1}\sqrt{\frac{x+2}{5}}+C$

d) None of these

336. $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx = a\log\left(\frac{x+1}{x-1}\right)+b\tan^{-1}\frac{x}{2}$, then (a,b) is

a) $(-1/2, 1/2)$ b) $(1/2, 1/2)$ c) $(-1, 1)$ d) $(1, -1)$

337. $\int e^x \frac{(x-1)}{x^2}\, dx$ is equal to

a) $\frac{e^x}{x^2}+c$ b) $\frac{-e^x}{x^2}+c$ c) $\frac{e^x}{x}+c$ d) $\frac{-e^x}{x}+c$

338. The value of the integral $\int_0^{3\alpha} \text{cosec}\,(x-\alpha)\,\text{cosec}(x-2\alpha)dx$, is

a) $2\sec\alpha\log\left(\frac{1}{2}\text{cosec}\,\alpha\right)$ b) $2\sec\alpha\log\left(\frac{1}{2}\sec\alpha\right)$ c) $2\,\text{cosec}\,\alpha\log(\sec\alpha)$ d) $2\,\text{cosec}\,\alpha\log\left(\frac{1}{2}\sec\alpha\right)$

339. $\int_0^3 \frac{3x+1}{x^2+9}\, dx$ is equal to

a) $\log(2\sqrt{2})+\frac{\pi}{12}$ b) $\log(2\sqrt{2})+\frac{\pi}{2}$ c) $\log(2\sqrt{2})+\frac{\pi}{6}$ d) $\log(2\sqrt{2})+\frac{\pi}{3}$

340. $\int \frac{dx}{\sqrt{(1-x)(x-2)}}$ is equal to

a) $\sin^{-1}(2x-3)+c$ b) $\sin^{-1}(2x+5)+c$ c) $\sin^{-1}(3-2x)+c$ d) $\sin^{-1}(5-2x)+c$

341. If $f(x)=\lim_{n\to\infty}[2x+4x^3+\ldots+2nx^{2n-1}](0<x<1)$, then $\int f(x)dx$ is equal to

a) $-\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{x^2-1}$ d) $\frac{1}{1-x^2}$

342. $\int \frac{dx}{\sin x-\cos x+\sqrt{2}}$ equals

a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ b) $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

343. $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\, dx$ is equal to

a) $\log 2$ b) $2\log 2$ c) $(\log 2)^2$ d) $\frac{1}{2}\log 2$

344. The integral $\int_0^1 \frac{2\sin^{-1}\frac{x}{2}}{x}dx$ equals

a) $\int_0^{\pi/6} \frac{x}{\tan x}dx$ b) $\int_0^{\pi/6} \frac{2x}{\tan x}dx$ c) $\int_0^{\pi/2} \frac{2x}{\tan x}dx$ d) $\int_0^{\pi/6} \frac{x}{\sin x}dx$

345. If $\int_2^e \left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx = a+\frac{b}{\log 2}$, then

a) $a=e, b=-2$ b) $a=e, b=2$ c) $a=-e, b=2$ d) None of these

346. The value of $\int_0^8 |x-5|\, dx$ is

a) 17 b) 12 c) 9 d) 18

347. $\int_0^1 \frac{x\, dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}}$ is equal to

a) 0 b) 1 c) $\frac{\pi}{4}$ d) $\frac{\pi^2}{2}$

348. $\int_0^1 \cot^{-1}(1-x+x^2)\ dx$ is equal to

a) $\pi - \log 2$ b) $\pi + \log 2$ c) $\frac{\pi}{2} + \log 2$ d) $\frac{\pi}{2} - \log 2$

349. $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$ is equal to

a) $\frac{1}{2}\log\frac{5}{3}$ b) $\frac{1}{3}\log\frac{5}{3}$ c) $\frac{1}{2}\log\frac{3}{5}$ d) $\frac{1}{5}\log\frac{3}{5}$

350. $\int\{1+2\tan x(\tan x+\sec x)\}^{1/2}dx$ is equal to

a) $\log\sec x(\sec x-\tan x)+C$

b) $\log \text{cosec}(\sec x+\tan x)+C$

c) $\log\sec x(\sec x+\tan x+C)$

d) $\log(\sec x+\tan x)+C$

351. If $I_1=\int_0^\infty \frac{1}{1+x^4}dx$ and $I_2=\int_0^\infty \frac{x^2}{1+x^4}dx$. Then $\frac{I_1}{I_2}=$

a) 1 b) 2 c) 1/2 d) None of these

352. $\int \frac{\sin x\, dx}{3+4\cos^2 x}$ is equal to

a) $\log(3+4\cos^2 x)+c$

b) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right)+c$

c) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$

d) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$

353. For any integer n, the integral

$\int_0^\pi e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value

a) π b) 1 c) 0 d) None of these

354. If $\frac{d}{dx}\{f(x)=\frac{1}{1+x^2}$, then $\frac{d}{dx}\{f(x^3)\}$ is

a) $\frac{3x}{1+x^3}$ b) $\frac{3x^2}{1+x^6}$ c) $\frac{-6x^5}{(1+x^6)^2}$ d) $\frac{-6x^5}{1+x^6}$

355. $\int_0^\pi[\cot x]\,dx$, [.]denotes the greatest integer function, is equal to

a) $\frac{\pi}{2}$ b) 1 c) -1 d) $-\frac{\pi}{2}$

356. $\int_{-3}^{2}\{|x+1|+|x+2|+|x-1|\}\,dx$ is equal to

a) $\frac{31}{2}$ b) $\frac{35}{2}$ c) $\frac{47}{2}$ d) $\frac{39}{2}$

357. $\int_0^3 |x^3+x^2+3x|dx$ is equal to

a) $\frac{171}{2}$ b) $\frac{171}{4}$ c) $\frac{170}{4}$ d) $\frac{170}{3}$

358. $\int\frac{dx}{\sin x\cos x}$ is equal to

a) $\log|\sin x|+c$ b) $\log|\tan x|+c$ c) $\log|\sec x|+c$ d) None of these

359. $\int_0^{2n\pi}\left\{|\sin x|-\left|\frac{1}{2}\sin x\right|\right\}dx$ equals

a) n b) $2n$ c) $-2n$ d) None of these

360. If $\int_a^b x^3\,dx=0$ and if $\int_a^b x^2\,dx=\frac{2}{3}$, then the values of a and b are respectively

a) 1,1 b) $-1,-1$ c) $1,-1$ d) $-1,1$

361. The value of $\int_0^{\pi/2}\text{cosec}(x-\pi/3)\,\text{cosec}(x-\pi/6)dx$, is

a) $2\log 3$ b) $-2\log 3$ c) $\log 3$ d) None of these

362. The primitive function of the function $f(x)=\frac{\sqrt{(a^2-x^2)}}{x^4}$ is

a) $c+\frac{\sqrt{a^2-x^2}}{3a^2x^3}$ b) $c-\frac{(a^2-x^2)^{3/2}}{2a^2x^2}$ c) $c-\frac{(a^2-x^2)^{3/2}}{3a^2x^3}$ d) None of these

363. If $f(x)=\begin{cases} x, & \text{for } x<1 \\ x-1, & \text{for } x\geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)dx$ is equal to

a) 1 b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) $\frac{5}{2}$

364. $\int \frac{x^2+x-6}{(x-2)(x-1)}dx$

a) $x+2\log(x-1)+c$ b) $2x+2\log(x-1)+c$ c) $x+4\log(1-x)+c$ d) $x+4\log(x-1)+c$

365. $\int \frac{x^3}{(1+x^2)^{1/3}}dx$ is equal to

a) $\frac{20}{3}(1+x^2)^{2/3}(2x^2-3)+C$

b) $\frac{3}{20}(1+x^2)^{2/3}(2x^2-3)+C$

c) $\frac{3}{20}(1+x^2)^{2/3}(2x^2+3)+C$

d) None of these

366. The equation $\int_{-\pi/4}^{\pi/4}\left\{a|\sin x|+\frac{b\sin x}{1+\cos x}+c\right\}dx=0$, where a,b,c are constants, gives a relation between

a) a,b and c b) a and c c) a and b d) b and c

367. The value of $\int_2^4\{|x-2|+|x-3|\}dx$ is

a) 1 b) 2 c) 3 d) 5

368. If $\mathrm{g}(x)=\int_0^x \cos^4 t\, dt$, then $\mathrm{g}(x+\pi)$ equals

a) $\mathrm{g}(x)+\mathrm{g}(\pi)$ b) $\mathrm{g}(x)-\mathrm{g}(\pi)$ c) $\mathrm{g}(x)\mathrm{g}(\pi)$ d) $\frac{\mathrm{g}(x)}{\mathrm{g}(\pi)}$

369. $\int \cos^3 x\ e^{\log(\sin x)}dx$ is equal to

a) $-\frac{\sin^4 x}{4}+C$ b) $-\frac{\cos^4 x}{4}+C$ c) $\frac{e^{\sin x}}{4}+C$ d) None of these

370. $\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}$ is equal to

a) $\frac{1}{2}\log\frac{5}{3}$ b) $\frac{1}{3}\log\frac{5}{3}$ c) $\frac{1}{5}\log\frac{3}{5}$ d) $\frac{1}{2}\log\frac{3}{5}$

371. The value of $\int \frac{ax^2-b}{x\sqrt{c^2x^2-(ax^2+b)^2}}dx$, is

a) $\sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right)+k$ b) $\sin^{-1}\left(\frac{ax^2+\frac{b}{x^2}}{c}\right)+k$ c) $\cos^{-1}\left(\frac{ax+b/x}{c}\right)+k$ d) $\cos^{-1}\left(\frac{ax^2+\frac{b}{x^2}}{c}\right)+k$

372. If $f(x)=\int_{-1}^x|t|dt$, then for any $x\geq 0, f(x)$ equals

a) $\frac{1}{2}(1-x^2)$ b) $\frac{1}{2}x^2$ c) $\frac{1}{2}(1+x^2)$ d) None of these

373. Let $I_1=\int_1^2\frac{1}{\sqrt{1+x^2}}dx$ and $I_2=\int_1^2\frac{1}{x}dx$. Then

a) $I_1>I_2$ b) $I_2>I_1$ c) $I_1=I_2$ d) $I_1>2I_2$

374. The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\, dx$ is

a) 0 b) $\pi-\frac{\pi^3}{3}$ c) $2\pi-\pi^3$ d) $\frac{7}{2}-2\pi^3$

375. If $I_n=\int_0^{\pi/2}x^n\sin x\, dx$, then I_4+12I_2 is equal to

a) 4π b) $3\left(\frac{\pi}{2}\right)^3$ c) $\left(\frac{\pi}{2}\right)^2$ d) $4\left(\frac{\pi}{2}\right)^3$

376. The value of the integral $\int_0^2 x[x]\, dx$, is

a) $\frac{7}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) None of these

377. $\int_{-1}^{0} \frac{dx}{x^2+2x+2}$ is equal to

a) 0 b) $\pi/4$ c) $\pi/2$ d) $-\pi/4$

378. The value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{8}$ d) 0

379. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf\{x(1-x)\}$, $I_2 = \int_{1-k}^{k} f\{x(1-x)\}$ dx where $2k-1>0$.Then, $\frac{I_1}{I_2}$ is

a) 2 b) k c) $\frac{1}{2}$ d) 1

380. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\lim_{n\to\infty} n\,(I_{n+1} + I_{n-1})$ equals

a) 1 b) 2 c) $\pi/4$ d) π

381. $\int_1^e \frac{1}{x} dx$ is equal to

a) ∞ b) 0 c) 1 d) $\log(1+e)$

382. $\lim_{n\to\infty} \frac{1^{99} + 2^{99} + \cdots + n^{99}}{n^{100}} =$

a) $\frac{99}{100}$ b) $\frac{1}{100}$ c) $\frac{1}{99}$ d) $\frac{1}{101}$

383. $\int \frac{\cos 2x}{\cos x} dx$ is equal to

a) $2\sin x + \log(\sec x - \tan x) + C$

b) $2\sin x - \log(\sec x - \tan x) + C$

c) $2\sin x + \log(\sec x + \tan x) + C$

d) None of these

384. $\int f'(ax+b)\,\{f(ax+b)\}^n \; dx$ is equal to

a) $\frac{1}{n+1}\{f(ax+b)\}^{n+1} + C$, for all n except $n=-1$

b) $\frac{1}{n+1}\{f(ax+b)\}^{n+1} + C$, for all n

c) $\frac{1}{a(n+1)}\{f(ax+b)\}^{n+1} + C$, for all n except $n=-1$

d) $\frac{1}{a(n+1)}\{f(ax+b)\}^{n+1} + C$, for all n

385. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} \; dx$ and $I_4 = \int_1^2 2^{x^3} dx$, then

a) $I_3 > I_4$ b) $I_3 = I_4$ c) $I_1 > I_2$ d) $I_2 > I_1$

386. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} \; dx = \ldots + c; x \neq \frac{k\pi}{2}$ and $\tan x > 0$

a) $\frac{1}{2\sqrt{\tan x}}$ b) $\sqrt{2\tan x}$ c) $2\sqrt{\tan x}$ d) $\sqrt{\tan x}$

387. $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals

a) $1+\sqrt{5}$ b) $-1+\sqrt{5}$ c) $-1+\sqrt{2}$ d) $1+\sqrt{2}$

388. $\int_0^\infty \frac{x \log x \, dx}{(1+x^2)^2}$ is equal to

a) 0 b) 1 c) ∞ d) None of these

389. The value of $\int_{-1}^{1} [x[1+\sin \pi x]\, dx$ is ([·] denotes the greatest integer)

a) 2 b) 0 c) 1 d) None of these

390. If $\int_0^\infty e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$ then $\int_0^\infty e^{-ax^2} dx, a > 0$ is

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{2a}$ c) $2\frac{\sqrt{\pi}}{a}$ d) $\frac{1}{2}\frac{\sqrt{\pi}}{a}$

391. If $\int \frac{2^x}{\sqrt{1-4^x}} dx = K \sin^{-1}(2^x) + C$, then K is equal to

a) $\log 2$ b) $\frac{1}{2}\log 2$ c) $\frac{1}{2}$ d) $\frac{1}{\log 2}$

392. The value of the integral $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$, where ($a$ and b integers), is

a) $-\pi$ b) 0 c) π d) 2π

393. $\int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1}\, dx$ is equal to

a) $\frac{x^m}{x^{m+n}+1} + c$ b) $\frac{x^n}{x^{m+n}+1} + c$ c) $\frac{x^{m+n}-1}{x^{m+n}+1} + c$ d) $-\frac{x^n}{x^{m+n}+1} + c$

394. The value of $\int_0^{16\pi/3} |\sin x| dx$, is

a) 21 b) 21/2 c) 10 d) 11

395. If $f(x)$ and $g(x), x\epsilon R$ are continuous functions, then value of integral $\int_{-\pi/2}^{\pi/2} [\{f(x) + f(-x)\}\{g(x) - g(-x)\}]\; dx$ is

a) π b) $\frac{\pi}{2}$ c) 1 d) 0

396. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to

a) $2\sqrt{\tan x} + C$ b) $2\sqrt{\cot x} + C$ c) $\frac{\sqrt{\tan x}}{2} + C$ d) None of these

397. $\int_0^1 \sin\left\{2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right\} dx =$

a) $\pi/6$ b) $\pi/4$ c) $\pi/2$ d) π

398. If $\int_{-\pi/3}^{\pi/3} \left(\frac{a}{3}|\tan x| + \frac{b \tan x}{1 + \sec x}\right) dx = 0$ where a, b, c are constants, then $c =$

a) $a \ln 2$ b) $\frac{a}{\pi}\ln 2$ c) $-\frac{a}{\pi}\ln 2$ d) $\frac{2a}{\pi}\ln 2$

399. If the tangent to the graph function $y = f(x)$ makes angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with the x-axis is at the point $x = 2$ and $x = 4$ respectively, the value of $\int_2^4 f'(x) f''(x)\; dx$

a) $f(4)f(2)$ b) $f(4)$ c) $f(2)$ d) 1

400. $\int \frac{dx}{\cos^3 \sqrt{2 \sin 2x}}$ is equal to

a) $\sqrt{\tan x} + \frac{\tan^{5/2} x}{5} + c$ b) $\sqrt{\tan x} + \frac{2}{5}\tan^{5/2} x + c$

c) $2\sqrt{\tan x} + \frac{2}{5}\tan^{5/2} x + c$ d) None of these

401. Let $f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0) = 0$. Then, $f(1)$ is

a) $\log(1+\sqrt{2})$ b) $\log(1+\sqrt{2}) - \frac{\pi}{4}$ c) $\log(1+\sqrt{2}) + \frac{\pi}{4}$ d) None of these

402. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{3/2}} dx$, is

a) 1/2 b) $1/\sqrt{2}$ c) 1 d) $\sqrt{2}$

403. If $I(m, n) = \int_0^1 t^m (1+t)^n\, dt$, then the expression for $I(m, n)$ in terms of $I(m+1, n-1)$ is

a) $\frac{2^n}{m+1} - \frac{n}{m+1}\; I(m+1, n-1)$ b) $\frac{n}{m+1}\; I(m+1, n-1)$

c) $\frac{2^n}{m+1} + \frac{n}{m+1}\; I(m+1, n-1)$ d) $\frac{m}{n+1}\; I(m+1, n-1)$

404. The value of $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi^2}{32}$ c) 1 d) None of these

405. The value of $\int_3^5 \frac{x^2}{x^2-4}\,dx$ is

a) $2-\log_e\left(\frac{15}{7}\right)$ b) $2+\log_e\left(\frac{15}{7}\right)$

c) $2+4\log_e 3-4\log_e 7+4\log_e 5$ d) $2-\tan^{-1}\left(\frac{15}{7}\right)$

406. $\int \sqrt{x^2+a^2}\,dx$ equals

a) $\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log\left\{x+\sqrt{x^2+a^2}\right\}+c$ b) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left\{x+\sqrt{x^2+a^2}\right\}+c$

c) $\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log\left\{x-\sqrt{x^2+a^2}\right\}+c$ d) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left\{x-\sqrt{x^2+a^2}\right\}+c$

407. $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then $\lim_{n\to\infty} n[I_n+I_{n+2}]$ is equal to

a) $\frac{1}{2}$ b) 1 c) ∞ d) zero

408. $\int (x^2+1)\sqrt{x+1}\,dx$ is equal to

a) $\frac{(x+1)^{7/2}}{7}-2\frac{(x+1)^{5/2}}{5}+2\frac{(x+1)^{3/2}}{3}+c$ b) $2\left[\frac{(x+1)^{7/2}}{7}-2\frac{(x+1)^{5/2}}{5}+2\frac{(x+1)^{3/2}}{3}\right]+c$

c) $\frac{(x+1)^{7/2}}{7}-2\frac{(x+1)^{5/2}}{5}+5$ d) $\frac{(x+1)^{7/2}}{7}-3\frac{(x+1)^{5/2}}{5}+11(x+1)^{1/2}+c$

409. $\int \frac{f'(x)}{f(x)\log[f(x)]}dx$ is equal to

a) $\frac{f(x)}{\log f(x)}+c$ b) $f(x)\log f(x)+c$ c) $\log[\log f(x)]+c$ d) $\frac{1}{\log[\log f(x)]}+c$

410. Consider the integrals

$I_1=\int_0^1 e^{-x}\cos^2 x\,dx$, $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$, $I_3=\int_0^1 e^{-x^2}\,dx$ and $I_4=\int_0^1 e^{-(1/2)x^2}\,dx$. The greatest of these integral is

a) I_1 b) I_2 c) I_3 d) I_4

411. $\int \frac{1}{(1+x^2)\sqrt{1-x^2}}dx$ is equal to

a) $\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$ b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)$ c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$ d) None of these

412. The value of the integral $\int_{1/n}^{(an-1)/n}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}\,dx$ is

a) $\frac{a}{2}$ b) $\frac{na+2}{2n}$ c) $\frac{na-2}{2n}$ d) None of these

413. The value of $\int e^x(x^5+5x^4+1)dx$ is

a) $e^x.x^5+c$ b) $e^x.x^5+e^x+c$ c) $e^{x+1}.x^5+c$ d) $5x^4.e^x+c$

414. If $\int_{-1/2}^{1/2}\cos x\log\left(\frac{1+x}{1-x}\right)dx=k.\log 2$, then k equals

a) 0 b) -1 c) -2 d) $\frac{1}{2}$

415. $\int \sin\sqrt{3}\,dx$ is equal to

a) $\sin\sqrt{x}-\sqrt{x}\cos\sqrt{x}$ b) $2\left(\sin\sqrt{x}-\sqrt{x}\cos\sqrt{x}\right)+c$

c) $\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}+c$ d) $2\left(\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}\right)+c$

416. If $f(x)=\int_x^{x^2}(t-1)dt, 1\le x\le 2$, then global maximum value of $f(x)$ is

a) 1 b) 2 c) 3 d) 4

417. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{\sqrt{n^2+r^2}}$ is equal to

a) $1-\sqrt{2}$ b) $\sqrt{2}-1$ c) $\sqrt{2}$ d) $-\sqrt{2}$

418. The value of $\int_0^{2\pi}\cos^{99}x\,dx$, is

a) 1 b) -1 c) 99 d) 0

419. The value of $\int_{-2}^{2}(ax^3+bx+c)dx$ depends on the

a) Value of b b) Value of c c) Value of a d) Values of a and b

420. The value of integral $\int_0^{\pi}x\,f(\sin x)\,dx$ is

a) 0 b) $\pi\int_0^{\pi/2}f\,(\sin x)\,dx$ c) $\frac{\pi}{4}\int_0^{\pi}f\,(\sin x)\,dx$ d) None of these

421. The value of the integral $\int\frac{\log(x+1)-\log x}{x(x+1)}dx$ is

a) $-\frac{1}{2}[\log(x+1)]^2-\frac{1}{2}(\log x)^2+\log(x+1)\log x+c$

b) $-[\{\log(x+1)\}^2-(\log x)^2]+\log(x+1)\log x+c$

c) $\left[\log\left(1+\frac{1}{x}\right)^2\right]+c$

d) None of the above

422. The value of $I=\int_0^1 x\left|x-\frac{1}{2}\right|dx$ is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) None of these

423. $\int\frac{a^{x/2}}{\sqrt{a^{-x}-a^{x}}}dx=$

a) $\frac{1}{\log a}\sin^{-1}(a^x)$ b) $\frac{1}{\log a}\tan^{-1}(a^x)$ c) $2\sqrt{a^{-x}-a^{x}}$ d) $\log(a^x-1)$

424. If $\int_{\log 2}^{x}\frac{du}{(e^u-1)^{1/2}}=\frac{\pi}{6}$, then e^x is equal to

a) 1 b) 2 c) 4 d) -1

425. $\int\{f(x)g''(x)-f''(x)g(x)\}dx$ is equal to

a) $\frac{f(x)}{g'(x)}$

b) $f'(x)g(x)-f(x)g'(x)$

c) $f(x)g'(x)-f'(x)g(x)$

d) $f(x)g'(x)+f'(x)g(x)$

426. The value of $\int_{-\pi/4}^{\pi/4}x^3\sin^4x\,dx$, is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{8}$ d) 0

427. The value of $\int_0^{\pi/2}\sin^8x\,dx$ is

a) $\frac{105\pi}{32(4!)}$ b) $\frac{105\pi}{14(4!)}$ c) $\frac{105}{16\pi(4!)}$ d) None of these

428. $\int_{-\pi/2}^{\pi/2}\frac{|x|}{8\cos^2 2x+1}dx$ has the value

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{24}$ d) None of these

429. If $\int f(x)\cos x\,dx=\frac{1}{2}\{f(x)\}^2+c$, then $f(x)$ is

a) $x+c$ b) $\sin x+c$ c) $\cos x+c$ d) c

430. If $f(x)=\frac{x+2}{2x+3}$. Then, $\int\left(\frac{f(x)}{x^2}\right)^{1/2}dx$ is equal to $\frac{1}{\sqrt{2}}g\left(\frac{1+\sqrt{2f(x)}}{1-\sqrt{2f(x)}}\right)-\sqrt{\frac{2}{3}}h\left(\frac{\sqrt{3f(x)}+\sqrt{2}}{\sqrt{3f(x)}-\sqrt{2}}\right)+c$ where

a) $g(x)=\tan^{-1}x, h(x)=\log|x|$
b) $g(x)=\log|x|, h(x)=\tan^{-1}x$
c) $g(x)=h(x)=\tan^{-1}x$
d) $g(x)=\log|x|, h(x)=\log|x|$

431. $\int 5^{5^{5^x}}\cdot 5^{5^x}\cdot 5^x\,dx$ is equal to

a) $\frac{5^{5^x}}{(\log 5)^3}+C$ b) $5^{5^{5^x}}(\log 5)^3+C$ c) $\frac{5^{5^{5^x}}}{(\log 5)^3}+C$ d) None of these

432. If $f(t)=\int_{-t}^{t}\frac{e^{-|x|}}{2}dx$, then $\lim_{t\to\infty}f(t)$ is equal to

a) 1 b) $\frac{1}{2}$ c) 0 d) -1

433. $\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx$ is equal to

a) $\frac{-1}{\sin x+\cos x}+C$
b) $\log(\sin x+\cos x)+C$
c) $\log(\sin x-\cos x)+C$
d) $\log(\sin x+\cos x)^2+C$

434. If the primitive of $\sin^{-3/2}x\sin^{-1/2}(x+\theta)$ is – 2 $\text{cosec}\theta\sqrt{f(x)}+c$, then

a) $f(x)=\frac{\sin x}{\sin(x+\theta)}$ b) $f(x)=\tan(x+\theta)$ c) $f(x)=\frac{\sin(x+\theta)}{\sin x}$ d) $f(x)=\frac{\tan(x+\theta)}{\sin x}$

435. $\int\frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx$ is equal to

a) $\frac{x}{\sqrt{x^4+x^2+1}}+C$ b) $\frac{\sqrt{x^4+x^2+1}}{x}+C$ c) $\frac{2x}{\sqrt{x^4+x^2+1}}+C$ d) $\frac{\sqrt{x^4+x^2+1}}{2x}+C$

436. $\int 32x^3(\log x)^2\,dx$ is equal to

a) $8x^4(\log x)^2+c$
b) $x^4\{8(\log x)^2-4\log x+1\}+c$
c) $x^4\{8(\log x)^2-4\log x\}+c$
d) $x^3\{(\log x)^2+2\log x\}+c$

437. $\int\frac{e^x(1+\sin x)}{1+\cos x}dx$ is equal to

a) $e^x\tan\left(\frac{x}{2}\right)+c$ b) $e^x\tan x+c$ c) $e^x\left(\frac{1+\sin x}{1-\cos x}\right)+c$ d) $c-e^x\cot\left(\frac{x}{2}\right)$

438. If $\int\log\left(\sqrt{1-x}+\sqrt{1+x}\right)dx=x\,f(x)+Ax+B\sin^{-1}x+C$, then

a) $f(x)=\log\left(\sqrt{1-x}+\sqrt{1+x}\right)$
b) $A=1/3$
c) $B=2/3$
d) $B=-1/2$

439. $\int_0^{\pi/4}\log(1+\tan x)\,dx$ is equal to

a) $\frac{\pi}{8}\log_e 2$ b) $\frac{\pi}{4}\log_2 e$ c) $\frac{\pi}{4}\log_e 2$ d) $\frac{\pi}{8}\log_e\left(\frac{1}{2}\right)$

440. $\int_{-1}^{2}\sin^3 x\,|x|\,dx$ is equal to

a) 0 b) 1 c) 4 d) None of these

441. $\int_0^{1000}e^{x-[x]}dx$ is

a) $e^{1000}-1$ b) $\frac{e^{1000}-1}{e-1}$ c) $1000(e-1)$ d) $\frac{e-1}{1000}$

442. $\int\frac{(\tan^{-1}x)^3}{(1+x^2)}dx$ is equal to

a) $3(\tan^{-1}x)^2+c$ b) $\frac{(\tan^{-1}x)^4}{4}+c$ c) $(\tan^{-1}x)^4+c$ d) None of these

443. The value of $\int_0^{\pi}|\sin^3\theta|\,d\theta$ is

a) 0 b) π c) $\frac{4}{3}$ d) $\frac{3}{8}$

444. If $d[f(x)]=e^{\tan x}\sec^2 x\,dx$, then $f(x)$ is equal to

a) $e^{\tan x}+c$ b) $e^{\sec^2 x}+c$ c) $e^{\sin x}+c$ d) None of these

445. The value of $\int_1^2\{f(g(x))\}^{-1}f'(g(x))\,g'(x)dx$, where $g(1)=g(2)$, is equal to

a) 1 b) 2 c) 0 d) None of these

446. Let f be a function such that $f(1)=4$ and $f'(x)\geq 2$ for $1\leq x<4$. How small can $f(4)$ possibly be?

a) 8 b) 12 c) 16 d) 10

447. $\int\frac{f'(x)}{f(x)\log[f(x)]}dx$ is equal to

a) $\frac{f(x)}{\log f(x)}+c$ b) $f(x).\log f(x)+c$ c) $\log[\log f(x)]+c$ d) $\frac{1}{\log[\log f(x)]}+c$

448. $\int_0^{\pi/2}\frac{2\sqrt{\cos\theta}}{3(\sqrt{\sin\theta}+\sqrt{\cos\theta})}d\theta$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

449. The value of $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots+\frac{n}{n^2+n^2}\right]$

a) $\frac{\pi}{4}$ b) $\log 2$ c) 0 d) 1

450. The value of $\int_{-\pi}^{\pi}\sin x\;f(\cos x)dx$ is

a) π b) 2π c) $2f(1)$ d) None of these

451. If $\int\frac{\sin x}{\cos x(1+\cos x)}dx=f(x)+c$, then $f(x)$ is equal to

a) $\log\left|\frac{1+\cos x}{\cos x}\right|$ b) $\log\left|\frac{\cos x}{1+\cos x}\right|$ c) $\log\left|\frac{\sin x}{1+\sin x}\right|$ d) $\log\left|\frac{1+\sin x}{\sin x}\right|$

452. If $f(x)$ satisfies the requirements of Rolle's Theorem in $[1,2]$ and $f'(x)$ is continuous in $[1,2]$, then $\int_1^2 f'(x)\,dx$ is equal to

a) 0 b) 1 c) 3 d) -1

453. The value of $\int e^{2x}(2\sin 3x+3\cos 3x)dx$ is

a) $e^{2x}\sin 3x+c$ b) $e^{2x}\cos 3x+c$ c) $e^{2x}+c$ d) $e^{2x}\,(2\sin 3x)+c$

454. $\int_{-2}^{2}|[x]|\,dx$ is equal to

a) 1 b) 2 c) 3 d) 4

455. If $\int_0^{\pi/2}\cos^n x\sin^n x\;dx=\lambda\int_0^{\pi/2}\sin^n x\;dx$, then $\lambda=$

a) $\frac{1}{2^{n-1}}$ b) $\frac{1}{2^{n+1}}$ c) $\frac{1}{2^n}$ d) $\frac{1}{2}$

456. If x satisfies the equation

$$x^2\left(\int_0^1\frac{dt}{t^2+2t\cos\alpha+1}\right)-x\left(\int_{-3}^{3}\frac{t^2\sin 2t}{t^2+1}dt\right)-2=0$$

$(0<\alpha<\pi)$, then the value of x is

a) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$ b) $\pm\sqrt{\frac{\sin\alpha}{\alpha}}$ c) $\pm 4\sqrt{\frac{\sin\alpha}{\alpha}}$ d) None of these

457. $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to

a) $\frac{\pi}{12}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

458. $\int_0^2 [x^2] dx$ is

a) $2-\sqrt{2}$ b) $2+\sqrt{2}$ c) $\sqrt{2}-1$ d) $-\sqrt{2}-\sqrt{3}+5$

459. If $f(x)\sin x\cos x\, dx = \frac{1}{2(b^2-a^2)}\log f(x) + c$, the $f(x)$ is equal to

a) $\frac{1}{a^2\sin^2 x + b^2\cos^2 x}$ b) $\frac{1}{a^2\sin^2 x - b^2\cos^2 x}$ c) $\frac{1}{a^2\cos^2 x + b^2\sin^2 x}$ d) $\frac{1}{a^2\cos^2 x - b^2\sin^2 x}$

460. If $f(x) = f(a-x)$, then $\int_0^a f(x)dx$ is equal to

a) $\int_0^a f(x)dx$ b) $\frac{a^2}{2}\int_0^a f(x)dx$ c) $\frac{a}{2}\int_0^a f(x)dx$ d) $-\frac{a}{2}\int_0^a f(x)dx$

461. The value of the integral $\int_{1/e}^{e} |\log x|\, dx$, is

a) $2\left(\frac{e-1}{e}\right)$ b) $2\left(\frac{1-e}{e}\right)$ c) $2-\frac{1}{e}$ d) None of these

462. $\int_0^{\pi/8} \cos^3 4\theta d\theta$ is equal to

a) $\frac{5}{3}$ b) $\frac{5}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

463. $\int_{-1}^{1} \frac{\cosh x}{1+e^{2x}}\, dx$ is equal to

a) 0 b) 1 c) $\frac{e^2-1}{2e}$ d) $\frac{e^2+2}{2e}$

464. If $u_{10} = \int_0^{\pi/2} x^{10}\sin x\ dx$, then the value of $u_{10} + 90\, u_8$, is

a) $9\left(\frac{\pi}{2}\right)^8$ b) $\left(\frac{\pi}{2}\right)^9$ c) $10\left(\frac{\pi}{2}\right)^9$ d) $9\left(\frac{\pi}{2}\right)^9$

465. $\int \frac{x^3\sin[\tan^{-1}(x^4)]}{1+x^8} dx$ is equal to

a) $\frac{1}{4}\cos[\tan^{-1}(x^4)] + c$ b) $\frac{1}{4}\sin[\tan^{-1}(x^4)] + c$

c) $-\frac{1}{4}\cos[\tan^{-1}(x^4)] + c$ d) $\frac{1}{4}\sec^{-1}[\tan^{-1}(x^4)] + c$

466. $\left[\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27} x\, dx\right] + \left[\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27} x\, dx\right]$ equals

a) 27^2 b) -54 c) 54 d) 0

467. $\int \frac{1}{\sin^2 x.\cos^2 x}\ dx$ is equal to

a) $\sin x - \cos x + c$ b) $\tan x + \cot x + c$ c) $\cos x + \sin x + c$ d) $\tan x - \cot x + c$

468. The value of the integral $\int_{-1}^{1}(x-[2x])\, dx$, is

a) 1 b) 0 c) 2 d) 4

469. The function $F(x) = \int_0^x \log\left(\frac{1-x}{1+x}\right) dx$, is

a) An even function b) An odd function c) A periodic function d) None of these

470. If $\int \frac{1}{x\sqrt{1-x^3}}\ dx = a\log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + b$, then a is equal to

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{2}{3}$

471. $\frac{d}{dx}\left(\int_{f(x)}^{g(x)} \phi(t)dt\right)$ is equal to

a) $\phi(g(x)) - \phi(f(x))$

b) $\frac{1}{2}[\phi(g(x))]^2 - \frac{1}{2}[\phi(f(x))]^2$

c) $g'(x)\phi(g(x)) - f'(x)\phi(f(x))$

d) $\phi'(g(x))g'(x) - \phi'(f(x))f'(x)$

472. $\int_1^3 (x-1)(x-2)(x-3)dx$ is equal to

a) 3 b) 2 c) 1 d) 0

473. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x)dx$ is

a) 3 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) 0

474. If $\int g(x)dx = g(x)$, then $\int g(x)\{f(x) + f'(x)\}\, dx$ is equal to

a) $g(x)f(x) - g(x)f'(x) + C$

b) $g(x)f'(x) + C$

c) $g(x)f(x) + C$

d) $g(x)f^2(x) + C$

475. $\int \left(\frac{x+2}{x+4}\right)^2 e^x\, dx$ is equal to

a) $e^x\left(\frac{x}{x+4}\right) + c$ b) $e^x\left(\frac{x+2}{x+4}\right) + c$ c) $e^x\left(\frac{x-2}{x+4}\right) + c$ d) $\left(\frac{2xe^x}{x+4}\right) + c$

476. If $P = \int_0^{3\pi} f(\cos^2 x)\, dx$ and $Q = \int_0^{\pi} f(\cos^2 x)\, dx$, then

a) $P - Q = 0$ b) $P - 2\,Q = 0$ c) $P - 3\,Q = 0$ d) $P - 5\,Q = 0$

477. If $I_1 = \int_0^{3\pi} f(\cos^2 x)dx$ and $I_2 = \int_0^{\pi} f(\cos^2 x)dx$ then

a) $I_1 = I_2$ b) $I_1 = 2I_2$ c) $I_1 = 5I_2$ d) None of these

478. For any $n \in N$ and $x \in R^+$, the value of the integral $\int_0^{n[x]}(x - [x])dx$, is

a) $n[x]$ b) $[x]$ c) $\frac{n}{2}[x]$ d) None of these

479. $\int_0^1 \frac{x^3\, dx}{(x^2+1)^{3/2}}$ is equal to

a) $\left(\sqrt{2}-1\right)^2$ b) $\frac{\left(\sqrt{2}-1\right)^2}{2}$ c) $\frac{\sqrt{2}-1}{2}$ d) None of these

480. The value of $\int_0^{2\pi}|\cos x - \sin x|\, dx$, is

a) $\frac{4}{\sqrt{2}}$ b) $2\sqrt{2}$ c) $\frac{2}{\sqrt{2}}$ d) $4\sqrt{2}$

481. The value of $\int_{-\pi/2}^{\pi/2}(x^3 + x\cos x + \tan^5 x + 1)dx$is equal to

a) 0 b) 2 c) π d) None of these

482. If a function $f(x)$ satisfies $f'(x) = g(x)$ Then, the value of $\int_a^b f(x)g(x)dx$ is

a) $\frac{1}{2}\left[\left(f(b)\right)^2 - \left(F(a)\right)^2\right]$ b) $\frac{1}{2}\left[\left(f(b)\right)^2 + \left(f(a)\right)^2\right]$ c) $\frac{1}{2}[f(b) - f(a)]^2$ d) None of these

483. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax + B\log(9e^{2x} - 4) + C$, then

a) $A = -\frac{3}{2}, B = \frac{35}{36}, C = 0$

b) $A = \frac{35}{36}, B = -\frac{3}{2}, C \in R$

c) $A = -\frac{3}{2}, B = \frac{35}{36}, C \in R$

d) None of these

484. If $\int \frac{\cos 4x+1}{\cot x - \tan x}dx = k\cos 4x + c$, then

a) $k = -1/2$ b) $k = -1/8$ c) $k = -1/4$ d) None of these

485. The value of $\int_{-2}^{4} |x+1| dx$ is equal to

a) 12 b) 14 c) 13 d) 16

486. $\int_{2}^{3} \frac{dx}{x^2-x}$ is equal to

a) $\log\left(\frac{2}{3}\right)$ b) $\log\left(\frac{1}{4}\right)$ c) $\log\left(\frac{4}{3}\right)$ d) $\log\left(\frac{8}{3}\right)$

487. The value of $\int \frac{x^2+1}{x^4-x^2+1} dx$ is

a) $\tan^{-1}(2x^2-1)+c$ b) $\tan^{-1}\frac{x^2+1}{x}+c$ c) $\sin^{-1}\left(x-\frac{1}{x}\right)+c$ d) $\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$

488. $\int \frac{1+\tan x}{e^{-x}\cos x} dx$ is equal to

a) $e^{-x}\tan x+c$ b) $e^{-x}\sec x+c$ c) $e^{x}\sec x+c$ d) $e^{x}\tan x+c$

489. If $f(x)=\int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in

a) $(2, 2)$ b) No value of x c) $(0, \infty)$ d) $(-\infty, 0)$

490. $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}} dx$ is equal to

a) $\sec^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right)+c$ b) $\frac{1}{2}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)+c$ c) $\frac{1}{2}\sec^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right)+c$ d) None of these

491. If $\left(\int_0^a x\, dx\right) \le (a+4)$, then

a) $0 \le a \le 4$ b) $-2 \le a \le 4$ c) $-2 \le a \le 0$ d) $a \le -2$ or $a \ge 4$

492. If $u_n=\int_0^{\pi/4} \tan^n x\, dx$, then u_n+u_{n-2} is equal to

a) $\frac{1}{n-1}$ b) $\frac{1}{n+1}$ c) $\frac{1}{2n-1}$ d) $\frac{1}{2n+1}$

493. $\int_0^{\pi} x \sin^4 x\ dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{3\pi^2}{16}$ c) $\frac{16\pi}{3}$ d) $\frac{16\pi^2}{3}$

494. $\int \frac{\sin x-\cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x\ dx$ is equal to

a) $e^{\sin x}+C$ b) $e^{\sin x-\cos x}+C$ c) $e^{\sin x+\cos x}+C$ d) $e^{\cos x-\sin x}+C$

495. The value of $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ is

a) $3\sqrt{x}+3(\sqrt[3]{x})-6\sqrt[6]{x}+6\log(\sqrt[6]{x}+1)+c$ b) $2\sqrt{x}+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+c$

c) $2\sqrt{x}-3(\sqrt[3]{x})+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+c$ d) None of the above

496. $\int \sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}} dx$ is equal to

a) $\frac{21}{32}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}+C$ b) $\frac{32}{21}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}+C$ c) $\frac{7}{32}\left\{1+\sqrt[3]{x^4}\right\}^{8/7}+C$ d) None of these

497. If $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = f(x)-\log(1+x^2)+c$, then $f(x)$ is equal to

a) $2x\tan^{-1}x$ b) $-2x\tan^{-1}x$ c) $x\tan^{-1}x$ d) $-x\tan^{-1}x$

498. If $I_{10}=\int_0^{\pi/2} x^{10}\sin x\, dx$. Then, the value of $I_{10}+90I_8$ is

a) $10\left(\frac{\pi}{2}\right)^3$ b) $10\left(\frac{\pi}{2}\right)^9$ c) $\frac{\pi}{2}$ d) 0

499. $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x+b^2\sin^2 x}$ is equal to

a) $\frac{\pi}{ab}$ b) $\frac{\pi}{2ab}$ c) $\frac{\pi^2}{ab}$ d) $\frac{\pi^2}{2ab}$

500. Let $f(x)=x-[x]$, for every real x, where $[x]$is the greatest integer less than or equal to x. Then, $\int_{-1}^{1} f(x) dx$ is

a) 1 b) 2 c) 3 d) 0

501. $\int_{-\pi}^{\pi}[\cos px - \sin qx]^2 dx$, where p, q are integers is equal to

a) $-\pi$ b) 0 c) π d) 2π

502. $\lim_{n\to\infty} \sum_{r=1}^{n} \left|\frac{r^3}{r^4+n^4}\right|$ equals

a) $\log 2$ b) $\frac{1}{2}\log 2$ c) $\frac{1}{3}\log 2$ d) $\frac{1}{4}\log 2$

503. If $\int_{\pi/2}^{x} \sqrt{3-2\sin^2 u}\ du + \int_0^y \cos t\, dt = 0$ then, $\frac{dy}{dx}$ is equal to

a) $\frac{\sqrt{4-3\sin^2 x}}{\cos y}$ b) $-\frac{\sqrt{3-2\sin^2 x}}{\cos y}$ c) $\sqrt{3-2\sin^2 x} + \cos y$ d) None of these

504. If $I_n = \int \tan^n x\, dx$ for $n \geq 2$, then $I_n + I_{n-2}$ is equal to

a) $\tan^n x + c$ b) $\frac{(\tan^{n-1} x)}{n-1} + c$ c) $\frac{\tan^n x}{n} + c$ d) $n \tan^n x + c$

505. The value of $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$, is

a) $\frac{\pi}{2}$ b) 0 c) π d) 2π

506. $\int(\sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x)dx$ is equal to

a) $x + c$ b) $\frac{3}{2}\sin 2x + c$

c) $-\frac{3}{2}\cos 2x + c$ d) $\frac{1}{3}\sin 3x - \cos 3x + c$

507. $\int_a^b \frac{|x|}{x} dx, a < 0 < b$,is equal to

a) $|b| - |a|$ b) $|b| + |a|$ c) $|a - b|$ d) None of these

508. $\int_0^{10\pi} |\sin x| dx$ is

a) 20 b) 8 c) 10 d) 18

509. If $\int_0^1 f(x)\ dx = M, \int_0^1 g(x)dx = N$, then which of the following is correct?

a) $\int_0^1 (f(x) + g(x))dx = M + N$

b) $\int_0^1 f(x)g(x)dx = MN$

c) $\int_0^1 \frac{1}{f(x)} dx = \frac{1}{M}$

d) $\int_0^1 \frac{f(x)}{g(x)} dx = \frac{M}{N}$

510. $\int \frac{(\sec^2 x - 7)}{\sin^7 x}\ dx$ is equal to

a) $\frac{\tan x}{\sin^7 x} + c$ b) $\frac{\cos x}{\sin^7 x} + c$ c) $\frac{\sin x}{\cos^7 x} + c$ d) $\frac{\sin x}{\tan^7 x} + c$

511. The value of $\int_{-1/2}^{1/2} \left|x \cos\left(\frac{\pi x}{2}\right)\right| dx$, is

a) $\frac{\pi\sqrt{2} - 4\sqrt{2} - 8}{\pi^2}$ b) $\frac{\sqrt{2} + 4\pi\sqrt{2} - 8}{\pi^2}$ c) $\frac{\pi\sqrt{2} + 4\sqrt{2} + 8}{\pi^2}$ d) None of these

512. $\int \frac{\log_e(\tan x)}{\sin x \cos x} dx$ is equal to

a) $[\log_e(\tan x)]^2 + c$ b) $\frac{1}{2}(\log_e \tan x)^2 + c$ c) $\log_e(\log_e \tan x) + c$ d) $\log_e \tan x + c$

513. $\int \left(\frac{x+2}{x+4}\right)^2 e^x dx$ is equal to

a) $e^x\left(\frac{x}{x+4}\right)+c$ b) $e^x\left(\frac{x+2}{x+4}\right)+c$ c) $e^x\left(\frac{x-2}{x+4}\right)+c$ d) $\left(\frac{2xe^x}{x+4}\right)+c$

514. $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$ is equal to

a) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

b) $\frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

c) $\frac{1}{\sin(a+b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

d) $\frac{1}{\cos(a+b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

515. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx =$

a) 1 b) 0 c) −1 d) None of these

516. The value of $\int_{-1}^{3}\left[\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right] dx$ is

a) 2π b) π c) π/2 d) π/4

517. If for every integer n $\int_n^{n+1} f(x)dx = n^2$, then the value of $\int_{-2}^{4} f(x)dx$, is

a) 16 b) 14 c) 19 d) None of these

518. $\int_0^{2\pi} (\sin x + |\sin x|)dx$ is equal to

a) 4 b) 0 c) 1 d) 8

519. If $f(x) = \lim_{y\to x} \frac{\sin^2 y - \sin^2 x}{y^2 - x^2}$, then $\int 4x\, f(x)dx$ is equal to

a) $\cos 2x + c$ b) $2\cos 2x + c$ c) $-\cos 2x + c$ d) $-2\cos 2x + c$

520. $\int \frac{x^4+1}{(1-x^4)^{3/2}} dx$ is equal to

a) $\frac{x}{\sqrt{1-x^4}}+C$ b) $\frac{-x}{\sqrt{1-x^4}}+C$ c) $\frac{2x}{\sqrt{1-x^4}}+C$ d) $\frac{-2x}{\sqrt{1-x^4}}+C$

521. $\int \frac{x\, e^x}{(1+x)^2} dx$ is equal to

a) $\frac{e^x}{x+1}+C$ b) $e^x(x+1)+C$ c) $-\frac{e^x}{(x+1)^2}+C$ d) $\frac{e^x}{1+x^2}+C$

522. $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx$is equal to

a) 0 b) 1 c) 2 d) π

523. $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$ is equal to

a) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$ b) $\frac{a}{b}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$

c) $\frac{b}{a}\tan^{-1}\left(\frac{b\tan x}{b}\right)+c$ d) None of these

524. If $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\, \cos^2 x} dx = A\sin 2x + B$, then

a) $A = -\frac{1}{2}$ b) $A = \frac{1}{2}$ c) $A = -1$ d) $A = 1$

525. $\int \frac{4^{x+1}-7^{x-1}}{28^x} dx$ is equal to

a) $\frac{1}{7\,\log_e 4}4^{-x}-\frac{4}{\log_e 7}7^{-x}+c$
b) $\frac{1}{7\,\log_e 4}4^{-x}+\frac{4}{\log_e 7}7^{-x}+c$
c) $\frac{4^{-x}}{\log_e 7}-\frac{7^{-x}}{\log_e 4}+c$
d) $\frac{4^{-x}}{\log_e 4}-\frac{7^{-x}}{\log_e 7}+c$

526. If $\int \frac{dx}{5+4\cos x}=A\tan^{-1}(B\tan x/2)+C$, then
a) $A=1, B=1/3$
b) $A=2/3, B=1/3$
c) $A=-1, B=1/3$
d) $A=1/3, B=2/3$

527. Let $f(x)$ be a function such that, $f(0)=f'(0)=0, f''(x)=\sec^4 x+4$, then the function is
a) $\log|(\sin x)|+\frac{1}{3}\tan^3 x+x$
b) $\frac{2}{3}\log|(\sec x)|+\frac{1}{6}\tan^2 x+2x^2$
c) $\log|(\cos x)|+\frac{1}{6}\cos^2 x+\frac{x^2}{5}$
d) None of the above

528. $\int 32x^3(\log x)^2 dx$ is equal to
a) $8x^4(\log x)^2+c$
b) $x^4\{8(\log x)^2-4\log x+1\}+c$
c) $x^4\{8(\log x)^2-4\log x\}+c$
d) $x^3\{(\log x)^2-2\log x\}+c$

529. $\int_1^{\frac{4\sqrt{3}}{5}-1}\frac{x+2}{\sqrt{x^2+2x-3}}dx$ equals to
a) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log 3$
b) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log 3$
c) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log(\sqrt{3}+2)$
d) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log(\sqrt{3}+2)$

530. $\int_{-2}^{2}|[x]|\,dx$is equal to
a) 1
b) 2
c) 3
d) 4

531. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where
$f(x)=\int_1^x \frac{\log t}{1+t}dt$. Then, $F(e)$equals
a) 1/2
b) 0
c) 1
d) 2

532. $\int_{\pi/6}^{\pi/3}\frac{1}{1+\tan^3 x}dx$ is
a) $\frac{\pi}{12}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{6}$

533. If $\int f(x)dx=g(x)+c$, then $\int f^{-1}(x)dx$ is equal to
a) $xf^{-1}(x)+c$
b) $f\{g^{-1}(x)\}+c$
c) $xf^{-1}(x)-g\{f^{-1}(x)\}+c$
d) $g^{-1}(x)+c$

534. $\int x\log x\ \ dx$ is equal to
a) $\frac{x^2}{4}\ (2\log x-1)+c$
b) $\frac{x^2}{2}\ (2\log x-1)+c$
c) $\frac{x^2}{4}\ (2\log x+1)+c$
d) $\frac{x^2}{2}\ (2\log x+1)+c$

535. The value of the integral $\int_{1/3}^{1}\frac{(x-x^3)^{1/3}}{x^4}dx$, is
a) 6
b) 0
c) 3
d) 4

536. $\int_{1/3}^{3}\frac{1}{x}\left(\frac{1}{x}-x\right)dx$ is equal to
a) $\frac{\sqrt{3}}{2}$
b) $\frac{\sqrt{3}\pi}{2}$
c) 0
d) None of these

537. Let $I=\int_0^1\frac{e^x}{x+1}dx$, then the value of the integral $\int_0^1\frac{x\,e^{x^2}}{x^2+1}dx$, is
a) I^2
b) $\frac{1}{2}I$
c) $2I$
d) $\frac{1}{2}I^2$

538. $\int(\sin^4 x-\cos^4 x)dx$ is equal to
a) $-\frac{\cos 2x}{2}+c$
b) $-\frac{\sin 2x}{2}+c$
c) $\frac{\sin 2x}{2}+c$
d) $\frac{\cos 2x}{2}+c$

539. The value of the integral $\int_0^{\pi/2}\sin^6 x\,dx$, is

a) $\frac{3\pi}{4}$ b) $\frac{5}{32}\pi$ c) $\frac{3}{16}\pi$ d) None of these

540. Let $\int_0^a f(x)\,dx = \lambda$ and $\int_0^a (2a-x)\,dx = \mu$. Then, $\int_0^{2a} f(x)\,dx$ is equal to

a) $\lambda+\mu$ b) $\lambda-\mu$ c) $2\lambda-\mu$ d) $\lambda-2\mu$

541. $\int \frac{dx}{7+5\cos x}$ is equal to

a) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{1}{\sqrt{6}}\tan\frac{x}{2}\right)+c$ b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$

c) $\frac{1}{4}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$ d) $\frac{1}{7}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$

542. $\int_0^{\pi/3} \frac{\cos x+\sin x}{\sqrt{1+\sin 2x}}\,dx$ is equal to

a) $\frac{4\pi}{3}$ b) $\frac{2\pi}{3}$ c) π d) $\frac{\pi}{3}$

543. The value of $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ is, (where $[x]$ and $\{x\}$ denotes the integral part and fractional part functions of x and $n \in N$)

a) $n+2$ b) $n+1$ c) n d) $n-1$

544. The value of $\int_{\pi/4}^{\pi/2} e^x(\log\sin x+\cot x)dx$ is

a) $e^{\pi/4}\log 2$ b) $-e^{\pi/4}\log 2$ c) $\frac{1}{2}e^{\pi/4}\log 2$ d) $-\frac{1}{2}e^{\pi/4}\log 2$

545. If $\int_0^1 \cot^{-1}(1-x+x^2)dx = k\int_0^1 \tan^{-1}x\ dx$, then $k=$

a) 1 b) 2 c) π d) 2π

546. The value of $\int_0^{\pi/2} \frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}dx$ is

a) 0 b) 1 c) 2 d) 3

547. If $\int \frac{1}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}\,dx = \frac{1}{4}\tan^{-1}(B\tan x)+c$, then

a) $A=\frac{1}{4}, B=\frac{1}{2}, C=1$ b) $A=\frac{1}{2}, B=\frac{1}{4}, C=1$ c) $A=1, B=\frac{1}{2}, C=\frac{1}{4}$ d) $A=\frac{1}{4}, B=1, C=\frac{1}{2}$

548. $\int \sqrt{1+\sin\frac{x}{2}}\,dx$ is equal to

a) $\frac{1}{4}\left[\cos\frac{x}{4}-\sin\frac{x}{4}\right]+c$ b) $4\left[\cos\frac{x}{4}-\sin\frac{x}{4}\right]+c$ c) $4\left[\sin\frac{x}{4}-\cos\frac{x}{4}\right]+c$ d) $4\left[\sin\frac{x}{4}+\cos\frac{x}{4}\right]+c$

549. $\int \frac{(x+3)e^x}{(x+4)^2}\,dx$ is equal to

a) $\frac{1}{(x+4)^2}+c$ b) $\frac{e^x}{(x+4)^2}+c$ c) $\frac{e^x}{x+4}+c$ d) $\frac{e^x}{x+3}+c$

550. $\int_0^{\pi/2} x\sin^2 x\cos^2 x\,dx$ is equal to

a) $\frac{\pi^2}{32}$ b) $\frac{\pi^2}{16}$ c) $\frac{\pi}{32}$ d) None of these

551. If $\int uv''\,dx = uv'-vu'+a$, then a is equal to

a) $\int u''v\,dx$ b) $\int u'v\,dx$ c) $\int uv'\,dx$ d) $\int u''\ dx$

552. $\int(\sqrt[3]{x})\left(\sqrt[5]{1+\sqrt[3]{x^4}}\right)dx$ is equal to

a) $\left(1+x^{\frac{3}{4}}\right)^{\frac{6}{5}}+c$ b) $\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}}+c$ c) $\frac{5}{8}\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}}+c$ d) $\frac{1}{6}\left(1+x^{\frac{4}{3}}\right)^{6}+c$

553. $\int \frac{1}{\cos x-\sin x}dx$ is equal to

a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

b) $\frac{1}{\sqrt{2}}\log\left|\cot\frac{x}{2}\right|+C$

c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

554. The value of $\int_0^1 \left(1+e^{-x^2}\right) dx$, is

a) -1 b) 2 c) $1+e^{-1}$ d) None of these

555. The value of $\int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}\, dx$ is

a) 0 b) 1 c) 2 d) 3

556. $\int \sqrt{x} e^{\sqrt{x}}\, dx$ is equal to

a) $2\sqrt{x}-e^{\sqrt{x}}-4\sqrt{xe^{\sqrt{x}}}+c$

b) $(2x-4\sqrt{x}+4)e^{\sqrt{x}}+c$

c) $(2x+4\sqrt{x}+4)e^{\sqrt{x}}+c$

d) $(1-4\sqrt{x})e^{\sqrt{x}}+c$

557. The value of $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right\}^{1/n}$, is

a) $\frac{e^{\pi/2}}{2e^2}$ b) $2e^2e^{\pi/2}$ c) $\frac{2}{e^2}e^{\pi/2}$ d) None of these

558. The solution of the equation $\int^x \sqrt{2}\frac{1}{x\sqrt{x^2-1}}dx=\frac{\pi}{12}$, is

a) $x=3$ b) $x=4$ c) $x=1$ d) None of these

559. If $f(a+x)=f(x)$, then $\int_0^{na} f(x)dx$ is equal to $(n\in N)$

a) $(n-1)\int_0^a f(x)dx$ b) $n\int_0^a f(x)dx$ c) $\int_0^{(n-1)a} f(x)dx$ d) None of these

560. The value of the integral $\int_{-\pi/3}^{\pi/3}\frac{x\sin x}{\cos^2 x}dx$, is

a) $(\pi/3-\log\tan 3\pi/2)$

b) $2(2\pi/3-\log\tan 5\pi/12)$

c) $3(\pi/2-\log\sin\pi/12)$

d) None of these

561. $\int\frac{a^{x/2}}{\sqrt{a^{-x}-a^x}}dx$ is equal to

a) $\frac{1}{\log a}\sin^{-1}(a^x)+c$ b) $\frac{1}{\log a}\tan^{-1}(a^x)+c$ c) $2\sqrt{a^{-x}-a^x}+c$ d) $\log(a^x-1)+c$

562. $\int\frac{x+\sin x}{1+\cos x}dx$ is equal to

a) $x\tan\frac{x}{2}+c$ b) $\log(1+\cos x)+c$ c) $\cot\frac{x}{2}+c$ d) $\log(x+\sin x)+c$

563. $\int_{-1}^1 \sin^3 x\cos^2 x\ dx$ is equal to

a) -1 b) 1 c) 0 d) None of these

564. $\int x\{f(x^2)g''(x^2)-f''(x^2)g(x^2)\}dx$ is equal to

a) $f(x^2)g'(x^2)-g(x^2)f'(x^2)+c$

b) $\frac{1}{2}\{f(x^2)g(x^2)f'(x^2)\}+c$

c) $\frac{1}{2}\{f(x^2)g'(x^2)-g(x^2)f'(x^2)\}+c$

d) None of the above

565. If $I=\int_{-2}^2|1-x^4|\,dx$, then I equals

a) 6 b) 8 c) 12 d) 21

566. $\int_{-1}^1[x+[x+[x]]]\,dx$, where $[\cdot]$ denotes greatest integer function, is equal to

a) -2 b) -1 c) -3 d) -4

567. The value of $\int_0^{\pi}(\sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x)\, dx$ depends upon

a) a_1 and a_2 b) a_0 and a_3 c) a_2 and a_3 d) a_1 and a_3

568. The value of the integral $\int \frac{x \sin x^2\, e^{\sec x^2}}{\cos^2 x^2} dx$, is

a) $\frac{1}{2} e^{\sec x^2} + C$ b) $\frac{1}{2} e^{\sin x^2} + C$ c) $\frac{1}{2} \sin x^2\, e^{\cos^2 x^2} + C$ d) None of these

569. $\int \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1 + x}} dx =$

a) $\frac{1}{2}\sqrt{1 + x} + C$ b) $\frac{2}{3}(1 + x)^{3/2} + C$ c) $\sqrt{1 + x} + C$ d) $2(1 + x)^{3/2} + C$

570. $\int_0^{\infty} \frac{dx}{\left[x + \sqrt{x^2 + 1}\right]^3} =$

a) $\frac{3}{8}$ b) $\frac{1}{8}$ c) $-\frac{3}{8}$ d) None of these

571. If $f(y) = e^y, g(y) = y;\quad y > 0$ and $F(t) = \int_0^t f(t - y) g(y)\, dy$, then

a) $F(t) = 1 - e^{-t}(1 + t)$ b) $F(t) = e^t - (1 + t)$

c) $F(t) = te^t$ d) $F(t) = te^{-t}$

572. $\int_0^{\pi} [2 \sin x]\, dx =$

a) $2\pi/3$ b) $-5\pi/3$ c) $-\pi$ d) -2π

573. The value of $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$ is

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{2}$ d) $\ln 2$

574. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ is equal to

a) $\tan^{-1} \sqrt{4x + 3} + c$ b) $3 \tan^{-1} \sqrt{4x + 3} + c$

c) $2 \tan^{-1} \sqrt{4x + 3} + c$ d) $4 \tan^{-1} \sqrt{4x + 3} + c$

575. If $f(a + b - x) = f(x)$, then $\int_a^b x\, f(x) dx$ is equal to

a) $\frac{a + b}{2} \int_a^b f(b - x) dx$ b) $\frac{a + b}{2} \int_a^b f(x) dx$ c) $\frac{b - a}{2} \int_a^b f(x) dx$ d) $\frac{a + b}{2} \int_a^b f(a + b + x) dx$

576. If $f(x)$ is differentiable and $\int_0^{t^2} x\, f(x) dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals

a) $\frac{2}{5}$ b) $-\frac{5}{2}$ c) 1 d) $\frac{5}{2}$

577. Let $f: (0, \infty) \to R$ and $F(x) = \int_0^x f(x) dt$. If $F(x^2) = x^2(1 + x)$, then $f(4)$ equals

a) 5/4 b) 7 c) 4 d) 2

578. $\int_0^{\sqrt{2}} [x^2]\, dx$, is

a) $2 - \sqrt{2}$ b) $2 + \sqrt{2}$ c) $\sqrt{2} - 1$ d) $\sqrt{2} - 2$

579. $\int \frac{e^x}{(2+e^x)(e^x+1)} dx$ is equal to

a) $\log\left(\frac{e^x + 1}{e^x + 2}\right) + c$ b) $\log\left(\frac{e^x + 2}{e^x + 1}\right) + c$ c) $\left(\frac{e^x + 1}{e^x + 2}\right) + c$ d) $\left(\frac{e^x + 2}{e^x + 1}\right) + c$

580. The value of $\int_0^{\pi/2} \frac{\sin^3 x \cos x}{\sin^4 x + \cos^4 x} dx$, is

a) $\pi/8$ b) $\pi/4$ c) $\pi/2$ d) π

581. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then

a) $1 < \alpha < 2$ b) $\alpha < 0$ c) $0 < \alpha < 1$ d) $\alpha = 0$

582. $\int_0^{\pi/4} \sin(x-[x])\, dx$ is equal to

a) $\frac{1}{2}$ b) $1-\frac{1}{\sqrt{2}}$ c) 1 d) None of these

583. The value of $\int_{-1}^{2} \frac{|x|}{x} dx$, is

a) 0 b) 1 c) 3 d) None of these

584. $\int \frac{1+x^2}{x\sqrt{1+x^4}} dx$ is equal to

a) $-\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2-2}\right|+C$

b) $\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2+2}\right|+C$

c) $\log\left|x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2-2}\right|+C$

d) None of these

585. $\lim_{x\to 0} \frac{\int_0^{x^2} \tan\sqrt{t}\, dt}{x^3}$ is equal to

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) 1 d) None of these

586. If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is

a) An odd function b) An even function

c) Neither even nor odd d) 0

587. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$, is

a) 1 b) 0 c) -1 d) 2

588. The value of $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx$ is equal to

a) $(\sqrt{2}-1)\pi$ b) $(\sqrt{2}+1)\pi$ c) π d) None of these

589. Let $f(x)$ be a differentiable function such that $f(1)=2$. If $\lim_{x\to 1}\int_2^{f(x)} \frac{2t}{x-1} dt = 4$, then the value of $f'(1)$ is

a) 1 b) 2 c) 4 d) None of these

590. Let $f(x) = \frac{\sin^2 \pi x}{1+\pi^x}$. Then , $\int [f(x)+f(-x)]dx$ is equal to

a) 0 b) $x+c$

c) $\frac{x}{2}-\frac{\cos \pi x}{2\pi}+c$ d) $\frac{x}{2}-\frac{\sin 2\pi x}{4\pi}+c$

591. $\int_0^{2\pi} (\sin x + |\sin x)\, dx$ is equal to

a) 0 b) 4 c) 8 d) 1

592. If $f(x) = \int_0^x \sin^4 t\, dt$, then $f(x+2\pi)$ is equal to

a) $f(x)$ b) $f(x)+f(2\pi)$ c) $f(x)-f(2\pi)$ d) $f(x).f(2\pi)$

593. $\frac{d}{dx}\int_2^{x^2}(t-1)dt$ is equal to

a) x^2-1 b) $x(x^2-1)$ c) $x-1$ d) $2x(x^2-1)$

594. The value of $\int_0^2 \left|\cos\left(\frac{\pi x}{2}\right)\right|$, is

a) 2π b) $\pi/2$ c) $3/4\pi$ d) $4/\pi$

595. $\int_{-1}^{1}|1-x|\,dx$ is equal to

a) -2 b) 0 c) 2 d) 4

596. $\int_{-\pi}^{\pi}\frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) π

597. The value of $\int_{-\pi}^{\pi}\sin^3 x\cos^2 x\,dx$ is equal to

a) 1 b) 2 c) 3 d) 0

598. $\int\frac{x^2-1}{x^4+x^2+1}dx$ is equal to

a) $\log(x^4+x^2+1)+c$ b) $\log\frac{x^2-x+1}{x^2+x+1}+c$ c) $\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}+c$ d) $\frac{1}{2}\log\frac{x^2+x+1}{x^2-x+1}+c$

599. $\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx$ is equal to

a) $2\sec x+c$ b) $2\tan x+c$ c) $\tan x+c$ d) None of these

600. $\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$ is equal to

a) $-\frac{1}{4}\log 3$ b) $\frac{1}{4}\log 3$ c) $-\frac{1}{3}\log 4$ d) None of these

601. The value of integral $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx$ is

a) $\frac{\pi}{2}+1$ b) $\frac{\pi}{2}-1$ c) -1 d) 1

602. Let $p(x)$ be a function defined on R such that $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$, $p'(x)=p'(1-x)$, for all $x\epsilon[0,1]$, $p(0)=1$ and $p(1)=41$. Then, $\int_0^1 p(x)dx$ equals

a) $\sqrt{41}$ b) 21 c) 41 d) 42

603. $\int\frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}dx$ is equal to

a) $\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+C$

b) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+C$

c) $\log\left\{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}\right\}+C$

d) None of these

604. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)}xg[x(1-x)]dx$ and $I_2=\int_{f(-a)}^{f(a)}g[x(1-x)]dx$, then the value of $\frac{I_2}{I_1}$ is

a) 2 b) -3 c) -1 d) 1

605. Let $f: R\to R$ be a continuous function such that $f(x)$ is not identically equal to zero. If $\int_2^5 f(x)\,dx=\int_{-2}^{5}f(x)\,dx$, then $f(x)$ is

a) An even function b) An odd function c) A periodic function d) None of these

606. For $y=f(x)=\int_0^x 2|t|\,dt$, the tangent lines parallel to the bisector of the first quadrant angle are

a) $y=x\pm\frac{1}{4}$ b) $y=x\pm\frac{3}{2}$ c) $y=x\pm\frac{1}{2}$ d) None of these

607. $\int\cos\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]dx$ is equal to

a) $\frac{1}{2}x^2+c$

b) $\frac{1}{2}\sin\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]+c$

c) $-\frac{1}{2}x^2+c$

d) $\frac{1}{2}x+c$

608. The value of $\int_2^3 \frac{x+1}{x^2(x-1)}\, dx$ is

a) $\log\frac{16}{9}+\frac{1}{6}$ b) $\log\frac{16}{9}-\frac{1}{6}$ c) $2\log 2-\frac{1}{6}$ d) $\log\frac{4}{3}-\frac{1}{6}$

609. The value of $\int x(x^x)^x\,(2\log x+1)dx$ is

a) $(x^x)^x+c$ b) x^x+c c) $x^{\log x}+c$ d) None of these

610. $\int \frac{dx}{(\sin x-2\cos x)(2\sin x+\cos x)}$ is equal to

a) $\log_e \sqrt[5]{\frac{\tan x-2}{2\tan x+1}}+c$

b) $\frac{1}{4}\log\left(\frac{\tan x-2}{\tan x+2}\right)+c$

c) $-\frac{1}{4}\log\left(\frac{2\sin x+\cos x}{\sin x+2\cos x}\right)+c$

d) None of the above

611. The value of $\int_0^\infty \frac{dx}{(a^2+x^2)}$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{2a}$ c) $\frac{\pi}{a}$ d) $\frac{1}{2a}$

612. The value of the integral $\int_1^e (\log x)^3 dx$ is

a) $6+2e$ b) $6-2e$ c) $2e-6$ d) None of these

613. The value of $\int_1^{\sqrt[7]{2}} \frac{1}{x(2x^7+1)}dx$, is

a) $\log(6/5)$ b) $6\log(6/5)$ c) $(1/7)\log(6/5)$ d) $(1/12)\log(6/5)$

614. If $\int f(x)dx=F(x)$, then $\int x^3 f(x^2)dx$ is equal to

a) $\frac{1}{2}\{x^2F(x^2)-\int F(x^2)dx^2\}$

b) $\frac{1}{2}\{x^2F(x^2)-\int F(x^2)dx\}$

c) $\frac{1}{2}\left(x^2F(x)-\frac{1}{2}\int F(x^2)dx\right)$

d) None of the above

615. If $\int_0^{2a} f(x)dx=2\int_0^a f(x)dx$, then

a) $f(2a-x)=-f(x)$

b) $f(2a-x)=f(x)$

c) $f(x)$ is an odd function

d) $f(x)$ is an even function

616. The value of the integral $\int_0^\pi x\log\sin x\, dx$, is

a) $\frac{\pi}{2}\log 2$ b) $\frac{\pi^2}{2}\log 2$ c) $-\frac{\pi^2}{2}\log 2$ d) None of these

617. The value of $\int_1^2 [f\{g(x)\}]^{-1}\ f'\{g(x)\}g'(x)dx$, where $g(1)=g(2)$ is equal to

a) 1 b) 2 c) 0 d) None of these

618. If $f(t)$ is a continuous function defined on $[a,b]$ such that $f(t)$ is an odd function, then the function $\phi(x)=\int_a^x f(t)\,dt$

a) Is an odd function

b) Is an even function

c) Is an increasing function on $[a,b]$

d) None of these

619. If $\int \frac{x^5}{\sqrt{1+x^3}}dx$ is equal to

a) $\frac{2}{9}(1+x^3)^{5/2}+\frac{2}{3}(1+x^3)^{3/2}+C$

b) $\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+C$

c) $\log\left|\sqrt{x}+\sqrt{1+x^3}\right|+C$

d) $x^2\log(1+x^3)+C$

620. The value of the integral $\int_{-\pi/2}^{\pi/2}\sqrt{\cos x-\cos^3 x}\ dx$ is

a) 0 b) $\frac{4}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{5}$

621. The value of the integral $\int_0^{\pi}\frac{x\,dx}{1+\cos\alpha\sin x}(0<\alpha<\pi)$ is

a) $\frac{\pi\alpha}{\sin\alpha}$ b) $\frac{\pi\alpha}{\cos\alpha}$ c) $\frac{\pi\alpha}{1+\sin\alpha}$ d) $\frac{\pi\alpha}{1+\cos\alpha}$

622. The value of $\int_{-3}^{3}(ax^5+bx^3+cx+k)dx$, where a,b,c,k are constant, depends only on

a) a and k b) a and b c) a,b and c d) k

623. $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=1}^{n}r\,e^{r/n}=$

a) 0 b) 1 c) e d) $2e$

624. If $I_n=\int x^n.e^{cx}\,dx$ for $n\geq 1$, then $c.I_n+n.I_{n-1}$ is equal to

a) $x^n\,e^{cx}$ b) x^n c) e^{cx} d) x^n+e^{cx}

625. The value of $\int_0^{\pi/2}\frac{1+2\cos x}{(2+\cos x)^2}\,dx$ is

a) $-\frac{1}{2}$ b) 2 c) $\frac{1}{2}$ d) None of these

626. The value of $\int_0^{\pi/4}(\pi x-4x^2)\log(1+\tan x)\,dx$ is

a) $\frac{\pi^3}{192}\log_e 2$ b) $\frac{\pi^3}{192}\log\sqrt{2}$ c) $\frac{\pi^3}{36}\log 2$ d) $\frac{\pi^3}{48}\log\sqrt{2}$

627. $\int_2^{\pi/2}\frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$ is equal to

a) $\log\frac{4}{3}$ b) $\log\frac{1}{3}$ c) $\log\frac{3}{4}$ d) None of these

628. $\int\frac{x^2}{(a+bx^2)^{5/2}}dx$ is equal to

a) $-\frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2}+C$

b) $\frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2}+C$

c) $\frac{1}{2a}\left(\frac{x^2}{a+bx^2}\right)^{2/3}+C$

d) None of these

629. If $f(x)$ and $g(x)$ are two integrable functions defined on $[a,b]$ then $\left|\int_a^b f(x)g(x)dx\right|$, is

a) Less than $\sqrt{\int_a^b f(x)dx\int_a^b g(x)dx}$

b) Less than or equal to $\sqrt{\int_a^b f^2(x)dx\int_a^b g^2(x)dx}$

c) Less than or equal to $\sqrt{\left\{\int_a^b f^2(x)dx\right\}\left\{\int_a^b g^2(x)dx\right\}}$

d) None of these

630. $\int \frac{3^x}{\sqrt{9^x-1}}\,dx$ is equal to

a) $\frac{1}{\log 3}\log|3^x+\sqrt{9^x-1}|+c$
b) $\frac{1}{\log 3}\log|9^x+\sqrt{9^x-1}|+c$
c) $\frac{1}{\log 9}\log|3^x+\sqrt{9^x-1}|+c$
d) $\frac{1}{\log 9}\log|3^x-\sqrt{9^x-1}|+c$

631. $\int\{1+2\tan x(\tan x+\sec x)\}^{1/2}\,dx$ is equal to

a) $\log(\sec x+\tan x)+c$
b) $\log(\sec x+\tan x)^{1/2}+c$
c) $\log\sec x(\sec x+\tan x)+c$
d) None of the above

632. If $f(x)=\int_{-1}^{x}|t|dt$,then for any $x\geq 0, f(x)$is equal to

a) $1-x^2$
b) $\frac{1}{2}(1+x^2)$
c) $1+x^2$
d) $\frac{1}{2}(1-x^2)$

633. The value of the integral $\int_a^b \frac{|x|}{x}dx, a<b$ is

a) $|a|-|b|$
b) $|b|-|a|$
c) $|a|-b$
d) $|b|-a$

634. $\int \frac{1+\tan^2 x}{1-\tan^2 x}\,dx$ equals

a) $\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$
b) $\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$
c) $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$
d) $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

635. If $\int_0^a f(x)dx=\lambda\ and\ \int_0^a f(2a-x)\,dx=\mu$, then $\int_0^{2a} f(x)\,dx=$

a) $\lambda+\mu$
b) $\lambda-\mu$
c) $2\lambda+\mu$
d) $\lambda+2\mu$

636. $\int_2^3\{x\}dx$ is equal to (where $\{\cdot\}$ denotes fractional part of x)

a) $\frac{17}{2}$
b) $\frac{7}{2}$
c) $\frac{5}{2}$
d) $\frac{1}{2}$

637. The value of $\int_0^{\pi/2}\frac{1}{1+\tan^3 x}dx$, is

a) 0
b) 1
c) $\pi/2$
d) $\pi/4$

638. $\int_{-\pi/4}^{\pi/4} e^{-x}\sin x\ dx$ is

a) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$
b) $\frac{\sqrt{2}}{2}e^{-\pi/4}$
c) $-\sqrt{2}(e^{-\pi/4}-e^{\pi/4})$
d) Zero

639. The greatest value of $f(x)=\int_{-1/2}^{x}|t|dt$ on the interval $[-1/2,1/2]$ is

a) $\frac{3}{8}$
b) $\frac{1}{4}$
c) $-\frac{3}{8}$
d) $-\frac{1}{2}$

640. Let a,b,c be non-zero real numbers such that $\int_0^1(1+\cos^8 x)(ax^2+bx+c)dx$

$=\int_0^2(1+\cos^8 x)(ax^2+bx+c)dx$

Then, the quadratic equation $ax^2+bx+c=0$ has

a) No root in $(0,2)$
b) At least one root in $(1,2)$
c) A double root in $(0,2)$
d) Two imaginary roots

641. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B, f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)dx=\frac{2A}{\pi}$, then constant A and B are

a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$
b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$
c) 0 and $-\frac{4}{\pi}$
d) $\frac{4}{\pi}$ and 0

642. If $I=\int_0^1\frac{dx}{\sqrt{1+x^4}}$, then

a) $I>0.78$
b) $I<0.78$
c) $I>1$
d) None of these

643. If $I_k=\int_{-2k\pi}^{2k\pi}|\sin x|[\sin x]\,dx, \forall k\in N$, where $[\cdot]$ denotes the greatest integer function, then $\sum_{k=1}^{10} I_k$ is equal to

a) -110
b) -440
c) -330
d) -220

644. $\int_0^{10}|x-5|dx|$is equal to

a) 17 b) 9 c) 12 d) 18

645. $\int \frac{e^x}{(2+e^x)(e^x+1)} dx =$

a) $\log\left(\frac{e^x+1}{e^x+2}\right)+C$ b) $\log\left(\frac{e^x+2}{e^x+1}\right)+C$ c) $\frac{e^x+1}{e^x+2}+C$ d) $\frac{e^x+2}{e^x+1}+C$

646. $\int_{-1}^{2} |x|^3 \, dx$ is equal to

a) $\frac{5}{4}$ b) $\frac{17}{4}$ c) $\frac{15}{4}$ d) $\frac{4}{5}$

647. The value of $\int_{-1}^{1} x|x| \, dx$, is

a) 2 b) 1 c) 0 d) None of these

648. $\int \frac{(x+1)}{x(1+xe^x)^2} dx$ is

a) $\log\left(\frac{xe^x}{xe^x+1}\right)+\frac{1}{1+xe^x}+c$ b) $\log\left(\frac{xe^x}{xe^x+1}\right)-\frac{1}{1+xe^x}+c$

c) $\log\left(\frac{xe^x+1}{xe^x}\right)+\frac{1}{1+xe^x}+c$ d) None of these

649. The greater value of $F(x)=\int_1^x |t| \, dt$ on the interval $[-1/2, 1/2]$, is

a) $\frac{3}{8}$ b) $\frac{1}{2}$ c) $-\frac{3}{8}$ d) $-\frac{1}{2}$

650. The value of the integral $\int_0^{\pi/2} \frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)} dx$, is

a) $\pi/4$ b) $\pi/2$ c) π d) 0

651. The value of $\int \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$ is

a) $2\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$

b) $2\sqrt{2+\frac{2}{x^2}+\frac{1}{x^4}}+c$

c) $\frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$

d) None of the above

652. The value of the integral $I=\int_0^1 x(1-x)^n \, dx$ is

a) $\frac{1}{n+1}$ b) $\frac{1}{n+2}$ c) $\frac{1}{n+1}-\frac{1}{n+2}$ d) $\frac{1}{n+1}+\frac{1}{n+2}$

653. $\int x^2(ax+b)^{-2} \, dx$ is equal to

a) $\frac{2}{a^2}\left(x-\frac{b}{a}\log(ax+b)\right)+c$ b) $\frac{2}{a^2}\left(x-\frac{b}{a}\log(ax+b)\right)-\frac{x^2}{a(ax+b)}+c$

c) $\frac{2}{a^2}\left(x+\frac{b}{a}\log(ax+b)\right)+\frac{x^2}{a(ax+b)}+c$ d) $\frac{2}{a^2}\left(x+\frac{b}{a}\log(ax+b)\right)-\frac{x^2}{a(ax+b)}+c$

654. If $0<a<1$, then $\int_{-1}^{1} \frac{1}{\sqrt{1-2ax+a^2}} dx$ is equal to

a) $\frac{1}{a}$ b) $\frac{2}{a}$ c) $\frac{3}{a}$ d) None of these

655. $\int_{\pi}^{2\pi} [\sqrt{2}\cos x] dx =$

a) $-\pi/2$ b) $\pi/2$ c) π d) None of these

656. The value of the $\int \frac{\sin x+\cos x}{3+\sin 2x} dx$ is

a) $\frac{1}{4}\log\left(\frac{2-\sin x+\cos x}{2+\sin x-\cos x}\right)+c$

b) $\frac{1}{2}\log\left(\frac{2+\sin x}{2-\sin x}\right)+c$

c) $\frac{1}{4}\log\left(\frac{1+\sin x}{1-\sin x}\right)+c$

d) None of the above

657. The value of $\int_{e^{-1}}^{e} \frac{dt}{t(1+t)}$ is equal to

a) 0

b) $\log\left(\frac{e}{1+e}\right)$

c) $\log\left(\frac{1}{1+e}\right)$

d) 1

658. The value of $\int_0^{\pi/2} \frac{\cos 3x+1}{2\cos x-1} dx$, is

a) 2

b) 1

c) 1/2

d) 0

659. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, then

a) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx=\int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$

b) $I(m,n)=\int_0^\infty \frac{x^{m}}{(1+x)^{m+n}}dx=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n}}dx$

c) $I(m,n)=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n-1}}dx=\int_0^\infty \frac{x^{m}}{(1+x)^{m+n-1}}dx$

d) None of these

660. The value of $\int_{-1}^{15} \text{sgn}\,(\{x\})dx$, where $\{\cdot\}$ denotes the fractional part function, is

a) 8

b) 16

c) 24

d) 0

661. $\int_5^{10} \frac{1}{(x-1)(x-2)}\ dx$ is equal to

a) $\log\frac{27}{32}$

b) $\log\frac{32}{27}$

c) $\log\frac{8}{9}$

d) $\log\frac{3}{4}$

662. $\int_{-1}^{1}\left(e^{x^3}+e^{-x^3}\right)(e^x-e^{-x})dx$ is equal to

a) $\frac{e^2}{2}-2e$

b) e^2-2e

c) $2(e^2-e)$

d) 0

663. If $\int_0^1 \frac{e^t dt}{t+1}=a$, then $\int_{b-1}^{b} \frac{e^{-t}dt}{t-b-1}$ is equal to

a) ae^{-b}

b) $-ae^{-b}$

c) $-be^{-a}$

d) ae^{b}

664. The value of the integral $\int_\alpha^\beta \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx$, is

a) 0

b) $\pi/2$

c) π

d) None of these

665. $\int \frac{1}{\{(x-1)^3(x+2)^5\}^{1/4}}dx$ is equal to

a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$

c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$

666. $\int_{-\pi/3}^{\pi/3} \frac{x\sin x}{\cos^2 x}dx$ is equal to

a) $\frac{1}{3}(4\pi+1)$

b) $\frac{4\pi}{3}-2\log\tan\frac{5\pi}{12}$

c) $\frac{4\pi}{3}+\log\tan\frac{5\pi}{12}$

d) None of these

667. $\int \frac{(x^4-x)^{1/4}}{x^5}dx$ is equal to

a) $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4}+C$

b) $\frac{4}{5}\left(1-\frac{1}{x^3}\right)^{5/4}+C$

c) $\frac{4}{15}\left(1+\frac{1}{x^3}\right)^{5/4}+C$

d) None of these

668. If $f(x)=\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x)dx$ is

a) 3 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) 0

669. $\int \frac{(x^3+3x^2+3x+1)}{(x+1)^5}\, dx$ is equal to

a) $-\frac{1}{(x+1)}+c$ b) $\frac{1}{5}\log(x+1)+c$ c) $\log(x+1)+c$ d) $\tan^{-1}x+c$

670. If $\int \sqrt{\frac{x}{a^3-x^3}}\, dx = \mathrm{g}(x)+c$, then $\mathrm{g}(x)$ is equal to

a) $\frac{2}{3}\cos^{-1}x$ b) $\frac{2}{3}\sin^{-1}\left(\frac{x^3}{a^3}\right)$ c) $\frac{2}{3}\sin^{-1}\left(\sqrt{\frac{x^3}{a^3}}\right)$ d) $\frac{2}{3}\cos^{-1}\left(\frac{x}{a}\right)$

671. If $I_n=\int \sin^n x\ dx$,then $nI_n-(n-1)I_{n-2}$ equals

a) $\sin^{n-1}x\cos x$ b) $\cos^{n-1}x\sin x$ c) $-\sin^{n-1}x\cos x$ d) $-\cos^{n-1}x\sin x$

672. $\int_{-2}^{2} e^{\sin^2 x}\cos^3 x\, dx$ is equal to

a) -1 b) 0 c) 1 d) π

673. $\lim_{x\to\frac{\pi}{2}} \frac{\int_{\pi/2}^{x}(2^{-\cos t}-1)\, dt}{\int_{\pi^2/4}^{x^2}\left(\sqrt{t}-\frac{\pi}{2}\right)dt}$ is equal to

a) $\frac{\log_e 2}{2\pi}$ b) $\frac{\log_e 2}{\pi}$ c) $\frac{4\log_e 2}{\pi}$ d) None of these

674. $\int [f(x)\mathrm{g}''(x)-f''(x)\mathrm{g}(x)]\, dx$ is equal to

a) $\frac{f(x)}{\mathrm{g}'(x)}$

b) $f'(x)\mathrm{g}(x)-f(x)\mathrm{g}'(x)$

c) $f(x)\mathrm{g}'(x)-f'(x)\mathrm{g}(x)$

d) $f(x)\mathrm{g}'(x)+f'(x)\mathrm{g}(x)$

675. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B, f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)\, dx=\frac{2A}{\pi}$, then the constants A and B are respectively

a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ c) 0 and $-\frac{4}{\pi}$ d) $\frac{4}{\pi}$ and 0

676. $\int \frac{dx}{\cos x-\sin x}$ is equal to

a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+c$

b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+c$

c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+c$

d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)+c\right|$

677. The value of $\int \frac{(x+1)^2}{x(x^2+1)}\, dx$ is equal to

a) $\log_e x+c$

b) $\log_e x+2\tan^{-1}x+c$

c) $\log_e\left(\frac{1}{x^2+1}\right)+c$

d) None of these

678. If $\int \sqrt{2}\sqrt{1+\sin x}\, dx=-4\cos(ax+b)+c$, then the value of a,b are respectively

a) $\frac{1}{2},\frac{\pi}{4}$ b) $1,\frac{\pi}{2}$ c) 1, 1 d) None of these

679. If $f(x)=x(x-1), 0\le x\le 1$ and $f(x+1)=f(x), \forall\, x\in R$, then $\left|\int_2^4 f(x)dx\right|$ is

a) 1 sq unit b) $\frac{1}{3}$ sq unit c) $\frac{2}{3}$ sq unit d) $\frac{1}{6}$ sq unit

680. The value of $\int_0^{\pi/2}\frac{\sin 8x\log\cot x}{\cos 2x}dx$, is

a) 0 b) π c) $\frac{5\pi}{2}$ d) $\frac{3\pi}{2}$

681. If $[\cdot]$ denotes the greatest integer function and $f(x) = \begin{cases} 3[x] - \frac{5|x|}{x}, x \neq 0 \\ 2 \quad\quad , x = 0 \end{cases}$ then $\int_{-3/2}^{2} f(x)dx$ is equal to

a) $-\dfrac{11}{2}$ b) $-\dfrac{7}{2}$ c) -6 d) $-\dfrac{17}{2}$

682. $\int \frac{(1+x^4)}{(1-x^4)^{3/2}}\, dx$ is equal to

a) $\dfrac{x}{\sqrt{1-x^4}} + c$ b) $\dfrac{-x}{\sqrt{1-x^4}} + c$ c) $\dfrac{2x}{\sqrt{1-x^4}} + c$ d) $\dfrac{-2x}{\sqrt{1-x^4}} + c$

683. If $\int \frac{\sin\theta - \cos\theta}{(\sin\theta + \cos\theta)\sqrt{\sin\theta\cos\theta + \sin^2\theta\cos^2\theta}} d\theta = \operatorname{cosec}^{-1}(f(\theta)) + C$, then

a) $f(\theta) = \sin 2\theta + 1$ b) $f(\theta) = 1 - \sin 2\theta$ c) $f(\theta) = \sin 2\theta - 1$ d) None of these

684. If $f(x) = \cos x - \cos^2 x + \cos^3 x - \ldots\infty$, then $\int f(x)dx$ equals

a) $\tan\dfrac{x}{2} + c$ b) $x + \tan\dfrac{x}{2} + c$ c) $x - \dfrac{1}{2}\tan\dfrac{x}{2} + c$ d) $x - \tan\dfrac{x}{2} + c$

685. If $F(x) = \frac{1}{x^2}\int_4^x [4t^2 - 2F'(t)]dt$, then $F'(4)$ equals

a) 32 b) 32/3 c) 32/9 d) None of these

686. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. The, value of the integral $\int_0^1 f(x)g(x)dx$, is

a) $\dfrac{1}{4}(e-7)$ b) $\dfrac{1}{4}(e-2)$ c) $\dfrac{1}{2}(e-3)$ d) None of these

687. If $\int_0^{\pi} \frac{1}{a+b\cos x}dx, a > 0$ is equal to $\frac{\pi}{\sqrt{a^2-b^2}}$, then $\int_0^{\pi} \frac{1}{(a+b\cos x)^2}dx$ is equal to

a) $\dfrac{\pi a}{(a^2-b^2)^{3/2}}$ b) $\dfrac{\pi b}{(a^2-b^2)^{3/2}}$ c) $\dfrac{\pi}{(a^2-b^2)^{3/2}}$ d) None of these

688. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + c$, then $f(x)$ is equal to

a) $\sin\dfrac{x}{2}$ b) $\cos\dfrac{x}{2}$ c) $\tan\dfrac{x}{2}$ d) $\log\dfrac{x}{2}$

689. The value of the integral $\int \frac{dx}{(e^x+e^{-x})^2}$ is

a) $\dfrac{1}{2}(e^{2x}+1) + c$ b) $\dfrac{1}{2}(e^{-2x}+1) + c$ c) $-\dfrac{1}{2}(e^{2x}+1)^{-1} + c$ d) $\dfrac{1}{4}(e^{2x}-1) + c$

690. The value of $\int \frac{x\cos x + 1}{\sqrt{2x^3 e^{\sin x} + x^2}} dx$ is

a) $ln\left|\dfrac{\sqrt{2x\, e^{\sin x}+1}-1}{\sqrt{2x\, e^{\sin x}+1}+1}\right| + C$

b) $ln\left|\dfrac{\sqrt{2x\, e^{\sin x}-1}-1}{\sqrt{2x\, e^{\sin x}-1}+1}\right| + C$

c) $ln\left|\dfrac{\sqrt{2x\, e^{\sin x}-1}+1}{\sqrt{2x\, e^{\sin x}-1}-1}\right| + C$

d) $ln\left|\dfrac{\sqrt{2x\, e^{\sin x}+1}+1}{\sqrt{2x\, e^{\sin x}-1}+1}\right| + C$

691. If $\phi'(x) = \frac{\log_e|\sin x|}{x}, x \neq n\pi, n \in Z$ and $\int_1^3 \frac{3\log_e|\sin x^3|}{x} dx = \phi(1)$, then the possible value of k is

a) 27 b) 18 c) 9 d) None of these

692. The value of $\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}}\, dx$ is

a) $x^2 + c$ b) $\dfrac{x^2}{2} + c$ c) $\dfrac{x^3}{3} + c$ d) $\dfrac{x}{2} + c$

693. The value of the integral $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}dx$ is equal to

a) 0 b) $2a$ c) a d) None of these

694. If $I_1 = \int_0^1 \frac{1}{|x|}\, dx$ and $I_2 = \int_0^1 \frac{1}{\sqrt{1+x^2}}\, dx$, then

a) $I_1 = I_2$ b) $I_1 < I_2$ c) $I_1 > I_2$ d) $\frac{I_1}{I_2} = 2$

695. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1}\tan x| - |\sin^{-1}\sin x|}{|\tan^{-1}\tan x| + |\sin^{-1}\sin x|}\, dx$ is equal to

a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) None of these

696. $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$ is equal to

a) $\frac{2}{3}\sin^{-1}(\cos^{3/2} x) + C$ b) $\frac{3}{2}\sin^{-1}(\cos^{3/2} x) + C$ c) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + C$ d) None of these

697. Let $x^2 \neq n\pi - 1, n \in N$. The indefinite integral of

$$x\sqrt{\frac{2\sin(x^2+1) - \sin 2(x^2+1)}{2\sin(x^2+1) + \sin 2(x^2+1)}}$$

is but for an arbitrary constant

a) $\log\left|\frac{1}{2}\sec(x^2+1)\right|$ b) $\log\left|\frac{1}{2}\sec\left(\frac{x^2+1}{2}\right)\right|$ c) $\frac{1}{2}\log|\sec(x^2+1)|$ d) None of these

698. The value of the integral $\int_0^{\pi} \frac{x\, dx}{1+\cos\alpha \sin x}, 0 < \alpha < \pi$, is

a) $\frac{\pi\alpha}{\sin\alpha}$ b) $\frac{\pi\alpha}{1+\sin\alpha}$ c) $\frac{\pi\alpha}{\cos\alpha}$ d) $\frac{\pi\alpha}{1+\cos\alpha}$

699. $\int \tan(\sin^{-1} x)\, dx$ is equal to

a) $\frac{1}{\sqrt{1-x^2}} + c$ b) $\sqrt{1-x^2} + c$ c) $\frac{-x}{\sqrt{1-x^2}} + c$ d) $-\sqrt{1-x^2} + c$

700. The value of integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$ is

a) $\frac{\pi}{2} + 1$ b) $\frac{\pi}{2} - 1$ c) -1 d) 1

701. The value of $\int_{\pi}^{2\pi} [2\sin x] dx$ is

a) $\frac{\pi}{3}$ b) $-\frac{5\pi}{3}$ c) $\frac{4\pi}{3}$ d) $-\frac{\pi}{3}$

702. If $\int \frac{1}{(\sin x + 4)(\sin x - 1)} dx$

$= A\frac{1}{\tan\frac{x}{2} - 1} + B\tan^{-1}(f(x)) + C$, then

a) $A = \frac{1}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4\tan x + 3}{\sqrt{15}}$

b) $A = -\frac{1}{5}, B = \frac{1}{\sqrt{15}}, f(x) = \frac{4\tan\left(\frac{x}{2}\right) + 1}{\sqrt{15}}$

c) $A = \frac{2}{5}, B = \frac{-2}{5}, f(x) = \frac{4\tan x + 1}{5}$

d) $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4\tan x/2 + 1}{\sqrt{15}}$

703. If $\int e^x (1+x).\sec^2(xe^x)\, dx = f(x) +$ constant, then $f(x)$ is equal to

a) $\cos(xe^x)$ b) $\sin(xe^x)$ c) $2\tan^{-1}(x)$ d) $\tan(x\, e^x)$

704. The value of $\int_0^{1000} e^{x-[x]}\, dx$, is

a) $\frac{e^{1000}-1}{1000}$ b) $\frac{e^{1000}-1}{e-1}$ c) $1000(e-1)$ d) $\frac{e-1}{1000}$

705. $\int \cos^{-3/7} x \sin^{-11/7} x\, dx$ is equal to

a) $\log|\sin^{4/7} x| + c$ b) $\frac{4}{7}\tan^{4/7} x + c$ c) $-\frac{7}{4}\tan^{-4/7} x + c$ d) $\log|\cos^{3/7} x| + c$

706. If $\int \frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}\, dx = Ax + B\log(3e^{2x}+4) + c$, then

a) $A = -\frac{3}{4}, B = \frac{1}{24}$ b) $A = \frac{3}{4}, B = -\frac{1}{24}$ c) $A = \frac{1}{4}, B = \frac{1}{24}$ d) $A = -\frac{3}{4}, B = \frac{1}{4}$

707. $\int \frac{x^2}{1+x^6} dx$ is equal to

a) $x^3 + c$ b) $\frac{1}{3}\tan^{-1}(x^3) + c$ c) $\log(1+x^3) + c$ d) None of these

708. $\int_0^{\pi} x \sin x \cos^4 x \; dx =$

a) $\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $-\frac{\pi}{5}$ d) None of these

709. $\int \frac{(2x^{12}+5x^9)}{(x^5+x^3+1)^3}\, dx$ is equal to

a) $\frac{x^2+2x}{(x^5+x^3+1)^2} + c$ b) $\frac{x^{10}}{2(x^5+x^3+1)^2} + c$

c) $\log|x^5+x^3+1| + \sqrt{(2x^7+5x^4)} + c$ d) None of the above

710. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x + 2^{\cos x}}}\, dx$ is

a) 2 b) π c) $\frac{\pi}{4}$ d) 2π

711. $\int_0^{\pi} \sqrt{\frac{\cos 2x+1}{2}}\, dx$ is equal to

a) 0 b) 2 c) -2 d) None of these

712. $\int e^{x\log a} e^x \; dx$ is equal to

a) $\frac{a^x}{\log ae} + c$ b) $\frac{e^x}{1+\log_e a} + c$ c) $(ae)^x + c$ d) $\frac{(ae)^x}{\log_e ae} + c$

713. $\int_{-1}^{1} \log(x + \sqrt{x^2+1})\; dx$ is equal to

a) 0 b) $\log 2$ c) $\log\frac{1}{2}$ d) None of these

714. If $I_n = \int (\log x)^n\, dx$, then $I_n + nI_{n-1}$ is equal to

a) $(x\log x)^n$ b) $x(\log x)^n$ c) $n(\log x)^n$ d) $(\log x)^{n-1}$

715. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then

a) $\log_e 2 < I < \pi/4$ b) $\log_e 2 > I$ c) $I = \pi/4$ d) $I = \log_e 2$

716. The value of $\int_a^b \frac{x}{|x|} dx, a < b < 0$ is

a) $-(|a| + |d|)$ b) $|b| - |a|$ c) $|a| - |b|$ d) $|a| + |b|$

717. Given $f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$, $\int f(x) dx$ is equal to

a) $\frac{x^3}{3} - x^2 \sin x + \sin 2x + C$

b) $\frac{x^3}{3} - x^2 \sin x - \cos 2x + C$

c) $\frac{x^3}{3} - x^2 \cos x - \cos 2x + C$

d) None of these

718. If $f(x) = \lim_{n\to\infty} n^2(x^{1/n} - x^{1/(n+1)}, x > 0$, then $\int x\, f(x)\, dx$ is equal to

a) $x^2/2$ b) 0 c) $x^2 \log x - \frac{1}{2}x^2 + c$ d) None of these

719. $\int \frac{1}{x^{1/2}(1+x^2)^{5/4}} dx$ is equal to

a) $\frac{-2\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ b) $\frac{2\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ c) $\frac{-\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ d) $\frac{\sqrt{x}}{\sqrt[4]{1+x^2}} + C$

720. $\int_0^1 (\{x\}[x+1]) dx$ (where $\{\cdot\}$ denotes fractional part of x and $[\cdot]$ denotes greatest integer of x) is equal to

a) 1 b) 0 c) $\frac{1}{2}$ d) None of these

721. $\int_0^2 (x - \log_2 a)\, dx = 2\log_2\left(\frac{2}{a}\right)$, if

a) $a = 2$ b) $a > 2$ c) $a = 4$ d) $a = 8$

722. $\int_{-2}^{10} \operatorname{sgn}(x - [x] dx$ Equals, where [.]denotes greatest integer function

a) -12 b) 10 c) 8 d) 12

723. The value of the integral $\int_0^\infty \frac{1}{1+x^4} dx$, is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\pi}{2\sqrt{2}}$ d) None of these

724. $\int \frac{x^2}{x^2+4}\, dx$ is equal to

a) $x - 2\tan^{-1}\left(\frac{x}{2}\right) + c$ b) $x + 2\tan^{-1}\left(\frac{x}{2}\right) + c$ c) $x - 4\tan^{-1}\left(\frac{x}{2}\right) + c$ d) $x + 4\tan^{-1}\left(\frac{x}{2}\right) + c$

725. $\int \frac{1+x}{x+e^{-x}}\, dx$ is equal to

a) $\log|(x - e^{-x})| + c$ b) $\log|(x + e^{-x})| + c$ c) $\log|(1 + xe^x)| + c$ d) $(1 + xe^x)^2 + c$

726. $\int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$ is equal to

a) $4\left(\frac{x+1}{x+2}\right)^{1/4} + c$ b) $-4\left(\frac{x+1}{x+2}\right)^{-1/4} + c$ c) $-4\left(\frac{x+2}{x+1}\right)^{1/4} + c$ d) None of these

727. $\int_0^\infty \left[\frac{[x+1]}{e^x}\right] dx$ (where $[\cdot]$ denotes greatest integer function) is equal to

a) 0 b) 1 c) ∞ d) None of these

728. $\int \frac{(\tan^{-1} x)^3}{1+x^2}\, dx$ is equal to

a) $3(\tan^{-1} x)^2 + c$ b) $\frac{(\tan^{-1} x)^4}{4} + c$ c) $(\tan^{-1} x)^4 + c$ d) None of these

729. $\int x^x (1 + \log|x|) dx$ is equal to

a) $x^x \log|x| + c$ b) $e^{x^x} + c$ c) $x^x + c$ d) None of these

730. $\int \left\{\frac{\log x - 1}{1 + (\log x)^2}\right\}^2 dx$ is equal to

a) $\frac{x}{(\log x)^2 + 1} + c$ b) $\frac{xe^x}{1+x^2} + c$ c) $\frac{x}{x^2+1} + c$ d) $\frac{\log x}{(\log x)^2 + 1} + c$

731. $\lim_{n\to\infty} \frac{1+2^4+3^4+\ldots+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+\ldots+n^3}{n^5}$ is

a) $\frac{1}{30}$ b) 0 c) $\frac{1}{4}$ d) $\frac{1}{5}$

732. $\int \frac{dx}{\sqrt{1-e^{2x}}}$ is equal to

a) $\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + c$ b) $\log\left|e^{x} + \sqrt{e^{2x} - 1}\right| + c$

c) $-\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + c$ d) $-\log\left|e^{-2x} + \sqrt{e^{-2x} - 1}\right| + c$

733. If $I = \int_0^{\pi/2} \cos(\sin x)\, dx$,

$J = \int_0^{\pi/2} \sin(\cos x)\, dx$ and $K = \int_0^{\pi/2} \cos x\, dx$. Then,

a) $K > I > J$ b) $J > I > K$ c) $I > J > K$ d) $I > K > J$

734. $\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$ is

a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{4}{3}$ d) $\frac{3}{4}$

735. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π

736. $\int e^{2x}(2\sin 3x + 3\cos 3x)dx$ is equal to

a) $e^{2x}\sin 2x + c$ b) $e^{2x}\cos 3x + c$ c) e^{2x+c} d) $e^{2x}(2\sin 3x) + c$

737. The value of the integral $\int_0^2 |x^2 - 1| dx$ is

a) 0 b) 2 c) $-\frac{1}{3}$ d) -2

738. If

$$I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx \text{ and } I_4 = \int_1^2 2^{x^3} dx$$

Then

a) $I_1 > I_2$ and $I_4 > I_3$ b) $I_2 > I_1$ and $I_3 > I_4$ c) $I_1 > I_2$ and $I_3 > I_4$ d) None of these

739. $\int(\sin 2x - \cos 2x)dx = \frac{1}{\sqrt{2}}\sin(2x - a) + b$, then

a) $a = \frac{5\pi}{4}, b \in R$ b) $a = -\frac{5\pi}{4}, b \in R$ c) $a = \frac{\pi}{4}, b \in R$ d) None of these

740. $\int(1 + x - x^{-1})e^{x+x^{-1}} dx$ is equal to

a) $(1 + x)e^{x+x^{-1}} + c$ b) $(x - 1)e^{x+x^{-1}} + c$ c) $-xe^{x+x^{-1}} + c$ d) $xe^{x+x^{-1}} + c$

741. The value of the integral $\sum_{k=1}^{n} \int_0^1 f(k - 1 + x)dx$, is

a) $\int_0^1 f(x)dx$ b) $\int_0^2 f(x)dx$ c) $\int_0^n f(x)dx$ d) $n\int_0^1 f(x)dx$

742. $\int_0^{\pi} \frac{1}{1+3^{\cos x}} dx$ is equal to

a) π b) 0 c) $\pi/2$ d) None of these

743. The value of the integral $\int_0^1 \frac{1}{x^2+2x\cos\alpha+1} dx$ is equal to

a) $\sin\alpha$ b) $\alpha \sin\alpha$ c) $\frac{\alpha}{2\sin\alpha}$ d) $\frac{\alpha}{2}\sin\alpha$

744. The value of the integral $\int_0^{\pi/4} \frac{\sin\theta+\cos\theta}{9+16\sin 2\theta} d\theta$, is

a) $\log 3$ b) $\log 2$ c) $\frac{1}{20}\log 3$ d) $\frac{1}{20}\log 2$

745. $\int_0^{\infty} \frac{dx}{\left(x+\sqrt{x^2+1}\right)^3}$ is equal to

a) $\frac{3}{8}$ b) $\frac{1}{8}$ c) $\frac{-3}{8}$ d) None of these

746. $\int \frac{dx}{x^2+4x+13}$ is equal to

a) $\log(x^2 + 4x + 13) + c$ b) $\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + c$

c) $\log(2x+4)+c$

d) $\dfrac{2x+4}{(x^2+4x+13)^2}+c$

747. Let $I=\int \frac{e^x}{e^{4x}+e^{2x}+1}dx, J=\int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then, for an arbitrary constant c, the value of $J-I$ equals

a) $\frac{1}{2}\log\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right|+c$

b) $\frac{1}{2}\log\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right|+c$

c) $\frac{1}{2}\log\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right|+c$

d) $\frac{1}{2}\log\left|\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right|+c$

748. If $b>a$, then $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$ is equal to

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{2}(b-a)$ d) $\frac{\pi}{4}(b-a)$

749. If $\int_0^{\pi/2}\sin^6 x\ dx=\frac{5\pi}{32}$, then the value of $\int_{-\pi}^{\pi}(\sin^6 x+\cos^6 x)\,dx$ is

a) $5\pi/8$ b) $5\pi/16$ c) $5\pi/2$ d) $5\pi/4$

750. If $I_m=\int_1^x(\log x)^m\,dx$ satisfies the relation $I_m=k-lI_{m-1}$, then

a) $k=e$ b) $l=m$ c) $k=\frac{1}{e}$ d) None of these

751. $\int x^{-2/3}\left(1+x^{1/2}\right)^{-5/3}dx$ is equal to

a) $3\left(1+x^{-1/2}\right)^{-1/3}+C$ b) $3\left(1+x^{-1/2}\right)^{-2/3}+C$ c) $3\left(1+x^{1/2}\right)^{-2/3}+C$ d) None of these

752. $\int(1-\cos x)\,\text{cosec}^2 dx$ is equal to

a) $\tan\frac{x}{2}+c$ b) $-\cot\frac{x}{2}+c$ c) $2\tan\frac{x}{2}+c$ d) $-2\cot\frac{x}{2}+c$

753. If $\int_0^a f(2a-x)dx=m$ and $\int_0^a f(x)\ dx=n$, then $\int_0^{2a} f(x)\,dx$is equal to

a) $2m+n$ b) $m+2n$ c) $m-n$ d) $m+n$

754. If $I_n=\int(\log x)^n\ dx$, then I_n+nI_{n-1} is equal to

a) $x(\log x)^n$ b) $(x\log x)^n$ c) $(\log x)^{n-1}$ d) $n(\log x)^n$

755. The points of extremum of $\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$ are

a) $x=0,\pm1,\pm1$ b) $x=\pm1,\pm2,\pm3$ c) $x=0,1,2,3$ d) None of these

756. The value of $\int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}\ dx$ is

a) 2 b) π c) $\pi/4$ d) 2π

757. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n}$is

a) e b) $e-1$ c) $1-e$ d) $e+1$

758. Let $f:R\to R, g:R\to R$ be continuous functions. Then the value of the integral $\int_{-\pi/2}^{\pi/2}[f(x)+f(-x)][g(x)-g(-x)]dx$ is

a) π b) 1 c) -1 d) 0

759. The integral $\int\frac{2x-3}{(x^2+x+1)^2}dx$ is equal to

a) $-\frac{8x+7}{x^2+x+1}-\frac{16}{3\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{3}\right)+C$

b) $-\frac{1}{x^2+x+1}-\frac{4}{3}\tan^{-1}(4x+3)+C$

c) $\frac{1}{2(x^2+x+1)} - \frac{(2x+1)^2}{(x^2+x+1)^2} + C$

d) $\frac{1}{4(x^2+x+1)} + \frac{2}{3}\tan^{-1}(2x+1) + C$

760. $\int \frac{\sqrt{x}}{x+1}\, dx$ is equal to

a) $2(\sqrt{x} + \tan^{-1}\sqrt{x}) + c$ b) $2(\sqrt{x} + \cot^{-1}\sqrt{x}) + c$

c) $2(\sqrt{x} - \cot^{-1} - \sqrt{x}) + c$ d) $2(\sqrt{x} - \tan^{-1}\sqrt{x}) + c$

761. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is equal to

a) $\pi^2/4$ b) π^2 c) 0 d) $\pi/2$

762. The value of $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\dots(2)\right\}^{1/n}$, is

a) $4/e$ b) $e/4$ c) $4\,e$ d) None of these

763. $\int_{-2}^{2} |x|\, dx$ is equal to

a) 0 b) 1 c) 2 d) 4

764. The value of the definite integral $\int_{a+2\pi}^{a+5\pi/2} (\sin^{-1}(\cos x) + \cos^{-1}(\sin x))\, dx$ is equal to

a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{4}$ c) $\frac{\pi^2}{2}$ d) π^2

765. Let $T \neq 0$ be a fixed number suppose f is a continuous function such that for all $x \in R, f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is

a) $\frac{3}{2}I$ b) $2I$ c) $3I$ d) $6I$

766. $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\, dx$ is equal to

a) $\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 + c$ b) $x\log\left(x+\sqrt{1+x^2}\right) + c$

c) $\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right) + c$ d) $\frac{1}{2}\left[\log(x+\sqrt{1+x^2})\right]^2 + c$

767. If $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x-1}} dx = \frac{\pi}{6}$, then x is equal to

a) e^2 b) $1/e$ c) $\log 4$ d) None of these

768. $\int \frac{\cos x + x\sin x}{x^2 + x\cos x}\, dx$ is equal to

a) $\log\left|\frac{\sin x}{1+\cos x}\right| + c$ b) $\log\left|\frac{\sin x}{x+\cos x}\right| + c$ c) $\log\left|\frac{2\sin x}{x+\cos x}\right| + c$ d) $\log\left|\frac{x}{x+\cos x}\right| + c$

769. $\int \frac{dx}{x\sqrt{x^6-16}}$ is equal to

a) $\frac{1}{3}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ b) $\cos^{-1}\left(\frac{x^3}{4}\right) + c$ c) $\frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ d) $\sec^{-1}\left(\frac{x^3}{4}\right) + c$

770. If $F(x) = \int_{x^2}^{x^3} \log t\, dt, (x > 0)$, then $F'(x)$ is equal to

a) $(9x^2 - 4x)\log x$ b) $(4x - 9x^2)\log x$ c) $(9x^2 + 4x)\log x$ d) None of these

771. If $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = k\int_0^1 \frac{\log(1+x)}{1+x^2} dx$, then $k =$

a) 4 b) 8 c) π d) 2π

772. The value of integral $\int_0^4 |x-1|\, dx$ is

a) 4 b) 5 c) 7 d) 9

773. If f is a continuous function, then

a) $\int_{-2}^{2} f(x)dx = \int_0^2 [f(x) - f(-x)]dx$ b) $\int_{-3}^{5} 2f(x)dx = \int_{-6}^{10} f(x-1)dx$

c) $\int_{-3}^{5} f(x)dx = \int_{-4}^{4} f(x-1)dx$ d) $\int_{-3}^{5} f(x)dx = \int_{-2}^{6} f(x-1)dx$

774. $\int [\sin(\log x) + \cos(\log x)]dx$ is equal to

a) $x\cos(\log x) + c$ b) $\cos(\log x) + c$ c) $x\sin(\log x) + c$ d) $\sin(\log x) + c$

775. Let $f(x)$ be a continuous function such that $f(a-x) + f(x) = 0$ for all $x \in [0, a]$. Then, the value of the integral $\int_0^a \frac{1}{1+e^{f(x)}} dx$ is equal to

a) a b) $\frac{a}{2}$ c) $f(a)$ d) $\frac{1}{2}f(a)$

776. $\int_{-1}^{1} |1-x|\, dx$ is equal to

a) -2 b) 0 c) 2 d) 4

777. The integral $\int_{-1}^{3} \left[\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right] dx$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 2π

778. The value of $\int \frac{x^2}{1+x^6} dx$ is

a) $x^3 + c$ b) $\frac{1}{3}\tan^{-1}(x^3) + c$ c) $\log(1+x^3)$ d) None of these

779. $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx$ is equal to

a) $\tan x - x + c$

b) $x + \tan x + c$

c) $x - \tan x + c$

d) $-x - \cot x + c$

780. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$ is equal to

a) $\frac{1}{2}\sqrt{1+x} + c$ b) $\frac{2}{3}(1+x)^{3/2} + c$ c) $\sqrt{1+x} + c$ d) $2(1+x)^{3/2} + c$

781. The primitive of the function $f(x) = (2x+1)|\sin x|$, when $\pi < x < 2\pi$ is

a) $-(2x+1)\cos x + 2\sin x + C$

b) $(2x+1)\cos x - 2\sin x + C$

c) $(x^2 + x)\cos x + C$

d) None of these

782. If $k\int_0^1 x.f(3x)dx = \int_0^3 t.f(t)dt$, then the value of k is

a) 9 b) 3 c) $\frac{1}{9}$ d) $\frac{1}{3}$

783. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx$ is equal to

a) $\log(x^e + e^x) + c$ b) $e\ \log(x^e + e^x) + c$

c) $\frac{1}{e}\log(x^e + e^x) + c$ d) None of these

784. $\int \text{cosec}\,(x-a)\text{cosec}\,x\, dx$ is equal to

a) $\frac{-1}{\sin a}\log|\sin x\ \text{cosec}(x-a)| + c$ b) $\frac{-1}{\sin a}\log[\sin(x-a)\sin x] + c$

c) $\frac{1}{\sin a}\log[\sin(x-a)\text{cosec}\,x] + c$ d) $\frac{1}{\sin a}\log[\sin(x-a)\sin x] + c$

785. The value of $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}}\, d\theta$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{5}$

786. The value of the integral $\int_0^{\pi} \frac{1}{a^2 - 2a\cos x + 1} dx (a < 1)$ is

a) $\frac{\pi}{1-a^2}$ b) $\frac{\pi}{a^2-1}$ c) $\frac{2\pi}{a^2-1}$ d) $\frac{3\pi}{4}$

787. The value of integral $\int_{-1}^{1} \frac{|x+2|}{x+2}\,dx$ is

a) 1 b) 2 c) 0 d) −1

788. The value of $\int_1^5 \left(\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}\right)dx$ is

a) $\frac{8}{3}$ b) $\frac{16}{3}$ c) $\frac{32}{3}$ d) $\frac{64}{3}$

789. If $\int \frac{1}{(\sin x+4)(\sin x-1)}\,dx = A\,\frac{1}{\tan\frac{x}{2}-1} + B\tan^{-1}(f(x)) + C_1$. Then,

a) $A=\frac{1}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan x+3}{\sqrt{15}}$

b) $A=-\frac{1}{5}, B=\frac{1}{\sqrt{15}}, f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

c) $A=\frac{2}{5}, B=\frac{-2}{5}, f(x)=\frac{4\tan x+1}{5}$

d) $A=\frac{2}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$

790. $\int(\sqrt{\tan x}+\sqrt{\cot x})dx$ is equal to

a) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

b) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

c) $\sqrt{2}\tan^{-1}(\sin x-\cos x)+c$

d) None of the above

791. $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

a) $\frac{\pi^2}{4}$ b) π^2 c) zero d) $\frac{\pi}{2}$

792. If $\int\frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log_e\sin(x-\alpha)+C$, then

a) $A=\sin\alpha, B=\cos\alpha$

b) $A=\cos\alpha, B=-\sin\alpha$

c) $A=\cos\alpha, B=\sin\alpha$

d) None of these

793. The value of the integral $\int_0^{100}\sin(x-[x])\,\pi\,dx$, is

a) $100/\pi$ b) $200/\pi$ c) 100π d) 200π

794. If $f(x)=\cos x-\cos^2 x+\cos^3 x-\ldots\infty$, then $\int f(x)dx$ is equal to

a) $\tan\frac{x}{2}+c$ b) $x-\tan\frac{x}{2}+c$ c) $x-\frac{1}{2}\tan\frac{x}{2}+c$ d) $\frac{x-\tan\frac{x}{2}}{2}+c$

795. If $\int_0^a f(2a-x)dx=\mu$ and $\int_0^a f(x)\,dx=\lambda$, then $\int_0^{2a} f(x)dx$ equals

a) $2\lambda-\mu$ b) $\lambda+\mu$ c) $\mu-\lambda$ d) $\lambda-2\mu$

796. $\int\frac{f'(x)}{f(x)\log\{f(x)\}}dx=$

a) $\frac{f(x)}{\log\{f(x)\}}+C$ b) $f(x)\log f(x)+C$ c) $\log\{\log f(x)\}+C$ d) $\frac{1}{\log\{\log f(x)\}}+C$

797. The value of $\int_1^4|x-3|dx$ is equal to

a) 2 b) $\frac{5}{2}$ c) $\frac{1}{2}$ d) $\frac{3}{2}$

798. $\int e^x\left(\log x+\frac{1}{x}\right)dx$ is equal to

a) $e^x\log x+c$ b) $\frac{e^x}{\log x}+c$ c) $\frac{\log x}{x}+c$ d) $\frac{e^x}{x}+c$

799. $\int_{-3\pi/2}^{-\pi/2}[(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to

a) $\left(\frac{\pi^4}{32}\right)+\left(\frac{\pi}{2}\right)$ b) $\left(\frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4}\right)-1$ d) $\frac{\pi^4}{32}$

800. $\int_1^x \frac{\log(x^2)}{x}\,dx$ is equal to

a) $(\log x)^2$ b) $\frac{1}{2}(\log x)^2$ c) $\frac{\log x^2}{2}$ d) None of these

801. If $f'(x) = x + \frac{1}{x}$, then the value of $f(x)$ is

a) $x^2 + \log x + c$ b) $\frac{x^2}{2} + \log x + c$ c) $\frac{x}{2} + \log x + c$ d) None of these

802. $\int e^{-\log x}\,dx$ is equal to

a) $e^{-\log x} + c$ b) $-xe^{-\log x} +$ c) $e^{\log x} + c$ d) $\log|x| + c$

803. $\int \frac{dx}{x(x+1)}$ equals

Where c is an arbitrary constant

a) $\log\left|\frac{x+1}{x}\right| + c$ b) $\log\left|\frac{x}{x+1}\right| + c$ c) $\log\left|\frac{x-1}{x}\right| + c$ d) $\log\left|\frac{x-1}{x+1}\right| + c$

804. $\int_0^{\pi^2/4} \sin\sqrt{x}\,dx$ is equals to

a) 0 b) 1 c) 2 d) 4

805. Limit of $\int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right] dt$ as $x \to \infty$ is

a) $\log_2 e$ b) $\log_e 2$ c) $\log_2\left(\frac{1}{e}\right)$ d) $\log_{1/e} 2$

806. If $I_1 = \int \sin^{-1} x\,dx$ and $I_2 = \int \sin^{-1}\sqrt{1-x^2}\,dx$, then

a) $I_1 = I_2$ b) $I_2 = \frac{\pi}{2} I_1$ c) $I_1 + I_2 = \frac{\pi}{2} x$ d) $I_1 + I_2 = \frac{\pi}{2}$

807. $\int \cos^3 x . e^{\log \sin x} dx$ is equal to

a) $-\frac{\sin^4 x}{4} + c$ b) $-\frac{\cos^4 x}{4} + c$ c) $\frac{e^{\sin x}}{4} + c$ d) None of these

808. The value of the integral $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\,dx$ is

a) -2 b) 2 c) 0 d) -3

809. $\int_0^{\pi/2} \sin 2x \log\tan x\,dx$ is equal to

a) π b) $\frac{\pi}{2}$ c) 0 d) 2π

810. The value of $\int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x\,dx$ is

a) 0 b) $\pi - \frac{\pi^2}{3}$ c) $2\pi - \pi^3$ d) $\frac{7}{2} - 2\pi^3$

811. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \ldots + \frac{n}{n^2}\sec^2 1\right]$ equals

a) $\frac{1}{2}\tan 1$ b) $\tan 1$ c) $\frac{1}{2}\operatorname{cosec} 1$ d) $\frac{1}{2}\sec 1$

812. $\int \frac{1}{1+\cos ax}\,dx$ is equal to

a) $\cot\frac{ax}{2} + c$ b) $\frac{1}{a}\tan\frac{ax}{2} + c$

c) $\frac{1}{a}(\operatorname{cosec} ax - \cot ax) + c$ d) $\frac{1}{a}(\operatorname{cosec} ax + \cot ax) + c$

813. $\int \sqrt{1+\cos x}\,dx$ is equal to

a) $2\sqrt{2}\cos\frac{x}{2} + c$ b) $2\sqrt{2}\sin\frac{x}{2} + c$ c) $\sqrt{2}\cos\frac{x}{2} + c$ d) $\sqrt{2}\sin\frac{x}{2} + c$

814. If $f(x) = \begin{cases} e^{\cos x}\sin x, & |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x)dx$ is equal to

a) 0 b) 1 c) 2 d) 3

815. If $\int \frac{x\,e^x}{\sqrt{1+e^x}}dx = f(x)\sqrt{1+e^x} - 2\log g(x) + C$, then

a) $f(x) = x - 1$ b) $g(x) = \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}-1}$ c) $g(x) = \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}$ d) $f(x) = 2(x+2)$

816. If a particle is moving with velocity $v(t) = \cos \pi t$ along a straight line such that at $t = 0, s = 4$ its position function is given by

a) $\frac{1}{\pi}\cos \pi t + 2$ b) $-\frac{1}{\pi}\sin \pi t + 4$ c) $\frac{1}{\pi}\sin \pi t + 4$ d) None of these

817. $\int \cos(\log_e x)dx$ is equal to

a) $\frac{1}{2}x[\cos(\log_e x) + \sin(\log_e x)]$ b) $x[\cos(\log_e x) + \sin(\log_e x)]$

c) $\frac{1}{2}x[\cos(\log_e x) - \sin(\log_e x)]$ d) $x[\cos(\log_e x) - \sin(\log_e x)]$

818. $\int \frac{(e^x - e^{-x})dx}{(e^x+e^{-x})\log(\cosh x)}$ is equal to

a) $\log(\tanh x) + c$ b) $2\log(e^x + e^{-x}) + c$

c) $2\log(e^x - e^{-x}) + c$ d) $\log[\log(\cosh x)] + c$

819. If $u = -f''(\theta)\sin\theta + f'(\theta)\cos\theta$ and $v = f''(\theta)\cos\theta + f'(\theta)\sin\theta$, then

$f\left[\left(\frac{du}{d\theta}\right)^2 + \left(\frac{dv}{d\theta}\right)^2\right]^{\frac{1}{2}} d\theta$ is equal to

a) $f(\theta) - f'(\theta) + c$ b) $f(\theta) + f''(\theta) + c$ c) $f'(\theta) + f''(\theta) + c$ d) $f'(\theta) - f''(\theta) + c$

820. The value of the integral $\int_0^{\pi} \frac{\sin k\,x}{\sin x}dx$ (k is an even integer) is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{k\,\pi}{2}$ d) 0

821. $\int \frac{\sin x}{\sin(x-\alpha)}dx$ is

a) $x\sin\alpha + \cos\alpha\log\sin(x+\alpha) + c$ b) $x\sin\alpha + \cos\alpha\log\sin(x-\alpha) + c$

c) $x\cos\alpha + \sin\alpha\log\cos(x+\alpha) + c$ d) $x\cos\alpha + \sin\alpha\log\sin(x-\alpha) + c$

822. To find the numerical value of $\int_{-2}^{2}(px^3 + qx + s)\,dx$ it is necessary to know the values of the constants:

a) p b) q c) s d) p and s

823. $\int x^3 \log x\,dx$ is equal to

a) $\frac{x^4 \log x}{4} + c$ b) $\frac{1}{16}[4x^4\log x - x^4] + c$

c) $\frac{1}{8}[x^4\log x - 4x^2] + c$ d) $\frac{1}{16}[4x^4\log x + x^4] + c$

824. $\int \frac{\sin^3 x}{(1+\cos^2 x)\sqrt{1+\cos^2 x+\cos^4 x}}dx$ is equal to

a) $\sec^{-1}(\sec x + \cos x) + C$

b) $\sec^{-1}(\sec x - \cos x) + C$

c) $\sec^{-1}(\sec x - \tan x) + C$

d) None of these

825. If $x = \int_2^{\sin t}\sin^{-1}\theta\,d\theta$ and $y = \int_n^{\sqrt{t}}\frac{\sin\theta^2}{\theta}d\,\theta$ then $\frac{dy}{dx}$ is equal to

a) $\frac{\tan t}{2t^2}$ b) $\frac{2t^2}{\tan t}$ c) $\frac{\tan t}{t^2}$ d) None of these

826. $\int_0^{\pi/2}\frac{200\sin x + 100\cos x}{\sin x + \cos x}dx$ is equal to

a) 50π b) $25\,\pi$ c) $75\,\pi$ d) $150\,\pi$

827. If $\frac{d}{dx}f(x) = g(x)$ for $a \le x \le b$ then, $\int_a^b f(x)g(x)dx$ equals

a) $f(b)-f(a)$ b) $g(b)-g(a)$ c) $\frac{[f(b)]^2-[f(a)]^2}{2}$ d) $\frac{[g(b)]^2-[g(a)]^2}{2}$

828. If $f(x)=\int_0^x(\sin^4 t+\cos^4 t)\,dt$, then $f(x+\pi)$ will be equal to

a) $f(x)+f\left(\frac{\pi}{2}\right)$ b) $f(x)+f(\pi)$ or $f(x)+2f\left(\frac{\pi}{2}\right)$

c) $f(x)-f(\pi)$ d) $f(x)-2f\left(\frac{\pi}{2}\right)$

829. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}}\,dx$ is equal to

a) 1 b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{\pi}{3}$

830. The value of $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\,dx$ is

a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+c$ b) $\frac{4}{3}\left(\frac{x+1}{x+2}\right)^{1/4}+c$ c) $\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4}+c$ d) $\frac{4}{3}\left(\frac{x-1}{x-2}\right)^{1/4}+c$

831. The value of $\int \frac{dx}{x+\sqrt{x-1}}$ is

a) $\log(x+\sqrt{x-1})+\sin^{-1}\left(\sqrt{\frac{x-1}{x}}\right)+c$ b) $\log(x+\sqrt{x-1})+c$

c) $\log(x+\sqrt{x-1})-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$ d) None of the above

832. If $2f(x)-3f(1/x)=x$, then $\int_1^2 f(x)dx$ is equal to

a) $(3/5)\log 2$ b) $(-3/5)(1+\log 2)$ c) $(-3/5)\log 2$ d) None of these

833. $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx$ is equal to

a) $\frac{e^x}{x}+c$ b) $\frac{e^x}{2x^2}+c$ c) $\frac{2e^x}{x}+c$ d) $\frac{2e^x}{x^2}+c$

834. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4}dx$, is

a) $\sqrt{\frac{3}{32}}$ b) $\frac{\sqrt{3}}{32}$ c) $\frac{32}{\sqrt{3}}$ d) $-\frac{\sqrt{3}}{32}$

835. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

a) $\log(x^4+1)+C$ b) $\frac{1}{4}\log(x^4+1)+C$ c) $-\log(x^4+1)$ d) None of these

836. If $\int_0^p \frac{dx}{1+4x^2}=\frac{\pi}{8}$, then the value of p is

a) $\frac{1}{4}$ b) $-\frac{1}{2}$ c) $\frac{3}{2}$ d) $\frac{1}{2}$

837. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx$ is equal to

a) $\frac{\pi}{2}$ b) $\sqrt{2}\log(\sqrt{2}+1)$ c) $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ d) None of these

838. $\int \frac{\sqrt{x^2+1}\,[\log(x^2+1)-2\log x]}{x^4}dx$ is equal to

a) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+C$

b) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$

c) $\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+C$

d) None of these

839. The value of $\int_3^5 \frac{x^2}{x^2-4}dx$, is

a) $2-\log_e\left(\frac{15}{7}\right)$

b) $2+\log_e\left(\frac{15}{7}\right)$

c) $2+4\log_e 3-4\log_e 7+4\log_e 5$

d) $2-\tan^{-1}\left(\frac{15}{7}\right)$

840. $\int_\pi^{10\pi}|\sin x|dx$ is equal to

a) 20 b) 8 c) 10 d) 18

841. Let $\Delta(y)=\begin{vmatrix} y+a & y+b & y+a-c \\ y+b & y+c & y-1 \\ y+c & y+d & y-b+d \end{vmatrix}$ and, $\int_0^2 \Delta(y)dy=-16$, where a, b, c, d are in A.P., then the common difference of the A.P. is equal to

a) ± 1 b) ± 2 c) ± 3 d) None of these

842. $\int \text{cosec}^4 x\, dx$ is equal to

a) $\cot x+\frac{\cot^3 x}{3}+c$ b) $\tan x+\frac{\tan^3 x}{3}+c$

c) $-\cot x-\frac{\cot^3 x}{3}+c$ d) $-\tan x-\frac{\tan^3 x}{3}+c$

843. The value of $\int_0^1(1+e^{-x^2})\,dx$ is

a) -1 b) 2 c) $1+e^{-1}$ d) None of these

844. The value of $\int e^{\sec x}\cdot\sec^3 x\,(\sin^2 x+\cos x+\sin x+\sin x\cos x)dx$ is

a) $e^{\sec x}(\sec^2 x+\sec x\tan x)+C$

b) $e^{\sec x}++C$

c) $e^{\sec x}(\sec x+\tan x)+C$

d) None of these

845. $\int\frac{\log\sqrt{x}}{3x}\,dx$ is equal to

a) $\frac{1}{3}\left(\log\sqrt{x}\right)^2+c$ b) $\frac{2}{3}\left(\log\sqrt{x}\right)^2+c$ c) $\frac{2}{3}(\log x)^2+c$ d) $\frac{1}{3}(\log x)^2+c$

846. If $I=\int_0^{2\pi}\sin^2 x\;dx$, then

a) $I=4\int_0^{\pi/4}\sin^2 x=4\int_0^{\pi/2}\sin^2 x\;dx$

b) $I=\int_0^{2\pi}\cos^2 x\;dx$

c) $I=8\int_0^{\pi/4}\sin^2 x\;dx$

d) None of these

847. The smallest interval $[a,b]$ such that $\int_0^1\frac{1}{\sqrt{1+x^4}}dx\in[a,b]$ is

a) $\left[\frac{1}{\sqrt{2}},1\right]$ b) $[0,1]$ c) $\left[\frac{1}{2},1\right]$ d) $\left[\frac{3}{4},1\right]$

848. The value of $\int\frac{3x+2}{(x-2)^2(x-3)}\;dx$ is

a) $11\;\log\frac{x-3}{x-2}-\frac{8}{x-2}+c$ b) $11\;\log\frac{x+3}{x+2}-\frac{8}{x-2}+c$

c) $11\log\frac{x-3}{x-2}+\frac{8}{x-2}+c$ d) $11\log\frac{x+3}{x+2}+\frac{8}{x-2}+c$

849. $\int\frac{\cos x-1}{\sin x+1}.e^x\,dx$ is equal to

a) $\frac{e^x\cos x}{1+\sin x}+c$ b) $c-\frac{e^x\sin x}{1+\sin x}$ c) $c-\frac{e^x}{1+\sin x}$ d) $c-\frac{e^x\cos x}{1+\sin x}$

850. $\int_0^{2\pi}\sin^6 x\cos^5 x\,dx$ is equal to

a) 2π b) $\pi/2$ c) 0 d) $-\pi$

851. $\int\cos^{-3/7}x\sin^{-11/7}x\,dx$ is equal to

a) $\log|\sin^{4/7}x|+c$ b) $\frac{4}{7}\tan^{4/7}x+c$ c) $-\frac{7}{4}\tan^{-4/7}x+c$ d) $\log|\cos^{3/7}x|+c$

852. $\lim_{n\to\infty}\left\{\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+\cdots+\frac{1}{nb}\right\}$ is equal to

a) $\log\left(\frac{b}{a}\right)$ b) $\log\left(\frac{a}{b}\right)$ c) $\log a$ d) $\log b$

853. The value of the integral $\int_{-a}^{a}\frac{xe^{x^2}}{1+x^2}dx$is

a) e^{a^2} b) 0 c) e^{-a^2} d) a

854. $\int\frac{(\sin\theta+\cos\theta)}{\sqrt{\sin 2\theta}}d\theta$ is equal to

a) $\log|\cos\theta-\sin\theta+\sqrt{\sin 2\theta}|+c$ b) $\log|\sin\theta-\cos\theta+\sqrt{\sin 2\theta}|+c$

c) $\sin^{-1}(\sin\theta-\cos\theta)+c$ d) $\sin^{-1}(\sin\theta+\cos\theta)+c$

855. If $\int\frac{e^x-1}{e^x+1}\,dx=f(x)+c$, then $f(x)$ is equal to

a) $2\log(e^x+1)$ b) $\log(e^{2x}-1)$ c) $2\log(e^x+1)-x$ d) $\log(e^{2x}+1)$

856. $\int e^{3\log x}(x^4+1)^{-1}\,dx$ equals

a) $\log(x^4+1)+c$ b) $\frac{1}{4}\log(x^4+1)+c$ c) $-\log(x^4+1)+c$ d) None of these

857. The anti-derivative of $f(x)=\frac{1}{3+5\sin x+3\cos x}$, whose graph passes through the point (0, 0) is

a) $\frac{1}{5}\left(\log\left|1-\frac{5}{3}\tan(x/2)\right|\right)$ b) $\frac{1}{5}\left(\log\left|1+\frac{5}{3}\tan(x/2)\right|\right)$

c) $\frac{1}{5}\left(\log\left|1+\frac{5}{3}\cot(x/2)\right|\right)$ d) None of these

858. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x)+g(x)=x^2$. Then, the value of the integral $\int_0^1 f(x)\;g(x)\,dx$, is

a) $e+\frac{e^2}{2}+\frac{5}{2}$ b) $e-\frac{e^2}{2}-\frac{5}{2}$ c) $e+\frac{e^2}{2}-\frac{3}{2}$ d) $e-\frac{e^2}{2}-\frac{3}{2}$

859. If $f(x)=\int_0^{x^2}\sqrt{1+t^2}dt$, then $f'(x)$ equals

a) $\sqrt{1+t^2}$ b) $\sqrt{1+x^4}$ c) $2x\sqrt{1+x^4}$ d) None of these

860. If $I=\int_3^4\frac{1}{\sqrt[3]{\log x}}dx$, then

a) $0.92<I<1$ b) $I>1$ c) $I<0.92$ d) None of these

861. If $f(y)=e^y, g(y)=y; y>0$ and $F(t)=\int_0^t f(t-y)\,g(y)dy$, then

a) $F(t)=t\,e^{-t}$

b) $F(t)=1-e^{-t}(t+1)$

c) $F(t)=e^t-(1+t)$

d) $F(t)=t\,e^t$

862. The value of the integral $I=\int_1^\infty\frac{x^2-2}{x^3\sqrt{x^2-1}}dx$, is

a) 0 b) $\frac{2}{3}$ c) $\frac{4}{3}$ d) None of these

863. $\int(\sin 2x - \cos 2x)\,dx = \frac{1}{\sqrt{2}}\sin(2x - a) + b$, then

a) $a = \frac{5\pi}{4}, b \in R$
b) $a = -\frac{5\pi}{4}, b \in R$
c) $a = \frac{\pi}{4}, b \in R$
d) None of these

864. $\int \frac{\sec x}{\sec x + \tan x}\,dx$ is equal to

a) $\tan x - \sec x + c$
b) $\log(1 + \sec x) + c$
c) $\sec x + \tan x + c$
d) $\log \sin x + \log \cos x + c$

865. Primitive of $\cos^{-1} x$ w.r.t x is

a) $x\cos^{-1} x - \frac{1}{2}\sqrt{1 - x^2} + c$
b) $x\cos^{-1} x - \sqrt{1 - x^2} + c$
c) $x\cos^{-1} x + \sqrt{1 - x^2} + c$
d) $x\cos^{-1} + \frac{1}{2}\sqrt{1 - x^2} + c$

866. If $\int_0^{36} \frac{1}{2x+9}\,dx = \log k$, then k is equal to

a) 3
b) 9/2
c) 9
d) 81

867. $\int \log 2x\,dx$ is equal to

a) $x\log 2x - \frac{x^2}{2} + c$
b) $x\log 2x - \frac{x}{2} + c$
c) $x^2\log 2x - \frac{x}{2} + c$
d) $x\log 2x - x + c$

868. If $f(0) = f'(0) = 0$ and $f''(x) = \tan^2 x$, then $f(x)$ is

a) $\log \sec x - \frac{x^2}{2}$
b) $\log \cos x + \frac{x^2}{2}$
c) $\log \sec x + \frac{x^2}{2}$
d) None of these

869. $\int \frac{dx}{x(x^5+1)}$ is equal to

a) $\frac{1}{5}\log x^5(x^5 + 1) + c$
b) $\frac{1}{5}\log\left(\frac{x^5 + 1}{x^5}\right) + c$
c) $\frac{1}{5}\log\left(\frac{x^5}{x^5 + 1}\right) + c$
d) None of these

870. If $\int \frac{1}{1+\sin x}\,dx = \tan\left(\frac{x}{2} + a\right) + b$, then

a) $a = -\frac{\pi}{4}, b \in R$
b) $a = \frac{\pi}{4}, b \in R$
c) $a = \frac{5\pi}{4}, b \in R$
d) None of these

871. $\int \sqrt{e^x - 1}\,dx$ is equal to

a) $2\left[\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}\right] + c$
b) $\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1} + c$
c) $\sqrt{e^x - 1} + \tan^{-1}\sqrt{e^x - 1} + c$
d) $2\left[\sqrt{e^x - 1} + \tan^{-1}\sqrt{e^x - 1}\right] + c$

872. If $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2} + \frac{B}{x} + \log\left|\frac{x}{x+1}\right| + C$, then

a) $A = \frac{1}{2}, B = 1$
b) $A = 1, B = -\frac{1}{2}$
c) $A = -\frac{1}{2}, B = 1$
d) $A = 1, B = 1$

873. The value of $\int_\alpha^\beta x\,|x|\,dx$, where $\alpha < 0 < \beta$, is

a) $\frac{1}{2}(\alpha^2 + \beta^2)$
b) $\frac{1}{3}(\beta^2 - \alpha^2)$
c) $\frac{1}{3}(\alpha^3 + \beta^3)$
d) None of these

874. The value of $\alpha \in (-\pi, 0)$ satisfying $\sin\alpha + \int_\alpha^{2\alpha} \cos 2x\,dx = 0$, is

a) $-\pi/2$
b) $-\pi$
c) $-\pi/3$
d) 0

875. If $I_1 = \int_0^{\pi/2} x\sin x\;dx$ and $I_2 = \int_0^{\pi/2} x\cos x\;dx$, then which one of the following is true?

a) $I_1 + I_2 = \frac{\pi}{2}$
b) $I_2 - I_1 = \frac{\pi}{2}$
c) $I_1 + I_2 = 0$
d) $I_1 = I_2$

876. $\int_{1/2}^{2} |\log_{10} x|\,dx$ is equals to

a) $\log_{10}(8/e)$
b) $\frac{1}{2}\log_{10}(8/e)$
c) $\log_{10}(2/e)$
d) None of these

877. $\int_0^1 |\sin 2\pi x|\,dx$ is equal to

a) 0
b) $-\frac{1}{\pi}$
c) $\frac{1}{\pi}$
d) $\frac{2}{\pi}$

878. If $I=\int_{-\pi}^{\pi}\frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx$, then I equals

a) $\frac{\pi}{2}$ b) 2π c) π d) $\frac{\pi}{4}$

879. $\int\frac{1}{1+\cos x+\sin x}\,dx$ is equal to

a) $\log\left|1+\tan\frac{x}{2}\right|+c$

b) $\frac{1}{2}\log\left|1+\tan\frac{x}{2}\right|+c$

c) $2\log\left|1+\tan\frac{x}{2}\right|+c$

d) $\frac{1}{2}\log\left|1-\tan\frac{x}{2}\right|+c$

880. $\int\frac{x^4-1}{x^2(x^4+x^2+1)^{1/2}}\,dx$ is equal to

a) $\sqrt{\frac{x^4+x^2+1}{c}}+c$

b) $\frac{x^2}{\sqrt{x^4+x^2+1}}+c$

c) $x(x^4+x^2+1)^{3/2}+c$

d) $\frac{\sqrt{x^4+x^2+1}}{x}+c$

881. The value of $\int_0^1\frac{x^3}{1+x^8}dx$, is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{8}$ c) $\frac{\pi}{16}$ d) None of these

882. $\int\frac{\sin x+8\cos x}{4\sin x+6\cos x}dx$ is equal to

a) $x+\frac{1}{2}\log(4\sin x+6\cos x)+c$

b) $2x+\log(2\sin x+3\cos x)+c$

c) $x+2\log(2\sin x+3\cos x)+c$

d) $\frac{1}{2}\log(4\sin x+6\cos x)+c$

883. $\int x^2(ax+b)^{-2}\,dx$ is equal to

a) $\frac{2}{a^2}\left(x-\frac{b}{a}\log(ax+b)\right)+c$

b) $\frac{2}{a^2}\left(x-\frac{b}{a}\log(ax+b)\right)-\frac{x^2}{a(ax+b)}+c$

c) $\frac{2}{a^2}\left(x+\frac{b}{a}\log(ax+b)\right)+\frac{x^2}{a(ax+b)}+c$

d) $\frac{2}{a^2}\left(x+\frac{b}{a}\log(ax+b)\right)-\frac{x^2}{a(ax+b)}+c$

884. The value of $\int_1^4 e^{\sqrt{x}}\,dx$ is

a) e^2 b) $2e^2$ c) $4e^2$ d) $3e^2$

885. If $\int\frac{1}{x+x^5}dx=f(x)+c$, then the value of $\int\frac{x^4}{x+x^5}\,dx$ equals

a) $\log x-f(x)+c$ b) $f(x)+\log x+c$ c) $f(x)-\log x+c$ d) None of these

886. $\int e^x(1-\cot x+\cot^2 x)dx$ equals

a) $e^x\cot x+c$ b) $e^x\operatorname{cosec}x+c$ c) $-e^x\cot x+c$ d) $-e^x\operatorname{cosec}x+c$

887. If $f(x)$ is defined $[-2,2]$ by $f(x)=4x^2-3x+1$ and $g(x)=\frac{f(-x)-f(x)}{x^2+3}$, then $\int_{-2}^{2}g(x)dx$ is equal to

a) 64 b) -48 c) 0 d) 24

888. $\int\sqrt{1+\sin\frac{x}{4}}\,dx$ is equal to

a) $8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ b) $\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)+c$ c) $\frac{1}{8}\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ d) $8\left(\cos\frac{x}{8}-\sin\frac{x}{8}\right)+c$

889. If $h(a)=h(b)$, the value of the integral $\int_a^b\left[f\left(g\left(h(x)\right)\right)\right]^{-1}f'\left(g(h(x))\right)g'(h(x))h'(x)dx$ is equal to

a) 0 b) $f(a)-f(b)$ c) $f(g(a))-f(g(b))$ d) None of these

890. $\int_0^{\pi/2}\frac{\sin^2 x}{\sin^2 x+\cos^2 x}dx$ is equal to

a) $\frac{\pi}{4}$ b) 2π c) π^2 d) $\frac{1}{2}\pi^2$

891. The value of the integral $\int_0^1 \log\sin\left(\frac{\pi x}{2}\right) dx$, is

a) $\log 2$ b) $-\log 2$ c) $\frac{\pi}{2}\log 2$ d) $-\frac{\pi}{2}\log 2$

892. Let $\frac{d}{dx} F(x) = \left(\frac{e^{\sin x}}{x}\right), x > 0$, If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k, is

a) 15 b) 16 c) 63 d) 64

893. $\int_0^{10} |x(x-1)(x-2)| dx$ is equal to

a) 160.05 b) 1600.5 c) 16.005 d) None of these

894. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals

a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ b) $\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

895. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx$, is

a) $-\frac{8}{3}$ b) $\frac{3}{2}$ c) $\frac{8}{3}$ d) None of these

896. If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then $I_8 + I_6$ is equal to

a) $\frac{1}{7}$ b) $\frac{1}{4}$ c) $\frac{1}{5}$ d) $\frac{1}{6}$

897. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to

a) $\frac{a^{\sqrt{x}}}{\log a}+C$ b) $\frac{2a^{\sqrt{x}}}{\log a}+C$ c) $2a^{\sqrt{x}}\cdot\log a + C$ d) None of these

898. $\int_a^b \sqrt{(x-a)(b-x)} \, dx, (b > a)$ is equal to

a) $\frac{\pi(b-a)^2}{8}$ b) $\frac{\pi(b+a)^2}{8}$ c) $(b-a)^2$ d) $(b+a)^2$

899. If $\int_0^{x^2} f(t) \, dt = x\cos\pi x$, then the value of $f(4)$ is

a) 1 b) $\frac{1}{4}$ c) -1 d) $\frac{-1}{4}$

900. If $f'(x) = \frac{dx}{(1+x^2)^{3/2}}$ and $f(0) = 0$, then $f(1)$ is equal to

a) $\sqrt{2}$ b) $-\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) None of these

901. The value of $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$, is

a) π b) 2π c) $\pi/2$ d) $3\pi/2$

902. If $\int f(x)\sin x\cos x \, dx = \frac{1}{2(b^2-a^2)}\log\{f(x)\} + C$, then $f(x)$ is equal to

a) $\frac{1}{a^2\sin^2 x + b^2\cos^2 x}$ b) $\frac{1}{a^2\sin^2 x - b^2\cos^2 x}$ c) $\frac{1}{a^2\cos^2 x + b^2\sin^2 x}$ d) $\frac{1}{a^2\cos^2 x - b^2\sin^2 x}$

903. $\int \sqrt{\frac{1+x}{1-x}} dx$ is equal to

a) $-\sin^{-1} x - \sqrt{1-x^2} + c$ b) $\sin^{-1} x + \sqrt{1-x^2} + c$

c) $\sin^{-1} x - \sqrt{1-x^2} + c$ d) $-\sin^{-1} x - \sqrt{x^2-1} + c$

904. The value of $\int_1^2 \frac{dx}{x(1+x^4)}$ is

a) $\frac{1}{4}\log\frac{17}{32}$ b) $\frac{1}{4}\log\frac{32}{17}$ c) $\log\frac{17}{2}$ d) $\frac{1}{4}\log\frac{17}{2}$

905. $\int \frac{1}{(x+1)^2\sqrt{x^2+2x+2}}dx$ is equal to

a) $\frac{\sqrt{x^2+2x+2}}{x+1}+C$ b) $\frac{\sqrt{x^2+2x+2}}{(x+1)^2}+C$ c) $\frac{-\sqrt{x^2+2x+2}}{x+1}+C$ d) None of these

906. If $I=\int_{-1}^{1}\left\{[x^2]+\log\left(\frac{2+x}{2-x}\right)\right\}dx$ where $[x]$ denotes the greatest integer less than or equal to x, the I equals

a) -2 b) -1 c) 0 d) 1

907. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right) dx$ is equal to

a) $-e^x\tan\left(\frac{x}{2}\right)+c$ b) $-e^x\cot\left(\frac{x}{2}\right)+c$ c) $-\frac{1}{2}e^x\tan\left(\frac{x}{2}\right)+c$ d) $\frac{1}{2}e^x\cot\left(\frac{x}{2}\right)+c$

908. $\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)} dx$ is equal to

a) $\tan^{-1}\left(x+\frac{1}{x}\right)+c$ b) $\cot^{-1}\left(x+\frac{1}{x}\right)+c$

c) $\log\left(x+\frac{1}{x}\right)+c$ d) $\log\left[\tan^{-1}\left(x+\frac{1}{x}\right)\right]+c$

909. $\int_0^1 \frac{x}{(1-x)^{3/4}}dx=$

a) $\frac{12}{5}$ b) $\frac{16}{5}$ c) $-\frac{16}{5}$ d) None of these

910. Least value of the function $f(x)=\int_0^x(1-\cos t)\,dt$ on the interval $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ is

a) $\frac{3\pi}{2}+1$ b) $\frac{\pi}{2}-1$ c) $\frac{\pi}{2}+1$ d) None of these

911. $\int \frac{x^2-2}{x^3\sqrt{x^2-1}} dx$ is equal to

a) $\frac{x^2}{\sqrt{x^2-1}}+c$ b) $-\frac{x^2}{\sqrt{x^2-1}}+c$ c) $\frac{\sqrt{x^2-1}}{x^2}+c$ d) $-\frac{\sqrt{x^2-1}}{x^2}+c$

912. The value of $\int \frac{(x-x^3)^{1/3}}{x^4}dx$ is

a) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$ b) $-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$ c) $\frac{1}{8}\left(1-\frac{1}{x^2}\right)^{4/3}+1$ d) None of these

913. $\int_0^\infty \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$ is equal to

a) $\pi\log 2$ b) $-\pi\log 2$ c) $\left(\frac{\pi}{2}\right)\log 2$ d) $-\left(\frac{\pi}{2}\right)\log 2$

914. If $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx=x[f(x)-g(x)]+c$, then

a) $f(x)=\log(\log x)$; $g(x)=\frac{1}{\log x}$ b) $f(x)=\log x$; $g(x)=\frac{1}{\log x}$

c) $f(x)=\frac{1}{\log x}$; $g(x)=\log(\log x)$ d) $f(x)=\frac{1}{x\log x}$; $g(x)=\frac{1}{\log x}$

915. Integration of $\frac{1}{\sqrt{x^2+9}}$ with respect to (x^2+1) is equal to

a) $\sqrt{x^2+9}+C$ b) $\frac{1}{\sqrt{x^2+9}}+C$ c) $2\sqrt{x^2+9}+C$ d) None of these

916. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx$ is equal to

a) $\frac{4}{3}[1+x^{3/4}+\log_e(1+x^{3/4})]+C$

b) $\frac{4}{3}[1 + x^{3/4} - \log_e(1 + x^{3/4})] + C$

c) $\frac{4}{3}[1 - x^{3/4} + \log_e(1 + x^{3/4})] + C$

d) None of these

917. $\int e^{x \log a} \cdot e^x \, dx$ is equal to

a) $(ae)^x$ b) $\frac{(ae)^x}{\log(ae)}$ c) $\frac{e^x}{1 + \log a}$ d) None of these

918. If $\int f(x)dx = g(x)$, then $f(x)g(x)$ is equal to

a) $\frac{1}{2}f^2(x)$ b) $\frac{1}{2}g^2(x)$ c) $\frac{1}{2}[g'(x)]^2$ d) $f'(x)g(x)$

919. If $\int \frac{\cos 4x+1}{\cos x - \tan x} \, dx = A\cos 4x + B$,then

a) $A = -\frac{1}{2}$ b) $A = -\frac{1}{8}$ c) $A = -\frac{1}{4}$ d) None of these

920. $\int_0^{\pi} k(\pi x - x^2)^{100} \sin 2x \, dx$ is equal to

a) π^{100} b) $\frac{1}{2}(\pi^{100} - \pi^{97})$ c) $\frac{1}{2}(\pi^{100} + \pi^{97})$ d) 0

921. The value of $\int_{\pi}^{2\pi}[2\sin x] \, dx$, where [ ] represents the greatest integer function, is

a) $\frac{-5\pi}{3}$ b) $-\pi$ c) $\frac{5\pi}{3}$ d) -2π

922. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx$ is equal to

a) $\frac{-2}{\sqrt{\tan x}} + C$ b) $2\sqrt{\tan x} + c$ c) $\frac{2}{\sqrt{\tan x}} + C$ d) $-2\sqrt{\tan x} + c$

923. $\int_0^{1.5}[x^2]dx$ is

a) $4 + 2\sqrt{2}$ b) $2 + \sqrt{2}$ c) $2 - \sqrt{2}$ d) None of these

924. $\int_0^{\pi} |\cos x| dx$ is equal to

a) $\frac{1}{2}$ b) -2 c) 1 d) 2

925. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx$

a) $\frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$

b) $\frac{3}{2}x^{2/3} - 6\tan^{-1} x^{1/6} + C$

c) $-\frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$

d) None of these

926. For any integer n, the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3(2n-1)x \, dx$ has the value

a) π b) 1 c) 0 d) None of these

927. The value of $\int_0^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1} \, dx$ is

a) 2 b) 1 c) $\frac{1}{2}$ d) 0

928. $\int_1^3 \frac{\cos(\log x)}{x} \, dx$ is equal to

a) 1 b) $\cos(\log 3)$ c) $\sin(\log 3)$ d) $\pi/4$

929. $\int (e^x + e^{-x}).(e^x - e^{-x})dx$ is equal to

a) $e^x + c$ b) $\frac{1}{2}(e^x - e^{-x})^2 + c$ c) $\frac{1}{2}(e^x + e^{-x})^2 + c$ d) $\frac{1}{3}(e^x + e^{-x})^3 + c$

930. $\int \frac{dx}{\sin x \cos x}$ is equal to

a) $\log|\sin x| + c$ b) $\log|\tan x| + c$ c) $\log|\cos x| + c$ d) None of these

931. The value of the integral $\int_0^{\pi/2} \sin^5 x \ dx$ is

a) $\frac{4}{15}$ b) $\frac{8}{5}$ c) $\frac{8}{15}$ d) $\frac{4}{5}$

932. $\int_{-\pi/2}^{\pi/2} \log_e \left\{\left(\frac{ax^2+bx+c}{ax^2-bx+c}\right)(a+b)|\sin x|\right\} dx$ is equal to

a) $\pi \log_e \ (a+b)$ b) $\pi \log_e \left(\frac{a+b}{2}\right)$ c) $\frac{\pi}{2}\log_e(a+b)$ d) None of these

933. The value of $\int_{1/e}^{e} \frac{|\log x|}{x^2} dx$, is

a) 2 b) $\frac{2}{e}$ c) $2\left(1-\frac{1}{e}\right)$ d) 0

934. The value of $\int_{-1}^{1} x|x| \, dx$ is

a) 2 b) 1 c) 0 d) None of these

935. If $\int \frac{1}{x^3+x^4} dx = \frac{A}{x^2} + \frac{B}{x} + \log\left|\frac{x}{x+1}\right| + C$, then

a) $A = \frac{1}{2}, B = 1$ b) $A = 1, B = -\frac{1}{2}$ c) $A = -\frac{1}{2}, B = 1$ d) None of these

936. If $\int \frac{1}{(x^2+1)(x^2+4)} dx = A \tan^{-1} x + B \tan^{-1} \frac{x}{2} + C$, then

a) $A = 1/3, B = -2/3$ b) $A = -1/3, B = 2/3$ c) $A = -1/3, B = \frac{1}{3}$ d) $A = \frac{1}{3}, B = -\frac{1}{6}$

937. The value of the integral $\int_{-1/2}^{1/2} \left\{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right\}^{1/2} dx$, is

a) $\log\left(\frac{4}{3}\right)$ b) $4\log\left(\frac{3}{4}\right)$ c) $4\log\left(\frac{4}{3}\right)$ d) $\log\left(\frac{3}{4}\right)$

938. If $\int \operatorname{cosec} x \, dx = f(x) + \text{constant}$, then $f(x)$ is equal to

a) $\tan x/2$ b) $\log|\tan(x/2)|$ c) $\log|\sin x|$ d) $\log|\cos x|$

939. $\int \frac{dx}{x(x^n+1)}$ is equal to

a) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$ b) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right) + c$ c) $\log\left(\frac{x^n}{x^n+1}\right) + c$ d) None of the above

940. $\int \frac{ax^3+bx^2+c}{x^4} \ dx$ equals

a) $a\log x + \frac{b}{x^2} + \frac{c}{3x^3} + k$ b) $a\log x + \frac{b}{x} - \frac{c}{3x^3} + k$

c) $a\log x - \frac{b}{x} - \frac{c}{3x^3} + k$ d) None of these

941. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$ is equal to

a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$ b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$ c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{x+1}}\right)$ d) None of these

942. The value of $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

943. $\int \frac{\sin 2x}{1+\cos^2 x} dx$ is equal to

a) $-\frac{1}{2}\log(1+\cos^2 x) + c$ b) $2\log(1+\cos^2 x) + c$

c) $\frac{1}{2}\log(1+\cos 2x) + c$ d) $c - \log(1+\cos^2 x)$

944. For any integer n, the integral $\int_0^{\pi} e^{\sin^2 x} \cos^3(2n+1)\, x \, dx$ has the value

a) π b) 1 c) 0 d) None of these

945. If $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx = gof(x) + \text{constant}$, then

a) $f(x) = \log(x + \sqrt{x^2+1})$
b) $f(x) = \log(x + \sqrt{x^2+1})$ and $g(x) = x^2$
c) $f(x) = \log(x + \sqrt{x^2+1})$ and $g(x) = \frac{x^2}{2}$
d) $f(x) = \frac{x^2}{2}$ and $g(x) = \log(x + \sqrt{x^2+1})$

946. $\int \frac{1+\tan^2 x}{1-\tan^2 x}\,dx$ is equal to

a) $\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ b) $\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$ c) $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ d) $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

947. $\int (x+1)(x+2)^7(x+3)\,dx$ is equal to

a) $\frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + c$
b) $\frac{(x+1)^2}{2} - \frac{(x+2)^8}{8} - \frac{(x+3)^2}{2} + c$
c) $\frac{(x+2)^{10}}{10} + c$
d) $\frac{(x+1)^2}{2} + \frac{(x+2)^8}{8} + \frac{(x+3)^2}{2} + c$

948. The value of $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$ is

a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{-2}{\sqrt{3}}$

949. $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) 0 b) 1 c) -1 d) $\frac{1}{2\sqrt{2}}$

950. $\int_0^{a} \sqrt{a^2-x^2}\,dx$ is equal to

a) πa^2 b) $\frac{1}{2}\pi a^2$ c) $\frac{1}{3}\pi a^2$ d) $\frac{1}{4}\pi a^2$

951. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}\,dx$ is equal to

a) $\tan^{-1}\frac{\sqrt{x^2+x+1}}{x}+C$
b) $2\tan^{-1}\frac{\sqrt{x^2+x+1}}{x}$
c) $3\tan^{-1}\frac{\sqrt{x^2+x+1}}{x}+C$
d) None of these

952. If $k \neq 0$ is a constant and $n \in N$, then, $\lim_{n\to\infty}\left\{\frac{n!}{(k\,n)^n}\right\}$ is equal to

a) ke b) $\frac{e}{k}$ c) $\frac{k}{e}$ d) $\frac{1}{ke}$

953. $\int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}\,dx$ is equal to

a) $10^x - x^{10} + c$ b) $10^x + x^{10} + c$ c) $(10^x - x^{10})^{-1} + c$ d) $\log(10^x + x^{10}) + c$

954. $\int e^{\sin\theta}[\log\sin\theta + \mathrm{cosec}^2\theta]\cos\theta\,d\theta$ is equal to

a) $\int e^{\sin\theta}[\log\sin\theta + \mathrm{cosec}^2\,\theta] + c$
b) $e^{\sin\theta}[\log\sin\theta + \mathrm{cosec}\,\theta] + c$
c) $e^{\sin\theta}[\log\sin\theta - \mathrm{cosec}\,\theta] + c$
d) $e^{\sin\theta}[\log\sin\theta - \mathrm{cosec}^2\theta] + c$

955. Let $f(x) = \int \frac{1}{(1+x^2)^{3/2}}\,dx$ and $f(0)=0$, then $f(1) =$

a) $-\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) None of these

MATHS (QUESTION BANK)

7.INTEGRALS

: ANSWER KEY :

1)	b	2)	c	3)	c	4)	b	145)	a	146)	d	147)	a	148)	b
5)	b	6)	b	7)	d	8)	d	149)	a	150)	c	151)	b	152)	c
9)	c	10)	a	11)	c	12)	c	153)	d	154)	b	155)	d	156)	c
13)	a	14)	b	15)	a	16)	c	157)	c	158)	b	159)	d	160)	b
17)	d	18)	b	19)	b	20)	d	161)	a	162)	c	163)	b	164)	c
21)	c	22)	a	23)	b	24)	b	165)	c	166)	a	167)	d	168)	b
25)	a	26)	c	27)	c	28)	a	169)	b	170)	a	171)	b	172)	d
29)	a	30)	a	31)	b	32)	d	173)	b	174)	a	175)	b	176)	a
33)	a	34)	c	35)	a	36)	c	177)	a	178)	a	179)	a	180)	c
37)	a	38)	a	39)	b	40)	a	181)	c	182)	b	183)	c	184)	c
41)	c	42)	d	43)	b	44)	b	185)	b	186)	a	187)	d	188)	d
45)	a	46)	d	47)	d	48)	d	189)	d	190)	a	191)	d	192)	d
49)	c	50)	b	51)	a	52)	a	193)	a	194)	a	195)	c	196)	b
53)	c	54)	a	55)	d	56)	c	197)	a	198)	a	199)	b	200)	b
57)	b	58)	c	59)	d	60)	b	201)	b	202)	a	203)	c	204)	d
61)	b	62)	d	63)	b	64)	b	205)	b	206)	b	207)	c	208)	d
65)	c	66)	d	67)	a	68)	a	209)	c	210)	c	211)	c	212)	b
69)	c	70)	c	71)	d	72)	b	213)	b	214)	d	215)	b	216)	a
73)	c	74)	c	75)	d	76)	c	217)	a	218)	b	219)	b	220)	b
77)	c	78)	b	79)	a	80)	b	221)	b	222)	d	223)	a	224)	c
81)	d	82)	a	83)	c	84)	d	225)	c	226)	a	227)	a	228)	c
85)	b	86)	a	87)	a	88)	c	229)	d	230)	b	231)	a	232)	c
89)	b	90)	d	91)	c	92)	d	233)	d	234)	b	235)	a	236)	c
93)	c	94)	c	95)	c	96)	b	237)	d	238)	b	239)	c	240)	b
97)	b	98)	c	99)	b	100)	a	241)	b	242)	c	243)	c	244)	a
101)	b	102)	a	103)	a	104)	c	245)	c	246)	b	247)	d	248)	c
105)	b	106)	b	107)	b	108)	a	249)	c	250)	a	251)	a	252)	a
109)	b	110)	b	111)	c	112)	c	253)	b	254)	c	255)	c	256)	b
113)	a	114)	c	115)	b	116)	d	257)	c	258)	c	259)	b	260)	a
117)	a	118)	b	119)	a	120)	c	261)	d	262)	b	263)	c	264)	d
121)	b	122)	a	123)	c	124)	b	265)	a	266)	b	267)	a	268)	a
125)	b	126)	d	127)	b	128)	c	269)	b	270)	c	271)	b	272)	b
129)	a	130)	a	131)	d	132)	b	273)	d	274)	c	275)	c	276)	b
133)	d	134)	c	135)	c	136)	d	277)	c	278)	c	279)	c	280)	c
137)	c	138)	b	139)	d	140)	c	281)	d	282)	b	283)	b	284)	c
141)	b	142)	d	143)	a	144)	a	285)	c	286)	a	287)	c	288)	b

289) d	290) b	291) c	292) d
293) a	294) c	295) c	296) b
297) d	298) d	299) d	300) a
301) a	302) b	303) c	304) d
305) d	306) c	307) c	308) c
309) a	310) d	311) b	312) c
313) b	314) c	315) a	316) a
317) a	318) a	319) c	320) c
321) b	322) a	323) c	324) b
325) d	326) b	327) d	328) a
329) b	330) a	331) d	332) a
333) b	334) a	335) b	336) a
337) c	338) d	339) a	340) a
341) d	342) c	343) a	344) b
345) a	346) a	347) c	348) d
349) a	350) c	351) a	352) c
353) c	354) b	355) d	356) c
357) b	358) b	359) b	360) d
361) b	362) c	363) c	364) d
365) b	366) b	367) c	368) a
369) b	370) a	371) a	372) c
373) b	374) a	375) c	376) b
377) b	378) d	379) c	380) a
381) c	382) b	383) a	384) c
385) c	386) c	387) b	388) a
389) a	390) d	391) d	392) d
393) d	394) b	395) d	396) a
397) b	398) c	399) d	400) a
401) b	402) b	403) a	404) b
405) b	406) b	407) b	408) b
409) c	410) d	411) c	412) c
413) b	414) a	415) b	416) d
417) b	418) d	419) b	420) b
421) a	422) c	423) a	424) c
425) c	426) d	427) a	428) b
429) b	430) d	431) c	432) a
433) b	434) c	435) b	436) b
437) a	438) a	439) a	440) a
441) c	442) b	443) c	444) a
445) c	446) d	447) c	448) a
449) a	450) d	451) a	452) a
453) a	454) d	455) c	456) a
457) a	458) d	459) a	460) c
461) a	462) d	463) c	464) c
465) c	466) d	467) d	468) a
469) a	470) a	471) c	472) d
473) c	474) c	475) a	476) c

477) d	478) c	479) d	480) d
481) c	482) a	483) c	484) b
485) c	486) c	487) d	488) c
489) d	490) c	491) b	492) a
493) b	494) a	495) c	496) a
497) a	498) b	499) d	500) a
501) d	502) d	503) b	504) b
505) c	506) a	507) a	508) a
509) a	510) a	511) a	512) b
513) a	514) b	515) a	516) a
517) c	518) a	519) c	520) a
521) a	522) c	523) a	524) a
525) a	526) b	527) b	528) b
529) b	530) d	531) a	532) a
533) c	534) a	535) a	536) c
537) b	538) b	539) b	540) a
541) a	542) d	543) d	544) c
545) b	546) a	547) d	548) c
549) c	550) d	551) a	552) c
553) a	554) d	555) c	556) b
557) c	558) d	559) b	560) b
561) a	562) a	563) c	564) c
565) c	566) c	567) d	568) a
569) b	570) a	571) b	572) a
573) a	574) c	575) b	576) a
577) c	578) c	579) a	580) a
581) c	582) b	583) b	584) b
585) d	586) b	587) b	588) a
589) a	590) d	591) b	592) b
593) d	594) d	595) c	596) d
597) d	598) c	599) c	600) b
601) b	602) b	603) a	604) a
605) d	606) a	607) c	608) b
609) a	610) a	611) b	612) b
613) c	614) a	615) b	616) c
617) c	618) b	619) b	620) b
621) a	622) d	623) b	624) a
625) c	626) a	627) a	628) b
629) c	630) a	631) c	632) b
633) b	634) d	635) b	636) d
637) d	638) a	639) b	640) b
641) d	642) a	643) d	644) a
645) b	646) b	647) c	648) a
649) c	650) a	651) c	652) c
653) b	654) d	655) a	656) a
657) d	658) b	659) a	660) b
661) b	662) d	663) b	664) c

665)	a	666)	b	667)	a	668)	c
669)	a	670)	c	671)	c	672)	b
673)	b	674)	c	675)	d	676)	d
677)	b	678)	a	679)	b	680)	a
681)	a	682)	a	683)	a	684)	d
685)	c	686)	d	687)	a	688)	c
689)	c	690)	a	691)	a	692)	c
693)	c	694)	c	695)	d	696)	c
697)	b	698)	a	699)	d	700)	b
701)	b	702)	d	703)	d	704)	c
705)	c	706)	b	707)	b	708)	b
709)	b	710)	c	711)	b	712)	d
713)	a	714)	b	715)	a	716)	b
717)	d	718)	d	719)	b	720)	c
721)	a	722)	c	723)	c	724)	a
725)	c	726)	c	727)	a	728)	b
729)	c	730)	a	731)	d	732)	c
733)	d	734)	c	735)	b	736)	a
737)	b	738)	a	739)	b	740)	d
741)	c	742)	c	743)	c	744)	c
745)	a	746)	b	747)	c	748)	b
749)	d	750)	b	751)	b	752)	a
753)	d	754)	a	755)	a	756)	c
757)	b	758)	d	759)	a	760)	d
761)	b	762)	a	763)	d	764)	b
765)	c	766)	d	767)	c	768)	d
769)	c	770)	a	771)	b	772)	b
773)	b	774)	c	775)	b	776)	c
777)	d	778)	b	779)	c	780)	b
781)	b	782)	a	783)	c	784)	a
785)	b	786)	a	787)	b	788)	c
789)	d	790)	a	791)	b	792)	c
793)	b	794)	b	795)	b	796)	c
797)	b	798)	a	799)	b	800)	a
801)	b	802)	d	803)	b	804)	c
805)	b	806)	c	807)	b	808)	c
809)	c	810)	a	811)	a	812)	b

813)	b	814)	c	815)	d	816)	c
817)	a	818)	d	819)	b	820)	d
821)	d	822)	c	823)	b	824)	a
825)	a	826)	c	827)	c	828)	b
829)	b	830)	a	831)	c	832)	b
833)	c	834)	b	835)	b	836)	d
837)	c	838)	b	839)	b	840)	d
841)	b	842)	c	843)	d	844)	c
845)	a	846)	b	847)	a	848)	c
849)	a	850)	c	851)	c	852)	a
853)	b	854)	c	855)	c	856)	b
857)	b	858)	d	859)	c	860)	a
861)	d	862)	a	863)	b	864)	a
865)	b	866)	a	867)	d	868)	d
869)	c	870)	a	871)	a	872)	c
873)	c	874)	c	875)	a	876)	b
877)	d	878)	c	879)	a	880)	d
881)	c	882)	a	883)	b	884)	b
885)	a	886)	c	887)	c	888)	a
889)	a	890)	a	891)	b	892)	d
893)	b	894)	d	895)	a	896)	a
897)	b	898)	a	899)	b	900)	c
901)	c	902)	a	903)	c	904)	b
905)	c	906)	c	907)	b	908)	d
909)	b	910)	b	911)	d	912)	b
913)	a	914)	a	915)	c	916)	b
917)	b	918)	b	919)	b	920)	d
921)	a	922)	a	923)	c	924)	d
925)	a	926)	c	927)	b	928)	c
929)	d	930)	b	931)	c	932)	b
933)	c	934)	c	935)	c	936)	a
937)	c	938)	b	939)	a	940)	c
941)	b	942)	d	943)	d	944)	c
945)	c	946)	d	947)	a	948)	b
949)	a	950)	d	951)	b	952)	d
953)	d	954)	c	955)	b		

MATHS (QUESTION BANK)

7.INTEGRALS

: HINTS AND SOLUTIONS :

1 **(b)**

We have,

$$I=\int_0^{\pi/2}\frac{1}{9\cos x+12\sin x}dx$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{1+\tan^2 x/2}{9-9\tan^2 x/2+24\tan x/2}dx$$

$$\Rightarrow I=\int_0^{1}\frac{2dt}{9-9t^2+24t}, \text{where } t=\tan\frac{x}{2}$$

$$\Rightarrow I=-\frac{2}{9}\int_0^1\frac{dt}{t^2-\frac{8}{3}t-1}$$

$$\Rightarrow I=-\frac{2}{9}\int_0^1\frac{1}{\left(t-\frac{4}{3}\right)^2-\left(\frac{5}{3}\right)^2}dt$$

$$\Rightarrow I=\frac{2}{9}\int_0^1\frac{1}{\left(\frac{5}{3}\right)^2-\left(t-\frac{4}{3}\right)^2}dt$$

$$\Rightarrow I=\frac{2}{9}\times\frac{3}{10}\left[\log\left(\frac{t-\frac{4}{3}+\frac{5}{3}}{\frac{5}{3}-t+\frac{4}{3}}\right)\right]_0^1$$

$$\Rightarrow I=\frac{1}{15}\left[\log\left(\frac{3t+1}{9-3t}\right)\right]_0^1=\frac{1}{15}\left(\log\frac{4}{6}-\log\frac{1}{9}\right)$$

$$=\frac{1}{15}\log_e 6$$

2 **(c)**

$$I=\int_{-2}^{0}[x^3+3x^2+3x+3 + (x+1)\cos(x+1)]dx$$

$$=\int_{-2}^{0}[(x+1)^3+2+(x+1)\cos(x+1)]dx$$

Put, $x+1=t\Rightarrow dx=dt$

$$\therefore\ I=\int_{-1}^{1}t^3dt+2\int_{-1}^{1}dt+\int_{-1}^{1}t\cos t\,dt$$

$$=0+2[1-(-1)]+0$$

$$\Rightarrow I=4\left[\begin{array}{c}\because t^3 \text{and } t\cos t \text{ are odd functions.}\\ \therefore \int_{-1}^{1}t^3dt=\int_{1}^{1}t\cos t\,dt=0\end{array}\right]$$

3 **(c)**

We have,

$$I=\int\frac{e^{2x}-2e^x}{e^{2x}+1}dx$$

$$\Rightarrow I=\int\frac{e^{2x}}{e^{2x}+1}dx-\int\frac{2e^x}{e^{2x}+1}dx$$

$$\Rightarrow I=\frac{1}{2}\int\frac{1}{e^{2x}+1}d(e^{2x}+1) - 2\int\frac{1}{(e^x)^2+1^2}d(e^x)$$

$$\Rightarrow I=\frac{1}{2}\log(e^{2x}+1)-2\tan^{-1}(e^x)+C$$

4 **(b)**

Let $I=\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$$=\int_0^1\tan^{-1}x\,dx+\int_0^1\tan^{-1}(x-1)dx$$

$$=\int_0^1\tan^{-1}x\,dx+\int_0^1\tan^{-1}(1-x-1)dx$$

$$=\int_0^1\tan^{-1}x\,dx-\int_0^1\tan^{-1}x\,dx=0$$

5 **(b)**

$$\int\sqrt{1+\sin\left(\frac{x}{4}\right)}\,dx=\int\sqrt{1+2\sin\frac{x}{8}\cos\frac{x}{8}}\,dx$$

$$=\int\sqrt{\sin^2\frac{x}{8}+\cos^2\frac{x}{8}+2\sin\frac{x}{8}\cos\frac{x}{8}}\,dx$$

$$=\int\sqrt{\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)^2}\,dx$$

$$=\int\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)dx$$

$$=\frac{-\cos\frac{x}{8}}{1/8}+\frac{\sin\frac{x}{8}}{1/8}+c$$

$= 8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$

6 **(b)**

Let $I = \int_a^b \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{a+b-x}}$ ... (i)

$\therefore I = \int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{x}}$ ... (ii)

On adding Eqs. (i) and (ii),we get

$2I = \int_a^b dx = [x]_a^b = b - a$

$\Rightarrow I = \frac{b-a}{2}$

7 **(d)**

We have,

$I = \int_0^3 \frac{1}{\sqrt{x+1}+\sqrt{5x+1}} dx$

$= \int_0^3 \frac{\sqrt{x+1}-\sqrt{5x+1}}{-4x} dx$

$\Rightarrow I = -\frac{1}{4}\int_0^3 \frac{\sqrt{x+1}}{x} dx + \frac{1}{4}\int_0^3 \frac{\sqrt{5x+1}}{x} dx$

$\Rightarrow I = I_1 + I_2$, where

$I_1 = -\frac{1}{4}\int_0^3 \frac{\sqrt{x+1}}{x} dx$ and $I_2 = \frac{1}{4}\int_0^3 \frac{\sqrt{5x+1}}{x} dx$

Putting $x + 1 = t^2$ in I_1, we get

$I_1 = -\frac{1}{2}\int_1^2 \frac{t^2}{t^2-1} dt$

$\Rightarrow I_1 = -\frac{1}{2}\int_1^2 \frac{t^2-1+1}{t^2-1} dt$

$\Rightarrow I_1 = -\frac{1}{2}\int_1^2 \left(1 + \frac{1}{t^2-1}\right) dt$

$\Rightarrow I_1 = -\frac{1}{2}\left[t + \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right]_1^2$

$\Rightarrow I_1 = -\frac{1}{2}\left[1 - \frac{1}{2}\lim_{t\to 1}\log\left|\frac{t-1}{t+1}\right|\right]$

Now, putting $5x + 1 = t^2$ in, I_2 we get

$I_2 = \frac{1}{4}\int_1^4 \frac{2t^2}{t^2-1} = \frac{1}{2}\int_1^4 \frac{t^2}{t^2-1} dt$

$= \frac{1}{2}\int_1^4 \frac{t^2-1+1}{t^2-1} dt$

$= \frac{1}{2}\left[t + \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right]_1^4$

$= \frac{1}{2}\left[3 + \frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\lim_{t\to 1}\left|\frac{t-1}{t+1}\right|\right]$

$\therefore I = I_1 + I_2 = 1 + \frac{1}{4}\log\frac{3}{5}$

8 **(d)**

Let $I = \int_0^{12a} \frac{f(x)}{f(x)+f(12a-x)} dx$ ...(i)

$\Rightarrow I = \int_0^{12a} \frac{f(12a-x)}{f(12a-x)+f(x)} dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{12a} 1dx = [x]_0^{12a}$

$\Rightarrow I = 6a$

9 **(c)**

We have,

$I = \int_{-2}^{2} \left[p\log\left(\frac{1+x}{1-x}\right) + q\log\left(\frac{1-x}{1+x}\right)^{-2} + r\right] dx$

Since, $\log\left(\frac{1+x}{1-x}\right)$ is an odd function

$\therefore \int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$

$\Rightarrow I = \int_{-2}^{2} r dx = r(2+2) = 4r$

10 **(a)**

We have,

$I = \int \frac{3^x}{\sqrt{1-9^x}} dx = \frac{1}{\log_e 3}\int \frac{1}{\sqrt{1^2-(3^x)^2}} d(3^x)$

$= (\log_3 e)\sin^{-1}(3^x) + C$

11 **(c)**

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore \int \frac{dx}{x\log x} = \int \frac{dt}{t} = \log t +$constant

$= \log(\log x) +$constant$= f(x) +$constant

$\therefore f(x) = \log(\log x)$

12 **(c)**

We have,

$\int_{\log 2}^{x} \frac{1}{e^x-1} dx = \log\left(\frac{3}{2}\right)$

$\Rightarrow \int_{\log 2}^{x} \frac{e^{-x}}{1-e^{-x}} dx = \log\left(\frac{3}{2}\right)$

$\Rightarrow [\log(1-e^{-x})]_{\log 2}^{x} = \log\frac{3}{2}$

$\Rightarrow \log(1-e^{-x}) - \log(1-e^{-\log 2}) = \log\frac{3}{2}$

$\Rightarrow \log(1-e^{-x}) - \log\left(1-\frac{1}{2}\right) = \log\frac{3}{2}$

$\Rightarrow \log(1-e^{-x}) = \log\frac{3}{2} + \log\frac{1}{2}$

$\Rightarrow \log(1-e^{-x}) = \log\frac{3}{4}$

$\Rightarrow 1-e^{-x} = \frac{3}{4} \Rightarrow e^{-x} = \frac{1}{4} \Rightarrow e^{x} = 4 \Rightarrow x = \log 4$

13 **(a)**

$I = \sum_{r=1}^{10} \int_0^1 f(r-1+x)dx$

$I_t = \int_0^1 f(t-1+x)dx$

Put $t-1+x = y \Rightarrow dx = dy$

$I_t = \int_{t-1}^{t} f(y)dy \Rightarrow I_t = \int_{t-1}^{t} f(x)dx$

$\therefore I_1 = \int_0^1 f(x)dx, I_2 = \int_1^2 f(x)dx,$

$I_3 = \int_2^3 f(x)dx, \ldots, I_{10} = \int_9^{10} f(x)dx$

So, $I = I_1 + I_2 + \ldots + I_{10} = \int_9^{10} f(x)dx$

14 **(b)**

We have,

$I = \int_{-1}^{1} [x^2 + \{x\}]\, dx$

$\Rightarrow I = \int_{-1}^{0} [x^2 + \{x\}]\, dx + \int_{0}^{1} [x^2 + \{x\}]\, dx$

$\Rightarrow I = \int_{-1}^{0} [x^2 + x + 1]\, dx + \int_{0}^{1} [x^2 + x]\, dx$

$\Rightarrow I = \int_{-1}^{0} ([x^2 + x] + 1)\, dx + \int_{0}^{1} [x^2 + x]\, dx$

$\Rightarrow I = \int_{-1}^{0} [x^2 + x]\, dx + \int_{-1}^{0} dx + \int_{0}^{1} [x^2 + x]\, dx$

$\Rightarrow I = \int_{-1}^{0} 0\, dx + \int_{-1}^{0} dx + \int_{0}^{2} [x^2 + x]\, dx$

$+ \int_{\frac{\sqrt{5}-1}{2}}^{1} [x^2 + x]\, dx$

$\Rightarrow I = 1 + \int_{0}^{\frac{\sqrt{5}-1}{2}} 0\, dx + \int_{\frac{\sqrt{5}-1}{2}}^{1} 1 dx = 1 + 1 - \frac{\sqrt{5}-1}{2}$

$= \frac{\sqrt{5}-5}{2}$

15 **(a)**

Here, $ff(x) = \frac{f(x)}{[1+f(x)^n]^{1/n}} = \frac{x}{(1+2x^n)^{1/n}}$

and $fff(x) = \frac{x}{(1+3x^n)^{1/n}}$

$g(x) = \underbrace{(fof o \ldots of)}_{n \text{ times}}(x).$

$\therefore \qquad = \frac{x}{(1+nx^n)^{1/n}}$

Let $I = \int x^{n-2}\, g(x)dx = \int \frac{x^{n-1}dx}{(1+nx^n)^{1/n}}$

$= \frac{1}{n^2} \int \frac{n^2 x^{n-1} dx}{(1+nx^n)^{1/n}}$

$= \frac{1}{n^2} \int \frac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}\, dx$

$\therefore\ I = \frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + c$

16 **(c)**

We have,

$I_n = \int_0^{\pi/2} \sin^n x\, dx$

$\Rightarrow I_n = \int_0^{\pi/2} \sin^{n-1} x \sin x\, dx$

$\qquad\qquad I \qquad\quad II$

$\Rightarrow I_n = [-\sin^{n-1} x \cos x]_0^{\pi/2}$

$+ (n-1) \int_0^{\pi/2} \sin^{n-2} x \cos^2 x\, dx$

$\Rightarrow I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x\, (1-\sin^2 x) dx$

$\Rightarrow I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x\, dx - (n$

$-1) \int_0^{\pi/2} \sin^n x\, dx$

$\Rightarrow I_n = (n-1)I_{n-2} - (n-1)I_n$

$\Rightarrow nI_n = (n-1)I_{n-2}$

$\Rightarrow I_{n-2} = n(I_{n-2} - I_n)$

17 **(d)**

We have,

$I = \int_0^3 x\sqrt{1+x}\, dx$

$\Rightarrow I = \left[\frac{2x}{3}(1+x)^{3/2}\right]_0^3 - \frac{2}{3}\int_0^3 (1+x)^{3/2} x$

$$\Rightarrow I=\left[\frac{2x}{3}(1+x)^{3/2}\right]_0^3-\frac{4}{15}\left[(1+x)^{5/2}\right]_0^3=\frac{116}{15}$$

18 **(b)**

Let $I=\int_0^1 \log\left\{\sin\left(\frac{\pi x}{2}\right)\right\}dx$

Put $\frac{\pi x}{2}=\theta \Rightarrow dx=\frac{2}{\pi}\, d\theta$

$\therefore I=\frac{2}{\pi}\int_0^{\pi/2}\log\sin\theta\, d\theta$

$=\frac{2}{\pi}\left(-\frac{\pi}{2}\log 2\right)=-\log 2$

19 **(b)**

Let $I=\int \frac{dx}{x(x^7+1)}$

Put $x^7=t \Rightarrow dx=\frac{1}{7x^6}dt$

$\therefore I=\frac{1}{7}\int\frac{dt}{t(t+1)}=\frac{1}{7}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$

$=\frac{1}{7}[\log t-\log(t+1)]+c$

$=\frac{1}{7}\log\left(\frac{t}{t+1}\right)+c$

$=\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right)+c$

20 **(d)**

Let $I=\int\cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

Put $x=\cos\theta$, then

$I=\int\cos\left\{2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\}dx$

$=\int\cos\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}dx$

$=\int\cos\theta\, dx=\int x\, dx=\frac{x^2}{2}+c$

21 **(c)**

Let $I=\int\frac{x^2+1}{x^2-1}\, dx$

$\Rightarrow I=\int\frac{x^2+1-1+1}{x^2-1}\, dx$

$\Rightarrow I=\int\frac{x^2-1}{x^2-1}\, dx+\int\frac{2}{x^2-1}\, dx$

$\Rightarrow I=\int 1\, dx+2\int\frac{1}{x^2-1}\, dx$

$\Rightarrow I=x+2\,\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+c$

$\Rightarrow I=x+\log\left(\frac{x-1}{x+1}\right)+c$

23 **(b)**

Let $I=\int\frac{dx}{x+\sqrt{x}}=\int\frac{1}{\sqrt{x}(1+\sqrt{x})}dx$

Put $1+\sqrt{x}=t \Rightarrow \frac{1}{\sqrt{x}}dx=2dt$

$\therefore I=2\int\frac{1}{t}\, dt=2\log t+c$

$=2\log(1+\sqrt{x})+c$

24 **(b)**

We have,

$I=\int\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$

$\Rightarrow I=\int\frac{1}{\sqrt{\sin^4 x\,(\cos\alpha+\cot x\sin\alpha)}}dx$

$\Rightarrow I=\int\frac{1}{\sqrt{\cos\alpha+\cot x\sin\alpha}}\operatorname{cosec}^2 x\, dx$

$\Rightarrow I=-\frac{1}{\sin\alpha}\int\frac{1}{\sqrt{\cos\alpha+\cot x\sin\alpha}}d(\cos\alpha+\cot x\sin\alpha)$

$\Rightarrow I=-\frac{2}{\sin\alpha}\sqrt{\cos\alpha+\cot x\sin\alpha}+C$

$\Rightarrow I=-2\operatorname{cosec}\alpha\sqrt{\cos\alpha+\cot x\sin\alpha}+C$

25 **(a)**

Let $I=\int_0^{\pi/2}\log\sin x\, dx$ ...(i)

$I=\int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi/2}\log\cos x\, dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I=\int_0^{\pi/2}\log\sin x\ \cos x\, dx$

$=\int_0^{\pi/2}\log\sin 2x\, dx-\int_0^{\pi/2}\log 2\ dx$

$=\frac{1}{2}\int_0^{\pi}\log\sin t\, dt-\frac{\pi}{2}\log 2$

$=\frac{1}{2}.2\int_0^{\pi/2}\log\sin t\, dt-\frac{\pi}{2}\log 2$

$\Rightarrow 2I=I-\frac{\pi}{2}\log 2 \Rightarrow I=-\frac{\pi}{2}\log 2$

26 **(c)**

Let $I=\int_0^1\frac{dx}{x+\sqrt{x}}=\int_0^1\frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

Put $\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$

$\Rightarrow \frac{1}{\sqrt{x}}dx=2dt$

$I=\int_0^1\frac{2dt}{t+1}=2[\log(t+1)]_0^1$

$=2(\log 2-\log 0)=\log 4$

27 **(c)**

It is given that

$f'(3)=\tan\frac{\pi}{4}, f'(2)=\tan\frac{\pi}{3}, f'(1)=\tan\frac{\pi}{6}$

Now,

$$\int_1^3 f''(x)f'(x)dx + \int_2^3 f''(x)dx$$

$$= \left[\frac{\{f'(x)\}^2}{2}\right]_1^3 + [f'(x)]_2^3$$

$$= \frac{1}{2}[\{f'(3)\}^2 - \{f'(1)\}^2] + [f'(3) - f'(2)]$$

$$= \frac{1}{2}\left[1 - \frac{1}{3}\right] + \left[1 - \frac{1}{\sqrt{3}}\right] = \frac{4}{3} - \frac{1}{\sqrt{3}}$$

28 **(a)**

Given, $f(x) = 2x^2 - 4x + 1$

$$\therefore\ g(x) = \frac{2x^2 - 4x + 1 - 2x^2 - 4x - 1}{2}$$

$$= -4x$$

$$\therefore \int_{-3}^{3} g(x)dx = -4\int_{-3}^{3} x\,dx$$

$$= -4\left[\frac{x^2}{2}\right]_{-3}^{3} = -2[9 - 9] = 0$$

29 **(a)**

$$I = \int \frac{x + \sin x}{1 + \cos x}dx = \int \frac{x}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}dx$$

$$\Rightarrow I = \int x\frac{1}{2}\sec^2\frac{x}{2}dx + \int \tan\frac{x}{2}dx$$

$$\Rightarrow I = x\tan\frac{x}{2} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx + C$$

$$= x\tan\frac{x}{2} + C$$

30 **(a)**

Let $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ ...(i)

$$= \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-9+x} + \sqrt{9-x}}dx$$

$\Longrightarrow I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_3^6 1\,dx = [x]_3^6 = 6 - 3 \Longrightarrow I = \frac{3}{2}$$

31 **(b)**

Let $I = \int \frac{dx}{(a^2+x^2)^{3/2}}$

Put $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\,d\theta$

$$\therefore I = \int \frac{a\sec^2\theta}{(a^2 + a^2\tan^2\theta)^{3/2}}d\theta$$

$$= \int \frac{a\sec^2\theta}{a^3(\sec^2\theta)^{3/2}}d\theta$$

$$\Rightarrow I = \frac{1}{a^2}\int \frac{d\theta}{\sec\theta} = \frac{1}{a^2}\int \cos\theta\,d\theta$$

$$= \frac{1}{a^2}\sec\theta + c$$

$$\Rightarrow \frac{x}{a^2(x^2 + a^2)^{1/2}} + c$$

32 **(d)**

Let $I = \int_0^{\pi/2}\sin^{100} x dx - \int_0^{\pi/2}\cos^{100} x dx$

$= 0 - 0 = 0$

$= 0$

33 **(a)**

$$\int [e^x f(x) + f'(x)e^x]dx$$

$$= e^x f(x) - \int e^x f'(x)dx + \int e^x f'(x)dx$$

$$= e^x f(x) + c$$

34 **(c)**

Let $I = \int \frac{dx}{1-\cos x-\sin x}$

Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore I = \int \frac{dx}{1 - \frac{\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} - \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$= \int \frac{\sec^2\frac{x}{2}dx}{\left[1 + \tan^2\frac{x}{2} - 1 + \tan^2\frac{x}{2} - 2\tan\frac{x}{2}\right]}$$

$$= \int \frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2} - 2\tan\frac{x}{2}} = \int \frac{\frac{1}{2}\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2} - \tan\frac{x}{2}}$$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$\therefore I = \int \frac{dt}{t^2 - t} = \int \frac{dt}{(t-1)}$$

$$= \int \left[\frac{1}{t-1} - \frac{1}{t}\right]dt = \int \frac{dt}{t-1} - \int \frac{dt}{t}$$

$$= \log(t-1) - \log t + c = \log\left|\frac{t-1}{t}\right| + c$$

$$= \log\left|\frac{\tan\frac{x}{2} - 1}{\tan\frac{x}{2}}\right| + c = \log\left|1 - \cot\frac{x}{2}\right| + c$$

35 **(a)**

We have,

$$\int_{\pi/2}^{\alpha} \sin x\,dx = \sin 2\alpha$$

$\Rightarrow -\cos\alpha = \sin 2\alpha$

$\Rightarrow \cos\alpha\,(2\sin\alpha + 1) = 0$

$\Rightarrow \cos\alpha = 0$ or, $\sin\alpha = -\frac{1}{2} \Rightarrow \alpha = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow \alpha = \frac{7\pi}{6}, \frac{11\pi}{6}$

36 **(c)**

We have,

$$\int_2^4 \{3 - f(x)\}dx = 7 \Rightarrow 6 - \int_2^4 f(x)\,dx = 7$$

$$\Rightarrow \int_2^4 f(x)\,dx = -1$$

$$\therefore I = \int_2^{-1} f(x)\,dx$$

$$\Rightarrow I = -\int_{-1}^2 f(x)\,dx$$

$$\Rightarrow I = -\left\{\int_{-1}^4 f(x)dx + \int_4^2 f(x)dx\right\}$$

$$\Rightarrow I = -\left\{\int_{-1}^4 f(x)dx - \int_2^4 f(x)\,dx\right\} = -(4+1)$$

$$= -5$$

37 **(a)**

$$\int f(x)g(x)dx = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, e^{\sin^{-1} x}\, dx$$

Put $\sin^{-1} x = t \quad \Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$$\therefore \int f(x)\,g(x)dx = \int te^t\,dt = te^t - e^t + c$$

$= e^{\sin^{-1} x}(\sin^{-1} x - 1) + c$

38 **(a)**

We have,

$$I = \int_{-1}^3 \{|x-2| + [x]\}dx = \int_{-1}^3 |x-2|dx + \int_{-1}^3 [x]\,dx$$

Now,

$$\int_{-1}^3 |x-2|\,dx = \int_{-1}^2 |x-2|\,dx + \int_2^3 |x-2|\,dx$$

$$= \int_{-1}^2 (2-x)\,dx + \int_2^3 (x-2)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{-1}^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= \left(2 + \frac{5}{2}\right) + \left(\frac{-3}{2} + 2\right) = 5$$

and,

$$\int_{-1}^3 [x]\,dx = \int_{-1}^0 -1\,dx + \int_0^1 0\,dx + \int_1^2 1.\,dx + \int_2^3 2.\,dx$$

$$\Rightarrow \int_{-1}^3 [x]\,dx = -1 + 1 + 2 = 2$$

$$\therefore I = \int_{-1}^3 (|x-2| + [x])\,dx = 5 + 2 = 7$$

39 **(b)**

Here, $g(2) = \int_0^2 f(t)dt$

$$= \int_0^1 f(t)dt + \int_1^2 f(t)dt$$

As $\frac{1}{2} \le f(t) \le 1$ for $0 \le t \le 1$

$$\Rightarrow \int_0^1 \frac{1}{2}dt \le \int_0^1 f(t)dt \le \int_0^1 dt$$

$\Rightarrow \frac{1}{2} \le \int_0^1 f(t)dt \le 1 \quad$ ...(i)

As $0 \le f(t) \le \frac{1}{2}$ for $1 < t \le 2$

$$\Rightarrow \int_1^2 0\,dt \le \int_1^2 f(t)dt \le \int_1^2 \frac{1}{2}dt$$

$\therefore \quad 0 \le \int_1^2 f(t)dt \le \frac{1}{2} \quad$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$\frac{1}{2} \le g(2) \le \frac{3}{2}$$

$\therefore$ g(2) satisfies the inequality $0 \le g(2) < 2$

40 **(a)**

$$\int \frac{x^2}{\sqrt{1-x}}dx = p\sqrt{(1-x)}(3x^2 + 4x + 8)$$

Put $1 - x = t^2$ in LHS, we get

$$I = -\int \frac{2t}{t}(1-t^2)^2 dt$$

$$= -2\int (1 + t^4 - 2t^2)\,dt$$

$$= -2\left[t + \frac{t^5}{5} - \frac{2t^3}{3}\right]$$

$$= -2\sqrt{1-x}\left[1 + \frac{(1-x)^2}{5} - \frac{2}{3}(1-x)\right]$$

$$= -2\sqrt{1-x}\left[\frac{15 + 3(1 + x^2 - 2x) - 10(1-x}{15}\right.$$

$$= \frac{-2}{15}\sqrt{1-x}(3x^2 + 4x + 8)$$

But $I = p\sqrt{1-x}(3x^2 + 4x + 8)$

$$\therefore p = \frac{-2}{15}$$

41 **(c)**

$\{x\} = x, 0 \le x < 1$

$$I = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

42 **(d)**

Given, $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$

Let $1 + x^3 = t \Rightarrow 3x^2dx = dt$

$\therefore I=\int \frac{(t-1)}{\sqrt{t}}.\frac{dt}{3}=\frac{1}{3}\int\left(\sqrt{t}-t^{-1/2}\right)dt$

$=\frac{1}{3}\left[\frac{2t^{3/2}}{3}-2t^{1/2}\right]+c$

$=\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+c$

44 **(b)**

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}|\sin x|dx$

$=2\int_0^{\frac{\pi}{2}}\sin x\,dx=-2[\cos x\,]_0^{\frac{\pi}{2}}$

$=-2\left(\cos\frac{\pi}{2}-\cos 0\right)=2$

45 **(a)**

$\int_{-2}^{3}|1-x^2|dx=\int_{-2}^{-1}(x^2-1)dx$

$+\int_{-1}^{1}(1-x^2)dx+\int_{1}^{3}(x^2-1)dx$

$=\left[\frac{x^3}{3}-x\right]_{-2}^{-1}+\left[x+\frac{x^3}{3}\right]_{-1}^{1}+\left[\frac{x^3}{3}-x\right]_1^3$

$=\frac{4}{3}+\frac{4}{3}+\frac{20}{3}=\frac{28}{3}$

46 **(d)**

We know that, $|x|=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$

$\therefore \int_{-2}^{2}(x-|x|)dx$

$=\int_{-2}^{0}\{x-(-x)\}dx$

$+\int_0^2(x-x)dx$

$=\int_{-2}^{0}2x\,dx+0$

$=2\left[\frac{x^2}{2}\right]_{-2}^{0}$

$=2\left[0-\frac{(-2)^2}{2}\right]=-4$

47 **(d)**

Given, $\int_0^1\log[x]\,dx=\int_0^1\log 0dx$

Since, log 0 does not exist, so we cannot find the value of given integral.

48 **(d)**

Let $I=\int_0^1\frac{dx}{x+\sqrt{1-x^2}}$

Put $x=\cos\theta\Rightarrow dx=-\sin\theta d\theta$

$\therefore I=\int_{\pi/2}^{0}\frac{-\sin\theta d\theta}{\cos\theta+\sqrt{1-\cos^2\theta}}$

$=\int_0^{\pi/2}\frac{\sin\theta}{\sin\theta+\cos\theta}d\theta$......(i)

$\Rightarrow I=\int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}d\theta$

$=\int_0^{\pi/2}\frac{\cos\theta}{\sin\theta+\cos\theta}d\theta$...(ii)

On adding Eqs. (i) and (ii), we get

$2I=\int_0^{\pi/2}d\theta=\frac{\pi}{2}$

$\Rightarrow I=\frac{\pi}{4}$

49 **(c)**

Let $I=\int\frac{\sin\theta+\cos\theta}{\sqrt{1+\sin 2\theta-1}}d\theta$

$=\int\frac{\sin\theta+\cos\theta}{\sqrt{1-(\sin\theta-\cos\theta)^2}}d\theta$

Put $\sin\theta-\cos\theta=t$

$\Rightarrow(\cos\theta+\sin\theta)d\theta=dt$

$\therefore\ I=\int\frac{1}{\sqrt{1-t^2}}dt$

$=\sin^{-1}t+c$

$=\sin^{-1}(\sin\theta-\cos\theta)+c$

50 **(b)**

We have,

$\frac{\cos x+x\sin x}{x(x+\cos x)}=\frac{(x+\cos x)-x+x\sin x}{x(x+\cos x)}$

$=\frac{1}{x}-\frac{(1-\sin x)}{x+\cos x}$

$\therefore I=\int\frac{\cos x+x\sin x}{x(x+\cos x)}dx=\int\frac{1}{x}-\frac{1-\sin x}{x+\cos x}dx$

$\Rightarrow I=\log x-\log(x+\cos x)+C$

$=\log\left(\frac{x}{x+\cos x}\right)+C$

51 **(a)**

Let $I=\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{1+(x+1)^2}$

$=\tan^{-1}(x+1)+c=f(x)+c$ [given]

$\therefore\ f(x)=\tan^{-1}(x+1)$

52 **(a)**

Let $I=\int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\cos x\sin x}$ ...(i)

$\Rightarrow I=\int_0^{\pi/2}\frac{\cos\left(\frac{\pi}{2}-x\right)-\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I=-\int_0^{\pi/2}\left(\frac{\cos x-\sin x}{1+\cos x\sin x}\right)dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 0dx = 0 \Rightarrow I = 0$$

53 **(c)**

$$\int \operatorname{cosec}^4 x\,dx = \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x\,dx$$

$$= \int \operatorname{cosec}^2 x(1+\cot^2 x)dx$$

$$= \int \operatorname{cosec}^2 x\,dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x\,dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

54 **(a)**

Let $I = \int_0^{\pi/2} \frac{\sin^{2/3} x}{\sin^{2/3} x + \cos^{2/3} x} dx$ ...(i)

and $I = \int_0^{\pi/2} \frac{\sin^{2/3}\left(\frac{\pi}{2}-x\right)}{\sin^{2/3}\left(\frac{\pi}{2}-x\right) + \cos^{2/3}\left(\frac{\pi}{2}-x\right)} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^{2/3} x}{\cos^{2/3} x + \sin^{2/3} x} dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin^{2/3} x + \cos^{2/3} x)}{(\sin^{2/3} x + \cos^{2/3} x)} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} 1\,dx \Rightarrow I = \frac{1}{2}[x]_0^{\pi/2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

55 **(d)**

Let $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ ...(ii)

$\left[\text{put } x = \left(\frac{\pi}{2} - x\right)\right]$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

56 **(c)**

Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ ...(i)

$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^{-x}} dx$ ...(ii)

On adding Eqs. (i)and (ii), we get

$$2I = \int_{-\pi}^{\pi} \cos^2 x dx$$

$$= \int_{-\pi}^{\pi} \frac{(\cos 2x + 1)}{2} dx$$

$$\Rightarrow 2I = \frac{1}{2}\left[\left(\frac{\sin 2x}{2} + x\right)\right]_{-\pi}^{\pi} = \frac{1}{2}(\pi + \pi)$$

$$\Rightarrow I = \frac{\pi}{2}$$

57 **(b)**

Let $I = \int_0^{\pi} x \sin^3 dx$ ...(i)

Also, $I = \int_0^{\pi} (\pi - x) \sin^3 x dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \sin^3 x\,dx$$

$$= \frac{\pi}{4} \int_0^{\pi} (3\sin x - \sin 3x)dx$$

$$= \frac{\pi}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi} = \frac{4\pi}{3}$$

Hence, $I = \frac{2\pi}{3}$

58 **(c)**

Let $I = \int_{-2}^{2} (px^3 + qx + s)dx = 0 + 0 + \int_{-2}^{2} sdx$

[$\because px^3$ and qx are odd functions]

$= 2s[x]_0^2 = 4s$

Hence, it is necessary to know the value of s

59 **(d)**

Let $I = \int e^{\tan^{-1} x} dx + \int e^{\tan^{-1} x} . \frac{x}{(1+x^2)}\,dx$

$$= \int \frac{d}{dx}\left(x\,e^{\tan^{-1} x}\right)dx + c$$

$$= xe^{\tan^{-1} x} + c$$

60 **(b)**

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4 - \sin^2\theta}} d\theta = \left[\sin^{-1}\left(\frac{\sin\theta}{2}\right)\right]_0^{\pi/2}$$

$$= \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

61 **(b)**

Let

$$A = \lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)...\left(1+\frac{n}{n}\right)\right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log A = \int_0^1 \log(1+x)\,dx$$

$$\Rightarrow \log A = [x\log(1+x) - x + \log(x+1)]_0^1$$

$$= \log\left(\frac{4}{e}\right)$$

$$\Rightarrow A = \frac{4}{e}$$

62 **(d)**

Let $I = \int_2^k (2x+1)\,dx$

$$= \left[\frac{2x^2}{2} + x\right]_2^k$$

$$= k^2 + k - 4 - 2$$

$= k^2 + k - 6$

But given, $I = 6$

$\Rightarrow k^2 + k - 6 = 6$

$\Rightarrow (k+4)(k-3) = 0$

$\Rightarrow k = 3 \quad [\because k \neq -4]$

63 **(b)**

We have,

$$I = \int_0^1 \cos\left\{2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$$

Putting $x = \cos\theta$ and $dx = -\sin\theta\, d\theta$, we get

$$I = \int_{\pi/2}^{0} \cos\left\{2\cot^{-1}\left(\tan\frac{\theta}{2}\right)\right\}(-\sin\theta)d\theta$$

$$\Rightarrow I = \int_{\pi/2}^{0} \cos\left\{2\cot^{-1}\left(\cot\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right)\right\}(-\sin\theta)d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \cos\left\{2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right\}\sin\theta\, d\theta$$

$$\Rightarrow I = -\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta$$

$$\Rightarrow I = -\frac{1}{2}\int_0^{\pi/2} \sin 2\theta\, d\theta = \frac{1}{4}[\cos 2\theta]_0^{\pi/2}$$

$$= \frac{1}{4}(-1-1) = -\frac{1}{2}$$

64 **(b)**

Let $I = \int(\sin x - \cos x)^4(\sin x + \cos x)dx$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

$$\therefore\ I = \int t^4\ dt = \frac{t^5}{5} + c = \frac{(\sin x - \cos x)^5}{5} + c$$

65 **(c)**

Let $I = \int \frac{1}{\sqrt{x}(4+\sqrt{x})}dx$

Put $4 + \sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

$\therefore\ I = 2\int\frac{1}{t}dt = 2\log|4+\sqrt{x}| + c$

66 **(d)**

Let $I = \int_0^{\pi/2}\frac{1}{1+\tan^3 x}dx = \int_0^{\pi/2}\frac{1}{1+\frac{\sin^3 x}{\cos^3 x}}dx$

$= \int_0^{\pi/2}\frac{\cos^3 x}{\cos^3 x+\sin^3 x}dx \quad$ ...(i)

$$= \int_0^{\pi/2}\frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right)+\sin^3\left(\frac{\pi}{2}-x\right)}dx$$

$\Rightarrow I = \int_0^{\pi/2}\frac{\sin^3 x}{\sin^3 x+\cos^3 x}dx \quad$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/3}\frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}dx$$

$$\Rightarrow\ 2I = \int_0^{\pi/2} 1\, dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

67 **(a)**

$$\int\frac{dx}{\cos x + \sqrt{3}\sin x} = \int\frac{dx}{2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)}$$

$$= \frac{1}{2}\int\sec\left(x - \frac{\pi}{3}\right)dx$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + c$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c$$

68 **(a)**

The integrand is an odd function. So, the value of given integral is zero

69 **(c)**

Since $\sqrt{3+x^3}$ is strictly increasing function on (1, 3)

$\therefore \sqrt{3+1} < \sqrt{3+x^3} < \sqrt{3+(3^3)}$ for all $x \in (1,3)$

$\Rightarrow 2 < \sqrt{3+x^3} < \sqrt{30}$ for all $x \in (1,3)$

$$\Rightarrow 2\int_1^3 dx < \int_1^3\sqrt{3+x^3}\,dx < \int_1^3\sqrt{30}\,dx$$

$$\Rightarrow 4 < \int_1^3\sqrt{3+x^3}\,dx < 2\sqrt{30}$$

$$\Rightarrow I = \int_1^3\sqrt{3+x^3}\,dx \in \left(4, 2\sqrt{30}\right)$$

70 **(c)**

Since x^3, $x\cos x$ and $\tan^5 x$ are odd functions

$$\therefore \int_{-\pi/2}^{\pi/2} x^3 dx = 0,\ \int_{-\pi/2}^{\pi/2} x\cos x\ dx$$

$$= 0,\ \int_{-\pi/2}^{\pi/2}\tan^5 x\ dx = 0$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\,dx = \int_{-\pi/2}^{\pi/2} dx$$

$$= \pi$$

71 **(d)**

$\int_{-1}^{1}[x \sin \pi x]\,dx = 2\int_0^1 [x\sin \pi x]\,dx$ ...(i)

(as $[x \sin \pi x]$ is a even function)

Now, $0 \le x \le 1$

$\Rightarrow\ 0 \le \pi x \le \pi$

$\Rightarrow\ 0 \le \sin \pi x \le 1$ ....(ii)

$\Rightarrow\ 0 \le x \sin \pi x < 1$

$\Rightarrow\ [x \sin \pi x] = 0$

From Eqs. (i) and (ii), we get

$$\int_0^1 [x \sin \pi x] dx = 0$$

72 **(b)**

Let $I = \int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$

$$= \int_{e^{-1}}^{1} \left|\frac{\log_e x}{x}\right| dx + \int_1^{e^2} \left|\frac{\log_e x}{x}\right| dx$$

$$= \int_{e^{-1}}^{1} \left(-\frac{\log_e x}{x}\right) dx + \int_1^{e^2} \frac{\log_e x}{x} dx$$

Put $\log_e x = z\ \Rightarrow\ \frac{1}{x}dx = dz$

$$\therefore\ I = \int_{-1}^{0} (-z)dz + \int_0^2 z\,dz$$

$$= \left[-\frac{z^2}{2}\right]_{-1}^{0} + \left[\frac{z^2}{2}\right]_0^2$$

$$= -\frac{1}{2}[0-1] + \frac{1}{2}[4-0]$$

$$= \frac{1}{2} + 2 = \frac{5}{2}$$

73 **(c)**

We have, $f(x) = -\cos^5 x - \sin^2 x$

$$\therefore I = \int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} (-\cos^5 x - \sin^2 x)\,dx$$

$\Rightarrow I = -\int_0^1 (1-t^2)^2\,dt - \int_0^{\pi/2} \frac{1-\cos 2x}{2} dx$, where $t = \sin x$

$$\Rightarrow I = -\frac{\pi}{4} - \frac{8}{15}$$

74 **(c)**

Putting $\sin x = t$ in the given integral, we get

$$I = \int \frac{1 - t^2 + (1-t^2)^2}{t^2 + t^4} dt$$

$$= \int \frac{(1-t^2)(2-t^2)}{t^2+t^4} dt$$

$$\Rightarrow I = \left(1 + \frac{2}{t^2} - \frac{6}{1+t^2}\right) dt$$

$$= t - \frac{2}{t} - 6\tan^{-1}(t) + C$$

$$\Rightarrow I = \sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$$

75 **(d)**

Let $I = \int_0^1 x^{3/2}\sqrt{1-x}dx$

Put $x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta d\,\theta$

$$\therefore I = \int_0^{\pi/2} \sin^3\theta.\sqrt{1-\sin^2\theta}\,2\sin\theta\cos\theta d\theta$$

$$= 2\int_0^{\pi/2} \sin^4\theta\cos^2\theta d\theta$$

$= 2\left[\frac{3.1.1}{6.4.2}.\frac{\pi}{2}\right]$ [using Walli's formula]

$$= \frac{\pi}{16}$$

76 **(c)**

Let $I = \int_0^1 x\left|x - \frac{1}{2}\right| dx$

$$= -\int_0^{1/2} x\left(x - \frac{1}{2}\right) dx + \int_{1/2}^1 x\left(x-\frac{1}{2}\right) dx$$

$$= \int_0^{1/2} \left(\frac{1}{2}x - x^2\right) dx + \int_{1/2}^1 \left(x^2 - \frac{1}{2}x\right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{1/2}^1$$

$$= \left(\frac{1}{16} - \frac{1}{24}\right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16}\right)$$

$$= \left(\frac{6-4}{96}\right) + \left(\frac{32-24-4+6}{96}\right) = \frac{12}{96} = \frac{1}{8}$$

77 **(c)**

Since $\phi(x) = \sin\{\log(x + \sqrt{x^2+1})\}$ is the composition of $f(x) = \log(x + \sqrt{x^2+1})$ and $g(x) = \sin x$ i.e. $\phi(x) = gof(x)$ and $f(x)$ and $g(x)$ both are odd functions. Therefore, $\phi(x)$ is an odd function

Hence, $\int_{-\pi/2}^{\pi/2} \phi(x)\,dx = 0$

78 **(b)**

We have,

$$\frac{x^3-1}{x^3+x} = 1 - \frac{1}{x} + \frac{x}{x^2+1} - \frac{1}{x^2+1}$$

$$\therefore \int \frac{x^3-1}{x^3+x} dx = x - \log x + \frac{1}{2}\log(1+x^2)$$

$$- \tan^{-1} x + C$$

79 **(a)**

We have,

$$\int \sqrt{\frac{1 + \sin\theta - \sin^2\theta - \sin^3\theta}{2\sin\theta - 1}} d\theta$$

$$= \int \sqrt{\frac{(1+\sin\theta)(1-\sin^2\theta)}{2\sin\theta - 1}}\, d\theta$$

$$= \int \sqrt{\frac{1+\sin\theta}{2\sin\theta - 1}} \cos\theta \; d\theta$$

$$= \int \sqrt{\frac{x+1}{2x-1}}\, dx, \text{where } x = \sin\theta$$

$$= \int \frac{x+1}{\sqrt{2x^2+x-1}}\, dx$$

$$= \frac{1}{4}\int \frac{4x+1}{\sqrt{2x^2+x-1}}\, dx + \frac{3}{4\sqrt{2}} \int \frac{1}{\sqrt{\left(x+\frac{1}{4}\right)^2 - \frac{9}{16}}}\, dx$$

$$= \frac{1}{2}\sqrt{2x^2+x-1} + \frac{3}{4\sqrt{2}}\left\{\log\left|x+\frac{1}{4}\right| + \sqrt{\left(x+\frac{1}{4}\right)^2 - \frac{9}{16}}\right\} + C$$

$$= \frac{1}{2}\sqrt{2x^2+x-1} + \frac{3}{4\sqrt{2}}\left\{\log\left|4x+1 + 2\sqrt{2}\sqrt{2x^2+x-1}\right|\right\} + C$$

$$= \frac{1}{2}\sqrt{2\sin^2\theta + \sin\theta - 1} + \frac{3}{4\sqrt{2}}\left|(4\sin\theta + 1) + 2\sqrt{2}\sqrt{2\sin^2\theta + \sin\theta - 1}\right| + C$$

$$= \frac{1}{2}\sqrt{\sin\theta - \cos 2\theta} + \frac{3}{4\sqrt{2}}\log\left|(4\sin\theta + 1) + 2\sqrt{2}\sqrt{\sin\theta - \cos 2\theta}\right| + C$$

80 **(b)**

$$\int_{-1}^{1} \max(\{x\}, \{-x\})dx$$

$$= \int_{-1}^{-0.5} \{-x\}dx + \int_{-0.5}^{0} \{x\}dx + \int_{0}^{0.5} \{-x\}dx + \int_{0.5}^{1} \{x\}dx$$

$$= \int_{-1}^{-0.5} \{-x\}dx + \int_{-0.5}^{0} \{x - [x]\}dx + \int_{0}^{0.5} (-x - [-x])dx + \int_{0.5}^{1} (x - [x])dx$$

$$= \left[-\frac{x^2}{2}\right]_{-1}^{-0.5} + \left[\frac{x^2}{2} + x\right]_{-0.5}^{0} + \left[x - \frac{x^2}{2}\right]_{0}^{0.5} + \left[\frac{x^2}{2}\right]_{0.5}^{1}$$

$$= -\frac{0.25}{2} + \frac{1}{2} + \left(-\frac{0.25}{2} + 0.5\right) + 0.5 - \frac{0.25}{2} + \frac{1}{2} - \frac{0.25}{2}$$

$$= 2 - \frac{1}{2} = \frac{3}{2}$$

81 **(d)**

$$I = \int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1}\, dx$$

$$\Rightarrow I = \int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1}\, dx + \int_{-1}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1}\, dx$$

$$\Rightarrow I = 0 + 2\int_{0}^{1} \frac{|x| + 1}{(|x| + 1)^2}\, dx$$

$$\left[\because \frac{x^3}{x^2 + 2|x| + 1} \text{ is an odd function}\right]$$

$$\Rightarrow I = 2\int_{0}^{1} \frac{x+1}{(x+1)^2}\, dx = 2[\log_e(x+1)]_0^1$$

$$= 2\log_e 2 = 2\ln 2$$

82 **(a)**

Let $I = \int_0^2 \frac{2x-2}{2x-x^2}\, dx$

Put $2x - x^2 = t \;\Rightarrow\; (2-2x)dx = dt$

$$I = -\int_0^0 \frac{dt}{t} = 0$$

83 **(c)**

Let $I = \int_0^{\pi} \frac{x\, dx}{1+\sin x}$ ...(i)

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)dx}{1 + \sin x} \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\pi\, dx}{1 + \sin x}$$

$$= \pi \int_0^{\pi} \frac{1 - \sin x}{1 - \sin^2 x}\, dx$$

$$\Rightarrow\ 2I = \pi \int_0^{\pi} (\sec^2 x - \sec x \tan x) dx$$

$$\Rightarrow\ 2I = \pi[\tan x - sec\ x]_0^{\pi} = 2\pi$$

$$\Rightarrow\ I = \pi$$

84 **(d)**

Let $I_n = \int_0^{\pi/2} \frac{\sin^2 n\,x}{\sin^2 x} dx$. Then,

$I_{n+1} - 2I_n + I_{n-1}$

$$= \int_0^{\pi/2} \frac{\sin^2(n+1)x - 2\sin^2 nx + \sin^2(n-1)\,x}{\sin^2 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n+1)x \sin x - \sin(2n-1)\,x \sin x}{\sin^2 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n+1) - \sin(2n-1)x}{\sin x} dx$$

$$= \int_0^{\pi/2} \frac{2\sin x \cos 2n\,x}{\sin x} dx$$

$$= 2\int_0^{\pi/2} \cos 2n\ x\,dx = \frac{1}{n}[\sin 2n\ x]_0^{\pi/2}$$

$$= \frac{1}{n}[\sin n\pi - 0] = 0$$

$\therefore I_{n+1} + I_{n-1} = 2I_n$ for all $n \geq 1$

$\Rightarrow I_1, I_2, I_3,$ are in A.P.

So, option (c) is true

Clearly, $I_1 = \int_0^{\pi/2} 1\,.\,dx = \pi/2$

and, $I_2 = \int_0^{\pi/2} \frac{\sin^2 2x}{\sin^2 x} = 4\int_0^{\pi/2} \cos^2 x\,dx = 4 \times \pi/4 = \pi$

$\therefore I_2 - I_1 = \pi - \pi/2 = \pi/2$

Thus, $I_1, I_2, I_3,$ ……. Are in AP with first term $\pi/2$ and common difference $\pi/2$

$\therefore I_n = \pi/2 + (n-1)\pi/2 - n\,\pi/2$

So, option (a) is true

Now, $\sin(I_{15}) = \sin(15\pi/2) = 1$

Hence, option (d) is not true

86 **(a)**

We have,

$f(x) = a\,e^{2x} + b\,e^x + c\,x, f(0) = -1, f'(\log 2) = 31$

$$\int_0^{\log 4} \{f(x) - cx\}\,dx = \frac{39}{2}$$

Now, $f(0) = -1$

$\Rightarrow a + b = -1$ ...(i)

and, $f'(x) = 2ae^{2x} + be^x + c$

$$\Rightarrow f'(\log 2) = 2a\,e^{2\log 2} + b\,e^{\log 2} + c = 8a + 2b + c$$

$\therefore f'(\log 2) = 31$

$\Rightarrow 8a + 2b + c = 31$ ...(ii)

Now,

$$\int_0^{\log 4} \{f(x) - cx\}\,dx = \frac{39}{2}$$

$$\Rightarrow \int_0^{\log 4} (ae^{2x} + be^x)\,dx = \frac{39}{2}$$

$$\Rightarrow \left[\frac{a}{2}e^{2x} + b\,e^x\right]_0^{\log 4} = \frac{39}{2}$$

$$\Rightarrow \left(\frac{a}{2}e^{2\log 4} + b\,e^{\log 4}\right) - \left(\frac{a}{2} + b\right) = \frac{39}{2}$$

$$\Rightarrow \frac{a}{2}(16) + b(4) - \frac{a}{2} - b = \frac{39}{2}$$

$$\Rightarrow \frac{15}{2}a + 3b = \frac{39}{2}$$

$\Rightarrow 15a + 6b = 39$

$\Rightarrow 5a + 2b = 13$ ...(iii)

Solving (i), (ii) and (iii), we get

$a = 5, b = -6, c = -7$

87 **(a)**

Let $I = \int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$

$$= \frac{1}{5}\left(\int_0^{\infty} \frac{1}{(x^2+4)} dx - \int_0^{\infty} \frac{1}{(x^2+9)} dx\right)$$

$$= \frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^{\infty} - \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^{\infty}\right]$$

$$= \frac{1}{5}\left[\frac{1}{2}.\frac{\pi}{2} - 0 - \left(\frac{1}{3}.\frac{\pi}{2} - 0\right)\right]$$

$$= \frac{1}{5}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\pi}{60}$$

88 **(c)**

We have,

$$I_{m,n} = \int_0^1 x^m (\log x)^n dx$$

$$\Rightarrow I_{m,n} = \left[(\log x)^n \frac{x^{m+1}}{m+1}\right]_0^1 - \int_0^1 n\,(\log x)^{n-1}.\frac{1}{x}.\frac{x^{m+1}}{m+1} dx$$

$$\Rightarrow I_{m,n} = 0 - \frac{n}{m+1}\int_0^1 x^m (\log x)^{n-1} dx = -\frac{n}{m+1} I_{m,n-1}$$

89 **(b)**

$$\int e^{3\log x}\,(x^4+1)^{-1}dx$$

$$=\int \frac{x^3}{1+x^4}\,dx=\frac{1}{4}\log(x^4+1)+c$$

90 **(d)**

Put $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$

$$\therefore \int_0^{\infty}\frac{x\,dx}{(1+x)(1+x^2)}$$

$$=\int_0^{\pi/2}\frac{\tan\theta\sec^2\theta d\theta}{(1+\tan\theta)(1+\tan^2\theta)}$$

$$=\int_0^{\pi/2}\frac{\tan\theta}{1+\tan\theta}d\theta$$

$$=\int_0^{\pi/2}\frac{\sin\theta}{\cos\theta+\sin\theta}d\theta$$

Let $I=\int_0^{\pi/2}\frac{\sin\theta}{\cos\theta+\sin\theta}d\,\theta \quad \ldots(i)$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)+\sin\left(\frac{\pi}{2}-\theta\right)}d\theta$$

$=\int_0^{\pi/2}\frac{\cos\theta}{\sin\theta+\cos\theta}d\theta \quad \ldots(\text{ii})$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^{\pi/2}1d\theta=[\theta]_0^{\pi/2}=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\therefore \int_0^{\infty}\frac{x\,dx}{(1+x)(1+x^2)}=\frac{\pi}{4}$$

91 **(c)**

$f(x)=\tan x-\tan^3 x+\tan^5 x-\ldots\infty$

$f(x)=\frac{\tan x}{1+\tan^2 x}=\frac{\tan x}{\sec^2 x}=\frac{\sin 2x}{2}$

$$\therefore \int_0^{\pi/4}f(x)dx=\int_0^{\pi/4}\frac{\sin 2x}{2}dx$$

$$=\left[-\frac{\cos 2x}{4}\right]_0^{\pi/4}=\frac{1}{4}$$

92 **(d)**

Let $I=\int_0^1\frac{x^4+1}{x^2+1}\,dx$

$$=\int_0^1\frac{x^4+1-1+1}{x^2+1}\,dx$$

$$=\int_0^1\left[\frac{x^4-1}{x^2+1}+\frac{2}{x^2+1}\right]dx$$

$$=\int_0^1\left(x^2-1+\frac{2}{x^2+1}\right)dx$$

$$=\left[\frac{x^3}{3}-x+2\tan^{-1}x\right]_0^1$$

$$=\left[\frac{1}{3}-1+2\tan^{-1}(1)-0\right]$$

$$=-\frac{2}{3}+2.\frac{\pi}{4}=\frac{3\pi-4}{6}$$

93 **(c)**

We have,

$I=\int_0^{\pi/2}\log\tan x\,dx \quad \ldots(\text{i})$

$\Rightarrow I=\int_0^{\pi/2}\log\tan\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi/2}\log\cot x\,dx$

...(ii)

Adding (i) and (ii), we get

$2I=0\Rightarrow I=0$

94 **(c)**

$$\frac{d}{dx}\int_{f(x)}^{g(x)}h(t)dt=h[g(x)]g'(x)-h[f(x)f'(x)$$

95 **(c)**

Differentiate both sides n times with respect to a

96 **(b)**

We have,

$$I=\int_{-1}^{1}\frac{|x+2|}{x+2}dx$$

$$=\int_{-1}^{1}\frac{x+2}{x+2}dx\left[\begin{array}{c}\because x+2\geq 0 \text{ for } -1\leq x\leq 1\\ \therefore |x+2|=x+2\end{array}\right]$$

$$\Rightarrow I=\int_{-1}^{1}1\cdot dx=2$$

97 **(b)**

Put $1+\log x=t \;\Rightarrow\; \frac{1}{x}dx=dt$

$$\int\frac{dx}{x(1+\log x)^2}=\int t^{-2}\,dt=\frac{t^{-1}}{-1}+c$$

$$=\frac{-1}{(1+\log x)}+c$$

99 **(b)**

$$\int_0^{\pi/2}x\sin x\,dx=[x(-\cos x)]_0^{\frac{\pi}{2}}$$

$$-\int_0^{\frac{\pi}{2}}1(-\cos x)dx$$

$$=0+[\sin x]_0^{\pi/2}=1$$

100 **(a)**

Let $\sin x=z \;\Rightarrow d\,(\sin x)=dz$

$$\therefore \int\frac{dz}{\sqrt{1-z^2}}=\sin^{-1}z+c$$

$=\sin^{-1}(\sin x)+c=x+c$

101 **(b)**

$$\int 1\cdot\sin^{-1}x\,dx=(\sin^{-1}x)x-\int\frac{1}{\sqrt{1-x^2}}\cdot x\,dx$$

$= x\sin^{-1}x — \frac{t\,dt}{t}$ (put $1 - x^2 = t \Rightarrow$ $-2x\,dx = 2t\,dt$)

$x\sin^{-1}x + \sqrt{1-x^2} + c$

102 **(a)**

Let $I_m = \int_0^\pi \frac{\sin 2m\,x}{\sin x}dx$. Then,

$$I_m - I_{m-1} = \int_0^\pi \frac{\sin 2m\,x - \sin 2(m-1)x}{\sin x}dx$$

$$\Rightarrow I_m - I_{m-1} = \int_0^\pi 2\cos(2m-1)\,x\,dx$$

$$\Rightarrow I_m - I_{m-1} = \frac{2}{2m-1}[\sin(2m-1)]_0^\pi = 0$$

$\Rightarrow I_m - I_{m-1}$ for all $m \in N$

$\Rightarrow I_m - I_{m-1} = I_{m-2} = \cdots = I_1$

But, $I_1 = \int_0^\pi \frac{\sin 2\,x}{\sin x}dx = 2\int_0^\pi \cos x\,dx = 0$

$\therefore I_m = 0$ for all $m \in N$

103 **(a)**

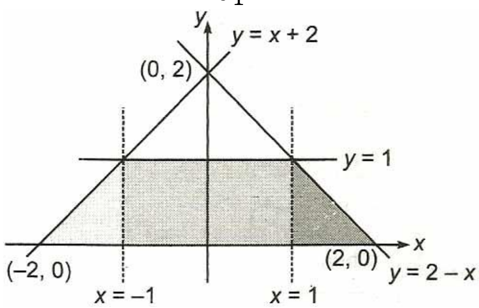
$\sin x > \sin^3 x$

$$\Rightarrow \int_0^{\pi/2}\sin x\,dx > \int_0^{\pi/2}\sin^3 x\,dx$$

$\Rightarrow I_1 > I_2$

104 **(c)**

$$\int_{-2}^{2} f(x)dx = \int_{-2}^{-1}(x+2)dx + \int_{-1}^{1}dx + \int_{1}^{2}(2-x)dx$$



$= \frac{1}{2} + 2 + \frac{1}{2} = 3$

105 **(b)**

Let $f(x) = x^3 + x^2 + 3x = \left(x(x^2+x+3)\right)$

Since $x^2 + x + 3 > 0$ for all x. Therefore,

$f(x) = x(x^2+x+3) > 0$ for all $x > 0$

$$\therefore I = \int_0^3 |x^3 + x^2 + 3x|\,dx = \int_0^3 (x^3 + x^2 + 3x)\,dx$$

$$\Rightarrow I = \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3 = \frac{81}{4} + 9 + \frac{27}{2} = \frac{171}{4}$$

106 **(b)**

$$\int_{-1/7}^{20/7} e^{5\{x\}}dx = \int_{-1/7}^{3-\frac{1}{7}} e^{5\{x\}}dx$$

$= 3\int_0^1 e^{5\{x\}}dx = 3\int_0^1 e^{5x}dx$ $(\{x\} = x$ as $0 \le x < 1)$

$$= 3\left[\frac{e^{5x}}{5}\right]_0^1 = 3\left(\frac{e^5 - 1}{5}\right)$$

107 **(b)**

$$\int_{\pi/4}^{\pi/2}\operatorname{cosec}^2 x\,dx = [-\cot x]_{\pi/4}^{\pi/2}$$

$$= \left(-\cot\frac{\pi}{2} + \cot\frac{\pi}{4}\right) = 1$$

108 **(a)**

Let $f(x) = C_2\frac{x^3}{3} + C_1\frac{x^2}{2} + C_0 x$

Clearly, $f(x)$, being a polynomial, is continuous on $[0, 1]$ and differentiable on $(0, 1)$

Also, $f(0) = 0$ and $f(1) = \frac{C_2}{3} + \frac{C_1}{2} + C_0 = 0$ [Given]

$\Rightarrow f(0) = f(1)$

Hence, $f(x)$ satisfies conditions of Rolle's theorem. Consequently there exists $\alpha \in (0,1)$ such that

$f'(\alpha) = 0$

$\Rightarrow C_2\alpha^2 + C_1\alpha + C_0 = 0$

$\Rightarrow C_2\alpha^2 + C_1\alpha + C_0 = 0 \Rightarrow \alpha$ is a root of $C_2x^2 + C_1x + C_0 = 0$

109 **(b)**

Let $I = \int \frac{dx}{2\sqrt{x}(1+x)}$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$$\therefore I = \int \frac{dt}{1+t^2} = \tan^{-1}t + c = \tan^{-1}\sqrt{x} + c$$

111 **(c)**

Let $I = \int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\cot x}}$

$\Rightarrow I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(i)

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$\Rightarrow I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx = [x]_{\pi/6}^{\pi/3}$$

$$\Rightarrow I = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{12}$$

112 **(c)**

$$I = \int_{-1/7}^{20/7} \sin(x - [x])dx = \int_{-1/7}^{3-\frac{1}{7}} \sin(\{x\})dx$$

$$= 3\int_0^1 \sin x\, dx = 3[-\cos x]_0^1 = 3(1 - \cos 1)$$

113 **(a)**

$$f\left(\frac{1}{x}\right) + x^2 f(x) = 0$$

Let $I = \int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx$

Put $x = \frac{1}{t} \;\Rightarrow\; dx = -\frac{1}{t^2}\, dt$

$$\therefore\; I = -\int_{\operatorname{cosec}\theta}^{\sin\theta} f\left(\frac{1}{t}\right).\frac{1}{t^2}\, dt$$

$$= \int_{\sin\theta}^{\operatorname{cosec}\theta} \frac{1}{x^2} f\left(\frac{1}{x}\right) dx$$

$$\Rightarrow\; I = -\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx \;\Rightarrow\; I = 0$$

114 **(c)**

$$I = \int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x\, dx$$

This is an even function.

$$\therefore I = 2\int_0^{\pi/2} \sin^4 x \cos^6 x\, dx$$

Applying Gamma function

$$\Rightarrow\; I = \frac{\sqrt{\frac{4+1}{2}}\,\sqrt{\frac{6+1}{2}}}{2|6} = \frac{2\sqrt{\frac{5}{2}}\,\sqrt{\frac{7}{2}}}{2|6}$$

$$= \frac{\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{5\cdot4\cdot3\cdot2\cdot1} = \frac{3\pi}{2^8} = \frac{3\pi}{256}$$

115 **(b)**

We have,

$$I = \int_0^1 \cot^{-1}(1 - x + x^2)dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1}\left\{\frac{1}{1 - x(1-x)}\right\} dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1}\left\{\frac{x + (1-x)}{1 - x(1-x)}\right\}$$

$$\Rightarrow I = \int_0^1 [\tan^{-1} x - \tan^{-1}(1-x)]\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1-x)\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1-(1-x))\, dx$$

$$\Rightarrow I = 2\int_0^1 \tan^{-1} x\, dx = \frac{\pi}{2} - \log 2$$

116 **(d)**

Put $x = -t$

$\Rightarrow dx = -dt$

$$\therefore \int_{-100}^{100} f(x)dx$$

$$= -\int_{100}^{-100} f(-t)dt$$

$$= \int_{-100}^{100} f(-t)dt$$

$$= \int_{-100}^{100} f(-x)dx$$

117 **(a)**

$$\int \frac{\cos x - 1}{\sin x + 1} e^x\, dx$$

$$= \int \frac{\cos x}{1 + \sin x} e^x\, dx - \int \frac{1}{1 + \sin x} e^x\, dx$$

$$= \frac{(\cos x)e^x}{1 + \sin x} + \int \frac{-(1 + \sin x)\sin x - \cos^2 x}{(1 + \sin x)^2} e^x dx$$

$$- \int \frac{e^x}{\sin x + 1} dx + c$$

$$= \frac{e^x \cos x}{1 + \sin x} + \int \frac{1}{1 + \sin x} e^x dx$$

$$- \int \frac{e^x}{1 + \sin x} dx + c$$

$$= \frac{e^x \cos x}{1 + \sin x} + c$$

118 **(b)**

$\text{LHS} = \int (\log x)^2\, dx$

$$= x\,(\log x)^2 - \int x\,.\,2\log x \frac{1}{x}\, dx$$

$$= x\,(\log x)^2 - 2\left[x\log x - \int x\,.\frac{1}{x} dx\right] + c$$

$$= x\,(\log x)^2 - 2[x\log x - x]$$

$$= x\,(\log x)^2 - 2x[\log x - 1] + c$$

But RHS is given by

$x[f(x)]^2 + Ax[f(x) - 1] + c$

$\therefore f(x) = \log x$ and $\;A = -2$

119 **(a)**

We have,

$$I_1 - I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)(\sec^2\theta - \text{cosec}^2\theta)d\theta$$

$$\Rightarrow I_1 - I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)d(\tan\theta + \cot\theta)$$

$$\Rightarrow I_1 - I_2 = \int_{4/\sqrt{3}}^{4/\sqrt{3}} f(t)\,dt, \text{where } t = \tan\theta + \cot\theta$$

$$\Rightarrow I_1 - I_2 = 0 \Rightarrow I_1 = I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

120 **(c)**

We have,

$$I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx$$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x}\,|\sin x|\,dx$$

$$\Rightarrow I = 2\int_{0}^{\pi/2} \sqrt{\cos x}\,|\sin x|\,dx$$

$$\Rightarrow I = 2\int_{0}^{\pi/2} \sqrt{\cos x}\sin x\,dx = -2\int_{1}^{0} \sqrt{t}\,dt, \text{where } t = \cos x$$

$$\Rightarrow I = -\frac{4}{3}(0-1) = \frac{4}{3}$$

121 **(b)**

We have,

$$I = \int \frac{1+x^2}{1+x^4}dx = \int \frac{(1+1/x^2)}{x^2+(1/x^2)}dx$$

$$\Rightarrow I = \int \frac{1+(1/x^2)}{\left(x-\frac{1}{x^2}\right)^2 + C}dx$$

$$= \int \frac{1}{\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2} d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$$

122 **(a)**

We have,

$g(x) = f'(x), f(-1) = 2$ and $f(2) = 4$

$$\therefore \int_{-1}^{2} g(x)\,dx = \int_{-1}^{2} f'(x)\,dx = f(2) - f(-1) = 4 - 2 = 2$$

124 **(b)**

We have,

$$I = \int \frac{1}{x^4\sqrt{a^2+x^2}}dx$$

$$\Rightarrow I = \int \frac{1}{x^5\sqrt{\left(\frac{a}{x}\right)^2+1}}dx = \int \frac{\frac{1}{x^2}}{x^3\sqrt{\left(\frac{a}{x}\right)^2+1}}dx$$

$$\Rightarrow I = -\frac{1}{2a^4}\int \frac{\left(\frac{a}{x}\right)^2}{\sqrt{\left(\frac{a}{x}\right)^2+1}} \times \frac{-2a^2}{x^3}dx$$

$$\Rightarrow I = -\frac{1}{2a^4}\int \frac{t}{\sqrt{t+1}}dt, \text{where } t = \frac{a^2}{x^2}$$

$$\Rightarrow I = -\frac{1}{2a^4}\int \frac{(t+1)-1}{\sqrt{t+1}}dt$$

$$\Rightarrow I = -\frac{1}{2a^4}\int \left(\sqrt{t+1} - \frac{1}{\sqrt{t+2}}\right)dt$$

$$\Rightarrow I = -\frac{1}{2a^4}\left\{\frac{2}{3}(t+1)^{3/2} - 2(t+1)^{1/2}\right\} + C$$

$$\Rightarrow I = \frac{1}{a^4}\left\{\frac{\sqrt{a^2+x^2}}{x} - \frac{(a^2+x^2)^{3/2}}{3x^3}\right\} + C$$

125 **(b)**

Let $I = \int_0^{\pi/2} \log|\tan x|\,dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \log\left|\tan\left(\frac{\pi}{2} - x\right)\right|$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \log|\tan x \cot x|\,dx$$

$$= \int_0^{\pi/2} \log 1\,dx = 0$$

$$\Rightarrow I = 0$$

126 **(d)**

We have,

$[\sin(-x)]^{11} = (-\sin x)^{11} = -\sin^{11} x$

$\therefore \sin^{11} x$ is an odd function x

Hence, $\int_{-1}^{1} \sin^{11} x\;dx = 0$

127 **(b)**

$$\int \cos^4 x\,dx = \frac{1}{4}(2\cos^2 x)^2\,dx$$

$$= \frac{1}{4}\int (1+\cos 2x)^2\,dx$$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)dx$$

$$= \frac{1}{8}\int \{2 + 4\cos 2x + (1+\cos 4x)\}dx$$

$$= \frac{1}{8}\left\{3x + 2\sin 2x + \frac{1}{4}\sin 4x\right\} + D$$

$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + D$$

On comparing, we get

$$A = \frac{3}{8}, B = \frac{1}{4}, C = \frac{1}{32}$$

128 **(c)**

$$\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$$

$$=\int e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$$

$$=\frac{1}{2}\int e^x\left(\operatorname{cosec}^2\frac{x}{2}\right)dx-\int e^x\cot\frac{x}{2}\,dx$$

$$=\frac{1}{2}\left[-e^x\cot\frac{x}{2}.2+\int e^x\cot\frac{x}{2}.2\,dx\right]$$

$$-\int e^x\cot\frac{x}{2}\,dx+c$$

$$=-e^x\cot\frac{x}{2}+c$$

129 **(a)**

Since, $\int_1^a [x]f'(x)dx=\int_1^2 f'(x)dx$

$$+\int_2^3 2f'(x)dx+\ldots+\int_{[a]}^a [a]f'(x)dx$$

$$=[f(x)]_1^2+2[f(x)]_2^3+\ldots+[a][f(x)]_{[a]}^a$$

$$=f(2)-f(1)+2f(3)-2f(2)+\ldots$$

$$+[a]f(a)-[a]f([a])$$

$$=[a]f(a)-\{f(1)+f(2)+\ldots+f([a])\}$$

130 **(a)**

We have,

$\int_a^b x\,f(x)dx=k\int_a^b f(x)dx$ ...(i)

$$\Rightarrow\int_a^b(a+b-x)\,(a+b-x)=k\int_a^b f(x)\,dx$$

$$\Rightarrow\int_a^b(a+b-x)\,f(x)dx$$

$$=k\int_a^b f(x)\,dx\ [\because f(a+b-x)$$

$$=f(x)]$$

$$\Rightarrow(a+b)\int_a^b f(x)\,dx-\int_a^b xf(x)\,dx=k\int_a^b f(x)dx$$

$$\Rightarrow(a+b)\int_a^b f(x)\,dx-k\int_a^b f(x)\,dx$$

$$=k\int_a^b f(x)dx \quad \text{[Using (i)]}$$

$$\Rightarrow(a+b)\int_a^b f(x)\,dx=2k\int_a^b f(x)\,dx\Rightarrow k=\frac{a+b}{2}$$

131 **(d)**

Let $I=\int|x|\cdot 1dx$

$$=|x|x-\int\frac{|x|}{x}x\,dx=x|x|-\int|x|dx$$

$$\Rightarrow 2I=x|x|$$

$$\Rightarrow I=\frac{x|x|}{2}+c$$

132 **(b)**

Let

$$I_1=\int_{-1}^{1}\sin^{-1}\left[x^2+\frac{1}{2}\right]dx$$

$$I_1=2\int_0^1\sin^{-1}\left[x^2+\frac{1}{2}\right]dx$$

$$I_1=2\left[\int_0^{1/\sqrt{2}}\sin^{-1}\left[x^2+\frac{1}{2}\right]dx+\int_{1/\sqrt{2}}^{1}\sin^{-1}\left[x^2+\frac{1}{2}\right]dx\right]$$

$$I_1=2\left[\int_0^{1/\sqrt{2}}\sin^{-1}0\,dx+\int_{1/\sqrt{2}}^{1}\sin^{-1}1\,dx\right]$$

$$I_1=2\left[\frac{\pi}{2}\int_{1/\sqrt{2}}^{1}dx\right]=\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

and,

$$I_2=\int_{-1}^{1}\cos^{-1}\left[x^2-\frac{1}{2}\right]dx$$

$$\Rightarrow I_2=2\int_0^1\cos^{-1}\left[x^2-\frac{1}{2}\right]dx$$

$$\Rightarrow I_2=2\left[\int_0^{1/\sqrt{2}}\cos^{-1}\left[x^2-\frac{1}{2}\right]dx+\int_{1/\sqrt{2}}^{1}\cos^{-1}\left[x^2-\frac{1}{2}\right]dx\right]$$

$$\Rightarrow I_2=2\left[\int_0^{1/\sqrt{2}}\cos^{-1}(-1)\,dx+\int_{1/\sqrt{2}}^{1}\cos^{-1}0\,dx\right]$$

$$\Rightarrow I_2=2\left[\int_0^{1/\sqrt{2}}\pi dx+\int_{1/\sqrt{2}}^{1}\frac{\pi}{2}dx\right]$$

$$\Rightarrow I_2=2\left[\pi\left(\frac{1}{\sqrt{2}}-0\right)+\frac{\pi}{2}\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$\Rightarrow I_2=2\left[\pi\left(\frac{1}{2\sqrt{2}}+\frac{1}{2}\right)\right]=\pi\left(\frac{1}{\sqrt{2}}+1\right)$$

$$\therefore \int_{-1}^{1} \sin^{-1}\left[x^2+\frac{1}{2}\right]dx + \int_{-1}^{1} \cos^{-1}\left[x^2-\frac{1}{2}\right]dx$$

$$= I_1 + I_2 = \pi\left(1-\frac{1}{\sqrt{2}}\right) + \pi\left(\frac{1}{\sqrt{2}}+1\right) = 2\pi$$

133 **(d)**

We have,

$$I = \int \frac{x^2-2}{x^3\sqrt{x^2-1}}dx$$

$$\Rightarrow I = \int \frac{x^2}{x^3\sqrt{x^2-1}}dx - 2\int \frac{1}{x^3\sqrt{x^2-1}}dx$$

$$\Rightarrow I$$

$$= \int \frac{1}{x\sqrt{x^2-1}}dx$$

$$-2\int \frac{1}{\sec^3\theta\tan\theta}\sec\theta\tan\theta\ d\theta, \text{(where } x = \sec\theta)$$

$$\Rightarrow I = \sec^{-1}x - \int (1+\cos 2\theta)d\theta$$

$$\Rightarrow I = \sec^{-1}x - \left(\theta + \frac{\sin 2\theta}{2}\right) + C$$

$$\Rightarrow I = \sec^{-1}x - \sec^{-1}x - \frac{\sqrt{x^2-1}}{x^2} + C$$

$$= -\frac{\sqrt{x^2-1}}{x^2} + C$$

134 **(c)**

Using L' Hospital's rule, we have

$$\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3} = \lim_{x\to 0}\frac{2x\sin x}{3x^2} = \frac{2}{3}\lim_{x\to 0}\frac{\sin x}{x} = \frac{2}{3}$$

135 **(c)**

Let $I = \int e^x \frac{x^2+1}{(x+1)^2}\,dx$

$$= \int e^x\frac{(x-1)}{(x+1)}dx + 2\int\frac{e^x}{(x+1)^2}dx$$

$$= \int e^x\,dx - 2\int\frac{e^x}{(x+1)}dx + 2\left[-\frac{e^x}{(x+1)} - \int -\frac{e^x}{(x+1)}\,dx\right]$$

$$= e^x - \frac{2e^x}{x+1} + c = e^x\left(\frac{x-1}{x+1}\right) + c$$

136 **(d)**

We have,

$$I = \int x\log\left(1+\frac{1}{x}\right)dx$$

$$\Rightarrow I = \int x\log(x+1)\,dx - \int x\log x\ dx$$

$$\Rightarrow I = \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int\frac{x^2}{x+1}dx - \frac{x^2}{2}\log x + \frac{1}{2}\int\frac{x^2}{2}dx$$

$$\Rightarrow I = \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int\left(x-1+\frac{1}{x}+1\right)dx - \frac{x^2}{2}\log x + \frac{1}{4}x^2$$

$$\Rightarrow I = \frac{x^2}{2}\log(x+1)\frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}-x\right) - \frac{1}{2}\log(x+1) + \frac{1}{4}x^2 + C$$

$$\Rightarrow I = \frac{x^2}{2}\log(x+1) - \frac{x^2}{2}\log x - \frac{1}{2}\log(x+1) + \frac{1}{2}x + C$$

Hence, $f(x) = \frac{x^2}{2} - \frac{1}{2}, g(x) = -\frac{1}{2}\log x$ and $A = \frac{1}{2}$

137 **(c)**

Let $I = \int_0^1 f(x)dx = 0$ and $f(x) \geq 0 \Rightarrow f(x) = 0$

138 **(b)**

Given, $f\left(\frac{3x-4}{3x+4}\right) = x+2$

Let $\frac{3x-4}{3x+4} = X$

$$\Rightarrow \frac{3x-4+3x+4}{3x-4-(3x+4)} = \frac{X+1}{X-1}$$

$$\Rightarrow \frac{6x}{-8} = \frac{X+1}{X-1}$$

$$\Rightarrow x = \frac{4(X+1)}{3(X-1)}$$

$$\therefore\ f(x) = \frac{4(X+1)}{3(X-1)} + 2 = \frac{10-2X}{3(1-X)}$$

$$= \frac{2}{3}\left[\int dx + \int\frac{4}{1-x}\ dx\right]$$

$$= \frac{2}{3}[x - 4\log(1-x)] + c$$

$$= \frac{2}{3}x - \frac{8}{3}\log(1-x) + c$$

139 **(d)**

Let $I = \int \frac{e^x 1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx$

$$= \int e^x\left\{\frac{\sqrt{1-x^{2n}}}{1-x^n} + \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right\}dx$$

Let $f(x) = \frac{\sqrt{1-x^{2n}}}{(1-x^n)}$

$$\Rightarrow f'(x) = \frac{\frac{(1-x^n)(-2n\,x^{2n-1})}{2\sqrt{1-x^{2n}}} + \sqrt{1-x^{2n}}(nx^{n-1})}{(1-x^n)^2}$$

$$\Rightarrow f'(x) = \frac{(1-x^{2n})(nx^{n-1}) - n(1-x^n)x^{2n-1}}{(1-x^n)^2\sqrt{1-x^{2n}}}$$

$$= \frac{n(1-x^n)x^{n-1}\{(1+x^n)-x^n\}}{(1-x^n)^2\sqrt{1-x^{2n}}}$$

$$= \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}$$

$$\therefore I = \frac{e^x\sqrt{1-x^{2n}}}{(1-x^n)} + c$$

140 **(c)**

An anti-derivative of $f(x)$

$= F(x)$

$$= \int [\log(\log x) + (\log x)^{-2}]dx + c$$

$$= x\log(\log x) - \int \frac{x}{x\log x}dx + \int (\log x)^{-2}\,dx + c$$

(integrating by parts the first integral)

$$= x\log(\log x) - [x(\log x)^{-1} + \int (\log x)^{-2}\,dx] + \int (\log x)^{-2}\,dx + c$$

[again integrating by parts $(\log x)^{-1}$]

$= x\log(\log x) - x(\log x)^{-1} + c$

On putting $x = e$, we have, $e = 0 - e + c$ so, $c = 2e$

Thus, $F(x) = x[\log(\log x) - (\log x)^{-1}] + 2e$

141 **(b)**

We have,

$$\int_a^b \frac{x^n}{x^n + (16-x)^n}dx = 6$$

$$\Rightarrow \frac{b-a}{2} = 6 \text{ and } a + b = 16 \Rightarrow a = 2, b = 14$$

142 **(d)**

Let $I = \int \underset{\text{I}}{\tan^{-1} x} \cdot \underset{\text{II}}{1}\, dx$

$$= \tan^{-1} x\,.\,x - \int \frac{1}{1+x^2}.x\,dx$$

$= x\tan^{-1} x - I_1$ ...(i)

Where $I_1 = \int \frac{x}{1+x^2}dx$

Put $1 + x^2 = t \Rightarrow x\,dx = \frac{1}{2}dt$

$$\therefore\ I_1 = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\log t$$

$$= \frac{1}{2}\log|1+x^2|$$

$\therefore$ From eq . (i), we get

$I = x\tan^{-1} x - \frac{1}{2}\log|1+x^2| + c$

143 **(a)**

$$\int x^{51}(\tan^{-1} x + \cot^{-1} x)dx = \int x^{51}.\frac{\pi}{2}dx$$

$$= \frac{\pi x^{52}}{104} + c$$

$$= \frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c$$

144 **(a)**

We have,

$$\int_0^2 |x-1|dx = \int_0^1 -(x-1)dx + \int_1^2 (x-1)dx$$

$$= -\left[\frac{x^2}{2} - x\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left[-\left(\frac{1}{2} - 1\right) - (0-0)\right] + \left[\left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right] = 1$$

145 **(a)**

Let $I = \int \frac{3-x^2}{(1-x)^2}.e^x dx = \int \left(\frac{2}{(1-x)^2} + \frac{1+x}{1-x}\right)e^x dx$

$$\Rightarrow\ I = e^x\left(\frac{1+x}{1-x}\right) + c$$

But $I = e^x f(x)$+c [given]

$$\therefore\ f(x) = \frac{1+x}{1-x}$$

146 **(d)**

$$\int_0^1 \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right)dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{x-(x-1)}{1+x(x-1)}\right)$$

$$= \int_0^1 \tan^{-1}x\,dx - \int_0^1 \tan^{-1}(x-1)dx$$

$$= \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-1+x)dx$$

$$= \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1} x\,dx$$

$$= 2\int_0^1 \tan^{-1} x\,dx$$

$$= 2\left[x\tan^{-1} x - \int \frac{x}{1+x^2}dx\right]_0^1$$

$$= 2\left[x\tan^{-1} x - \frac{1}{2}\log(1+x^2)\right]_0^1$$

$$= 2\left[\left\{1\tan^{-1} 1 - \frac{1}{2}\log(2)\right\} - \left\{0 - \frac{1}{2}\log 1\right\}\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2 - 0\right] = \frac{\pi}{2} - \log 2$$

147 **(a)**

$\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt=\lim_{x\to 1}\frac{\int_4^{f(x)}2t\,dt}{x-1}$ (using L' Hospital's rule)

$=\lim_{x\to 1}\frac{2f(x)\cdot f'(1)}{1}$

$=2f(1)\cdot f'(1)$, where $f(1)=4$

$=8f'(1)$

148 **(b)**

We know, $\int e^x[f(x)+f'(x)]dx=e^xf(x)+c$

$\therefore$ If $f(x)=\log\sin x$

$\Rightarrow f'(x)=\frac{1}{\sin x}.\cos x=\cot x$

$\therefore \int e^x(\log\sin x+\cot x)dx=e^x\log\sin x+c$

149 **(a)**

Let $f(x)=(1-x^2)\sin x\cos^2 x$. Then,

$f(-x)=\{1-(x)^2\}\sin(-x)\cos^2(-x)$

$\Rightarrow f(-x)=-(1-x^2)\sin x\cos^2 x$

$\Rightarrow f(-x)=-f(x)$

Hence, $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx=0$

150 **(c)**

Let $I=\int\frac{xdx}{(x^2-a^2)(x^2-b^2)}$

Put $x^2=t \Rightarrow xdx=\frac{1}{2}dt$

$\therefore \quad I=\frac{1}{2}\int\frac{dt}{(t-a^2)(t-b^2)}$

$=\frac{1}{2}\int\left\{\frac{dt}{(a^2-b^2)(t-a^2)}-\frac{dt}{(a^2-b^2)(t-b^2)}\right\}$

$=\frac{1}{2(a^2-b^2)}\log\left|\frac{t-a^2}{t-b^2}\right|+c$

$\Rightarrow I=\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+c$

151 **(b)**

Since $\cos^2 x$ is a periodic function with period π

Therefore, so is $f(\cos^2 x)$. Hence,

$\int_0^{n\pi}f(\cos^2 x)dx=n\int_0^{\pi}f(\cos^2 x)dx\Rightarrow k=n$

152 **(c)**

Let $I=\int(1-\cos^2 x)\cos^2 x\,.\sin x\ dx$

Put $\cos x=t$

$\Rightarrow -\sin x\ dx=dt$

$\therefore I=-\int(1-t^2)t^2dt=\int(t^4-t^2)\,dt$

$=\frac{t^5}{5}-\frac{t^3}{3}+c$

$=\frac{(\cos x)^5}{5}-\frac{(\cos x)^3}{3}+c$

153 **(d)**

$\int_{\pi/2}^{\alpha}\sin x\,dx=\sin 2\alpha\,,\alpha\in(0,2\pi)$

$\Rightarrow -[\cos x]_{\pi/2}^{\alpha}=\sin 2\alpha$

$\Rightarrow -\cos\alpha=\sin 2\alpha$

$\Rightarrow \sin\left(\alpha-\frac{\pi}{2}\right)=\sin 2\alpha$

Hence, $\alpha=\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{6}$

154 **(b)**

Let $\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x}{1+a^x}dx$

Using $\int_{-a}^{a}f(x)\,dx=\int_0^a\{f(x)+f(-x)\}\,dx$, we have

$$I=\int_0^{\pi/4}\left(\frac{\tan^2 x}{1+a^x}+\frac{\tan^2 x}{1+a^{-x}}\right)dx$$

$$\Rightarrow I=\int_0^{\pi/4}\tan^2 x\,dx=\int_0^{\pi/4}(\sec^2 x-1)\,dx$$

$$=[\tan x-x]_0^{\pi/4}=\frac{4-\pi}{4}$$

155 **(d)**

We have,

$$I=\int e^x\frac{1+n\,x^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx$$

$$\Rightarrow I=\int e^x\frac{(1-x^{2n})+n\,x^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}dx$$

$$\Rightarrow I=\int e^x\left\{\sqrt{\frac{1+x^n}{1-x^n}}+n\,x^{n-1}\times\frac{1}{(1-x^n)^2}\times\sqrt{\frac{1-x^n}{1+x^n}}\right\}dx$$

$$\Rightarrow I=e^x\sqrt{\frac{1+x^n}{1-x^n}}+C=e^x\sqrt{\frac{1-x^{2n}}{1-x^n}}+C$$

156 **(c)**

(A) $\int_0^1 e^x\,dx=[e^x]_0^1=e-1$

(B) $\int_0^1 2^x\,dx=\left[\frac{2^x}{\log_e 2}\right]_0^1=\frac{1}{\log 2}.(2-2^0)=\frac{1}{\log 2}$

(C) $\int_0^1\sqrt{x}\,dx=\left[\frac{x^{3/2}}{3/2}\right]_0^1=\frac{2}{3}$

(D) $\int_0^1 x\,dx=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}$

158 **(b)**

We have,

$$\int_1^a(a-4x)\,dx\geq 6-5a$$

$\Rightarrow [ax-2x^2]_1^a\geq 6-5a$

$\Rightarrow -a^2-a+2\geq 6-5a$

$\Rightarrow -a^2+4a-4\geq 0$

$\Rightarrow a^2 - 4a + 4 \leq 0 \Rightarrow (a-2)^2 \leq 0 \Rightarrow a = 2$

159 **(d)**

$$\int_0^2 (3x)dx = \int_0^2 (3x - [3x])dx$$

$$= 3 \cdot \int_0^2 x\,dx - \int_0^2 [3x]dx$$

$$= 3\left[\frac{x^2}{2}\right]_0^2 - \frac{1}{3}\int_0^6 [x]dx \quad \text{(by property)}$$

$$= 6 - \frac{1}{3}\sum_{r=0}^{5}\int_r^{r+1}[x]dx$$

$$= 6 - \frac{1}{3}\sum_{r=0}^{5} r$$

$$= 6 - \frac{1}{3}(0 + 1 + 2 + \dots + 5)$$

$$= 6 - \frac{1}{3}\cdot 15 = 1$$

160 **(b)**

We have,

$$I_n = \int_0^{\pi/4} \tan^n x\,dx \Rightarrow I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x\,dx$$

$$\therefore I_2 + I_{n-2} = \int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)\,dx$$

$$\Rightarrow I_2 + I_{n-2} = \int_0^1 \tan^{n-2} x\,d(\tan x)$$

$$\Rightarrow I_2 + I_{n-2} = \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4} = \frac{1}{n-1}$$

161 **(a)**

Let, $I = \int [e^{a\log x} + e^{x\log a}]dx$

$$= \int [e^{\log_e x^a} + e^{\log_e a^x}]dx$$

$$= \int [x^a + a^x]dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

162 **(c)**

We have,

$$I\left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right|$$

$$\Rightarrow I \leq \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx = \int_{10}^{19} \frac{|\sin x|}{1+x^8}dx$$

$$\Rightarrow I \leq \int_{10}^{19} \frac{1}{1+x^8}dx$$

$$\Rightarrow I < \int_{10}^{19} \frac{1}{x^8}dx = \frac{1}{7}(10^{-7} - 19^{-7}) < \frac{10^{-7}}{7}$$

$$< 10^{-7}$$

163 **(b)**

We have,

$7 \leq 10 + 3\cos x \leq 13$

$$\Rightarrow \frac{1}{13} \leq \frac{1}{10 + 3\cos x} \leq \frac{1}{7} \Rightarrow \frac{2\pi}{13}$$

$$\leq \int_0^{2\pi} \frac{1}{10 + 3\cos x}dx \leq \frac{2\pi}{7}$$

164 **(c)**

Let $I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$. Then,

$$\Rightarrow I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow I = -\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \Rightarrow I = -I \Rightarrow 2I$$

$$= 0 \Rightarrow I = 0$$

165 **(c)**

We have,

$$I = \int \frac{x\tan^{-1} x}{\sqrt{1+x^2}}dx$$

$$\Rightarrow I = \frac{1}{2}\int \tan^{-1} x \frac{2x}{\sqrt{1+x^2}}dx$$

$$\Rightarrow I = \frac{1}{2}\Big[\tan^{-1} x \; 2\sqrt{1+x^2}$$

$$- \int \frac{1}{1+x^2} 2\sqrt{1+x^2}\,dx\Big]$$

$$\Rightarrow I = \frac{1}{2}\left[2\sqrt{1+x^2}\tan^{-1} x - 2\int \frac{1}{\sqrt{1+x^2}}dx\right]$$

$$\Rightarrow I = \sqrt{1+x^2}\tan^{-1} x - \log\left(x + \sqrt{1+x^2}\right) + C$$

Hence, $f(x) = \tan^{-1} x$ and $A = 1$

166 **(a)**

We have,

$$I = \int_{-2}^{4} x[x]dx$$

$$\Rightarrow I = \int_{-2}^{-1} -2x\,dx + \int_{-1}^{0} -x\,dx + \int_0^1 0.x\,dx$$

$$+ \int_1^2 x\,dx + \int_2^3 2x\,dx + \int_3^4 3x\,dx$$

$$\Rightarrow I = -(1-4) + \left(0 + \frac{1}{2}\right) + 0 + \left(2 - \frac{1}{2}\right)$$

$$+ (9-4) + \frac{3}{2}(16-9)$$

$$\Rightarrow I = 3 + \frac{1}{2} + \frac{3}{2} + 5 + \frac{21}{2} = \frac{6+1+3+10+21}{2}$$
$$= \frac{41}{2}$$

167 **(d)**

$$\int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x)dx$$
$$= [[\sin x + \cos x]_0^{\pi/4} - [\sin x + \cos x]_{\pi/4}^{5\pi/4} + [\sin x + \cos x]_{2\pi}^{\pi/4}]$$
$$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 0 + \cos 0)\right] - \left[\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} - \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)\right]$$
$$+ \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 2\pi + \cos 2\pi)\right]$$
$$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] + \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$$
$$= [\sqrt{2} - 1] - [-\sqrt{2} - \sqrt{2}] + [\sqrt{2} - 1]$$
$$= [\sqrt{2} - 1 + 2\sqrt{2} + \sqrt{2} - 1] = 4\sqrt{2} - 2$$

168 **(b)**

We have,

$$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\ dt, x\epsilon(-1,1)$$

On differentiating w.r.t x, we get

$$e^{-x}(f'(x) - f(x)) = \sqrt{x^4+1}$$
$$\Rightarrow f'(x) = f(x) + \sqrt{x^4+1}\, e^x$$

$\because f^{-1}$ is the inverse of f

$$\therefore f^{-1}(f(x)) = x$$
$$\Rightarrow f^{-1'}(f(x))f'(x) = 1$$
$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f'(x)}$$
$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f(x) + \sqrt{x^4+1}\, e^x}$$

As $x = 0,\ f(x) = 2$

and $f^{-1}(2) = \frac{1}{2+1} = \frac{1}{3}$

169 **(b)**

Let $I = \int \frac{f'(x)}{\sqrt{f(x)}}\, dx$

Put $f(x) = t \Rightarrow f'(x)dx = dt$

$$\therefore I = \int \frac{1}{\sqrt{t}} dt = 2\sqrt{t} + c = 2\sqrt{f(x)} + c$$

170 **(a)**

$$3a \int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx = \frac{3a}{(a-1)^2}\left[\frac{(ax-1)^3}{3} \times \frac{1}{a}\right]_0^1$$
$$= \frac{1}{(a-1)^2}[(a-1)^3 + 1]$$
$$= (a-1) + (a-1)^{-2}$$

171 **(b)**

Let $I = \int_1^e 10^{\log_e x} dx$

Again, let $I_1 = \int 10^{\log_e x}\ dx$

$$\Rightarrow I_1 = x.10^{\log_e x} - \int x.10^{\log_e x}.\frac{\log_e 10}{x} dx$$
$$\Rightarrow I_1 = x.10^{\log_e x} - \int 10^{\log_e x}\ \log_e 10 dx$$
$$\Rightarrow (1 + \log_e 10)I_1 = x.10^{\log_e x}$$
$$\Rightarrow I_1 = \frac{x.10^{\log_e x}}{1 + \log_e 10}$$
$$\therefore I = \left[\frac{x.10^{\log_e x}}{1 + \log_e 10}\right]_1^e$$
$$= \left[\frac{10_e - 1}{1 + \log_e 10}\right] = \frac{10e - 1}{\log_e 10e}$$

172 **(d)**

Let $I = \sqrt{2}\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx$

Put $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$$\therefore\ I = \sqrt{2}\int \frac{\sin\left(\frac{\pi}{4} + t\right)\ dt}{\sin t}$$
$$= \sqrt{2}\int \frac{\sin\frac{\pi}{4}\cos t + \cos\frac{\pi}{4}\sin t}{\sin t}\ dt$$
$$= \sqrt{2}\int \left(\frac{1}{\sqrt{2}}\frac{\cos t}{\sin t} + \frac{1}{\sqrt{2}}\right)\ dt$$
$$= \int (\cot t + 1)\, dt$$
$$= \log|\sin t| + t + c_1$$
$$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c, \left(\because c_1 - \frac{\pi}{4} = c\right)$$

173 **(b)**

We have,

$$I = \int \frac{1}{x(x^4-1)} dx = \int \frac{x^3}{x^4(x^4-1)} dx$$
$$\Rightarrow I = \frac{1}{4}\int \frac{1}{x^4(x^4-1)} d(x^4)$$
$$= \frac{1}{4}\int \left(\frac{1}{x^4-1} - \frac{1}{x^4}\right) d(x^4)$$

$\Rightarrow I = \frac{1}{4}\{\log_e(x^4-1) - \log_e x^4\} + C$

$= \frac{1}{4}\log_e\left(\frac{x^4-1}{x^4}\right) + C$

174 **(a)**

$g(x+\pi) = \int_0^{\pi+x} \cos^4 t\, dt$

$= \int_0^{\pi} \cos^4 t\, dt + \int_{\pi}^{\pi+x} \cos^4 t\, dt$

$I_1 = I_2$

where, $I_1 = g(\pi)$ and $I_2 = \int_{\pi}^{\pi+x} \cos^4 t\, dt$

$\Rightarrow I_2 = \int_0^x \cos^4(y+\pi) dy$

$= \int_0^x \cos^4 y\, dy = g(x)$

$\therefore g(\pi + x) = g(x) + g(\pi)$

175 **(b)**

Let $I = \int_0^{\pi/2} f(\sin 2x) dx$. Then, ...(i)

$I = \int_0^{\pi/2} f(\sin 2x)\cos x\, dx$, $\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)dx\right]$ ...(ii)

$\therefore 2I = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x)dx$

$\Rightarrow 2I = \sqrt{2}\int_0^{\pi/2} f(\sin 2x)\cos\left(x - \frac{\pi}{4}\right)dx$

$\Rightarrow 2I = \sqrt{2}\int_{-\pi/4}^{\pi/4} f\left\{\sin\left(\frac{\pi}{2} + 2t\right)\right\}\cos t\, dt$, where $x - \frac{\pi}{4} = t$,

$\Rightarrow 2I = \sqrt{2}\int_{-\pi/4}^{\pi/4} f(\cos 2t)\cos t\, dt$

$\Rightarrow 2I = \sqrt{2}\int_0^{\pi/4} f(\cos 2t)\cos t\, dt$ $\left[\begin{matrix}\because \text{Integrand} \\ \text{is an even function}\end{matrix}\right]$

$\Rightarrow I = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\, dx$

176 **(a)**

Given, $f(x) = \int_1^x \sqrt{2-t^2}\, dt$

$\Rightarrow f'(x) = \sqrt{2-x^2}$

Now, $x^2 - f'(x) = 0$

$\Rightarrow x^2 - \sqrt{2-x^2} = 0$

$\Rightarrow x^4 + x^2 - 2 = 0$

$\Rightarrow (x^2+2)(x^2-1) = 0$

$\Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

177 **(a)**

Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}dx}{\sqrt{\cos x}+\sqrt{\sin x}}$ ...(i)

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ ...(ii)

$\left[\text{put } x = \frac{\pi}{2} - x\right]$

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} dx \quad \Rightarrow I = \frac{\pi}{4}$

178 **(a)**

$\int_0^3 \frac{3x+1}{x^2+9}dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}dx + \int_0^3 \frac{dx}{x^2+9}$

$= \left[\frac{3}{2}\log(x^2+9) + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$

$= \frac{3}{2}[(\log 3^2 + 9)$

$- \log(0+9)$

$+ \frac{1}{3}\left[\tan^{-1}\left(\frac{3}{3}\right) - \tan^{-1} 0\right]$

$= \frac{3}{2}(\log 18 - \log 9) + \frac{1}{3}\left(\frac{\pi}{4}\right)$

$= \frac{3}{2}\log 2 + \frac{\pi}{12} = \log(2\sqrt{2}) + \frac{\pi}{12}$

179 **(a)**

$\phi(x) = \cos x - \int_0^x (x-t)\phi(t)dt$

$= \cos x - x\int_0^x \phi(t)dt + \int_0^x t\,\phi(t)dt$

$\Rightarrow \phi'(x) = -\sin x - \int_0^x \phi(t)dt - x\phi(x) + x\phi(x)$

$\Rightarrow \phi''(x) = -\cos x - \phi(x)$

$\Rightarrow \phi(x) + \phi''(x) = -\cos x$

180 **(c)**

Putting $\lambda x = t, \lambda\, dx = dt$, we get

$\int_0^\infty e^{-\lambda x}x^{n-1}dx = \frac{1}{\lambda^n}\int_0^\infty e^{-t}t^{n-1}\,dt$

$= \frac{1}{\lambda^n}\int_0^\infty e^{-x}x^{n-1}\,dx = \frac{I_n}{\lambda^n}$

181 **(c)**

Let $I = \int x^{49}\frac{\tan^{-1}(x^{50})}{(1+x^{50})^2}dx$

Put $x^{50} = t \Rightarrow 50x^{49}\,dx = dt$

$\therefore I = \frac{1}{50}\int \frac{\tan^{-1} t}{1+t^2}dt$

Again, put $\tan^{-1} t = u \Rightarrow \frac{1}{1+t^2}dt = du$

$\therefore\ I = \frac{1}{50}\int u\,du = \frac{u^2}{100} + c$

$= \frac{(\tan^{-1} x^{50})^2}{100} + c$

But $I = k(\tan^{-1} x^{50})^2 + c$ [given]

$\therefore\ k = \frac{1}{100}$

182 **(b)**

We have,

$$I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1 + a^x} dx$$

$$\Rightarrow I = \int_0^{\pi} \left\{\frac{\sin^2 x}{1 + a^x} + \frac{\sin^2(-x)}{1 + a^{-x}}\right\} dx$$

$$\left\{\text{Using : } \int_{-a}^{a} f(x)dx = \int_0^{a} \{f(x) + f(-x)\}dx\right\}$$

$$\Rightarrow I = \int_0^{\pi} \sin^2 x\ dx$$

$$\Rightarrow I = 2\int_0^{\pi/2} \sin^2 x\ dx$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \text{, if } f(2a - x) = f(x)\right]$$

$$\Rightarrow I = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$$

183 **(c)**

Let $I = \int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$

Put $x = \tan\theta \quad \Rightarrow dx = \sec^2\theta\ d\theta$

$$\therefore I = \int \frac{\tan\theta \tan^{-1}(\tan\theta)}{(1 + \tan^2\theta)^{3/2}} \sec^2\theta\ d\theta$$

$$\Rightarrow I = \int \frac{\theta \tan\theta \sec^2\theta}{\sec^3\theta} d\theta = \int \theta \sin\theta\ d\theta$$

$$\Rightarrow I = \theta(-\cos\theta) + \int \cos\theta\ d\theta$$

$$= -\theta\cos\theta + \sin\theta + c$$

$$\Rightarrow\ I = -\tan^{-1} x . \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

$$\Rightarrow\ I = \frac{(x - \tan^{-1} x)}{\sqrt{1+x^2}} + c$$

184 **(c)**

Let $f(x) = ax^2 + bx + c$. Then,

$f(0) = c, f(1) = a + b + c$ and $f\left(\frac{1}{2}\right) = \frac{a}{4} + \frac{b}{2} + c$

$$\therefore 6\int_0^1 f(x)\,dx - \left\{f(0) + 4f\left(\frac{1}{2}\right)\right\}$$

$$= 6\left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx\right]_0^1 - \{c + a + 2b + 4c\}$$

$$= 2a + 3b + 6c - (a + 2b + 5c) = a + b + c = f(1)$$

$\therefore k = 1$

185 **(b)**

We have,

$$I = \int_0^{a} f(x)\ g(x)dx$$

$$\Rightarrow I = \int_0^{a} f(a - x)\ g(a - x)dx$$

$$\Rightarrow I = \int_0^{a} f(x)\ \{2 - g(x)\}dx \quad \left[\begin{array}{l}\because f(a - x) = f(x),\\ g(a - x) = 2 - g(x)\end{array}\right]$$

$$\Rightarrow I = 2\int_0^{a} f(x)\,dx - \int_0^{a} f(x)\,g(x)dx$$

$$\Rightarrow I = 2\int_0^{a} f(x)\,dx - I$$

$$\therefore 2I = 2\int_0^{a} f(x)\,dx \Rightarrow I = \int_0^{a} f(x)\,dx$$

187 **(d)**

Let $I = \int_0^{\pi/8} \cos^3 4\theta\,d\theta$

$$= \int_0^{\pi/8} \cos^2 4\theta \cdot \cos 4\theta\ d\theta$$

$$= \int_0^{\pi/8} \left(\frac{1 + \cos 8\theta}{2}\right) \cos 4\theta\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/8} \cos 4\theta\,d\theta + \frac{1}{2}\int_0^{\pi/8} \cos 8\theta \cos 4\theta\,d\theta$$

$$= \frac{1}{2}\left[\frac{\sin 4\theta}{4}\right]_0^{\pi/8} + I_1$$

$$I = \frac{1}{8} + I_1$$

Where, $I_1 = \frac{1}{2}\int_0^{\pi/8} \cos 8\theta \cos 4\theta\,d\theta$

$$= \frac{1}{4}\int_0^{\pi/8} 2\cos 8\theta \cos 4\theta\,d\theta$$

$$= \frac{1}{4}\left[\int_0^{\pi/8} \cos 12\theta\,d\theta + \int_0^{\pi/8} \cos 4\theta\,d\theta\right]$$

$$= \frac{1}{4}\left[\frac{\sin 12\theta}{12} + \frac{\sin 4\theta}{4}\right]_0^{\pi/8}$$

$$= \frac{1}{4}\left[-\frac{1}{21} + \frac{1}{4}\right] = \frac{1}{4}\left[\frac{2}{12}\right]0$$

$I_1 = \frac{1}{24}$ ...(iii)

From Eqs.(i) and (ii), we get

$$I=\frac{1}{8}+\frac{1}{24}-\frac{3+1}{24}=\frac{1}{6}$$

188 **(d)**

$$\int_{[x]}^{[x]+1} f(t)dt=[x]$$

$\int_{-2}^{-2+1} f(x)dx=-2, \int_{-1}^{0} f(x)dx=$ $-1, \dots . \int_{3}^{4} f(x)dx=3$

$$\therefore \int_{-2}^{4} f(x)dx=-2-1+1+2+3=3$$

189 **(d)**

Case I If $x>0$, then $|x|=x$

$$\therefore \int |x|\log|x|dx=\int x\log x\,dx$$

$$=\log x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2}{2}\cdot\log x-\frac{x^2}{4}+c$$

$$=-\frac{x^2}{2}\log|x|-\frac{x^2}{4}+c$$

Case II If $x<0$, then $|x|=-x$

$$\therefore \int |x|\log|x|dx=-\int x\log(-x)dx$$

$$=-\left\{\log(-x)\cdot\frac{x^2}{2}-\frac{x^2}{4}\right\}+c$$

$$=-\frac{x^2}{2}\log|x|+\frac{x^2}{2}+c$$

On combining both cases, we get

$$\int |\,x\,|\log|x|dx=\frac{1}{2}x|x|\log|x|-\frac{1}{4}x|x|+c$$

190 **(a)**

We have,

$$fof(x)=f\big(f(x)\big)=\frac{f(x)}{[1+\{f(x)\}^n]^{1/n}}$$
$$=\frac{x}{(1+2x^n)^{1/n}}$$

Similarly,

$$fofof(x)=\frac{x}{(1+3x^n)^{1/n}},(fofofo\dots\cdot of)(x)$$
$$=\frac{x}{(1+nx^n)^{1/n}}$$
$$n-\text{times}$$

$$\therefore I=\int x^{n-2}\,g(x)=\int\frac{x^{n-1}}{(1+nx^n)^{1/n}}dx$$

$$\Rightarrow I=\frac{1}{n^2}\int(1+nx^n)^{-1/n}d(1+nx^n)$$

$$\Rightarrow I=\frac{1}{n^2}\frac{(1+nx^n)^{-\frac{1}{n}+1}}{\left(-\frac{1}{n}+1\right)}+k$$

$$=\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+k$$

191 **(d)**

Let $I=\int_0^{2\pi}\frac{\sin 2\theta}{a-b\cos\theta}d\theta$. Then,

$$I=\int_0^{2\pi}\frac{\sin 2(2\pi-\theta)}{a-b\cos(2\pi-\theta)}d\theta,$$

$$\Rightarrow I=\int_0^{2\pi}\frac{-\sin 2\theta}{a-b\cos\theta}d\theta\Rightarrow I=-I\Rightarrow 2I=0\Rightarrow I$$
$$=0$$

192 **(d)**

Given, $f(-x)=-f(x), \forall$ values of real x

We know that,

$$\int_{-a}^{a} f(x)dx=0=\int_{-a}^{0} f(x)dx+\int_0^a f(x)dx \quad [\because f(-x)=-f(x)]$$

$$\Rightarrow \int_{-1}^{0} f(x)\,dx+\int_0^1 f(x)dx=0$$

$$\Rightarrow \int_{-1}^{0} f(x)dx=-5\quad [\because \int_0^1 f(x)dx=5]$$

$$\Rightarrow \int_{-1}^{0} f(t)dt=-5$$

193 **(a)**

Let $I=\int_0^a f(x)g(x)dx$

$$=\int_0^a f(a-x)g(a-x)dx$$

$$\Rightarrow\ I=\int_0^a f(x)[2-g(x)]dx$$

$[\because f(x)=f(a-x), g(x)+g(a-x)=2]$

$$=\int_0^a 2f(x)dx-\int_0^a f(x)g(x)dx$$

$$\Rightarrow\ I=\int_0^a 2f(x)dx-I$$

$$\Rightarrow\ I=\int_0^a f(x)dx$$

194 **(a)**

Let $f(x)=\log\left(\frac{a+x}{a-x}\right)$

$$\Rightarrow f(-x)=\log\left(\frac{a-x}{a+x}\right)$$

$$=-\log\left(\frac{a+x}{a-x}\right)$$

$$=-f(x)$$

$\Rightarrow f(x)$ is an odd function.

$\therefore \int_{-10}^{10} f(x)dx = 0$

195 **(c)**

Let $I = \int_0^\pi |\sin^3 \theta|\, d\theta$

Since, $\sin\theta$ is positive in interval $(0, \pi)$

$\therefore\ I = \int_0^\pi \sin^3 \theta\, d\theta = \int_0^\pi \sin\theta \cdot \sin^2\theta\, d\theta$

$= \int_0^\pi \sin\theta(1 - \cos^2\theta)\, d\theta$

$= \int_0^\pi \sin\theta\, d\theta + \int_0^\pi (-\sin\theta)\cos^2\theta\, d\theta$

$= [-\cos\theta]_0^\pi + \left[\frac{\cos^3\theta}{3}\right]_0^\pi$

$= -[\cos\pi - \cos 0] + \frac{1}{3}[\cos^3\pi - \cos^3 0]$

$= -[-1-1] + \frac{1}{3}[-1-1] = 2 - \frac{2}{3} = \frac{4}{3}$

196 **(b)**

Let $I = \int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\therefore I = -\int_1^{\sqrt{3}/2} \frac{1}{t_3} = \left[\frac{1}{2t^2}\right]_1^{\sqrt{3}/2} = \frac{1}{6}$

197 **(a)**

Let $I = \int_0^2 \sqrt{\frac{2+x}{2-x}} dx = \int_0^2 \frac{2+x}{\sqrt{4-x^2}} dx$

$= \int_0^2 \frac{2}{\sqrt{4-x^2}} dx + \int_0^2 \frac{x}{\sqrt{4-x^2}} dx$

Put $x = 2\sin\theta$ in the first integral and $4 - x^2 = t$ in the second integral

$I = 2\int_0^{\pi/2} \frac{2\cos\theta d\theta}{2\cos\theta} + \int_4^0 -\frac{dt}{2\sqrt{t}}$

$= 2\int_0^{\pi/2} d\theta + \frac{1}{2}\int_0^4 t^{-1/2}\, dt$

$= 2[\theta]_0^{\pi/2} + \frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right]_0^4$

$= \pi + 2$

198 **(a)**

Given, $\int f(x)dx = f(x) + c$

$\Rightarrow f(x) = f'(x)$

$\therefore \int (f(x)^2)dx = \int f(x) \cdot f'(x)dx$

$= \frac{1}{2}(f(x))^2 + c$

199 **(b)**

Let $I = \int \sqrt{\frac{x-1}{x+1}} dx = \int \frac{x-1}{\sqrt{x^2-1}}\, dx$

$= \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}}\, dx$

$= \sqrt{x^2-1} - \sin^{-1} x + c$

200 **(b)**

Let $I = \int \sin x\, d(\cos x)$

Put $\cos x = t \Rightarrow d(\cos x) = dt$

$\therefore I = \int \sqrt{1-t^2} dt$

$= \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}(t) + c$

$= \frac{\cos x}{2} \cdot \sin x + \frac{1}{2}\sin^{-1}(\cos x) + c$

$= \frac{1}{4}\sin 2x + \frac{1}{2}\sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] + c$

$= \frac{1}{4}\sin 2x - \frac{x}{2} + c$

201 **(b)**

Let $I = \int_{-\pi/2}^{\pi/2} \cos x\, dx$

Since, $\cos(-x) = \cos x$

$\therefore I = 2\int_0^{\pi/2} \cos x\, dx = 2[\sin x]_0^{\pi/2} = 2$

202 **(a)**

Given, $f(x) = f(a+x)$and $\int_0^a f(x)dx = k$

$\int_a^{na} f(x)dx = n\int_0^a f(x) = nk$

$[\because f(x)$ is periodic function$]$

203 **(c)**

Let $I = \int_0^\pi \frac{1}{5+3\cos x} dx$ ...(i)

Then,

$I = \int_0^\pi \frac{1}{5-3\cos x} dx$ ...(ii) $\left[\because \int_0^a f(x)\, dx = \int_0^a f(a-x)dx\right]$

Adding (i) and (ii), we get

$2I = 10\int_0^\pi \frac{1}{25 - 9\cos^2 x} dx$

$\Rightarrow I = 5 \times 2\int_0^{\pi/2} \frac{1}{25 - 9\cos^2 x} dx$

$\Rightarrow I = 10\int_0^{\pi/2} \frac{\sec^2 x}{25 + 16\tan^2 x} dx$

$\Rightarrow I = 10 \times \frac{1}{20}\left[\tan^{-1}\left(\frac{4\tan x}{5}\right)\right]_0^{\pi/2} = \frac{\pi}{4}$

204 **(d)**

We have,

$$I_1 = \int_0^x e^{zx} e^{-z^2} dz \text{ and } I_2 = \int_0^x e^{-\frac{z^2}{4}} dz$$

Putting $z = \frac{x+t}{2}$ and $dt = 2\,dz$ in I_1, we get

$$I_1 = \frac{1}{2}\int_{-x}^{x} e^{\left(\frac{x+t}{2}\right)x} . e^{-\left(\frac{x+t}{2}\right)^2} dt$$

$$\Rightarrow I_1 = \frac{1}{2}\int_{-x}^{x} e^{\frac{1}{2}\left\{x^2+tx-\frac{1}{2}(x+t)^2\right\}} dt$$

$$\Rightarrow I_1 = \frac{1}{2}\int_{-x}^{x} e^{\frac{1}{2}\left\{\frac{1}{2}x^2-\frac{1}{2}t^2\right\}} dt$$

$$\Rightarrow I_1 = \frac{1}{2}\int_{-x}^{x} e^{\frac{1}{4}x^2} e^{-\frac{1}{4}t^2} dt$$

$$\Rightarrow I_1 = \frac{1}{2}e^{\frac{1}{4}x^2}\int_{-x}^{x} e^{-\frac{1}{4}t^2} dt$$

$$\Rightarrow I_1 = \frac{1}{2}e^{\frac{1}{4}x^2} \times 2\int_0^x e^{-\frac{1}{4}t^2} dt \quad \left[\because e^{-\frac{1}{4}t^2} \text{ is an even function}\right]$$

$$\Rightarrow I_1 = e^{\frac{1}{4}x^2}\int_0^x e^{-\frac{1}{4}t^2} dt = e^{\frac{1}{4}x^2} I_2$$

206 **(b)**

$$\int_0^\pi f(x)\sin x\,dx + \int_0^\pi f''(x)\sin x\,dx = 5$$

$$\Rightarrow [f(x)(-\cos x)]_0^\pi + \int_0^\pi f'(x)\cos x\,dx + \int_0^\pi f''(x)\sin x\,dx = 5$$

$$\Rightarrow [-f(x)\cos x]_0^\pi + [f'(x)\sin x]_0^\pi - \int_0^\pi f''(x)\sin x\,dx + \int_0^\pi f''(x)\sin x\,dx = 5$$

$$\Rightarrow f(\pi) + f(0) = 5 \Rightarrow f(0) = 5 - f(\pi) = 3$$

207 **(c)**

$$\int \frac{x+2}{2x^2+6x+5}\,dx = \frac{1}{4}\int \frac{4x+6+2}{2x^2+6x+5}\,dx$$

$$= \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}\,dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$$

$$\Rightarrow P = \frac{1}{4}$$

208 **(d)**

$$\int (1+x-x^{-1})e^{x+x^{-1}}dx$$

$$\int \left[xe^{x+x^{-1}}\left(1-\frac{1}{x^2}\right) + e^{x+x^{-1}}\right]dx$$

$$= \frac{x^{x+x^{-1}}}{e} - \int e^{x+x^{-1}}dx + \int e^{x+x^{-1}}dx$$

$$= xe^{x+x^{-1}} + c$$

209 **(c)**

$$\because \cos^2(\pi+x) = \cos^2 x$$

$$\Rightarrow I_1 = \int_0^{3\pi} f(\cos^2 x)dx = 3\int_0^\pi f(\cos^2 x)dx$$

$$= 3I_2$$

210 **(c)**

Let $I = \int_0^{\pi/2}(x\sqrt{\tan x} + \sqrt{\cot x})\,dx$ ...(i)

Then, $I = \int_0^{\pi/2}\left(\frac{\pi}{2} - x\right)(\sqrt{\cot x} + \sqrt{\tan x})\,dx$

$$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi/2} (\sqrt{\cot x} + \sqrt{\tan x})\,dx - I$$

$$\Rightarrow 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\cos x + \sin x}{\sqrt{(\sin x - \cos x)}}dx$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}}\int_0^{\pi/2} \frac{\cos x + \sin x}{\sqrt{1-(\sin x - \cos x)^2}}dx$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}}\sin^{-1}[(\sin x - \cos x)]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}}\{\sin^{-1}(1) - \sin^{-1}(-1)\}$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}} \times \pi \Rightarrow I = \frac{\pi^2}{2\sqrt{2}}$$

211 **(c)**

Let $I = \int_0^\pi \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ ... (i)

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x)}{a^2\cos^2 x + b^2\sin^2 x}dx \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii) , we get

$$2I = 2\pi\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$\Rightarrow 2I = 2\pi\int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}dx$$

Now, put $\tan x = t \Rightarrow dx = \frac{dt}{\sec^2 x}$

$$\therefore 2I = 2\pi\int_0^\infty \frac{dt}{a^2 + b^2t^2}$$

$$= \frac{1}{b^2} \times 2\pi\int_0^\infty \frac{dt}{\frac{a^2}{b^2} + t^2}$$

$$= \left[\frac{2\pi}{b^2}.\frac{b}{a}\tan^{-1}\frac{bt}{0}\right]_0^\infty$$

$$= \frac{2\pi}{ab}[\tan^{-1}\infty - \tan^{-1}0] = \frac{2\pi}{ab} \times \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi^2}{2ab}$$

212 **(b)**

We have,

$f(x) = ae^{2x} + be^x + cx \Rightarrow f'(x)$
$= 2ae^{2x} + be^x + c$

Now,

$f(0) = -1 \Rightarrow a + b = -1$ ...(i)

$f'(\log 2) = 31 \Rightarrow 8a + 2b + c = 31$ ...(ii)

and $\int_0^{\log 4}(f(x) - cx)\,dx = \frac{39}{2}$

$$\Rightarrow \int_0^{\log 4} (ae^{2x} + be^x) = \frac{39}{2}$$

$\Rightarrow \left[\frac{1}{2}ae^{2x} + be^x\right]_0^{\log 4} = \frac{39}{2} \Rightarrow \frac{15}{2}a + 3b = \frac{39}{2}$ ...(iii)

Solving (i), (ii) and (iii), we get $a = 5, b = -6, c = 3$

213 **(b)**

Putting $x = \tan\theta$ and $dx = \sec^2\theta\, d\theta$, we get

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}}dx = \int_0^{\pi/4} \frac{\sec^2\theta\, d\theta}{\sec^3\theta} = \int_0^{\pi/4} \cos\theta\, d\theta$$
$$= \frac{1}{\sqrt{2}}$$

214 **(d)**

Let us assume $f(x) = \frac{e^{x^2}}{2}$

$$\therefore \int x.\frac{e^{x^2}}{2}dx = \int 2x.\frac{e^{x^2}}{2}dx$$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{e^t}{4}dt = \frac{e^t}{4} + c = \frac{1}{2}\left(\frac{e^{x^2}}{2}\right) + c$$

Hence, $\int x\,f(x)dx = \frac{1}{2}f(x) + c$, when $f(x) = \frac{e^{x^2}}{2}$

215 **(b)**

Let $I = \int_0^{\pi} \frac{1}{1+\sin x}dx$

$$= \int_0^{\pi} \frac{1}{1 + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}dx$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx$$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$\therefore I = \int_0^{\infty} \frac{2dt}{(1+t)^2} = \left[-\frac{2}{1+t}\right]_0^{\infty} = 2$$

216 **(a)**

$$\int_{-1}^1 f(x)dx = \int_{-1}^1 (x - [x])dx$$

$$= \int_{-1}^1 x\,dx - \int_{-1}^1 [x]dx$$

$= 0 - \int_{-1}^1 [x]dx$ ($\because x$ is an odd function)

But $[x] = \begin{cases} -1, & \text{if } -1 \le x < 0 \\ 0, & \text{if } 0 \le x < 1 \\ 1, & \text{if } x = 1 \end{cases}$

$$\therefore \int_{-1}^1 [x]\,dx = \int_{-1}^0 [x]\,dx + \int_0^1 [x]\,dx$$

$$= \int_{-1}^0 [-1]\,dx + \int_0^1 0\,dx = -[x]_{-1}^0 + 0 = 1$$

Thus, $\int_{-1}^1 f(x)dx = 1$

217 **(a)**

Let $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^2.x^3\left(1+\frac{1}{x^4}\right)^{3/4}}$

Put $1 + x^{-4} = t \Rightarrow \frac{-4}{x^5}dx = dt$

$$\therefore I = -\frac{1}{4}\int \frac{dt}{t^{3/4}} = -\frac{1}{4}.\frac{t^{1/4}}{1/4} + c$$

$$= -\left(1 + \frac{1}{x^4}\right)^{1/4} + c$$

$$= \frac{-(x^4+1)^{1/4}}{x} + c$$

218 **(b)**

We have,

$I_1 = \int_{1-k}^{k} x\sin\{x(1-x)\}\,dx$ and $I_2 = \int_{1-k}^{k} \sin\{x(1-x)\}\,dx$

$$\Rightarrow I_1 = \int_{1-k}^{k} (1-x)\sin\{x(1-x)\}\,dx$$

$$\Rightarrow I_1 = \int_{1-k}^{k} x\sin\{x(1-x)\}\,dx$$
$$- \int_{1-k}^{k} x\sin\{x(1-x)\}\,dx$$

$\Rightarrow I_1 = I_2 - I_1 \Rightarrow 2I_1 = I_2$

219 **(b)**

Given that, $\int \frac{1}{f(x)}dx = \log\{f(x)\}^2 + c$

$$\Rightarrow \frac{1}{f(x)} = \frac{1}{\{f(x)\}^2}2f(x)f'(x)$$

$$\Rightarrow f'(x) = \frac{1}{2}$$

$$\Rightarrow f(x) = \frac{x}{2} + d$$

(where d is a integration constant)

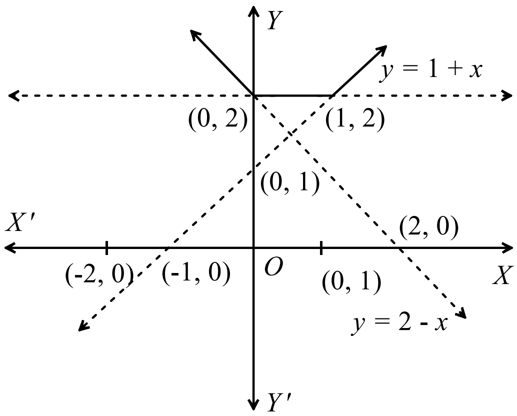
220 **(b)**

$$\int \frac{x^4+x^2+1}{x^2-x+1}\,dx = \int (x^2+x+1)\,dx$$
$$= \frac{x^3}{3}+\frac{x^2}{2}+x+c$$

221 **(b)**

Let $f(x) = \max.\{2-x, 2, 1+x\}$

The graph of $f(x)$ is shown by continuous lines in Fig.



Clearly,
$$f(x) = \begin{cases} 2-x, & x \le 0 \\ 2, & 0 \le x \le 1 \\ 1+x, & x \ge 1 \end{cases}$$
$$\therefore \int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} (2-x)\,dx + \int_{0}^{1} 2dx$$
$$\Rightarrow \int_{-1}^{1} f(x)\,dx = \left[2x - \frac{x^2}{2}\right]_{-1}^{0} + [2x]_0^1 = \frac{9}{2}$$

222 **(d)**
$$\int_0^1 \frac{x^2}{1+x^2}\,dx = \int_0^1 \frac{1+x^2}{1+x^2}\,dx - \int_0^1 \frac{1}{1+x^2}\,dx$$
$$= \int_0^1 1\,dx - \int_0^1 \frac{1}{1+x^2}\,dx$$
$$= [x]_0^1 - [\tan^{-1}x]_0^1 = 1 - \frac{\pi}{4}$$

223 **(a)**

Let $I = \int \frac{e^x}{(2+e^x)(e^x+1)}\,dx$

Put $e^x = t \Rightarrow e^x dx = dt$
$$\therefore I = \int \frac{1}{(2+t)(1+t)}\,dt$$
$$= \int \left[\left(\frac{1}{1+t}\right) - \left(\frac{1}{2+t}\right)\right] dt$$
$$= \log(1+t) - \log(2+t) + c$$
$$= \log\left(\frac{1+e^x}{2+e^x}\right) + c$$

224 **(c)**

We have,
$$\phi(x) \begin{cases} \int_0^x (1-t)dt, & 0 \le x \\ \int_0^1 (1-t)dt + \int_1^x 0\,dt, & 1 \le x \\ \int_0^1 (1-t)dt + \int_1^2 0\,dt + \int_2^x (2-t)^2 dt, & 2 \le x \end{cases}$$
$$\Rightarrow \phi(x) \begin{cases} \left(x - \frac{x^2}{2}\right), & 0 \le x \le 1 \\ \frac{1}{2}, & 1 \le x \le 2 \\ \frac{1}{2} + \frac{(x-2)^3}{3}, & 2 \le x \le 3 \end{cases}$$

226 **(a)**

We know that $\int_a^{a+T} f(x)\,dx$ is independent of a, if $f(x)$ is a periodic function with period T

Since $\sin^4 x + \cos^4 x$ is a periodic function with period $\pi/2$

$\therefore \int_a^{a+\pi/2} (\sin^4 x + \cos^4 x)\,dx$ is independent of a

227 **(a)**

Let $I = \int_{-1/2}^{1/2} \cos x \log\left(\frac{1-x}{1+x}\right) dx$ ...(i)

On replacing x by $-x$, we get
$$I = \int_{-1/2}^{1/2} \cos(-x) \log\left(\frac{1+x}{1-x}\right) dx$$

$\Rightarrow I = -\int_{-1/2}^{1/2} \cos x \log\left(\frac{1-x}{1+x}\right) dx$ ...(ii)

On adding Eqs.(i) and (ii), we get
$$2I = \int_{-1/2}^{1/2} \cos x \log\left(\frac{1-x}{1+x}\right) dx - \int_{-1/2}^{1/2} \cos x \log\left(\frac{1-x}{1+x}\right) dx$$
$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

228 **(c)**
$$\int \frac{x^2-1}{x^4+x^2+1}\,dx$$
$$\Rightarrow I = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}\,dx$$
$$= \int \frac{1-\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2 - 1}\,dx$$

Put $x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right)dx = dt$
$$\therefore I = \int \frac{dt}{t^2-1^2} = \frac{1}{2\times 1}\log\frac{t-1}{t+1} + c$$
$$= \frac{1}{2}\log\frac{x^2+1-x}{x^2+1+x} + c$$

229 **(d)**

Let $I = \int \frac{\sqrt{x}}{x+1} dx = \int \frac{\sqrt{x}\, dx}{(\sqrt{x})^2+1}$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \quad dx = 2\sqrt{x} dt$

$\therefore \; I = \int \frac{2t^2 dt}{t^2+1} = 2\left[\int \frac{t^2+1}{t^2+1} dt - \int \frac{1}{t^2+1} dt\right]$

$= 2t - 2\tan^{-1}(t^2+1) + c$

$= 2\sqrt{x} - 2\tan^{-1}(x+1) + c$

But $I = A\sqrt{x} + B\tan^{-1}(x+1) + c$

$\Rightarrow \quad A = 2, B = -2$

230 **(b)**

Let $I = \int x\, e^{x^2} dx$

Put $x^2 = t \Rightarrow 2x\, dx = dt \Rightarrow x\, dx = \frac{dt}{2}$

$\therefore I = \frac{1}{2}\int e^t dt = \frac{e^t}{2} + c = \frac{e^{x^2}}{2} + c$

231 **(a)**

Let $\; f(\theta) = \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)$

Now, $f(-\theta = \log\left(\frac{2+\sin\theta}{2-\sin\theta}\right)$

$= -\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) = -f(\theta)$

$\therefore f(\theta)$is an odd function.

So, $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) d\theta = 0$

232 **(c)**

Let $I = \int \frac{dx}{\left[(\sqrt{x+99})^2+1\right]\sqrt{x+99}}$

Put $\sqrt{x+99} = t \quad \Rightarrow \quad \frac{1}{\sqrt{x+99}} dx = 2dt$

$\therefore \; I = \int \frac{2dt}{t^2+1} = 2\tan^{-1} t + c$

$= 2\tan^{-1}\sqrt{x+99} + c$

But $2\tan^{-1}\sqrt{x+99} + c = f(x) + c \quad$ [given]

$\therefore \; f(x) = 2\tan^{-1}\sqrt{x+99}$

233 **(d)**

For $n \le 1$ and $-1 \le x \le 1$, we have

$\Rightarrow 1 \le \frac{1}{\sqrt{1-x^{2n}}} \le \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \int_0^{1/2} 1 \cdot dx \le \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx \le \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$

$\Rightarrow \frac{1}{2} \le \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx \le [\sin^{-1} x]_0^{1/2}$

$\Rightarrow \frac{1}{2} \le I \le \frac{\pi}{6}$

234 **(b)**

Let $I = \int \frac{\operatorname{cosec} x}{\cos^2(1+\log\tan\frac{x}{2})} dx$

Put $1 + \log\tan\frac{x}{2} = t$

$\Rightarrow \frac{1}{\tan\frac{x}{2}} . \sec^2\frac{x}{2} . \frac{1}{2} dx = dt$

$\Rightarrow \operatorname{cosec} x\, dx = dt$

$\therefore I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t \; dx = \tan t + c$

$= \tan\left(1 + \log\tan\frac{x}{2}\right) + c$

235 **(a)**

Here, $I(m,n) = \int_0^1 t^m (1+t)^n dt$

$\Rightarrow I(m,n) = \left\{(1+t)^n . \frac{t^{m+1}}{m+1}\right\}_0^1$

$- \int_0^1 n(1+t)^{n-1} . \frac{t^{m+1}}{m+1} dt$

$= \frac{2^n}{m+1} - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} . t^{m+1} dt$

$\therefore I(m,n) = \frac{2^n}{m+1} - \frac{n}{m+1} . I(m+1, n-1)$

236 **(c)**

$I = \int \frac{dx}{x(\log x)(\log\log x) \dots \underbrace{(\log\log \dots x)}_{8 \text{ times}}}$

Put $\underbrace{(\log\log \dots x)}_{8 \text{ times}} = t$

$\Rightarrow I = \frac{1}{\underbrace{(\log\log \dots x)}_{7 \text{ times}} \dots (\log\log x)(\log x) x} dx = dt$

$\therefore I = \int \frac{dt}{t} = \log t + c = \underbrace{(\log\log\log \dots}_{9 \text{ times}} x) + c$

237 **(d)**

Given, $I = \left|\int_2^5 \frac{\sin x dx}{(1+x^2)}\right|$

$\le \int_2^5 \left|\frac{\sin x}{1+x^2}\right| dx$

$\le \int_2^5 \frac{1}{1+x^2} dx \le \int_2^5 \frac{1}{x^2} dx$

$= \left[\frac{x^{-1}}{-1}\right]_2^5 = -\left[\frac{1}{5} - \frac{1}{2}\right]$

$\Rightarrow I \le \frac{3}{10}$

238 **(b)**

$\lim_{n\to\infty}\left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{n+2n}\right)$

$= \sum_{r=0}^{2n} \frac{1}{n+r} = \frac{1}{n}\sum_{r=0}^{2n} \frac{1}{1+\frac{r}{n}}$

$$= \int_0^2 \frac{1}{1+x} dx = [\log(1+x)]_0^2$$

$$= \log 3 - \log 1 = \log 3$$

239 **(c)**

We have,

$$\phi(x) = \int_0^x (t+1)dt \Rightarrow \phi'(x) = x+1$$

Clearly, $\phi'(x) = x+1 > 0$ for $x \in [2,3]$

Therefore, $\phi(x)$ is an increasing function on [2, 3]

So, its greatest and least values are $\phi(3)$ and $\phi(2)$ respectively. Therefore, required difference is given by

$$\phi(3) - \phi(2) = \int_0^3 (t+1)dt - \int_0^2 (t+1)\, dt = \frac{7}{2}$$

240 **(b)**

Putting $x = \tan\theta$, we have

$$I = \int_0^\infty \frac{x \log x}{(1+x^2)^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\tan\theta \log\tan\theta \sec^2\theta \, d\theta}{(1+\tan^2\theta)^2}$$

$$\Rightarrow I = \int_0^{\pi/2} \sin\theta\cos\theta \log\tan\theta \, d\theta \quad \text{...(i)}$$

$$\Rightarrow I = \int_0^{\pi/2} \sin\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)\log\tan\left(\frac{\pi}{2}-\theta\right) d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \cos\theta\sin\theta\log\cot\theta \, d\theta \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \sin\theta\cos\theta \, (\log\tan\theta + \log\cot\theta)\, d\theta$$

$$\Rightarrow 2I = \int_0^{\pi/2} \sin\theta\cos\theta \, . \log 1 \, d\theta = 0 \Rightarrow I = 0$$

241 **(b)**

Let

$$I = \int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x} dx = \int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{2^x . 5^x} dx$$

$$\Rightarrow I = 2\int_0^1 \left(\frac{2}{5}\right)^x dx - \frac{1}{2}\int_0^1 \left(\frac{5}{2}\right)^{x-1} dx,$$

$$\Rightarrow I = 2\left[\frac{(2/5)^x}{\log_e(2/5)}\right]_0^1 - \frac{1}{2}\left[\frac{(5/2)^{x-1}}{\log_e 5/2}\right]_0^1$$

$$\Rightarrow I = 2\left[\frac{2/5 - 1}{\log_e(2/5)}\right] - \frac{1}{2}\left[\frac{1 - 2/5}{\log_e 5/2}\right]$$

$$\Rightarrow I = -\frac{6}{5}\frac{1}{\log_e(2/5)} - \frac{3}{10\log_e(5/2)}$$

$$\Rightarrow I = -\frac{3}{5}\left\{\frac{2}{\log_e(2/5)} + \frac{1}{2\log_e(5/2)}\right\}$$

242 **(c)**

Let $I = \int_0^{\pi r} \sin^{2n} x \, dx$

Since $\sin^{2n} x$ is a periodic function with period $\pi/2$. Therefore,

$$\therefore I = \int_0^{\pi r} \sin^{2n} x \, dx = \int_0^{2r(\pi/2)} \sin^{2n} x \, dx$$

$$\Rightarrow I = 2r \int_0^{\pi/2} \sin^{2n} x \, dx$$

$$\Rightarrow I = r\int_0^{\pi} \sin^{2n} x \, dx \quad \left[\begin{array}{l} \because \sin^{2n}(\pi - x) = \sin^{2n} x \\ \therefore \int_0^{\pi} \sin^{2n} x\, dx = 2\int_0^{\pi/2} \sin^{2n} x \, \text{ɿ} \end{array}\right.$$

244 **(a)**

Let $I = \int_0^\lambda \frac{y\, dy}{\sqrt{y+\lambda}}$

$$= \int_0^\lambda \left[\sqrt{y+\lambda} - \frac{\lambda}{\sqrt{y+\lambda}}\right] dy$$

$$= \left[\frac{(y+\lambda)^{3/2}}{3/2}\right]_0^\lambda - \left[\frac{\lambda\sqrt{y+\lambda}}{1/2}\right]_0^\lambda$$

$$= \frac{2}{3}\left[(2\lambda)^{3/2} - \lambda^{3/2}\right] - 2\lambda\left[(2\lambda)^{1/2} - (\lambda)^{1/2}\right]$$

$$= 2\lambda\sqrt{\lambda}\left[\frac{2\sqrt{2}-1}{3} - \left(\sqrt{2}-1\right)\right]$$

$$= \frac{2}{3}\lambda\sqrt{\lambda}(2-\sqrt{2})$$

245 **(c)**

Given, $\int_0^x \sqrt{1 - \left(f'(t)\right)^2}\, dt = \int_0^x f(t)\, dt, 0 \le x \le 1$

Applying Leibnitz theorem, we get

$$\sqrt{1 - \left(f'(x)\right)^2} = f(x)$$

$$\Rightarrow 1 - \left(f'(x)\right)^2 = f^2(x)$$

$$\Rightarrow \left(f'(x)\right)^2 = 1 - f^2(x)$$

$$\Rightarrow f'(x) = \pm\sqrt{1 - f^2(x)}$$

$$\Rightarrow \frac{dy}{dx} = \pm\sqrt{1-y^2}, \text{where } y = f(x)$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \pm dx$$

On integrating both sides, we get

$\sin^{-1}(y) = \pm x + C$

$\because f(0) = 0 \Rightarrow C = 0$

$\therefore y = \pm \sin x$

$y = \sin x = f(x)$ given $f(x) \geq 0$ for $x \in [0,1]$

It is known that $\sin x < x, \forall\, x \in R^+$

$\therefore \sin\left(\frac{1}{2}\right) < \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $\sin\left(\frac{1}{3}\right) < \frac{1}{3}$

$\Rightarrow f\left(\frac{1}{3}\right) < \frac{1}{3}$

246 **(b)**

$$\int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

$$= \left[\frac{1}{x} e^x\right]_1^2 + \int_1^2 \frac{e^x}{x^2} dx - \int_1^2 \frac{e^x}{x^2} dx$$

$$= \frac{e^2}{2} - e$$

248 **(c)**

$\because \sin^{-1} x$ and $\cos^{-1} x$ are defined for $[-1, 1]$.

For $x > e, \log x > 1$

$\Rightarrow \sin^{-1}(\log x) + \cos^{-1}(\log x)$ is not defined.

Hence, the given integral does not exist.

249 **(c)**

We have,

$$I = \int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x} dx$$

$$\Rightarrow I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1}{3 + 4\sin x} d(3 + 4\sin x)$$

$$\Rightarrow I = \frac{1}{4}[\log(3 + 4\sin x)]_0^{\pi/4}$$

$$\Rightarrow I = \frac{1}{4}\log\left(3 + 4\frac{\sqrt{3}}{2}\right) - \frac{1}{4}\log 3$$

$$= \frac{1}{4}\log\left(\frac{3 + 2\sqrt{3}}{3}\right)$$

$$\therefore \int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x} dx = k\log\left(\frac{3 + 2\sqrt{3}}{3}\right)$$

$$\Rightarrow \frac{1}{4}\log\left(\frac{3 + 2\sqrt{3}}{3}\right) = k\log\left(\frac{3 + 2\sqrt{3}}{3}\right)$$

$$\Rightarrow k = \frac{1}{4}$$

250 **(a)**

We have,

$$F(x) = \int_{x^2}^{x^3} \log t\, dt, (x > 0)$$

$$\Rightarrow F'(x) = 3x^2(\log x^3) - 2x\log x^2$$

$$\Rightarrow F'(x) = 9x^2\log x - 4x\log x = (9x^2 - 4x)\log x$$

251 **(a)**

Let $I = \int \frac{\sin x \cos x}{\sqrt{1 - \sin^4 x}}\, dx$

Put $\sin^2 x = t \Rightarrow 2\sin x\cos x\; dx = dt$

$$\therefore I = \int \frac{dt}{2\sqrt{1 - t^2}} = \frac{1}{2}\sin^{-1} t + c$$

$$= \frac{1}{2}\sin^{-1}(\sin^2 x) + c$$

252 **(a)**

We have,

$$I = \int \frac{1 + x}{1 + x^{1/3}} dx = \int \frac{1^3 + \left(x^{1/3}\right)^3}{1 + x^{1/3}} dx$$

$$\Rightarrow I = \int \left(1 - x^{1/3} + x^{2/3}\right) dx$$

$$= x - \frac{3}{4}x^{4/3} + \frac{3}{5}x^{5/3} + C$$

253 **(b)**

We have,

$$I = \int_0^1 x(1 - x)^n dx$$

$$I = \int_0^1 (1 - x)\left(1 - (1 - x)\right)^n dx \left[\because \int_0^a f(x)\, dx = \int_0^a f(a - x) dx\right]$$

$$\Rightarrow I = \int_0^1 x^n (1 - x) dx$$

$$\Rightarrow I = \int_0^1 x^n - x^{n+1} dx \Rightarrow I = \frac{1}{n + 1} - \frac{1}{n + 2}$$

$$= \frac{1}{(n + 1)(n + 2)}$$

254 **(c)**

Let $I = \int_0^1 \frac{x\, dx}{[x + \sqrt{1 - x^2}]\sqrt{1 - x^2}}$

Put $x = \sin\theta \;\Rightarrow dx = \cos\theta\, d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\sin\theta \cdot \cos\theta\, d\theta}{(\sin\theta + \cos\theta) \cdot \cos\theta}$$

$I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta \quad$ ...(i)

Now, $I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\,\theta$

$\Rightarrow \; I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \quad$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}\right) d\theta$$

$$= \int_0^{\pi/2} 1\, d\theta = \frac{\pi}{2}$$
$$\Rightarrow\ I = \frac{\pi}{4}$$

255 **(c)**

$I = \int_{-1}^{1} |1-x| dx$

Here, $-1 \le x \le 1\ \Rightarrow\ 1-x \ge 0$

$$\therefore\ I = \int_{-1}^{1} (1-x0) dx = \left[x - \frac{x^2}{2}\right]_{-1}^{1}$$
$$= 1 - \frac{1}{2} + \frac{1}{2} + 1 = 2$$

256 **(b)**

$$\int \frac{\sec x\, dx}{\sqrt{\sin(2x+\theta) + \sin\theta}}$$
$$= \int \frac{\sec x\, dx}{\sqrt{2\sin(x+\theta)\cos x}}$$
$$= \frac{1}{\sqrt{2}} \int \frac{\sec^{3/2} x\, dx}{\sqrt{\sin x \cos\theta + \sin\theta \cos x}}$$
$$= \frac{1}{\sqrt{2}} \int \frac{\sec^2 x\, dx}{\sqrt{\tan x \cos\theta + \sin\theta}}$$

Put $\tan x = t\ \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{t\cos\theta + \sin\theta}}$$
$$= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{t\cos\theta + \sin\theta}}{\cos\theta} + c$$
$$= \sqrt{2(\tan x \sec\theta + \tan\theta \sec\theta)} + c$$

257 **(c)**

Given, $\int f(x)\, dx = e^x$ and $\int g(x)\, dx = \cos x$

$\Rightarrow f(x) = e^x$ and $g(x) = -\sin x$

$$\therefore \int f(x) \cos x\, dx + \int g(x)\, e^x dx$$
$$= \int e^x \cos x\, dx - \int e^x \sin x\, dx$$
$$= e^x \cos x + \int \sin x\, e^x dx - \int e^x \sin x\, dx$$
$$= e^x \cos x + c$$

258 **(c)**

Given, $P = \int_0^{3\pi} f(\cos^2 x) dx$ ...(i)

and $Q = \int_0^{\pi} f(\cos^2 x) dx$ ...(ii)

From Eq. (i),

$$P = 3\int_0^{\pi} f(\cos^2 x)\, dx = 3Q$$
$$\therefore\ P - 3Q = 0$$

259 **(b)**

We have,

$$f(x) = \int_{x^2}^{x^4} \sin\sqrt{t}\, dt$$
$$\Rightarrow f'(x) = \frac{d}{dx}(x^4)\left(\sin\sqrt{x^4}\right) - \frac{d}{dx}(x^2)\left(\sin\sqrt{x^2}\right)$$
$$\Rightarrow f'(x) = 4x^3 \sin x^2 - 2\, x \sin x$$

261 **(d)**

Since, $\int_2^4 \{3 - f(x)\} dx = 7$

$$\Rightarrow\ 3[x]_2^4 - \int_2^4 f(x)\, dx = 7$$
$$\Rightarrow\ \int_2^4 f(x)\, dx = 6 - 7 = -1$$
$$\therefore\ \int_{-1}^{2} f(x)\, dx = \int_{-1}^{4} f(x)\, dx - \int_2^4 f(x)\, dx$$
$$= 4 - (-1) = 5$$

262 **(b)**

$$\int \frac{(x+1)^2}{x(x^2+1)}\, dx = \int \frac{x^2+1+2x}{x(x^2+1)} dx$$
$$= \int \frac{x^2+1}{x(x^2+1)} dx + 2\int \frac{x}{x(x^2+1)} dx$$
$$= \int \frac{dx}{x} + 2\int \frac{dx}{x^2+1}$$
$$= \log_e x + 2\tan^{-1} x + c$$

263 **(c)**

We have,

$$\lim_{x\to\infty} \frac{\pi}{n}\left\{\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \cdots + \sin\frac{(n-1)\pi}{n}\right\}$$
$$= \lim_{n\to\infty} \frac{\pi}{n} \sum_{r=1}^{(n-1)} \sin\left(\frac{r\pi}{n}\right) = \pi \int_0^1 \sin x\,\pi dx$$
$$= \int_0^{\pi} \sin t\, dt = 2$$

264 **(d)**

$\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$ ...(i)

For $x = 1$

$\int_0^1 f(t)dt = \frac{1}{8} + \frac{1}{3} + a$ ...(ii)

On differentiating Eq.(i) w.r.t. x, we get

$f(x) = -x^2 f(x) + 2x^{15} + 2x^5$

$$\Rightarrow f(x) = \frac{2(x^{15}+x^5)}{1+x^2}$$

Now, from Eq. (ii)

$$2\int_0^1 \left(\frac{x^{15}+x^5}{1+x^2}\right) dx = \frac{1}{8} + \frac{1}{3} + a\ \Rightarrow\ a = -\frac{167}{840}$$

265 **(a)**

Let $\phi(x) = [f(x) + f(-x)][g(x) - g(-x)]$,

then $\phi(-x) = [f(-x) + f(x)][g(-x) - g(x)]$

$\phi(-x) = -\phi(x)$

$\Rightarrow$ It is an odd function.

$\therefore \int_{-\pi}^{\pi} \phi(x)dx = 0$

$\Rightarrow \int_{-\pi}^{\pi} [f(x) + f(-x)][g(x) - g(-x)]dx = 0$

266 **(b)**

Let $I = \int_0^{\pi} x\, f(\sin x)dx$ ...(i)

$= \int_0^{\pi} (\pi - x) f[\sin(\pi - x)]dx$

$\Rightarrow I = \int_0^{\pi} (\pi - x) f(\sin x)dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi} \pi f(\sin x)dx = 2\pi \int_0^{\pi/2} f(\sin x)dx$

$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$

$\therefore A = \pi$

267 **(a)**

$\because x = [x] + \{x\} = n + f$ (say)

Where $[x] = n, \{x\} = f (0 \le f < 1)$

Now, $\int_0^x [t^2]dt = 2(x - 1)$

$\Rightarrow \int_0^{n+f} [t]^2 dt = 2(n + f - 1)$

$\Rightarrow \sum_{r=0}^{n-1} \int_r^{r+1} [t]^2 dt + \int_n^{n+f} [t]^2 dt = 2(n + f - 1)$

$\Rightarrow \sum_{r=0}^{n-1} r^2 + n^2 f = 2(n + f - 1)$

$\Rightarrow \frac{(n-1)n(2n-1)}{6} - 2(n-1) = (2 - n^2)f$

For $n = 1, 0 - 0 = f \Rightarrow f = 0$

Then, $x = n + f = 1 + 0 = 1$

For $n = 2, 1 - 2 = -2f \Rightarrow f = \frac{1}{2}$

Then, $x = 2 + \frac{1}{2} = \frac{5}{2}$

For $n > 2, f < 0$ impossible

Hence, $x = 1, \frac{5}{2}$

268 **(a)**

We have,

$I = \int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^n}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx$

$\Rightarrow I = \pi \int_{-\pi/6}^{\pi/6} \frac{1}{1 - \sin\left(|x| + \frac{\pi}{6}\right)}$

$+ 4 \int_{-\pi/6}^{\pi/6} \frac{x^n}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx$

$\Rightarrow I = 2\pi \int_0^{\pi/6} \frac{1}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx + 4 \times 0$

$\Rightarrow I = 2\pi \int_0^{\pi/6} \frac{1}{1 + \cos\left(\frac{2\pi}{3} + x\right)} dx$

$\Rightarrow I = \frac{2\pi}{2} \int_0^{\pi/6} \sec^2\left(\frac{\pi}{3} + \frac{x}{2}\right) dx$

$\Rightarrow I = 2\pi \left[\tan\left(\frac{\pi}{3} + \frac{x}{2}\right)\right]_0^{\pi/6}$

$\Rightarrow I = 2\pi \left(\tan\frac{5\pi}{12} - \tan\frac{\pi}{3}\right) = \frac{2\pi \times \sin\left(\frac{5\pi}{12} - \frac{\pi}{3}\right)}{\cos\frac{5\pi}{12}\cos\frac{\pi}{3}}$

$\Rightarrow I = \frac{4\pi \sin 15°}{\cos 75°} = 4\pi$

269 **(b)**

For $x \epsilon \left(0, \frac{\pi}{4}\right) \cos^2 x > \sin^2 x$

$\Rightarrow \int_0^{\pi/4} \cos^2 x\ dx > \int_0^{\pi/4} \sin^2 x\ dx$

$\Rightarrow I_1 < I_2$

270 **(c)**

Since, $2e^{2\alpha} - 3e^{\alpha} - 2 > 0$

$\Rightarrow e^{\alpha} < -\frac{1}{2}$ or $e^{\alpha} > 2$

But $e^{\alpha} > 0, \forall \alpha \in R \Rightarrow e^{\alpha} > 2$

$\therefore \alpha \in (\log 2, \infty)$

271 **(b)**

Given, $\frac{d}{dx}\left[a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4 - 1}$

On integrating both sides, we get

$a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)$

$= \frac{1}{2} \int \left[\frac{1}{x^2 - 1} - \frac{1}{x^2 + 1}\right] dx$

$\Rightarrow a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)$

$= \frac{1}{4} \int \log\left(\frac{x-1}{x+1}\right) - \frac{1}{2} \tan^{-1} x$

$\Rightarrow a = -\frac{1}{2}, \quad b = \frac{1}{4}$

$\therefore a - 2b = -\frac{1}{2} - 2\left(\frac{1}{4}\right) = -1$

272 **(b)**

Let $I = \int_{\alpha}^{\beta} \sqrt{\frac{x-\alpha}{\beta-x}} dx$

Put $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$,

$\therefore x-\alpha=(\beta-\alpha)\sin^2\theta, \beta-x$
$=(\beta-\alpha)\cos^2\theta$
and $dx=2(\beta-\alpha)\sin\theta\cos\theta d\theta$
$\therefore I$

$$=\int_0^{\pi/2}\sqrt{\frac{(\beta-\alpha)\sin^2\theta}{(\beta-\alpha)\cos^2\theta}}\times 2(\beta-\alpha)\sin\theta\cos\theta$$

$$=(\beta-\alpha)\int_0^{\pi/2}(1-cos2\theta)d\theta$$

$$=(\beta-\alpha)\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/2}=(\beta-\alpha).\frac{\pi}{2}$$

273 **(d)**

$$\int\frac{x\,dx}{x^2+4x+5}=\int\frac{x}{x^2+4x+4+1}dx$$

$$=\int\frac{x+2-2}{(x+2)^2+1}dx$$

$$=\frac{1}{2}\int\frac{2(x+2)dx}{(x+2)^2+1}-2\int\frac{dx}{1+(x+2)^2}$$

$$=\frac{1}{2}\log[(x+2)^2+1]-2\tan^{-1}(x+2)+c$$

$$=\frac{1}{2}\log(x^2+4x+5)-2\tan^{-1}(x+2)+c$$

274 **(c)**

$$\int\cos\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]dx$$

$$=\int\cos[\cos^{-1}(-x)]dx$$

$$=\int(-x)dx=-\frac{x^2}{2}+c$$

275 **(c)**

Let $I=\int_0^a\sqrt{\frac{a-x}{x}}dx$

Put $x=a\sin^2\theta \Rightarrow dx=2a\sin\theta\cos\theta d\theta$

$$\therefore I=\int_0^{\pi/2}\sqrt{\frac{\cos^2\theta}{\sin^2\theta}}.2\,a\sin\theta\cos\theta d\theta$$

$$=2a\int_0^{\pi/2}\cos^2\theta\,d\theta$$

$$=2a\times\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi a}{2}$$

276 **(b)**

We have,

$$I=\int_{-\pi/2}^{\pi/2}\sin^2 x\cos^2 x\ (\sin x+\cos x)dx$$

$\Rightarrow I=\int_{-\pi/2}^{\pi/2}\sin^3 x\cos^2 x\,dx-$
$\int_{-\pi/2}^{\pi/2}\sin^2 x\cos^3 x\,dx$ ...(i)

Since $\sin^3 x\cos^2 x$ is an odd function and $\sin^2 x\cos^3 x$ is an even function

$$\therefore\int_{-\pi/2}^{\pi/2}\sin^3 x\cos^2 x\,dx=0$$

and, $\int_{-\pi/2}^{\pi/2}\sin^2 x\cos^2 x\,dx=$
$2\int_0^{\pi/2}\sin^2 x\cos^3 x\,dx$

$$\therefore I=2\int_0^{\pi/2}\sin^2 x\cos^3 x\,dx$$

$$\Rightarrow I=2\int_0^1 t^2(1-t^2)dt\text{, where } t=\sin x$$

$$\Rightarrow I=2\int_0^1(t^2-t^4)dt=2\left[\frac{1}{3}-\frac{1}{5}\right]=\frac{4}{15}$$

277 **(c)**

Let $f(x)=\log\left(\frac{x-1}{x+1}\right)$

$$f(-x)=\log\left(\frac{-x-1}{-x+1}\right)=-\log\left(\frac{x-1}{x+1}\right)$$

$=-f(x)$

$\Rightarrow f(x)$is an odd function

$$\therefore\int_{-1}^1\log\left(\frac{x-1}{x+1}\right)dx=0$$

278 **(c)**

Given, $\int_n^{n+1}f(x)dx=n^2$

On putting $n=-2,-1,0,1,2,3.$ we get

$$\int_{-2}^{-1}f(x)dx=4,\quad\int_{-1}^0 f(x)dx=1,$$

$$\int_0^1 f(x)dx=0,\quad\int_1^2 f(x)dx=1,$$

$$\int_2^3 f(x)dx=4,\quad\int_3^4 f(x)dx=9,$$

$$\therefore\int_{-2}^4 f(x)dx=4+1+0+1+4+9=19$$

279 **(c)**

The function $f(x)=x|x|$ is an odd function

$$\therefore\int_{-1}^1 x|x|dx=0$$

280 **(c)**

Using $C_1\to C_1-C_2-C_3$, we get

$$f(x)=\begin{vmatrix}\sin x & \sin 2x & \sin 3x\\ 0 & 3 & 4\sin x\\ 0 & \sin x & 1\end{vmatrix}$$

$$=3\sin x-4\sin^3 x=\sin 3x$$

$$\therefore \int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} \sin 3x\,dx = \left[-\frac{\cos 3x}{3}\right]_0^{\pi/2} = \frac{1}{3}$$

281 **(d)**

We have,

$$I = \int_\alpha^\beta \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx$$

$$\Rightarrow I = \int_\alpha^\beta \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2}}dx$$

$$\Rightarrow I = \left[\sin^{-1}\left(\frac{x - \frac{\alpha+\beta}{2}}{\frac{\beta-\alpha}{2}}\right)\right]_\alpha^\beta = \sin^{-1} 1 - \sin^{-1}(-1)$$

$$= \pi$$

283 **(b)**

Let $I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Put $\sin^{-1} x = t$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\therefore I = \int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

284 **(c)**

Let $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{2\cos^2\frac{x}{2}}$

$$= \int_0^{\pi/2} \sec^2\frac{x}{2}dx = \left[2\tan\frac{x}{2}\right]_0^{\pi/2} = 2$$

285 **(c)**

$$I_1 = \int_{1-k}^{k} (1-x) f[x(1-x)]\,dx$$

$$\Rightarrow 2I_1 = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

286 **(a)**

Putting $x = a\tan\theta$ and $dx = a\sec^2\theta\ d\theta$, we get

$$\int \frac{1}{(a^2+x^2)^{3/2}}dx = \int \frac{a\sec^2\theta}{a^3\sec^3\theta}d\theta$$

$$\Rightarrow I = \frac{1}{a^2}\int \cos\theta\ d\theta = \frac{1}{a^2}\sin\theta + C$$

$$= \frac{1}{a^2}\frac{x}{\sqrt{a^2+x^2}} + C$$

287 **(c)**

Let $I = \int \frac{\cos x - \sin x}{1 + 2\sin x\cos x}\,dx$

Put $\cos x + \sin x = t$

$$\Rightarrow (\cos x - \sin x)dx = dt$$

$$\therefore I = \int \frac{1}{t^2}dt$$

$$= -\frac{1}{t} + c$$

$$= -\frac{1}{\sin x + \cos x} + c$$

288 **(b)**

We have,

$$I = \int_0^{\pi/2} |\sin x - \cos x|dx$$

$$\Rightarrow I = \int_0^{\pi/4} -(\sin x - \cos x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$\Rightarrow I = [\cos x + \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$\Rightarrow I = \left(\frac{2}{\sqrt{2}} - 1\right) + \left(-1 + \frac{2}{\sqrt{2}}\right) = 2\left(\frac{2}{\sqrt{2}} - 1\right)$$

$$= 2(\sqrt{2} - 1)$$

289 **(d)**

Let $I = \int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{2\pi}^{\pi/4}(\cos x - \sin x)\,dx$

$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4} + [\cos x - \sin x]_{2\pi}^{\pi/4}$$

$$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] + \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$$

$$= \left[\frac{2}{\sqrt{2}} - 1\right] - \left[-\frac{4}{\sqrt{2}}\right] + \left[\frac{2}{\sqrt{2}} - 1\right]$$

$$= (\sqrt{2} - 1) + 2\sqrt{2} + (\sqrt{2} - 1) = 4\sqrt{2} - 2$$

290 **(b)**

We have,

$$I = \int_1^4 e^{\sqrt{x}}\,dx = 2\int_1^2 t\ e^t dt, \text{where } x = t^2$$

$$\Rightarrow I = 2[t\,e^t - e^t]_1^2 = 2e^2$$

291 **(c)**

We have,

$$I_n = \int_0^{\pi/2} \cos^n x \cos n\,x\,dx$$

$$\therefore I_{n+1} - I_n = \int_0^{\pi/2} (\cos^{n+1} x \cos(n+1)x - \cos^n x \cos n x)\, dx$$

$$\Rightarrow I_{n+1} - I_n = \int_0^{\pi/2} \cos^n x \{\cos(n+1) x \cos x - \cos nx\} dx$$

$$\Rightarrow I_{n+1} - I_n = \int_0^{\pi/2} \cos^n x \{\cos(n+1) x \cos x - \cos\{(n+1)x - x\}\} dx$$

$$\Rightarrow I_{n+1} - I_n = \int_0^{\pi/2} \cos^{\mathrm{n}} x \{-\sin(n+1) x \sin x\}\, dx$$

$$\Rightarrow I_{n+1} - I_n = \int_0^{\pi/2} \underset{I}{\sin(n+1) x} \underset{II}{\cos^n x (-\sin x)}\, dx$$

$$\Rightarrow I_{n+1} - I_n = \left[\sin(n+1) x \frac{\cos^{n+1} x}{n+1}\right]_0^{\frac{\pi}{2}} - \int_0^{\pi/2} \cos(n+1) x \cos^{n+1} x\, dx$$

$\Rightarrow I_{n+1} - I_n = -I_{n+1}$

$\therefore 2I_{n+1} = I_n$

292 **(d)**

Let $I = \int \frac{2x^2+3}{(x^2-1)(x^2+4)} dx = \int \frac{dx}{(x^2-1)} + \int \frac{dx}{(x^2+4)}$

$$\left[\because \frac{2x^2+3}{(x^2-1)(x^2+4)} = \frac{1}{x^2-1} + \frac{1}{x^2+4}\right]$$

$$\Rightarrow I = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

But $I = a\log\left(\frac{x-1}{x+1}\right) + b\tan^{-1}\left(\frac{x}{2}\right) + c$

$\therefore \quad a = \frac{1}{2}, \quad b = \frac{1}{2}$

293 **(a)**

Let $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x} dx$ ...(i)

On putting $x = \left(\frac{\pi}{2} - x\right)$ in Eq.(i), we get

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right)} dx$$

$= \int_0^{\pi/2} -\left(\frac{\cos x - \sin x}{1+\cos x \sin x}\right) dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} 0\, dx = 0 \Rightarrow I = 0$$

294 **(c)**

Let $I = \int_0^{\pi/2} \log \sin x\, dx$ ... (i)

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$= \int_0^{\pi/2} \log \cos x\ dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_0^{\pi/2} \log \sin 2x + dx - \log 2 \int_0^{\pi/2} dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

$$\therefore 2I = \int_0^{\pi} \frac{\log \sin t}{2} dt - \log 2[x]_0^{\pi/2}$$

$$= \frac{1}{2} \times 2 \int_0^{\pi/2} \log \sin t\, dt - \frac{\pi}{2}\log 2$$

$$= \int_0^{\pi/2} \log \sin x dx - \frac{\pi}{2}\log 2$$

$$= I - \frac{\pi}{2}\log 2$$

$$\Rightarrow I = -\frac{\pi}{2}\log 2$$

295 **(c)**

Given, $\int e^x(f(x) - f'(x))dx = \phi(x)$

$\Rightarrow e^x(f(x) - f'(x)) = \phi'(x)$

Let $I = \int e^x f(x) dx$

$$= f(x)e^x - \int f'(x)e^x dx$$

$$= f(x)e^x - \left[\int e^x f(x) dx - \int \phi'(x) dx\right]$$

$\Rightarrow 2I = f(x)e^x + \phi(x)$

$\therefore I = \frac{1}{2}[e^x f(x) + \phi(x)]$

296 **(b)**

$\because \phi'(x) = f(x)$

$\therefore \phi(x)$ is a periodic function with period T. Because $f(x)$ has the period T

297 **(d)**

Let $I = \int 2^{2^{2^x}}.2^{2^x}.2^x\, dx$

Put $2^{2^{2^x}} = t \Rightarrow 2^{2^{2^x}}.2^{2^x}.2^x(\log 2)^3 dx = dt$

$$\therefore I = \int \frac{dt}{(\log 2)^3} = \frac{t}{(\log 2)^3} = \frac{2^{2^{2^x}}}{(\log 2)^3} + c$$

298 **(d)**

Let $I = \int \frac{2dx}{\sqrt{1-4x^2}}$

Put $2x = \sin\theta \Rightarrow 2dx = \cos\theta d\theta$

$$\therefore I = \int \frac{\cos\theta d\theta}{1 - \sin^2\theta}$$

$$= \int \frac{\cos\theta}{\cos\theta} d\theta = \int 1\, d\theta$$

$\Rightarrow I = \theta + c$

$\therefore I = \sin^{-1}(2x) + c$

299 **(d)**

$$\int \frac{d(\cos\theta)}{\sqrt{1-\cos^2\theta}} = \sin^{-1}(\cos\theta) + c$$

300 **(a)**

Let $I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$

$$= \int_0^1 \frac{(x^4-1)(1-x)^4 + (1-x)^4}{(1+x^2)}\,dx$$

$$= \int_0^1 (x^2-1)(1-x)^4 + \int_0^1 \frac{(1+x^2-2x)^2}{(1+x^2)}\,dx$$

$$= \int_0^1 \left\{(x^2-1)(1-x)^4 + (1+x^2) - 4x + \frac{4x^2}{(1+x^2)}\right\} dx$$

$$= \int_0^1 \left((x^2-1)(1-x)^4 + (1+x^2) - 4x + 4 - \frac{4}{1+x^2}\right) dx$$

$$= \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}\right) dx$$

$$= \left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{5x^5}{5} - \frac{4x^3}{3} + 4x - 4\tan^{-1}x\right]_0^1$$

$$= \frac{1}{7} - \frac{4}{6} + \frac{5}{5} - \frac{4}{3} + 4 - 4\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{22}{7} - \pi$$

301 **(a)**

We have,

$$f(x) = \int_0^x |x-2|\,dx \Rightarrow f'(x) = |x-2|$$

Clearly, $f'(x)$ is everywhere continuous and differentiable except at $x = 2$

302 **(b)**

If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is an even function

303 **(c)**

$$\int \frac{\sec x \operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x}\,dx$$

$$= \int \frac{\frac{1}{\cos x \sin x}}{\frac{2\cos x}{\sin x} - \frac{1}{\sin x \cos x}}\,dx$$

$$= \int \frac{dx}{2\cos^2 x - 1}$$

$$= \int \frac{dx}{\cos 2x} = \int \sec 2x\,\,dx$$

$$= \frac{1}{2}\log|\sec 2x + \tan 2x| + c$$

304 **(d)**

Let $I = \int_0^\pi \frac{\theta\sin\theta}{1+\cos^2\theta}d\theta$ ...(i)

$$\Rightarrow I = \int_0^\pi \frac{(\pi-\theta)\sin\theta}{1+\cos^2\theta}d\theta \text{ ... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \frac{\pi\sin\theta}{1+\cos^2\theta}\,d\theta$$

Put $\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

$$\therefore 2I = -\pi\int_1^{-1}\frac{1}{1+t^2}dt = 2\pi\int_0^1\frac{1}{1+t^2}dt$$

$$= 2\pi[\tan^{-1}t]_0^1 = 2\pi.\frac{\pi}{4}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

305 **(d)**

Let $f(\theta) = \left[\int_0^{\sin^2\theta}\sin^{-1}\sqrt{\phi}\,\,d\phi + \int_0^{\cos^2\theta}\cos^{-1}\sqrt{\phi}\,\,d\phi\right]$

$$f'(\theta) = \left(\frac{d}{d\theta}\sin^2\theta\right)\left[\sin^{-1}\sqrt{\sin^2\theta}\right] + \left(\frac{d}{d\theta}\cos^2\theta\right)\left[\cos^{-1}\sqrt{\cos^2\theta}\right]$$

$(2\sin\theta\cos\theta)\theta - (2\sin\theta\cos\theta)\theta = 0$

$\therefore\ f(\theta) = \text{constant} = a$ [say]

$\therefore\ f\left(\frac{\pi}{4}\right) = a$

$$\Rightarrow \int_0^{1/2}(\sin^{-1}\sqrt{\phi} + \cos^{-1}\sqrt{\phi})d\phi = a$$

$$\Rightarrow \frac{\pi}{2}[\phi]_0^{1/2} = a \Rightarrow \frac{\pi}{4} = a$$

306 **(c)**

Let $I = \int_0^{\pi/2}\frac{dx}{1+\tan^3 x} = \int_0^{\pi/2}\frac{\cos^3 x}{\sin^3 x + \cos^3 x}dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2}\frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right) + \cos^3\left(\frac{\pi}{2}-x\right)}dx$$

$I = \int_0^{\pi/2}\frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$= \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

307 **(c)**

Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$

Because in $x \epsilon (0,1), x > \sin x$

$$I < \int_0^1 \sqrt{x}\, dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 \Rightarrow I < \frac{2}{3}$$

For $x \epsilon (0,1), \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

And $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-\frac{1}{2}} dx = 2 \Rightarrow J < 2$

308 **(c)**

$$\int_{-1}^{1} |x| dx + \int_1^3 |x - 2| dx$$

$$= 2\int_0^1 x\, dx + \int_1^2 (2 - x) dx + \int_2^3 (x - 2) dx$$

$$= 2 \times \frac{1}{2} + \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= 1 + \frac{1}{2} + \frac{1}{2} = 2$$

309 **(a)**

Putting $x^n + 1 = t$ and $n\, x^{n-1} dx = dt$, we get

$$I = \int \frac{1}{x\,(x^n + 1)} dx$$

$$= \frac{1}{n}\int \frac{1}{t(t-1)} dt$$

$$= \frac{1}{n}\int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$$

$$\Rightarrow I = \frac{1}{n}\log\left(\frac{t-1}{t}\right) + C = \frac{1}{n}\log\left(\frac{x^n}{x^n + 1}\right) + C$$

310 **(d)**

We have,

$$I = \int_0^{1/2} |\sin \pi\, x|\, dx = \frac{1}{\pi}\int_0^{\pi/2} |\sin t|\, dt, \text{where } t = \pi x$$

$$\Rightarrow I = \frac{1}{\pi}\int_0^{\pi/2} \sin t\, dt = \frac{1}{\pi}$$

312 **(c)**

We know that,

$$\left|\int_a^b f(x)g(x)dx\right| \le \sqrt{\int_a^b f^2(x)\, dx}.\sqrt{\int_a^b g^2(x)\, dx}$$

Putting $f(x) = 1$ and $g(x) = \sqrt{1 + x^3}$, we get

$$I \int_0^1 \sqrt{1 + x^3}\, dx \le \sqrt{\int_0^1 1 + x^3\, dx} = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$$

$\Rightarrow I < 2$ and $I < \frac{\sqrt{7}}{2}$

313 **(b)**

We have,

$$I = \frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right) dx$$

Putting $\frac{x}{c} = t$ and $dx = c\, dt$, we get

$$I = \frac{1}{c}\int_a^b f(t)\, c\, dt = \int_a^b f(t)\, dt = \int_a^b f(x)\, dx$$

314 **(c)**

$$\int e^x \left(\frac{1 - x}{1 + x^2}\right)^2 dx = \int e^x \frac{(1 + x^2 - 2x)}{(1 + x^2)^2} dx$$

$$= \int e^x \left(\frac{1}{1 + x^2} - \frac{2x}{(1 + x^2)^2}\right) dx$$

$$= e^x . \frac{1}{1 + x^2} + \int \frac{2xe^x}{(1 + x^2)^2} dx - \int \frac{2xe^x}{(1 + x^2)^2} dx$$

$$= \frac{e^x}{1 + x^2} + c$$

315 **(a)**

We have,

$$\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x} \Rightarrow \int \frac{e^{\sin x}}{x} dx = F(x)$$

$$\therefore \int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{e^{\sin x^3}}{x^3} . 3x^2 dx$$

$$= \int_1^{64} \frac{e^{\sin x^3}}{x^3} d(x^3)$$

$= F(64) - F(1)$

Hence, $k = 64$

317 **(a)**

We have,

$$I = \int \frac{\log(x + 1) - \log x}{x(x + 1)} dx = \int \frac{\log(1 + 1/x)}{(x^2 + x)} dx$$

$$\Rightarrow I = \int \frac{\log(1 + 1/x)}{(x^2(1 + 1/x)} dx$$

$$= -\int \log\left(1 + \frac{1}{x}\right) d\left\{\log\left(1 + \frac{1}{x}\right)\right\}$$

$$\Rightarrow I = -\frac{1}{2}\left\{\log\left(1 + \frac{1}{x}\right)\right\}^2 + C$$

$$\Rightarrow I = -\frac{1}{2}[\log(x + 1) - \log x]^2 + C$$

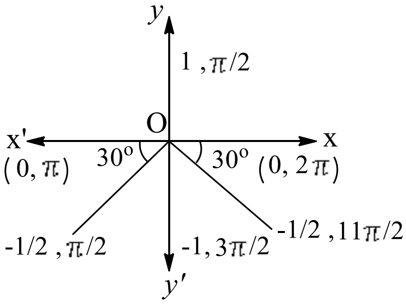
$$\Rightarrow I = -\frac{1}{2}[\{\log(x + 1)\}^2 + (\log x)^2 - 2\log(x + 1) \cdot \log x] + C$$

$\Rightarrow I = -\frac{1}{2}\{\log(x+1)\}^2 - \frac{1}{2}(\log x)^2 + \log(x+1)$

$\cdot \log x + C$

318 **(a)**

It is a question of greatest integer function. We have, subdivide the interval π to 2π as under keeping in view that we have to evaluate $[2\sin x]$



We know that, $\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6} = -\frac{1}{2}$

$\sin\frac{11\pi}{6} = \sin\left(2\pi - \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\sin\frac{9\pi}{6} = \sin\frac{3\pi}{6} = -1$

Hence, we divide the interval π to 2π as $\left(\pi, \frac{7\pi}{6}\right), \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

$\sin x = \left(0, -\frac{1}{2}\right), \left(-1, -\frac{1}{2}\right), \left(0, -\frac{1}{2}\right)$

$2\sin x = (0, -1), (-2, -1), (0, -1)$

$[2\sin x] = -1$

$\therefore\ I = I_1 + I_2 + I_3$

$= \int -1\,dx + \int -2\,dx \int -1\,dx$

Between proper limits

$= -\frac{\pi}{6} - 2\left(\frac{4\pi}{6}\right) - \frac{\pi}{6} = -\frac{10\pi}{6} = -\frac{5\pi}{3}$

319 **(c)**

$\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x'}{1+x^2}\right)\right] dx$

$= \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$

320 **(c)**

Let $F(x) = e^x$

Also, $f(x) + g(x) = x^2 \Rightarrow g(x) = x^2 - e^x$

Now, $\int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$

$= \int_0^1 x^2 e^2\,dx - \int_0^1 e^{2x} dx$

$= [x^2 e^x - \int 2xe^x dx\,]_0^1 - \frac{1}{2}[e^{2x}]_0^1$

$= [x^2 e^x - 2xe^x + 2e^x]_0^1 - \frac{1}{2}(e^2 - 1)$

$= [(1 - 2 + 2)e^1 - (0 - 0 + 2)e^0] - \frac{1}{2}e^2 + \frac{1}{2}$

$= e - \frac{e^2}{2} - \frac{3}{2}$

321 **(b)**

We have,

$I = \int_0^{\pi} \log(1 + \cos x)\,dx$

$\Rightarrow I = \int_0^{\pi} \log\left(2\cos^2\frac{x}{2}\right) dx$

$\Rightarrow I = \int_0^{\pi} \left(\log 2 + 2\log\cos\frac{x}{2}\right) dx$

$\Rightarrow I = \pi\log 2 + 2\int_0^{\pi} \log\cos\frac{x}{2} dx$

$\Rightarrow I = \pi\log 2 + 2 \times 2\int_0^{\pi/2} \log\cos t\,dt$, where $t = \frac{x}{2}$ and $dx = 2dt$

$\Rightarrow I = \pi\log 2 + 4 \times -\frac{\pi}{2}\log 2 = -\pi\log 2$

322 **(a)**

Putting $\tan^{-1} x = t$, we have

$I = \int e^{\tan^{-1} x}\left(\frac{1 + x + x^2}{1 + x^2}\right) dx$

$\Rightarrow I = \int e^t\,(\tan t + \sec^2 t)dt = e^t \tan t + C$

$= x\,e^{\tan^{-1} x} + C$

323 **(c)**

Given,

$I_1 = \int_a^{\pi - a} x\,f(\sin x)\,dx$

and $I_2 = \int_a^{\pi - a} f(\sin x)\,dx$

Now, $I_1 = \int_a^{\pi - a} x\,f(\sin x)\,dx$

$= \int_a^{\pi - a} (\pi - x) f[\sin(\pi - x)]\,dx$

$= \int_a^{\pi - a} (\pi - x) f\,(\sin x)\,dx$

$= \int_a^{\pi - a} \pi\,f(\sin x)dx - I_1$

$\Rightarrow 2I_1 = \pi\,I_2 \Rightarrow I_2 = \frac{2}{\pi}I_1$

324 **(b)**

$\int \cos^{-1}\left(\frac{1}{x}\right) dx = \int \sec^{-1} x \cdot 1\,dx$

$= \sec^{-1} x \int dx - \int \left[\frac{d}{dx}\sec^{-1} x \int dx\right] dx$

$= x\sec^{-1}x - \int \frac{1}{x\sqrt{x^2-1}} x\,dx$

$= x\sec^{-1}x - \int \frac{1}{\sqrt{x^2-1}}\,dx$

$= x\sec^{-1}x - \cosh^{-1}x + c$

325 **(d)**

Let $I = \int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\,d\theta$

$I = \int_0^{\pi/2} \frac{\cos\theta\,d\theta}{\sin\theta+\cos\theta}$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\theta\right)d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$

$= \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta+\sin\theta}\,d\theta$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_0^{\pi/2} 1\,d\theta \Rightarrow I = \frac{\pi}{4}$

326 **(b)**

Let

$I = \int_0^{\sqrt{n}} [x^2]dx$

$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} [x^2]\,dx$

$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} (r-1)\,dx$

$\Rightarrow I = \sum_{r=1}^{n} (r-1)\left(\sqrt{r}-\sqrt{r-1}\right)$

$\Rightarrow I = \left(\sqrt{2}-\sqrt{1}\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(\sqrt{4}-\sqrt{3}\right)$

$+ \cdots + (n-1)\left(\sqrt{n}-\sqrt{n-1}\right)$

$\Rightarrow I = n\sqrt{n} - \left(\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}\right)$

$= n\sqrt{n} - \sum_{r=1}^{n}\sqrt{r}$

327 **(d)**

Let $I = \int \frac{1}{x}(\log_{ex} e)dx = \int \frac{1}{x(1+\log_e x)}dx$

Put $\log_e x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int \frac{dt}{(1+t)} = \log_e(1+t) + c$

$= \log_e(1+\log_e x) + c$

328 **(a)**

$I_1 - I_2 = \int_0^{\pi/2} (\cos\theta - \sin 2\theta) f(\sin\theta + \cos^2\theta)d\theta$

Put $\sin\theta + \cos^2\theta = t$

$\Rightarrow (\cos\theta - \sin 2\theta)d\theta = dt$

Then, $I_1 - I_2 = \int_1^1 f(t)dt = 0$

$\therefore I_1 = I_2$

329 **(b)**

$(1) I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx$

$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x}\,\sin x\,dx$

$= 2\int_0^{\pi/2} \sqrt{\cos x}\sin x\,dx$

$= -\frac{4}{3}[\cos^{3/2}x]_0^{\pi/2} = \frac{4}{3}$

$(2) I = \int_0^1 |x-1|dx + \int_1^4 |x-1|dx + \int_0^3 |x-3|dx + \int_3^4 |x-3|dx$

$= \int_0^1 -(x-1)dx$

$+ \int_1^4 (x-1)dx + \int_0^3 -(x-3)dx$

$+ \int_3^4 (x-3)dx$

$= 10$

330 **(a)**

Let $I = \int_{-\pi/4}^{\pi/4} \sin^{-4}x\,dx = \int_{-\pi/4}^{\pi/4} \text{cosec}^{-4}x\,dx$

$= \int_{-\pi/4}^{\pi/4} (1+\cot^2 x)\,\text{cosec}^2 x\,dx$

Put $\cot x = t$

$\Rightarrow -\text{cosec}^2 x\,dx = dt$

$\therefore I = -\int_{-1}^{1}(1+t^2)\,dt = -2\int_0^1 (1+t^2)\,dt$

$= -2\left[t + \frac{t^3}{3}\right]_0^1 = -2\left[1+\frac{1}{3}\right] = -\frac{8}{3}$

331 **(d)**

We have,

$\lim_{x\to\infty} \frac{\left(\int_0^x e^x\,dx\right)^2}{\int_0^x e^{2x^2}\,dx} = \lim_{x\to\infty} \frac{(e^x-1)^2}{\int_0^x e^{2x^2}\,dx}$

$= \lim_{x\to\infty} \frac{2(e^x-1)e^x}{\int_0^x e^{2x^2}}$ [Using L'Hospital's rule]

$= 2\lim_{x\to\infty} \frac{e^x-1}{e^{2x^2-x}}$

$= 2\lim_{x\to\infty} \frac{e^x}{e^{2x^2-x}(4x-1)}$ [Using L'Hospital's rule]

$= 2\lim_{x\to\infty} \frac{1}{e^{2x^2-2x}(4x-1)} = 0$

332 **(a)**

Given, $\int_{\sin x}^1 t^2\ f(t)dt = 1 - \sin x$

Now, $\frac{d}{dx}\int_{\sin x}^1 t^2\ f(t)dt = \frac{d}{dx}(1-\sin x)$

$\Rightarrow [1^2 f(1)].(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

[by Leibnitz formula]

$\Rightarrow$ Put $\sin x = 1/\sqrt{3}$

$\therefore\ f\left(\frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 = 3$

333 **(b)**

We have,

$f(a-x) + f(a+x) = 0$

$\Rightarrow f(2a-x) + f(x) = 0$ [On replacing x by $x-a$]

$\Rightarrow f(2a-x) = -f(x)$

$\because \int_0^{2a} f(x)dx = \int_0^a \{f(x) + f(2a-x)\}\,dx$

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a \{f(x) - f(x)\}\,dx = 0$

334 **(a)**

$\int \frac{dx}{\sin(x-a)\sin(x-b)}$

$= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\,dx$

$= \frac{1}{\sin(a-b)}$

$\int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\,dx$

$= \frac{1}{\sin(a-b)}\left[\int \cot(x-a)dx - \int \cot(x-b)dx\right]$

$= \frac{1}{\sin(a-b)}\,[\log\sin(x-a) - \log\sin(x-b)] + c$

$= \frac{1}{\sin(a-b)} \log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$

335 **(b)**

Putting $2 + x = t^2$, we get

$I = \int \sqrt{\frac{5-x}{2+x}}dx = 2\int \sqrt{7-t^2}\ dt$

$\Rightarrow I = t\sqrt{7-t^2} + 7\sin^{-1}\frac{t}{\sqrt{7}} + C$

$\Rightarrow I = \sqrt{x+2}\,\sqrt{5-x} + 7\sin^{-1}\frac{\sqrt{x+2}}{7} + C$

336 **(a)**

We have,

$I = \int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx$

$= \int \frac{1}{x^2-1}dx + \int \frac{1}{x^2+4}dx$

$\Rightarrow I = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$\Rightarrow I = -\frac{1}{2}\log\left(\frac{x+1}{x-1}\right) + \frac{1}{2}\tan^{-1} x + C$

$\therefore a = -1/2$ and $b = 1/2$

337 **(c)**

$\int e^x \frac{(x-1)}{x^2}\ dx$

$= \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)\ dx$

$= \frac{e^x}{x} + c$

338 **(d)**

We have,

$I = \int_0^{3\alpha} \operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)dx$

$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\alpha}{\sin(x-\alpha)\sin(x-2\alpha)}dx$

$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\{(x-\alpha)-(x-2\alpha)\}}{\sin(x-\alpha)\sin(x-2\alpha)}dx$

$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \cot(x-2\alpha) - \cot(x-\alpha)\,dx$

$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin(x-2\alpha)}{\sin(x-\alpha)}\right]_0^{3\alpha}$

$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin\alpha}{\sin 2\alpha} - \log\frac{\sin 2\alpha}{\sin\alpha}\right]$

$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\left(\frac{\sin\alpha}{2\sin\alpha\cos\alpha}\right)\right]$

$= \frac{2}{\sin\alpha}\log\left(\frac{1}{2}\sec\alpha\right)$

$\Rightarrow I = 2\operatorname{cosec}\alpha\log\left(\frac{1}{2}\sec\alpha\right)$

339 **(a)**

$\int_0^3 \frac{3x+1}{x^2+9}\ dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}\ dx$

$+ \int_0^3 \frac{1}{x^2+9}\ dx$

$= \frac{3}{2}[\log(x^2+9)]_0^3 + \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$

$= \frac{3}{2}[\log 18 - \log 9] + \frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(0)]$

$= \frac{3}{2}[\log 2] + \frac{\pi}{12}$

$= \log(2\sqrt{2}) + \frac{\pi}{12}$

340 **(a)**

Let $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$

$$= \int \frac{dx}{\sqrt{-x^2 + 3x - 2)}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x - \frac{3}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\left(x - \frac{3}{2}\right)}{\frac{1}{2}}\right) + c = \sin^{-1}(2x - 3) + c$$

341 **(d)**

Let $g_n(x) = 1 + x^2 + x^4 + \ldots + x^{2n} = \frac{x^{2n+2} - 1}{x^2 - 1}$

$\therefore h_n(x) = g'_n(x) = \dfrac{2x(nx^{2n+2} - (n+1)x^{2n} + 1)}{(x^2 - 1)^2}$

Now, $f(x) = \lim_{n\to\infty} h_n(x) = \frac{2x}{(x^2-1)^2}$

As $0 < x < 1$

Thus, $\int f(x)dx = \int \frac{2x}{(x^2-1)^2} dx$

$$= -\frac{1}{x^2 - 1} = \frac{1}{1 - x^2}$$

342 **(c)**

$$\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}(\sin x \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= -\frac{1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + c = \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$$

343 **(a)**

Let $I = \int_0^{\pi/2} \frac{\cos x}{1 + \sin x}\, dx$

$= [\log(1 + \sin x)]_0^{\pi/2}$

$= \log 2 - \log 1 = \log 2$

344 **(b)**

Let $I = \int_0^1 \frac{2\sin^{-1}\frac{x}{2}}{x} dx$

Put $\sin^{-1}\frac{x}{2} = t \Rightarrow \frac{x}{2} = \sin t$

$\Rightarrow x = 2\sin I$

$\Rightarrow dx = 2\cos t\, dt$

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{2t}{2\sin t}.2\cos t\, dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{2t}{\tan t} dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{2x}{\tan x} dx$$

345 **(a)**

Given , $\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = a + \frac{b}{\log 2}$

Put $\log x = z \Rightarrow x = e^z \Rightarrow dx = e^z dz$

$$\therefore \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx$$

$$= \int_{\log 2}^1 \left(\frac{1}{z} - \frac{1}{z^2}\right) e^z\ dz$$

$$= \int_{\log 2}^1 e^2 \left(\frac{1}{z} + d\left(\frac{1}{z}\right)\right) dz$$

$$= \left[e^2 . \frac{1}{z}\right]_{\log 2}^1 = e - \frac{2}{\log 2}$$

$\therefore\ a = e$ and $b = -2$

347 **(c)**

Let $I = \int_0^1 \frac{x\, dx}{[x + \sqrt{1-x^2}]\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$

$$\therefore\quad I = \int_0^{\pi/2} \frac{\sin\theta\cos\theta d\theta}{(\sin\theta + \cos\theta).\cos\theta}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta \quad \ldots \text{(i)}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \qquad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}\right) d\theta$$

$$= \int_0^{\pi/2} 1 d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

348 **(d)**

Let $I = \int_0^1 \cot^{-1}(1 - x + x^2)dx$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{x - (x-1)}{1 + x(x-1)}\right) dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(x-1)dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1 - x - 1)dx$$

$$= 2\int_0^1 \tan^{-1} x\, dx$$

$$= 2\left[x\tan^{-1}x - \int \frac{x}{1+x^2}dx\right]_0^1$$

$$= 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1$$

$$= 2\left[\left(1\tan^{-1}1 - \frac{1}{2}\log 2\right) - \left(0 - \frac{1}{2}\log 1\right)\right]$$

$$= \frac{\pi}{2} - \log 2$$

349 **(a)**

Let $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$

Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\therefore I = \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\tan^2\theta - 3)\sqrt{\tan^2\theta + 1}}d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 1 - 3)\sec\theta}d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 4)\sec\theta}d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec\theta}{(\sec^2\theta - 4)}d\theta$$

$$= \left[\frac{1}{2}\log\frac{(\sec\theta - 2)}{(\sec\theta + 2)}\right]_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \quad \left[\because \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{\sec\tan^{-1}\sqrt{15} - 2}{\sec\tan^{-1}\sqrt{15} + 2}\right) - \log\left(\frac{\sec\tan^{-1}\sqrt{8} - 2}{\sec\tan^{-1}\sqrt{8} + 2}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{\sec\sec^{-1}4 - 2}{\sec\sec^{-1}4 + 2}\right) - \log\left(\frac{\sec\sec^{-1}3 - 2}{\sec\sec^{-1}3 + 2}\right)\right]$$

$$= \frac{1}{2}\left[\log\frac{2}{6} - \log\frac{1}{5} = \frac{1}{2}\log\frac{5}{3}\right]$$

350 **(c)**

We have,

$$\Rightarrow I = \int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2}dx$$

$$\Rightarrow I = \int \{1 + \tan^2 x + \tan^2 x + 2\tan x\sec x\}^{1/2}dx$$

$$\Rightarrow I = \int (\sec^2 x + \tan^2 x + 2\tan x\sec x)^{1/2}dx$$

$$\Rightarrow I = \int (\tan x + \sec x)dx$$

$$\Rightarrow I = \log\sec x + \log(\sec x + \tan x) + C$$

$$\Rightarrow I = \log\sec x\,(\sec x + \tan x) + C$$

351 **(a)**

We have,

$$I_1 = \int_0^\infty \frac{1}{1+x^4}dx \text{ and } I_2 = \int_0^\infty \frac{x^2}{1+x^4}dx$$

Putting $x = \frac{1}{t}$ in I_1, we get

$$I_1 = \int_\infty^0 \frac{t^4}{1+t^4} \times -\frac{1}{t^2}dt = \int_0^\infty \frac{t^2}{t+t^4}dt = I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

352 **(c)**

Let $I = \int \frac{\sin x}{3+4\cos^2 x}dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore I = \int \frac{-dt}{3+4t^2} = \frac{-1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I = -\frac{1}{4\cdot\frac{\sqrt{3}}{2}}\cdot\tan^{-1}\frac{t}{\left(\frac{\sqrt{3}}{2}\right)} + c$$

$$= -\frac{1}{2\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}} + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c$$

353 **(c)**

Let $f(x) = e^{\cos^2 x}.\cos^3(2n+1)x$

Then, $f(\pi - x) = e^{\cos^2(\pi - x)}.\cos^3[(2n+1)\pi - (2n+1)x]$

$= -e^{\cos^2 x}.\cos^3(2n+1)x$

$\Rightarrow f(\pi - x) = -f(x)$

$\therefore f(x)$ is an odd function.

Hence, $\int_0^\pi e^{\cos^2 x}.\cos^3(2n+1)x\, dx = 0$

354 **(b)**

Given , $\frac{d}{dx}\{f(x)\} = \frac{1}{1+x^2}$

On integrating both sides, we get

$f(x) = \tan^{-1} x$

$$\therefore \frac{d}{dx}f(x^3) = \frac{d}{dx}(\tan^{-1} x^3)$$

$$= \frac{1}{1+(x^3)^2}.3x^2$$

$$= \frac{3x^2}{1+x^6}$$

355 **(d)**

Let $I = \int_0^\pi [\cot x]dx$ ... (i)

$\Rightarrow I = \int_0^{\pi} [\cot(\pi - x)]dx$

$= \int_0^{\pi} [-\cot x]dx$ ... (ii)

On adding Eqs. (i) and (ii),

$2I = \int_0^{\pi} [\cot x]dx + \int_0^{\pi} [-\cot x]dx$

$= \int_0^{\pi} (-1)dx$

$[\because [x] + [-x] = -, \text{if} x \notin z \; and [x] + [-x] = 0, \text{if} x \in z]$

$= [-x]_0^{\pi} = -\pi$

$\therefore \; I = -\frac{\pi}{2}$

356 **(c)**

Let $I = \int_{-3}^{2} (|x+1| + |x+2| + |x-1|)dx$

Again, let $f(x) = |x+1| + |x+2| + |x-1|$

$$= \begin{cases} -(x+1) - (x+2) - (x-1), & -3 < x \le -2 \\ -(x+1) + x + 2 - (x-1), & -2 < x \le -1 \\ 1 + x + x + 2 - (x-1) & -1 < x \le 0 \\ 1 + x + x + 2 - (x-1) & 0 \le x < 1 \\ 1 + x + x + 2 + x - 1 & 1 \le x < 2 \end{cases}$$

$$= \begin{cases} -3x - 2, & -3 < x \le -2 \\ -x + 2, & -2 < x \le -1 \\ x + 4, & -1 \le x < 1 \\ 3x + 2, & 1 \le x < 2 \end{cases}$$

$\therefore I = \int_{-3}^{-2} (-3-2)dx + \int_{-2}^{-1} (-x+2)dx$

$+ \int_{-1}^{1} (x+4)dx + \int_{1}^{2} (3x+2)dx$

$= \left[-\frac{3x^2}{2} - 2x\right]_{-3}^{-2} + \left[-\frac{x^2}{2} + 2x\right]_{-2}^{-1}$

$+ \left[\frac{x^2}{2} + 4x\right]_{-1}^{1} + \left[\frac{3x^2}{2} + 2x\right]_{1}^{2}$

$= \left[-6 + 4 - \left(-\frac{27}{2} + 6\right)\right] + \left[-\frac{1}{2} - 2 - (-2-4)\right]$

$+ \left[\frac{1}{2} + 4 - \left(\frac{1}{2} - 4\right)\right]\left[6 + 4 - \left(\frac{3}{2} + 2\right)\right]$

$= \frac{11}{2} + \frac{7}{2} + 8 + \frac{13}{2}$

$= \frac{31}{2} + 8 = \frac{47}{2}$

Alternate

Let $I = \int_{-3}^{2} \{|x+1| + |x+2| + |x-1|\}dx$

$= \int_{-3}^{-1} |x+1|dx$

$+ \int_{-1}^{2} |x+1|dx + \int_{-3}^{-2} |x+2|dx$

$+ \int_{-2}^{2} |x+2|dx + \int_{-3}^{1} |x-1|dx$

$+ \int_{1}^{2} |x-1|dx$

$= -\int_{-3}^{-1} (x+1)dx$

$+ \int_{-1}^{2} (x+1)dx$

$- \int_{-3}^{-2} (x+2)dx$

$+ \int_{-2}^{2} (x+2)dx$

$- \int_{-3}^{1} (x-1)dx + \int_{1}^{2} (x-1)dx$

$= -\left(\frac{x^2}{2} + x\right)_{-3}^{-1} + \left(\frac{x^2}{2} + x\right)_{-1}^{2} - \left(\frac{x^2}{2} + 2x\right)_{-3}^{-2}$

$+ \left(\frac{x^2}{2} + 2x\right)_{-2}^{2} - \left(\frac{x^2}{2} - x\right)_{-3}^{1} - \left(\frac{x^2}{2} - x\right)_{1}^{2}$

$= \frac{47}{2}$

357 **(b)**

$\int_0^3 |x^3 + x^2 + 3x|dx = \int_0^3 (x^3 + x^2 + 3x)dx$

$= \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3$

$= \frac{81}{4} + \frac{27}{3} + \frac{27}{2} = \frac{171}{4}$

358 **(b)**

$\int \frac{dx}{\sin x \cos x} = \int \frac{\cos x}{\sin x \cos^2 x} dx$

$= \int \frac{1}{\tan x} \sec^2 x \, dx = \log|\tan x| + c$

359 **(b)**

Let $I = \int_0^{2n\pi} \left\{|\sin x| - \frac{1}{2}|\sin x|\right\} dx$

$= \int_0^{2n\pi} \frac{1}{2} |\sin x| dx$

$= \frac{1}{2}\left[\int_0^{2\pi} |\sin x| dx + \int_{2\pi}^{4\pi} |\sin x| \, dx + \ldots\right.$

$\left. + \int_{2(n-1)\pi}^{2n\pi} |\sin x\} dx \right]$

Now, $I_1 = \int_0^{2\pi} |\sin x| dx$

$= \int_0^{\pi} \sin x\, dx - \int_{\pi}^{2\pi} \sin x\, dx$

$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = 4$

$\therefore\ I = \frac{1}{2}[4 + 4 + 4 + \ldots + n \text{ times}]$

$= \frac{1}{2}(4n) = 2n$

$I = \int_0^{2n\pi} \frac{1}{2} |\sin x|\, dx$

Since $|\sin x|$ is periodic with period π.

$\therefore\ I = 2n \int_0^{\pi} \frac{|\sin x| dx}{2}$

$I = n \int_0^{\pi} \sin x\, dx$

$= [-\cos x]_0^{\pi}$

$\Rightarrow I = 2n$

Alternate

$I = \int_0^{2n\pi} \frac{1}{2} |\sin x| dx$

Since, $|\sin x|$is periodic with period π.

$\therefore I + 2n \int_0^{\pi} \frac{|\sin x|}{2} dx$

$= n \int_0^{\pi} \sin x\, dx = n[-\cos x]_0^{\pi}$

$\Rightarrow\ \ I = 2n$

360 **(d)**

Given, $\int_a^b x^3\, dx = 0$ and $\int_a^b x^2\, dx = \frac{2}{3}$

If we take $a = -1$ and $b = 1$, then it will satisfy the given integration.

361 **(b)**

Let

$I = \int_0^{\pi/2} \operatorname{cosec}(x - \pi/3) \operatorname{cosec}(x - \pi/6) dx$

$\Rightarrow I = 2 \int_0^{\pi/2} \frac{\sin[(x - \pi/6) - (x - \pi/3)]}{\sin(x - \pi/6)\sin(x - \pi/3)} dx$

$\Rightarrow I = 2 \int_0^{\pi/2} [\cot(x - \pi/3) - \cot(x - \pi/6)]\, dx$

$\Rightarrow I = 2\left[\log\sin\left(x - \frac{\pi}{3}\right) - \log\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\pi/2}$

$\Rightarrow I = 2\left[\log\left(\frac{\sin(x - \pi/3)}{\sin(x - \pi/6)}\right)\right]_0^{\pi/2}$

$\Rightarrow I = 2\left[\log\left(\frac{1/2}{\sqrt{3}/2}\right) - \log\left(\frac{\sqrt{3}/2}{1/2}\right)\right]$

$\Rightarrow I = 2[-\log\sqrt{3} - \log\sqrt{3}] = -4\log\sqrt{3}$

$= -2\log 3$

362 **(c)**

Primitive function means indefinite integral.

$\therefore$ Primitive function of $f(x)$

$I = \int \frac{\sqrt{(a^2 - x^2)}}{x^4} dx$ (say)

$= \int \frac{x\sqrt{\frac{a^2}{x^2} - 1}}{x \cdot x^3} dx$

$= \int \frac{1}{x^3} \cdot \sqrt{\left(\frac{a^2}{x^2} - 1\right)} dx$

Put $\frac{a^2}{x^2} - 1 = t^2$

$\Rightarrow\ -\frac{2a^2}{x^3} dx = 2t\, dt$

$\Rightarrow \frac{1}{x^3} dx = -\frac{1}{a^2} t\, dt$

Then, $I = -\frac{1}{a^2} \int t^2 dt$

$= -\frac{1}{3a^2} t^3 + c$

$= -\frac{1}{3a^2}\left(\frac{a^2}{x^2} - 1\right)^{3/2} + c$

$= -\frac{(a^2 - x^2)^{3/2}}{3a^2x^3} + c$

363 **(c)**

We have,

$(x) = \begin{cases} x, \text{for } x < 1 \\ x - 1 \text{ for } x \geq 1' \end{cases}$

$\Rightarrow x^2 f(x) = \begin{cases} x^3, & \text{for } x < 1 \\ x^3 - x^2 & \text{for } x \geq 1 \end{cases}$

$\Rightarrow \int_0^2 x^2 f(x)\, dx = \int_0^1 x^3 dx + \int_1^2 x^3 - x^2\, dx = \frac{5}{3}$

364 **(d)**

$\int \frac{x^2 + x - 6}{(x - 2)(x - 1)} dx = \int \frac{x + 3}{x - 1} dx$

$= \int 1\, dx + \int \frac{4}{(x - 1)} dx$

$= x + 4\log(x - 1) + c$

365 **(b)**

We have,

$I = \int \frac{x^3}{(1 + x^2)^{1/3}} dx = \frac{1}{2} \int \frac{x^2}{(1 + x^2)^{1/3}} \cdot 2x\, dx$

$\Rightarrow I = \frac{1}{2} \int \frac{(1 + x^2) - 1}{(1 + x^2)^{1/3}} d(1 + x^2)$

$\Rightarrow I = \frac{1}{2} \int \{(1 + x^2)^{2/3} - (1 + x^2)^{-1/3}\} d(1 + x^2)$

$$\Rightarrow I=\frac{1}{2}\left\{\frac{3}{5}(1+x^2)^{5/3}-\frac{3}{2}(1+x^2)^{2/3}\right\}+C$$
$$\Rightarrow I=\frac{1}{2}(1+x^2)^{2/3}\left\{\frac{3}{5}(1+x^2)-\frac{2}{3}\right\}+C$$
$$\Rightarrow I=\frac{1}{20}(1+x^2)^{2/3}(6x^2-9)+C$$
$$=\frac{3}{20}(1+x^2)^{2/3}(2x^2-3)+C$$

366 **(b)**

We have,

$$\int_{-\pi/4}^{\pi/4}\left\{a|\sin x|+\frac{b\sin x}{1+\cos x}+c\right\}dx=0$$
$$\Rightarrow a\int_{-\pi/4}^{\pi/4}|\sin x|\,dx+b\int_{-\pi/4}^{\pi/4}\frac{\sin x}{1+\cos x}dx+c\int_{-\pi/4}^{\pi/4}dx=0$$
$$\Rightarrow 2a\int_0^{\pi/4}|\sin x|\,dx+b\times 0+2c\int_0^{\pi/4}dx=0$$
$$\Rightarrow 2a\int_0^{\pi/4}\sin x\,dx+2c\times\frac{\pi}{4}=0$$
$$\Rightarrow -2a\left(\cos\frac{\pi}{4}-\cos 0\right)+\frac{\pi c}{2}=0$$
$$\Rightarrow -2a\left(\frac{1}{\sqrt{2}}-1\right)+\frac{\pi c}{2}=0\Rightarrow a(2-\sqrt{2})+\frac{\pi c}{2}=0$$

367 **(c)**

Let $I=\int_2^4\{|x-2|+|x-3|\}dx$

$$=\int_2^3\{(x-2)+(3-x)\}dx+\int_3^4\{(x-2)+(x-3)\}dx$$
$$=\int_2^3 dx+\int_3^4(2x-5)dx$$
$$=[x]_2^3+[x^2-5x]_3^4$$
$$=3-2+[16-20-(9-15)]$$
$$=1+2=3$$

368 **(a)**

$g(x)=\int_0^x\cos^4 t\,dt$ (given)

$$g(x+\pi)=\int_0^{\pi+x}\cos^4 t\,dt$$
$$=\int_0^{\pi}\cos^4 t\,dt+\int_{\pi}^{\pi+x}\cos^4 t\,dt=I_1+I_2$$
$$\Rightarrow I_1=g(\pi)$$
$$I_2=\int_{\pi}^{\pi+x}\cos^4 t\,dt$$

Put $t=\pi+y\ \Rightarrow dt=dy$

$$I_2=\int_0^x\cos^4(y+\pi)dy=\int_0^x[\cos(\pi+y)]^4dy$$
$$=\int_0^x(-\cos y)^4\,dy=\int_0^x\cos^4 y\,dy=g(x)$$
$$\therefore\ g(x+\pi)=g(x)+g(\pi)$$

369 **(b)**

We have,

$$\Rightarrow I=\int\cos^3 x\ e^{\log(\sin x)}dx=\int\cos^3 x\sin x\ dx$$
$$\Rightarrow I=-\int\cos^3 x\ d(\cos x)=-\frac{\cos^4 x}{4}+C$$

370 **(a)**

Put $x+1=t^2\ \Longrightarrow\ dx=2t\,dt$

At $x=8, t=3$ and $x=15, t=4$

$$\therefore I=\int_3^4\frac{2t\,dt}{(t^2-1-3)t}$$
$$=\int_3^4\frac{2dt}{(t^2-4)}=2.\frac{1}{4}\left[\log\frac{t-2}{t+2}\right]_3^4$$
$$=\frac{1}{2}\left[\log\frac{1}{3}-\log\frac{1}{5}\right]=\frac{1}{2}\log\frac{5}{3}$$

371 **(a)**

Using $\sin^2 x=\frac{1-\cos x}{2}$ and $\cos^2 x=\frac{1+\cos 2x}{2}$,

We get

$$I=\int\frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx=\int\frac{(1-\cos 2x)^2}{2(1+\cos^2 2x)}dx$$
$$\Rightarrow I=\frac{1}{2}\int 1-\frac{2\cos 2x}{2-\sin^2 2x}dx$$
$$\Rightarrow I=\frac{1}{2}\left\{x-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}\right|\right\}+C$$

372 **(c)**

We have,

$$f(x)=\begin{cases}\int_{-1}^{x}-t\,dt, & -1\le x\le 0\\ \int_{-1}^{0}-tdt+\int_0^x tdt, & x\ge 0\end{cases}$$
$$\Rightarrow f(x)=\begin{cases}\frac{1}{2}(1-x^2), & -1\le x\le 0\\ \frac{1}{2}(1+x^2), & x\ge 0\end{cases}$$

373 **(b)**

Since $\sqrt{1+x^2}>x^2$ for all $x\in[1,2]$. Therefore, $\frac{1}{\sqrt{1+x^2}}<\frac{1}{x}$ for all $x\in[1,2]$

$$\Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}} dx < \int_1^2 \frac{1}{x} dx \Rightarrow I_1 < I_2$$

374 **(a)**

Let $f(x) = (1-x^2)\sin x \cos^2 x$

$f(-x) = [1-(-x)^2][\sin(-x)]\cos^2(-x)$

$= -(1-x^2)\sin x \cos^2 x = -f(x)$

$\Rightarrow f(x)$ is an odd function.

$$\therefore \int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x \, dx = 0$$

375 **(c)**

We have,

$$I_n = \int_0^{\pi/2} x^n \sin x \, dx$$

I II

$$\Rightarrow I_n = [-x^n \cos x]_0^{\pi/2} + n\int_0^{\pi/2} x^{n-1}\cos x \, dx$$

I II

$$\Rightarrow I_n = n[x^{n-1}\sin x]_0^{\pi/2} - n(n-1)\int_0^{\pi/2} x^{n-2}\sin x \, dx$$

$$\Rightarrow I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Putting $n = 4$, we get

$$I_4 + 12I_2 = 4\left(\frac{\pi}{2}\right)^3$$

376 **(b)**

We have,

$$\int_0^2 x[x]\, dx = \int_0^1 x \times 0 \, dx + \int_1^2 x \, dx = \frac{3}{2}$$

377 **(b)**

$$\int_{-1}^0 \frac{dx}{x^2+2x+2} = \int_{-1}^0 \frac{dx}{(x+1)^2+1}$$

$= [\tan^{-1}(x+1)]_{-1}^0$

$= [\tan^{-1} 1 - \tan^{-1} 0] = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

378 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

Since, $x^3 \sin^4 x$ is an odd function

$\therefore \; I = 0$

379 **(c)**

$$I_1 = \int_{1-k}^{k} xf\{x(1-x)\}dx$$

$$= \int_{1-k}^{k} (1-x)f[(1-x)\{1-(1-x)\}]dx (\text{put} x = 1-x)$$

$$= \int_{1-k}^{k} (1-x)f\{x(1-x)\}dx$$

$$= \int_{1-k}^{k} f\{x(1-x)\}dx - \int_{1-k}^{k} xf\{x(1-x)\}dx$$

$= I_2 - I_1$

$\therefore 2I_1 = I_2$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

380 **(a)**

We have,

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

$$\therefore I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n+1} x + \tan^{x-1} x)\, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n-1} x \sec^2 x)\, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \left[\frac{\tan^n x}{n}\right]_0^{\pi/4} = \frac{1}{n}$$

$\Rightarrow n(I_{n+1} + I_{n-1}) = 1$

$\Rightarrow \lim_{n\to\infty} (I_{n+1} + I_{n-1}) = 1$

381 **(c)**

$$\int_1^e \frac{1}{x} dx = [\log x]_1^e = \log e - \log 1 = 1$$

382 **(b)**

We have,

$$\lim_{n\to\infty} \frac{1^{99} + 2^{99} + \cdots + n^{99}}{n^{100}} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^{99}}{n^{100}}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^{99}$$

$$= \int_0^1 x^{99}\, dx = \left[\frac{x^{100}}{100}\right]_0^1 = \frac{1}{100} - 0 = \frac{1}{100}$$

383 **(a)**

We have,

$$I = \int \frac{\cos 2x}{\cos x} dx = \int 2\cos x - \sec x \, dx$$

$\Rightarrow I = 2\sin x - \log(\sec x + \tan x) + C$

$\Rightarrow I = 2\sin x + \log(\sec x - \tan x) + C$

384 **(c)**

We have,

$$I=\int f'(ax+b)\,\{f(ax+b)\}^n\,dx$$

$$\Rightarrow I=\frac{1}{a}\int \{f(ax+b)\}^n\,d\{f(ax+b)\}$$

$$=\frac{1}{a}\times\frac{\{(ax+b)\}^{n+1}}{n+1}+C$$

385 **(c)**

$\because\ 2^{x^3}<2^{x^2}, 0<x<1$ and $2^{x^3}>2^{x^2}$, $x>1$

$\therefore I_4>I_3$ and $I_2<I_1$

386 **(c)**

Let $I=\int \frac{\sqrt{\tan x}}{\sin x\cos x}\,dx$

$$=\int \frac{\sqrt{\sin x}}{\sqrt{\cos x}\sin x\cos x}\,dx$$

$$=\int \frac{1}{\sqrt{\sin x}\cos^{3/2} x}\,dx$$

$$=\int \frac{1}{\sqrt{\tan x}\cos^2 x}\,dx$$

$$=\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$$

Put $\tan x=t\Rightarrow \sec^2 x\ dx=dt$

$$\therefore I=\int \frac{1}{\sqrt{t}}\,dt=2\sqrt{t}=2\sqrt{\tan x}$$

387 **(b)**

Let $I=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{n\sqrt{1+(r/n)^2}}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r/n}{\sqrt{1+(r/n)^2}}$$

Put $\frac{r}{n}=x, \frac{1}{n}dx, \lim_{n\to\infty}\ \sum_{r=1}^{2n}=\int_0^2$

$$\therefore I=\int_0^2\frac{x}{\sqrt{1+x^2}}dx=[\sqrt{1+x^2}\,]_0^2=\sqrt{5}-1$$

388 **(a)**

$$\because I=\int_0^\infty \frac{x\log x}{(1+x^2)^2}dx$$

Put $x=\tan\theta\ \Rightarrow dx=\sec^2\theta\,d\theta$

$$\therefore I=\int_0^{\pi/2}\frac{\tan\theta\log(\tan\theta)}{\sec^4\theta}\sec^2\theta\,d\theta$$

$$=\int_0^{\pi/2}\frac{\sin\theta\log(\tan\theta)}{\cos\theta\cdot\frac{1}{\cos^2\theta}}d\theta$$

$$=\int_0^{\pi/2}\sin\theta\cos\theta\log(\tan\theta)\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}\sin 2\theta\log(\tan\theta)\,d\theta$$

$$=0\ \left(\because \int_0^{\pi/2}\sin\ 2\theta\log\tan\theta\,d\theta=0\right)$$

389 **(a)**

$$\int_{-1}^{1}[x[1+\sin\pi x]+1]dx$$

$$=\int_{-1}^{0}[x[1+\sin\pi x]+1]dx+\int_0^1[x[1+\sin\pi x]+1]dx$$

Now, $-1<x<0\ \Rightarrow\ [1+\sin\pi x]=0$

$0<x<1\ \Rightarrow\ [1+\sin\pi x]=1$

$\Rightarrow\ [x[1+\sin\pi x]+1]=1$

$$\therefore\ \int_{-1}^{1}[x[1+\sin\pi x]+1]dx=2$$

390 **(d)**

We have,

$$I=\int_0^\infty e^{-ax^2}\,dx$$

$$\Rightarrow I=\frac{1}{\sqrt{a}}\int_0^\infty e^{-(\sqrt{a}\,x)^2}d(\sqrt{a}x)$$

$$\Rightarrow I=\frac{1}{\sqrt{a}}\times\frac{\sqrt{\pi}}{2}\Rightarrow I=\frac{1}{2}\frac{\sqrt{\pi}}{a}\quad\left[\because\int_0^\infty e^{-x^2}\,dx=\frac{\sqrt{\pi}}{2}\right]$$

391 **(d)**

Let $I=\int\frac{2x}{\sqrt{1-4^x}}dx\cdot$ Then,

$$I=\frac{1}{\log 2}\int\frac{1}{1-(2^x)^2}d(2^x)=\frac{1}{\log 2}\sin^{-1}(2^x)+C$$

$$\therefore K=\frac{1}{\log 2}$$

392 **(d)**

We have,

$$I=\int_{-\pi}^{\pi}(\cos ax-\sin bx)^2\,dx$$

$$\Rightarrow I=\int_{-\pi}^{\pi}(\cos^2 ax-\sin^2 bx-2\cos ax\sin bx)\,dx$$

$$\Rightarrow I=\int_{-\pi}^{\pi}(\cos^2 ax-\sin^2 bx)\,dx-2\int_{-\pi}^{\pi}\cos ax\sin bx\,dx$$

$$\Rightarrow I = 2\int_{-\pi}^{\pi}(\cos^2 ax + \sin^2 bx)\,dx - 0$$

$$\Rightarrow I = 2\int_0^{\pi}\frac{1+\cos 2ax}{2} + \frac{1-\cos 2bx}{2}\,dx$$

$$\Rightarrow I = \int_0^{\pi} 2 + \cos 2ax - \cos 2bx\,dx = 2\pi$$

393 **(d)**

Let $I = \int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1}\,dx$

$$= \int \frac{mx^{m+2n-1} - nx^{n-1}}{(1+x^{m+n})^2}\,dx$$

$$\therefore I = \int \frac{mx^{m-1} - \frac{n}{x^{n+1}}}{\left(x^m + \frac{1}{x^n}\right)^2}\,dx$$

Let $x^m + \frac{1}{x^n} = t$

$$\Rightarrow mx^{m-1} - \frac{n}{x^{n+1}}\,dx = dt$$

$$\therefore I = \int \frac{1}{t^2}\,dt = -\frac{1}{t} + c$$

$$= \frac{-1}{\left(x^m + \frac{1}{x^n}\right)} + c = \frac{-x^n}{x^{m+n}+1} + c$$

394 **(b)**

We have,

$$I = \int_0^{16\pi/3} |\sin x|dx = \int_0^{5\pi} |\sin x|dx + \int_{5\pi}^{16\pi/3} |\sin x|dx$$

$\Rightarrow I$

$$= 5\int_0^{\pi} |\sin x|\,dx$$

$$+ \int_0^{\pi/3} |\sin x|\,dx \quad [\because |\sin x| \text{is periodic with period } \pi$$

$$\Rightarrow I = 5\int_0^{\pi} \sin x\,dx + \int_0^{\pi/3} \sin x\,dx$$

$$= 5 \times 2 + \left(-\frac{1}{2} + 1\right) = \frac{21}{2}$$

395 **(d)**

Let $I = \int_{-\pi/2}^{\pi/2} \{f(x) + f(-x)\}\{g(x) - g(-x)\}\,dx$

Again, let

$$h(x) = \{f(x) + f(-x)\}\{g(x) - g(-x)\}$$

$$\Rightarrow h(-x) = \{f(-x) + f(x)\}\{g(-x) - g(x)\}$$

$$\Rightarrow h(-x) = -h(x)$$

Hence, $h(x)$ is an odd function

$\therefore\ I = 0$

396 **(a)**

We have,

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx = \int \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\,dx$$

$$\Rightarrow I = \int (\tan x)^{-1/2}\,d(\tan x) = 2\sqrt{\tan x} + C$$

398 **(c)**

We have,

$$\int_{-\pi/3}^{\pi/3} \left\{\frac{a}{3}|\tan x| + \frac{6\tan x}{1+\sec x} + c\right\}dx = 0$$

$$\Rightarrow \frac{a}{3}\int_{-\pi/3}^{\pi/3} |\tan x|\,dx + b\int_{-\pi/3}^{\pi/3} \frac{\tan x}{1+\sec x}\,dx + c\int_{-\pi/3}^{\pi/3} dx = 0$$

$$\Rightarrow 2a\int_0^{\pi/3} \tan x\,dx + b \times 0 + c\left(\frac{2\pi}{3}\right) = 0$$

$$\Rightarrow \frac{2a}{3}[\log_e|\sec x|]_0^{\pi/3} + \frac{2\pi}{3}c = 0$$

$$\Rightarrow \frac{2a}{3}\ln 2 + \frac{2\pi}{3}c = 0 \Rightarrow c = -\frac{a}{\pi}\ln 2$$

399 **(d)**

Given, $f'(2) = \tan\frac{\pi}{4} = 1, f'(4) = \tan\frac{\pi}{3} = \sqrt{3}$

$$\therefore \int_2^4 f'(x)f''(x)dx = \left[\frac{1}{2}[f'(x)]^2\right]_2^4$$

$$= \frac{1}{2}[f'(4)]^2 - \frac{1}{2}[f'(2)]^2$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

400 **(a)**

Let $I = \int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$= \int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \frac{1}{2}\int \frac{dx}{\cos^{7/2} x\sin^{1/2} x}$$

$$= \frac{1}{2}\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx = \frac{1}{2}\int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}\,dx$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore I = \frac{1}{2}\int \frac{1+t^2}{\sqrt{t}}\,dt$$

$$= \frac{1}{2}\int t^{-1/2}dt + \frac{1}{2}\int t^{3/2}dt = t^{1/2} + \frac{t^{5/2}}{5} + c$$

$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2}x + c$

401 **(b)**

On putting, $x = \tan\theta \Rightarrow dx = sce^2\theta\, d\theta$, we get

$f(x) = \int \frac{\tan^2\theta . \sec^2\theta}{\sec^2\theta(1+\sec\theta)} d\theta$

$= \int \frac{\sec^2\theta - 1}{1+\sec\theta} d\theta = \int (\sec\theta - 1)\, d\theta$

$= \log(\sec\theta + \tan\theta) - \theta + c$

$\Rightarrow f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1}x + c$

At $x = 0$, $f(0) = \log(1+0) - 0 + c \Rightarrow c = 0$

$\therefore f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1}x$

At $x = 1$, $f(1) = \log(1+\sqrt{2}) - \frac{\pi}{4}$

402 **(b)**

We have,

$I = \int_0^1 \frac{1}{(1+x^2)^{3/2}} dx = \int_0^{\pi/4} \cos\theta$, where $x = \tan\theta$

$\Rightarrow I = \frac{1}{\sqrt{2}}$

403 **(a)**

We have, $I(m,n) = I = \int_0^1 t^m(1+t)^n dt$

$\Rightarrow I(m,n) = \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1}\right]_0^1 - \frac{n}{m+1}\int_0^1 (1+t)^{n-1}t^{m+1}dt$

$\Rightarrow I(m,n) = \frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$

404 **(b)**

Let $I = \int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$

Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$\therefore I = \int_0^{\pi/4} t\, dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$

$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^4 - 0^2\right] = \frac{\pi^2}{32}$

405 **(b)**

Let $I = \int_3^5 \frac{x^2}{x^2-4} dx = \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4}\right) dx$

$= \int_3^5 \left(1 + \frac{4}{x^2-4}\right) dx$

$= \left[x + \frac{4}{2\times 2}\log_e\left(\frac{x-2}{x+2}\right)\right]_3^5$

$= \left[5 + \log_e\left(\frac{5-2}{5+2}\right) - 3 - \log_e\left(\frac{3-2}{3+2}\right)\right]$

$= 2 + \log_e\left(\frac{3}{7}\right) - \log_e\left(\frac{1}{5}\right)$

$= 2 + \log_e\left(\frac{3}{7}\times\frac{5}{1}\right) = 2 + \log_e\left(\frac{15}{7}\right)$

406 **(b)**

$\int \sqrt{x^2+a^2}\, dx = \int \sqrt{x^2+a^2} \cdot 1\, dx$

$= \sqrt{x^2+a^2}\int 1dx - \int \left[\frac{d}{dx}\left(\sqrt{x^2+a^2}\right)\int 1dx\right] dx$

$= x\sqrt{x^2+a^2} - \int \left[\frac{2x}{2\sqrt{x^2+a^2}}x\right] dx$

$= x\sqrt{x^2+a^2} - \int \left[\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\right] dx$

$= x\sqrt{x^2+a^2} - \int \left[\sqrt{x^2+a^2} - \frac{a^2}{\sqrt{x^2+a^2}}\right] dx$

$= x\sqrt{x^2+a^2} - \int \sqrt{x^2+a^2}dx + a^2\int \frac{dx}{\sqrt{x^2+a^2}}$

$= x\sqrt{x^2+a^2} - I + a^2\log[x + \sqrt{x^2+a^2}] + c$

$\Rightarrow 2I = x\sqrt{x^2+a^2} + a^2\log[x+\sqrt{x^2+a^2}] + c$

$\Rightarrow I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left[x\sqrt{x^2+a^2}\right] + c$

407 **(b)**

$I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1 + \tan^2 x)dx$

$= \int_0^{\pi/4} \tan^n x \sec^2 x\, dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I_n + I_{n+2} = \int_0^1 t^n\, dt = \frac{1}{n+1}$

$\Rightarrow \lim_{n\to\infty} n[I_n + I_{n+2}]$

$= \lim_{n\to\infty}\frac{n}{n+1}$

$= \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)} = 1$

408 **(b)**

Put $x + 1 = t^2$

$\Rightarrow dx = 2t\, dt$

$\Rightarrow x^2 + 1 = (t^2-1)^2 + 1$

$= t^4 - 2t^2 + 2$

$\therefore$ Given integral

$= \int (t^4 - 2t^2 + 2)t.2t\, dt$

$$= 2\int (t^6 - 2t^4 + 2t^2)dt$$

$$= 2\left[\frac{t^7}{7} - 2\frac{t^5}{5} + 2\frac{t^3}{3}\right] + c$$

$$= 2\left[\frac{(x+1)^{\frac{7}{2}}}{7} - 2\frac{(x+1)^{\frac{5}{2}}}{5} + 2\frac{(x+1)^{\frac{3}{2}}}{3}\right] + c$$

409 **(c)**

$$\int \frac{f'(x)}{f(x)\log[f(x)]}dx = \log[\log f(x)] + c$$

410 **(d)**

We have, $0 < x < 1$

$\therefore \frac{1}{2}x^2 < x^2 < x$

$\Rightarrow -x^2 > -x$

$\Rightarrow e^{-x^2} > e^{-x}$

$\Rightarrow e^{-x^2}\cos^2 x > e^{-x}\cos^2 x$

$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx > \int_0^1 e^{-x}\cos^2 x\;dx$

$\Rightarrow I_2 > I_1$ ...(i)

Also, $\cos^2 x \le 1$

$\Rightarrow e^{-x^2}\cos^2 x < e^{-x^2}$

$< e^{-(1/2)x^2}$ $\left[\because -\frac{1}{2}x^2 > -x^2 > -x\right]$

$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx < \int_0^1 e^{-x^2}dx < \int_0^1 e^{-(1/2)x^2}dx$

$\Rightarrow I_2 < I_3 < I_4$ ...(ii)

From (i) and (ii), we get $I_1 < I_2 < I_3 < I_4$

Hence, I_4 is the greatest integral

411 **(c)**

We have,

$I = \int \frac{1}{(1+x^2)\sqrt{1-x^2}}dx$

$\Rightarrow I = \int \frac{-t\,dt}{(t^2+1)\sqrt{t^2-1}}$, where $x = \frac{1}{t}, dx = -\frac{1}{t^2}dt$

$\Rightarrow I = \int \frac{-u\,du}{(u^2+2)\sqrt{u^2}}$, where $t^2 - 1 = u^2$

$\Rightarrow I = -\int \frac{1}{u^2 + (\sqrt{2})^2}du = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + K$

$\Rightarrow I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right) + K$

$\Rightarrow I = -\frac{1}{\sqrt{2}}\left\{\frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)\right\} + K$ $[\because \tan^{-1}x + \cot^{-1}x = \pi/2]$

$\Rightarrow I = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right) + \left(K - \frac{\pi}{2\sqrt{2}}\right)$

$\Rightarrow I = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right) + C$, where $C = K - \frac{\pi}{2\sqrt{2}}$

412 **(c)**

$I = \int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$

$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(i)

$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{\frac{1}{n}+a-\frac{1}{n}-x}dx}{\sqrt{a-\left(\frac{1}{n}+a-\frac{1}{n}-x\right)} + \sqrt{\left(\frac{1}{n}+a-\frac{1}{n}-\right.}}$

$\Rightarrow I = \int_{\frac{1}{n}}^{a-\frac{1}{n}} \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_{1/n}^{a-(1/n)} 1\,dx = [x]_{\frac{1}{n}}^{a-\frac{1}{n}}$

$\Rightarrow 2I = a - \frac{1}{n} - \frac{1}{n} = \frac{na-2}{n}$

$\Rightarrow I = \frac{na-2}{2n}$

413 **(b)**

Let $I = \int e^x(x^5 + 5x^4 + 1)dx$

$= \int e^x x^5 dx + 5\int e^x x^4 dx + \int e^x\,dx$

$= x^5 e^x - \int 5x^4 e^x dx + 5\int e^x x^4 dx + e^x$

$= x^5 e^x + e^x + c$

414 **(a)**

$\cos x \log\left(\frac{1+x}{1-x}\right)$is an odd function

$\therefore \int_{-1/2}^{1/2} \cos x\log\left(\frac{1+x}{1-x}\right)dx = 0$

$\therefore\ k = 0$

415 **(b)**

Let $I = \int \sin\sqrt{3}\;dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\therefore\ I = \int 2t\sin t\,dt$

$= 2[-t\cos t + \int \cos t\,dt]$

$= 2[-t\cos t + \sin t] + c$

$= 2[-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}] + c$

416 **(d)**

$f'(x) = (x^2-1)(2x) - (x-1)$

$= (x-1)(2x^2 + 2x - 1)$

Which is positive for $x > 1$. Hence, f increases in $[1, 2]$

Hence, global maximum of f is $f(2) = 4$

417 **(b)**

$$\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{\left(\frac{r}{n}\right)}{\sqrt{1+\left(\frac{r}{n}\right)^2}} = \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1+x^2}}dx$$

$$= \frac{1}{2}\left[\frac{\sqrt{1+x^2}}{1/2}\right]_0^1 = \sqrt{2}-1$$

418 **(d)**

We have,

$$I = \int_0^{2\pi} \cos^{99} x\, dx,$$

$$\Rightarrow I = 2\int_0^{\pi} \cos^{99} x\, dx \quad [\because \cos^{99}(2\pi - x) = \cos^{99} x]$$

But, $\int_0^{\pi} \cos^{99} x\, dx = 0 \quad [\because \cos^{99}(\pi - x) = -\cos^{99} x]$

$\therefore I = 2 \times 0 = 0$

419 **(b)**

Let $I = \int_{-2}^{2}(ax^3 + bx + c)dx$

In the given integral, $ax^3$and bx are odd functions. Hence, it depends only on the value of c.

420 **(b)**

Let $I = \int_0^{\pi} x\, f(\sin x)dx$

$$\Rightarrow I = \int_0^{\pi}(\pi - x)f\{\sin(\pi - x)\}dx$$

$$= \pi\int_0^{\pi} f(\sin x)dx - x\int_0^{\pi} f(\sin x)dx$$

$$\Rightarrow I = \pi\int_0^{\pi} f(\sin x)dx - I$$

$$\Rightarrow 2I = 2\pi\int_0^{\pi/2} f(\sin x)dx$$

$$\Rightarrow I = \pi\int_0^{\pi/2} f(\sin x)dx$$

421 **(a)**

Let $I = \int \frac{\log(x+1)-\log x}{x(x+1)}dx = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(x+1)}dx$

$$= \int \frac{\log\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)}dx$$

Put $1 + \frac{1}{x} = t \quad \Rightarrow \quad -\frac{1}{x^2}\, dx = dt$

$$\therefore\ I = -\int \frac{\log t}{t}dt = -\frac{1}{2}(\log t)^2 + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$= -\frac{1}{2}[\log(x+1) - \log x]^2 + c$$

$$= -\frac{1}{2}\{\log(x+1)\}^2 - \frac{1}{2}(\log x)^2 + \log(x+1).\log x + c$$

422 **(c)**

Let $I = \int_0^1 x\left|x - \frac{1}{2}\right|dx$

$$= -\int_0^{1/2} x\left(x - \frac{1}{2}\right)dx + \int_{1/2}^{1} x\left(x - \frac{1}{2}\right)dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{1/2}^{1}$$

$$= \left(\frac{1}{16} - \frac{1}{24}\right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16}\right) = \frac{1}{8}$$

423 **(a)**

Let

$$I = \int \frac{a^{x/2}}{\sqrt{a^{-x} - a^x}}dx = \int \frac{a^x}{\sqrt{1 - (a^x)^2}}dx$$

$$\Rightarrow I = \frac{1}{\log a}\int \frac{1}{\sqrt{1^2 - (a^x)^2}}d(a^x)$$

$$= \frac{1}{\log a}\sin^{-1}(a^x) + C$$

424 **(c)**

Let $I = \int_{\log 2}^{x} \frac{e^u}{e^u(e^u - 1)^{1/2}}du$

Put $e^u - 1 = t^2 \Rightarrow e^u du = 2t\, dt$

$$\therefore I = \int_1^{\sqrt{e^x-1}} \frac{2t}{(t^2+1)t}dt$$

$$= 2\int_1^{\sqrt{e^x-1}} \frac{dt}{(1+t^2)}$$

$$= 2[\tan^{-1} t\,]_1^{\sqrt{e^x-1}}$$

$$= 2\left[\tan^{-1}\sqrt{e^x - 1} - \frac{\pi}{4}\right] = \frac{\pi}{6} \quad \text{[given]}$$

$$\Rightarrow \tan^{-1}\sqrt{e^x - 1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{e^x - 1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\Rightarrow \quad e^x = 3 + 1 = 4$$

425 **(c)**

We have,

$$I = \int\{f(x)g''(x) - f''(x)g(x)\}dx$$

$$\Rightarrow I = \int f(x)g''(x)dx - \int f''(x)g(x)dx$$

$$\Rightarrow I = \left\{f(x)g'(x) - \int f'(x)g'(x)dx\right\}$$
$$-\left\{g(x)f'(x) - \int g'(x)f'(x)dx\right\}$$
$$\Rightarrow I = f(x)g'(x) - f'(x)g(x)$$

426 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

Since, $x^3 \sin^4 x$ is an odd function.

$\therefore I = 0$

427 **(a)**

Let $I = \int_0^{\pi/2} \sin^8 x \ dx = \frac{7.5.3.1}{8.6.4.2} . \frac{\pi}{2}$ [by Walli's formula]

$$= \frac{105\pi}{32(4.3.2.1)}$$
$$= \frac{105\pi}{32.4!}$$

428 **(b)**

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x + 1} dx$$
$$= 2\int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$
$$= 2I_1, \text{where } I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$

Now,

$I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x+1} dx$ ...(i)

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8\cos^2(\pi - 2x) + 1} dx$$

$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2}-x}{8\cos^2 2x+1} dx$ ...(ii)

Adding (i) and (ii), we get

$$2I_1 = \frac{\pi}{2}\int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$
$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$
$$= 2 \times \frac{\pi}{4}\int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$
$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/4} \frac{2\sec^2 2x}{9 + \tan^2 2x} dx$$
$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/4} \frac{1}{3^2 + \tan^2 2x} d(\tan 2x)$$
$$\Rightarrow I_1 = \frac{\pi}{4} \times \frac{1}{3}\left[\tan^{-1}\left(\frac{\tan 2x}{3}\right)\right]_0^{\pi/4} = \frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$$

Hence, $I = 2I_1 = \frac{\pi^2}{12}$

429 **(b)**

$$\because f(x)\cos x = \frac{1}{2} \cdot 2f(x)f'(x)$$

Then, $f'(x) = \cos x$

$\therefore f(x) = \sin x + c$

430 **(d)**

On putting $y^2 = f(x) = \frac{x+2}{2x+3}$, we have

$x = \frac{3y^2-2}{1-2y^2}$ and $dx = -\frac{2y}{(1-2y^2)^2} dy$

$$\therefore \int (f(x))^{1/2} \frac{dx}{x} = -\int y \frac{2y}{(1-2y^2)^2} \cdot \frac{1-2y^2}{3y^2-2} dy$$
$$= 2\int \frac{y^2}{(2y^2-1)(3y^2-2)} dy$$
$$= -2\int \left[\frac{1}{2y^2-1} - \frac{2}{3y^2-2}\right] dy$$
$$= \frac{1}{\sqrt{2}} \log\left|\frac{1+\sqrt{2}y}{1-\sqrt{2}y}\right| - \sqrt{\frac{2}{3}} \log\left|\frac{\sqrt{3}y + \sqrt{2}}{\sqrt{3}y - \sqrt{2}}\right| + c$$

Thus, $g(x) = \log|x|$ and $h(x) = \log|x|$

431 **(c)**

Let $5^{5^{5^x}} = t$ ∵ Then, $5^{5^{5^x}} 5^{5^x} 5^x (\log 5)^3 dx = dt$,

$$\therefore I = \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx = \frac{1}{(\log 5)^3}\int 1 \cdot dt$$
$$= \frac{5^{5^{5^x}}}{(\log 5)^3} + C$$

432 **(a)**

$$f(t) = \int_{-t}^{t} \frac{e^{-[x]}}{2} dx = 2\int_0^t \frac{e^{-x}}{2} dx$$
$$= -[e^{-x}]_0^t = -e^{-t} + 1$$

Now, $\lim_{t\to\infty} f(t) = -\lim_{t\to\infty} e^{-t} + 1 = 1$

433 **(b)**

We have,

$$I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$
$$\Rightarrow I = \log(\sin x + \cos x) + C$$

434 **(c)**

The primitive of the given function is

$$\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \theta)}}$$
$$= \int \frac{dx}{\sqrt{\sin^3 x(\sin x \cos\theta + \cos x \sin\theta)}}$$

$$= \int \frac{\text{cosec}^2 x\, dx}{\sqrt{\cos\theta + \cot x \sin\theta}}$$

Put $\cot x = t \Rightarrow -\text{cosec}^2 x\, dx = dt$

$$= -\frac{1}{\sqrt{\sin\theta}} \int \frac{dt}{\sqrt{\cot\theta + 1}}$$

$$= -\frac{2}{\sqrt{\sin\theta}} (\cot\theta + t)^{1/2} + c$$

$$= -\frac{2}{\sin\theta} (\cos\theta + t\sin\theta)^{1/2} + c$$

$$= \frac{-2\text{cosec}\,\theta(\sin x\cos\theta + \sin\theta\cos x)^{1/2}}{\sqrt{\sin x}} + c$$

$$= -2\,\text{cosec}\,\theta \left(\frac{\sin(\theta + x)}{\sin x}\right)^{1/2} + c$$

435 **(b)**

We have,

$$I = \int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}} dx$$

$$= \int \frac{(x^2 - 1)(x^2 + 1)}{x^2\sqrt{x^4 + x^2 + 1}} dx$$

$$\Rightarrow I = \int \frac{\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x^2}\right)}{\sqrt{x^2 + \frac{1}{x^2} + 1}} dx$$

$$= \int \frac{\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left\{\left(x + \frac{1}{x}\right)^2 - 1^2\right\}$$

$$\Rightarrow I = \sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2} + C = \sqrt{x^2 + \frac{1}{x^2} + 1} + C$$

$$\Rightarrow I = \sqrt{\frac{x^4 + x^2 + 1}{x}} + C$$

436 **(b)**

$$\int 32x^3(\log x)^2\ dx$$

$$= 32\{(\log x)^2 \frac{x^4}{4} - \int 2\log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx\}$$

$$= 8x^4(\log x)^2 - 16\int x^3 \log x\, dx$$

$$= 8x^4(\log x)^2 - 16\,\{\log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx\,\}$$

$$= 8x^4(\log x)^2 - 4x^4\log x + 4\int x^3 dx$$

$$= 8x^4(\log x)^2 - 4x^4 + \log x + x^4 + c$$

$$= x^4[\,8(\log x)^2 - 4\log x + 1\,] + c$$

437 **(a)**

Let $I = \int \frac{e^x(1+\sin x)}{1+\cos x} dx$

$$= \int \frac{1}{2} e^x \sec^2\frac{x}{2} dx + \int e^x \tan\frac{x}{2} dx$$

$$= \frac{1}{2}\left[2e^x\tan\frac{x}{2} - \int 2e^x \tan\frac{x}{2} dx\right] + \int e^x\tan\frac{x}{2} dx$$

$$= e^x \tan\frac{x}{2} + c$$

438 **(a)**

We have,

$$I = \int \log(\sqrt{1-x} + \sqrt{1+x}) \cdot 1\, dx$$

I II

$$\Rightarrow I = x\log(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{1}{2}\frac{1}{\sqrt{1-x^2}}\left(\frac{\sqrt{1-x^2}}{x} - \frac{1}{x}\right) \cdot x\, dx$$

$$\Rightarrow I = x\log(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int\left(1 - \frac{1}{\sqrt{1-x^2}}\right) dx$$

$$\Rightarrow I = x\log(\sqrt{1-x} + \sqrt{1+x}) - \frac{x}{2} + \frac{1}{2}\sin^{-1} x + c$$

Hence, $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$, $A = -\frac{1}{2}$ and $B = \frac{1}{2}$

439 **(a)**

Let $I = \int_0^{\pi/4} \log(1 + \tan x) dx$ ....(i)

$$\Rightarrow\ I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$$

$$= \int_0^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$$

$$= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$$

$$\Rightarrow\ I = \log 2[x]_0^{\pi/4} - I$$

$$\Rightarrow\ 2I = \frac{\pi}{4}\log_e 2\ \Rightarrow\ I = \frac{\pi}{8}\log_e 2$$

440 **(a)**

$f(x) = |x|\sin^3 x$ is an odd function

$$\therefore\ \int_{-2}^{2} |x|\sin^3 x\, dx = 0$$

441 **(c)**

Since, $e^{x-[x]}$ is a periodic function with period 1.

$\therefore \int_0^{1000} e^{x-[x]}dx$

$= 1000\int_0^1 e^x dx = 1000[e^x]_0^1$

$= 1000(e-1)$

442 **(b)**

Put $\tan^{-1}x = t \Rightarrow \frac{1}{(1+x^2)}dx = dt$

$\therefore \int \frac{(\tan^{-1}x)^3}{(1+x^2)}dx = \int t^3\,dt = \frac{t^4}{4}+c$

$= \frac{(\tan^{-1}x)^4}{4}+c$

443 **(c)**

Let $I = \int_0^\pi \sin^3\theta d\theta$

$\left[\begin{matrix}\because \sin\theta > 0 \\ \text{for } 0<\theta<\pi\end{matrix}\right]$

$= \int_0^\pi \sin\theta(1-\cos^2\theta)d\theta$

Put $\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

$\therefore I = \int_{-1}^1 (1-t^2)dt = \left[t-\frac{t^3}{3}\right]_{-1}^1$

$= 2-\frac{2}{3}$

$= \frac{4}{3}$

444 **(a)**

We have, $d[f(x)] = e^{\tan x}\sec^2 x\,dx$

On integrating w.r.t x, we get

$f(x) = e^{\tan x}+c$

445 **(c)**

We have,

$I = \int_1^2 \{f(g(x))\}^{-1} f'(g(x))\,g'(x)dx$

$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} f'(g(x))\,g'(x)dx$

$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} d(fog(x)) = [\log(fog\,(x))]_1^2$

$\Rightarrow I = \log\{f(g(2))\} - \log\{f(g(1))\}$

$= 0\ [\because g(1) = g(2)]$

446 **(d)**

Given, $f'(x) \geq 2$

On integrating both sides w.r.t. x, we get

$f(x) \geq 2x+c$

$f(1) \geq 2\times 1+c$

$\Rightarrow c \leq 4-2 \Rightarrow c \leq 2$

$\Rightarrow f(x) \geq 2x+2$

$\therefore f(4) \geq 2\times 4+2 = 10$

447 **(c)**

Let $I = \int \frac{f'(x)}{f(x)\log f(x)}dx$

Put $\log f(x) = t \Rightarrow \frac{f'(x)}{f(x)}dx = dt$

$\therefore I = \int \frac{1}{t}\,dt = \log t + c$

$= \log\log f(x) + c$

448 **(a)**

Let $I = \frac{2}{3}\int_0^{\pi/2} \frac{\sqrt{\cos\theta}}{(\sqrt{\sin\theta}+\sqrt{\cos\theta})}d\theta$ ...(i)

And $I = \frac{2}{3}\int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2-\theta)}}{\sqrt{\sin(\pi/2-\theta)}+\sqrt{\cos(\pi/2-\theta)}}d\theta$

$\Rightarrow I = \frac{2}{3}\int_0^{\pi/2}\frac{\sqrt{\sin\theta}}{\sqrt{\sin\theta}+\sqrt{\cos\theta}}d\theta$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \frac{2}{3}\int_0^{\pi/2} 1d\theta = \frac{2}{3}[\theta]_0^{\pi/2} \Rightarrow I = \frac{1}{3}.\frac{\pi}{2} = \frac{\pi}{6}$

449 **(a)**

$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots+\frac{n}{n^2+n^2}\right]$

$= \lim_{n\to\infty}\sum_{r=1}^n \frac{n}{n^2+r^2}$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^n \frac{1}{1+\left(\frac{r}{n}\right)^2}$

$= \int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]_0^1 = \frac{\pi}{4}$

450 **(d)**

Integrand is an odd function. So, the value of the integral is zero

451 **(a)**

Let $I = \int \frac{\sin x}{\cos x(1+\cos x)}dx$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$\therefore I = \int \frac{-dt}{t(1+t)} = -\int\left[\frac{1}{t}-\frac{1}{(1+t)}\right]dt$

$= \log\left(\frac{t+1}{t}\right)+c = f(x)+c$ [given]

$\therefore f(x) = \log\left(\frac{t+1}{t}\right) = \log\left(\frac{1+\cos x}{\cos x}\right)$

452 **(a)**

Since $f(x)$ satisfies conditions of Rolle's theorem on [1, 2]. Therefore, $f(1) = f(2)$

Hence, $\int_1^2 f'(x)\,dx = f(2)-f(1) = 0$

453 **(a)**

$\int e^{2x}(2\sin 3x + 3\cos 3x)dx$

$= 2\int e^{2x}\sin 3x + dx + 3\int e^{2x}\cos 3x\, dx$

$= e^{2x}\sin 3x - 3\int e^{2x}\cos 3x\, dx$

$+ 3\int e^{2x}\cos 3x\, dx$

$= e^{2x}\sin 3x + c$

454 **(d)**

$\int_{-2}^{2} |[x]|dx$

$= \int_{-2}^{-1} |[x]|dx + \int_{-1}^{0} |[x]|dx + \int_{0}^{1} |[x]|dx$

$+ \int_{1}^{2} |[x]|dx$

$= \int_{-2}^{-1} 2\, dx + \int_{-1}^{0} 1\, dx + \int_{0}^{1} 0\, dx + \int_{1}^{2} 1\, dx$

$= 2[x]_{-2}^{-1} + [x]_{-1}^{0} + 0 + [x]_{1}^{2}$

$= 2(-1+2) + (0+1) + (2-1) = 2+1+1 = 4$

455 **(c)**

We have,

$\int_0^{\pi/2} \cos^n x \sin^n x\ dx = \lambda \int_0^{\pi/2} \sin^n x\ dx$

$\Rightarrow \int_0^{\pi/2} (\sin 2x)^n\, dx = 2^n \lambda \int_0^{\pi/2} \sin^n x\ dx$

$\Rightarrow \int_0^{\pi} (\sin t)^n dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$

$\Rightarrow 2\int_0^{\pi/2} \sin^n t\, dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$

$\left[\because \int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx \text{ if } f(2a-x) = f(x)\right]$

$\Rightarrow 2 = 2^{n+1}\lambda \Rightarrow \lambda = \frac{1}{2^n}$

456 **(a)**

Clearly, $\frac{t^2 \sin 2t}{t^2+1}$ is an odd function

$\therefore \int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1} dt = 0$

Also,

$\int_0^1 \frac{1}{t^2 + 2t\cos\alpha + 1} dt = \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^1$

$= \frac{\alpha}{2\sin\alpha}$

Thus the given equation reduces to

$x^2 \frac{\alpha}{2\sin\alpha} - 2 = 0 \Rightarrow x = \pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

457 **(a)**

Let $I = \int_{\pi/3}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$\int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\right) dx \ \ldots$(i)

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$ ...(ii)

$\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

On adding Eqs. (i) and (ii), we get

$2I = \int_{\pi/6}^{\pi/3} 1\, dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$

$\Rightarrow I = \frac{\pi}{12}$

458 **(d)**

$\int_0^2 [x^2]dx$

$= \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx$

$= \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$

$= \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dx + \int_{\sqrt{3}}^{2} 3dx$

$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$

$= 5 - \sqrt{3} - \sqrt{2}$

459 **(a)**

Given, $\int f(x)\sin x\cos x\, dx = \frac{1}{2(b^2-a^2)}\log\{f(x)\} + c$

On differentiating both sides, we get

$f(x)\sin x\cos x = \frac{1}{2(b^2-a^2)} \cdot \frac{1}{f(x)} f'(x)$

$\Rightarrow\ 2(b^2-a^2)\sin x\cos x = \frac{f'(x)}{[f(x)]^2}$

$\Rightarrow\ \int (2b^2 \sin x\cos x - 2a^2 \sin x\cos x)dx$

$= \int \frac{f'(x)}{[f(x)]^2} dx$

$I_1 - I_2 = \int \frac{f'(x)}{[f(x)]^2}$ ...(i)

Where $I_1 = \int 2b^2 \sin x\cos x\, dx$ and $I_2 = \int 2a^2 \sin x\cos x\, dx$

Now, $I_1 = 2b^2\ \sin x\cos x\, dx$

Put $\cos x = t \ \Rightarrow\ -\sin x\, dx = dt$

$\therefore\ I_1 = -\int 2b^2 t dt = -b^2 t^2 = -b^2 \cos^2 x$

and $I_2 = \int 2a^2 \sin x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = t$

$\therefore\ I_2 = \int 2a^2\, t dt = a^2 t^2 = a^2 \sin^2 x$

$\therefore$ From Eq. (i),

$$-b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow\ f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

460 **(c)**

Let $I = \int_0^a x f(x) dx$ ...(i)

$$\Rightarrow I = \int_0^a (a-x) f(a-x) dx$$

$$I = \int_0^a (a-x) f(x) dx$$

$[\because f(x) = f(a-x)$given$]$

$$\Rightarrow I = a\int_0^a f(x)dx - \int_0^a x f(x) dx \quad \text{... (ii)}$$

On adding Eqs.(i) and (ii), we get

$$2I = a\int_0^a f(x)dx$$

$$\Rightarrow I = \frac{a}{2}\int_0^a f(x)dx$$

461 **(a)**

We have,

$$I = \int_{1/e}^{e} |\log x|\, dx$$

$$\Rightarrow I = \int_{1/e}^{e} -\log x\, dx + \int_{1}^{e} \log x\, dx$$

$$\Rightarrow I = [x - x\log x]_{1/e}^{1} + [x \log x - x]_1^e = 2\left(\frac{e-1}{e}\right)$$

462 **(d)**

Let $I = \int_0^{\pi/8} \cos^3 4\theta d\theta$

$$= \int_0^{\pi/8} \left(\frac{3\cos 4\theta + \cos 12\theta}{4}\right) d\theta$$

$$= \frac{1}{4}\left[\frac{3\sin 4\theta}{4} + \frac{\sin 12\theta}{12}\right]_0^{\pi/8}$$

$$= \frac{1}{4}\left[\frac{3}{4}\left(\sin\frac{\pi}{2} - 0\right) + \frac{1}{12}\left(\sin\frac{3\pi}{2} - 0\right)\right]$$

$$= \frac{1}{4}\left[\frac{3}{4} + \frac{1}{12}(-1)\right] = \frac{1}{4}\left(\frac{8}{12}\right) = \frac{1}{6}$$

463 **(c)**

$$I = \int_{-1}^{1} \frac{\cos\text{h}\, x}{1+e^{2x}} dx$$

$$= \int_{-1}^{1} \frac{e^x + e^{-x}}{2(1+e^{2x})} dx \quad \left[\because \cosh x = \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{1}{2}\int_{-1}^{1} \frac{1+e^{2x}}{(1+e^{2x})e^x} dx = \frac{1}{2}\int_{-1}^{1} e^{-x} dx$$

$$= -\frac{1}{2}[e^{-x}]_{-1}^{1} = -\frac{1}{2}(e^{-1} - e^1) = \frac{e^2 - 1}{2e}$$

464 **(c)**

We have,

$$u_{10} = \int_0^{\pi/2} x^{10} \sin x\, dx$$

$$\Rightarrow u_{10} = [-x^{10}\cos x]_0^{\pi/2} + 10\int_0^{\pi/2} x^9 \cos x\, dx$$

$$\Rightarrow u_{10} = 10[x^9 \sin x]_0^{\pi/2} - 90\int_0^{\frac{\pi}{2}} x^8 \sin x\, dx$$

$$= 10\left(\frac{\pi}{2}\right)^9 - 90\, u_8$$

$$\Rightarrow u_{10} + 90 u_8 = 10\left(\frac{\pi}{2}\right)^9$$

465 **(c)**

Let $I = \int \frac{x^3 \sin[\tan^{-1}(x^4)]dx}{1+x^8}$

Put $x^4 = t \Rightarrow 4x^3\, dx = dt$

$$\therefore\ I = \int \frac{1}{4}.\ \frac{\sin[\tan^{-1}(t)]dt}{1+t^2}$$

Again, put $\tan^{-1} t = u \Rightarrow \frac{1}{1+t^2} dt = du$

$$\therefore\ I = \int \frac{1}{4}\sin u\, du = -\frac{1}{4}\cos u + c$$

$$= -\frac{1}{4}\cos[\tan^{-1}(x^4)] + c$$

466 **(d)**

We have,

$$\sum_{n=1}^{10}\int_{-2n-1}^{-2n} \sin^{27} x\, dx + \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx$$

$$= -\sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} t\, dt$$

$$+ \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx, \text{ where } x = -t$$

$$= -\sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\ dx + \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx = 0$$

467 **(d)**

$$\int \frac{1}{\sin^2 x . \cos^2 x}\, dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$
$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx$$
$$= \int (\sec^2 x + \mathrm{cosec}^2\, x)\, dx$$
$$= \tan x - \cot x + c$$

468 **(a)**

We have,

$$I = \int_{-1}^{1} (x - [2x])\, dx = \int_{-1}^{1} x\, dx - \int_{-1}^{1} [2x]\, dx$$
$$\Rightarrow I = 0 - \left[\int_{-1}^{-1/2} [2x]\, dx + \int_{-1/2}^{0} [2x] dx + \int_{0}^{1/2} [2x] dx + \int_{1/2}^{1} [2x]\, dx\right]$$
$$\Rightarrow I = -\left[\int_{-1}^{-1/2} -2dx + \int_{-1/2}^{0} -1dx + \int_{0}^{1/2} 0dx + \int_{1/2}^{1} 1dx\right]$$
$$\Rightarrow I = -\left[-2\left(-\frac{1}{2}+1\right) - 1\left(0+\frac{1}{2}\right) + 0 + \left(1-\frac{1}{2}\right)\right]$$
$$\Rightarrow I = -\left[-1 - \frac{1}{2} + \frac{1}{2}\right] = 1$$

469 **(a)**

We know that if $f(t)$ is an odd function. Then, $\int_0^x f(t)\, dt$ is an even function.

Since the function $f(x) = \log\frac{1-x}{1+x}$ is an odd function. Therefore, $F(x)$ is an even function

471 **(c)**

We have

$$\frac{d}{dx}\left\{\int_{f(x)}^{g(x)} \phi(t) dt\right\} = g'(x).\phi(g(x)) - f'(x).\phi(f(x))$$

472 **(d)**

$$\int_1^3 (x-1)(x-2)(x-3) dx$$
$$= \int_1^3 (x-1)(x^2 - 5x + 6) dx$$
$$= \int_1^3 (x^3 - 6x^2 + 11x - 6) dx$$
$$= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{11x^2}{2} - 6x\right]_1^3$$
$$= \left[\frac{81}{4} - \frac{162}{3} + \frac{99}{2} - 18 - \left(\frac{1}{4} - \frac{6}{3} + \frac{11}{2} - 6\right)\right]$$
$$= \left[-\frac{27}{12} + \frac{27}{12}\right] = 0$$

473 **(c)**

Given, $f(x) =$

$$\begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - (C_2 + C_3)$, we get

$$= \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4\sin x \\ 0 & \sin x & 1 \end{vmatrix}$$
$$= \sin x(3 - 4\sin^2 x) = 3\sin x - 4\sin^3 x$$
$$f(x) = \sin 3x$$
$$\therefore \int_0^{\pi/2} f(x) dx = \int_0^{\pi/2} \sin 3x\, dx$$
$$= \left[-\frac{\cos 3x}{3}\right]_0^{\pi/2} = \left(-\frac{1}{3}\right)[\cos 3x]_0^{\pi/2}$$
$$= \left(-\frac{1}{3}\right)\left[\cos\left(3 \times \frac{\pi}{2}\right) - \cos 0\right]$$
$$= \left(-\frac{1}{3}\right)(0 - 1) = \frac{-1}{3}(-1) = \frac{1}{3}$$

474 **(c)**

We have,

$$I = \int g(x)\{f(x) + f'(x)\}\, dx$$
$$\Rightarrow I = \int g(x) f(x) dx + \int g(x) f'(x)\, dx$$
$$\Rightarrow I = f(x)\left(\int g(x) dx\right) - \int \left(f'(x) \int g(x) dx\right) dx + \int g(x) f'(x)\, dx$$
$$\Rightarrow I = f(x) g(x) - \int g(x) g'(x) dx + \int g(x) g'(x) dx + C$$
$$\Rightarrow I = f(x) g(x) + C \quad \left[\because \int g(x) dx = g(x)\right]$$

475 **(a)**

$$\int \left(\frac{x+2}{x+4}\right)^2 e^x\, dx = \int e^x \left[\frac{x^2 + 4x + 4}{(x+4)^2}\right] dx$$
$$= \int e^x \left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right] dx$$
$$= \int e^x \left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right] dx$$
$$= \int \frac{e^x\, x}{(x+4)} dx + \int \frac{4e^x}{(x+4)^2} dx$$
$$= e^x \left(\frac{x}{x+4}\right) - \int \frac{4e^x}{(x+4)^2} dx + \int \frac{4e^x}{(x+4)^2} dx$$

$= \frac{xe^x}{(x+4)} + c$

476 **(c)**

Given, $P = \int_0^{3\pi} f(\cos^2 x)dx$ ...(i)

and $Q = \int_0^{\pi} f(\cos^2 x)\,dx$ ...(ii)

From Eq. (i)

$P = 3\int_0^{\pi} f(\cos^2 x)dx$

$\Rightarrow P = 3Q \Rightarrow P - 3Q = 0$

477 **(d)**

Let $g(x) = f(\cos^2 x)$. Then,

$g(x+n\pi) = f\{\cos^2(x+n\pi)\} = f(\cos^2 x) = g(x)$

$\therefore \int_0^{n\pi} g(x)\,dx = n\int_0^{\pi} g(x)\,dx$

$\Rightarrow \int_0^{3\pi} g(x)\,dx = 3\int_0^{\pi} g(x)\,dx$ [Putting $n = 3$]

$\Rightarrow \int_0^{3\pi} f(\cos^2 x)\,dx = 3\int_0^{\pi} f(\cos^2 x)\,dx$

$\Rightarrow I_1 = 3I_2$

478 **(c)**

We know that $x - [x]$ is a periodic function with period 1 unit. Therefore,

$\int_0^{n[x]} (x-[x])dx = n[x]\int_0^1 (x-[x])\,dx$

$= n[x]\int_0^1 x dx = \frac{n}{2}[x]$

479 **(d)**

Put $t = x^2 + 1 \Rightarrow dt = 2x\,dx$

$\int_0^2 \frac{x^3}{(x^2+1)^{3/2}}dx = \frac{1}{2}\int_1^5 \frac{(t-1)}{t^{3/2}}dt$

$= \frac{1}{2}\int_1^5 [t^{-\frac{1}{2}} - t^{-3/2}]dt$

$= \frac{1}{2}\left[2\sqrt{t} + 2\frac{1}{\sqrt{t}}\right]_1^5$

$= \frac{1}{2}\left[2\sqrt{5} + \frac{2}{\sqrt{5}} - 2 - 2\right]$

$= \left[\sqrt{5} + \frac{1}{\sqrt{5}} - 2\right] = \frac{6-2\sqrt{5}}{\sqrt{5}}$

480 **(d)**

We have,

$I = \int_0^{2\pi} |\cos x - \sin x|\,dx$

$\Rightarrow I = \sqrt{2}\int_0^{2\pi} \left|\cos\left(x + \frac{\pi}{4}\right)\right| dx$

$\Rightarrow I = \sqrt{2}\int_{\pi/4}^{9\pi/4} |\cos t\,|\,dt$, where $t = x + \frac{\pi}{4}$

$\Rightarrow I = \sqrt{2}\left\{\int_{\pi/4}^{\pi/2} \cos t\,dt + \int_{\pi/2}^{3\pi/2} (-\cos t)dt + \int_{3\pi/2}^{9\pi/4} \cos t\,dt\right\}$

$\Rightarrow I = \sqrt{2}\left[\left(1 - \frac{1}{\sqrt{2}}\right) - (-1-1) + \left(\frac{1}{\sqrt{2}} + 1\right)\right]$

$= 4\sqrt{2}$

481 **(c)**

Let $I = \int_{-\pi/2}^{\pi/2}(x^3 + x\cos x + \tan^5 x + 1)dx$

Since, $x^3, x\cos x$ and $\tan^5 x$ are odd functions, therefore

$I = \int_{-\pi/2}^{\pi/2} 1dx = [x]_{-\pi/2}^{\pi/2} = \pi$

482 **(a)**

$\because f'(x) = g(x)$

$\Rightarrow \int_a^b f(x)g(x)dx = \int_a^b f(x)f'(x)dx$

$= \left[\frac{(f(x))^2}{2}\right]_a^b$

$= \frac{1}{2}[(f(b))^2 - (f(a))^2]$

483 **(c)**

We have,

$I = \int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}dx = \int \frac{4e^{2x} + 6}{9e^{2x} - 4}dx$

$\Rightarrow I = \int \frac{4\,e^{2x}}{9\,e^{2x} - 4}dx + 6\int \frac{1}{9\,e^{2x} - 4}dx$

$\Rightarrow I = \frac{2}{9}\int \frac{18\,e^{2x}}{9e^{2x} - 4}dx + 6\int \frac{e^{-2x}}{9 - 4e^{-2x}}dx$

$\Rightarrow I = \frac{2}{9}\log(9e^{2x} - 4) + \frac{6}{8}\log(9 - 4e^{-2x}) + C$

$\Rightarrow I = \frac{2}{9}\log(9\,e^{2x} - 4) + \frac{3}{4}\log(9\,e^{2x} - 4) - \frac{3}{4}\log e^{2x} + C$

$\Rightarrow I = \frac{-3}{2}x + \frac{35}{36}\log(9e^{2x} - 4) + C$

Hence, $A = -\frac{3}{2}, b = \frac{35}{36}$ and $C \in R$

484 **(b)**

Let $I = \int \frac{\cos 4x + 1}{\cot x - \tan x} dx$

$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx$

$= \int \frac{2\cos^2 2x}{\cos 2x} . \sin x \cos x \, dx$

$= \int \cos 2x . \sin 2x \, dx$

$= \frac{1}{2} \int 2 \cos 2x \sin 2x \, dx$

$= \frac{1}{2} \int \sin 4x \, dx = \frac{1}{2} \frac{(-\cos 4x)}{4} + c$

$\Rightarrow I = \frac{-\cos 4x}{8} + c = k \cos 4x + c$

$\therefore k = -\frac{1}{8}$

485 **(c)**

Let $I = \int_{-2}^{4} |x+1| dx$

$= \int_{-2}^{-1} -(x+1) dx + \int_{-1}^{4} (1+x) dx$

$= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[x + \frac{x^2}{2}\right]_{-1}^{4}$

$= -\left[\frac{1}{2} - 1 - (2-2)\right] + \left[4 + 8 — \left(1 + \frac{1}{2}\right)\right]$

$= \frac{1}{2} + \left(\frac{25}{2}\right) = 13$

486 **(c)**

$\int_2^3 \frac{dx}{x^2 - x}$

$= \int_2^3 \frac{dx}{x(x-1)} = \int_2^3 \left[\frac{1}{x-1} - \frac{1}{x}\right] dx$

$= [\log(x-1)]_2^3 - [\log x]_2^3$

$= [\log 2 - \log 1] - [\log 3 - \log 2]$

$= 2\log 2 - \log 3 = \log \frac{4}{3}$

487 **(d)**

Let $I = \int \frac{x^2+1}{x^4 - x^2 + 1} dx = \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 1} dx$

$= \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 1} dx$

Put $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$

$\therefore I = \int \frac{dt}{t^2 + 1} = \tan^{-1} t + c$

$= \tan^{-1}\left(x - \frac{1}{x}\right) + c$

$= \tan^{-1}\left(\frac{x^2 - 1}{x}\right) + c$

488 **(c)**

Let $I = \int e^x \sec x \, dx + \int e^x \sec x \tan x \, dx$

$= e^x \sec x - \int e^x \sec x \tan x \, dx$

$+ \int e^x \sec x \tan x \, dx + c$

$= e^x \sec x + c$

489 **(d)**

$f'(x) = 2xe^{-(x^2+1)^2} - 2xe^{-x^4}$

$= 2x\left(e^{-(x^2+1)^2} - e^{-x^4}\right)$ $\left[\text{here } e^{-x^4} > e^{-(x^2+1)^2}\right]$

So, $f'(x) > 0$, when $x < 0$ $[\text{as } x^4 < (x^2+1)^2]$

491 **(b)**

Given, $\int_0^a x \, dx \le a + 4$

$\Rightarrow \left[\frac{x^2}{2}\right]_0^a \le a + 4 \Rightarrow \frac{a^2}{2} \le a + 4$

$\Rightarrow a^2 \le 2a + 8 \Rightarrow a^2 - 2a - 8 \le 0$

$\Rightarrow (a-4)(a+2) \le 0 \Rightarrow -2 \le a \le 4$

492 **(a)**

Given, $u_n = \int_0^{\pi/4} \tan^n x \, dx$

or $u_n = \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1) \, dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x \, dx - \int_0^{\pi/4} \tan^{n-2} x \, dx$

On putting $\tan x = t, \sec^2 x \, dx = dt$ in Ist integral, we get

$u_n = \int_0^1 t^{n-2} dt - u_{n-2}$

$\Rightarrow u_n + u_{n-2} = \left[\frac{t^{n-1}}{n-1}\right]_0^1 = \left[\frac{1^{n-1}}{n-1} - 0\right] = \frac{1}{n-1}$

493 **(b)**

Let $I = \int_0^\pi x \sin^4 x \, dx$ ...(i)

$I = \int_0^\pi (\pi - x) \sin^4 x \, dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \pi \int_0^\pi \sin^4 x \, dx$

$= 2\pi \int_0^{\pi/2} \sin^4 x \, dx$

$= 2\pi . \frac{4-1}{4} . \frac{4-3}{4-2} . \frac{\pi}{2}$

$= 2\pi.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{3\pi^2}{8}$

$\Rightarrow I = \frac{3\pi^2}{16}$

494 **(a)**

We have,

$\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x \; dx$

$\Rightarrow I = \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x} \cos x \; dx$

$= \int e^{\sin x} \cos x \; dx = e^{\sin x} + C$

495 **(c)**

Let $I = \int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

Now, let $x = t^6 \Rightarrow dx = 6t^5 dt$

$\Rightarrow I = \int \frac{6t^5}{t^3+t^2} dt = 6\int \frac{t^3}{t+1} dt$

$= 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$

$= 2t^3 - 3t^2 + 6t - 6\log(t+1) + c$

$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x}+1) + c$

496 **(a)**

We have,

$I = \int \sqrt[3]{x} \sqrt[7]{1+\sqrt[3]{x^4}} dx$

$= \frac{3}{4}\int \sqrt[7]{1+x^{4/3}} \times \frac{4}{3}x^{1/3} dx$

$\Rightarrow I = \frac{3}{4}\int (1+x^4)^{1/7} d(1+x^{4/3})$

$\Rightarrow I = \frac{3}{4} \times \frac{7}{8} \times \left(1+x^{4/3}\right)^{8/7} + C$

$= \frac{21}{32}\left(1+x^{4/3}\right)^{8/7} + C$

497 **(a)**

Let $I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

$\therefore \quad I = \int \sin^{-1}(\sin 2\theta).\sec^2\theta \, d\theta$

$= 2\int \theta \sec^2\theta \, d\theta = 2[\theta\tan\theta - \int \tan\theta \, d\theta]$

$= 2[\theta\tan\theta + \log\cos\theta] + c$

$= 2\left[x\tan^{-1}x + \log\frac{1}{\sqrt{1+x^2}}\right] + c$

$= 2x\tan^{-1}x - \log(1+x^2) + c$

$= f(x) - \log(1+x^2) + c \quad$ [given]

$\therefore \; f(x) = 2x\tan^{-1}x$

498 **(b)**

Given $I_{10} = \int_0^{\pi/2} x^{10} \sin x \, dx$

$= [-x^{10}\cos x]_0^{\pi/2} + \int_0^{\pi/2} 10x^9.\cos x dx$

$= 0 + [10x^9 \sin x]_0^{\pi/2} - \int_0^{\pi/2} 90x^8 \sin x \, dx$

$= 10\left(\frac{\pi}{2}\right)^9 - 90I_8$

$\Rightarrow I_{10} + 90I_8 = 10\left(\frac{\pi}{2}\right)^9$

499 **(d)**

Let $I = \int_0^\pi \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ ...(i)

and $I = \int_0^\pi \frac{(\pi-x)dx}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}$

$\Rightarrow \; I = \int_0^\pi \frac{(\pi-x)dx}{a^2\cos^2 x+b^2\sin^2 x}$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_0^\pi \frac{(x+\pi-x)dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow \; 2I = \pi\int_0^\pi \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow \; I = \frac{\pi}{2}\int_0^\pi \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow \; I = 2\cdot\frac{\pi}{2}\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

On dividing numerator and denominator be $\cos^2 x$, we get

$I = \pi\int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$

Put $b\tan x = t \; \Rightarrow b\sec^2 x\,dx = dt$

$\therefore \quad I = \frac{\pi}{b}\int_0^\infty \frac{dt}{a^2+t^2} = \frac{\pi}{b}\cdot\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_0^\infty$

$= \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{2ab}$

500 **(a)**

$\int_{-1}^1 (x - [x])dx$

$= \int_{-1}^0 (x+1)dx + \int_0^1 (x-0)dx$

$= \left[\frac{x^2}{2} + x\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} + \frac{1}{2} = 1$

501 **(d)**

We have,

$I = \int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$

$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 px + \sin^2 qx - 2\cos px \sin qx)dx$

Since $\cos^2 px$ and $\sin^2 qx$ are even functions of x and $\cos px \sin qx$ an odd function. Therefore,

$$\int_{-\pi}^{\pi} \cos^2 px\, dx = 2\int_0^{\pi} \cos^2 px\, dx\,, \int_{-\pi}^{\pi} \sin^2 qx\, dx$$

$$= 2\int_0^{\pi} \sin^2 qx\, dx$$

and, $\int_{-\pi}^{\pi} \cos px \sin qx\ \ dx = 0$

$$\therefore I = 2\int_0^{\pi} \cos^2 px\, dx + 2\int_0^{\pi} \sin^2 qx\, dx$$

$$\Rightarrow I = 2\int_0^{\pi} \left(\frac{1+\cos 2\ px}{2}\right) dx$$

$$+ 2\int_0^{\pi} \left(\frac{1-\cos 2\,qx}{2}\right) dx$$

$$\Rightarrow I = \int_0^{\pi} (1+\cos 2px)\, dx + \int_0^{\pi} (1-\cos 2qx)\, dx$$

$$\Rightarrow I = \left[x + \frac{\sin 2\ px}{2\,p}\right]_0^{\pi} + \left[\frac{x - \sin 2\,qx}{2q}\right]_0^{\pi} = 2\pi$$

502 **(d)**

We have,

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^3}{r^4+n^4} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{(r/n)^3}{1+(r/n)^4} . \frac{1}{n}$$

$$= \int_0^1 \frac{x^3}{1+x^4} dx = \frac{1}{4}[\log(1+x^4)]_0^1 = \frac{1}{4}\log 2$$

503 **(b)**

We have,

$$\int_{\pi/2}^{x} \sqrt{3-2\sin^2 u}\ du + \int_0^y \cos t\, dt = 0$$

Differentiating w.r.t. x, we get

$$\sqrt{3-2\sin^2 x} + \frac{dy}{dx}\cos y = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{3-2\sin^2 x}}{\cos y}$$

504 **(b)**

$$I_n = \int \tan^n x\, dx = \int \tan^{n-2} x\,(\sec^2 x - 1) dx$$

$$= \int \tan^{n-2} x\,.\sec^2 x\, dx - I_{n-2}$$

$$\Rightarrow\ I_n + I_{n-2} = \int \tan^{n-1} x\,.\sec^2 x\, dx$$

Put $\tan x = t \quad \Rightarrow \quad \sec^2 x\, dx = dt$

$$\therefore\ I_n + I_{n-2} = \int t^{n-2}\, dt = \frac{t^{n-1}}{n-1} + c$$

$$\Rightarrow\ I_n + I_{n-2} = \frac{(\tan^{n-1} x)}{n-1} + c$$

505 **(c)**

We have,

$$I_{n+1} - I_n = 2\int_0^{\pi} \cos(n+1)\,x\, dx = 0$$

$\therefore I_{n+1} = I_n$

$\Rightarrow I_{n+1} = I_n = \cdots = I_0 \Rightarrow I_n = \pi$ for all $n \geq 0$

506 **(a)**

Let $\ I = \int(\sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x)dx$

$$= \int \{(\sin^2 x)^3 + (\cos^2 x)^3 + 3\sin^2 x \cos^2 x\}\, dx$$

$$= \int \begin{bmatrix}(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x \\ -\sin^2 x \cos^2 x) + 3\sin^2 x\cos^2 x\end{bmatrix} dx$$

$$= \int \begin{bmatrix}(\sin^2 x + \cos^2 x) - 3\sin^2 x \cos^2 x \\ +3\sin^2 x \cos^2 x\end{bmatrix} dx$$

$$= \int 1\ \ dx = x + c$$

507 **(a)**

$$\int_a^b \frac{|x|}{x} dx = \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx \quad [\because a < 0 < b]$$

$$= \int_a^0 -1\, dx + \int_0^b 1\, dx$$

$$= a + b = b - (-a)|b| - |a|$$

508 **(a)**

$$\int_0^{10\pi} |\sin x| dx$$

$$= 10\left[\int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \sin x\, dx\right]$$

$[\because |\sin x|$is periodic with period $\pi]$

$= 10[[-\cos x\,]_0^{\pi/2} + [-\cos x\,]_{\pi/2}^{\pi}]$

$= 10[1+1] = 20$

510 **(a)**

Let $\ I = \int \frac{(\sec^2 x - 7)}{\sin^7 x}\, dx$

$$= \int \frac{\sin^7 x \sec^2 x - 7\sin^7 x}{(\sin^7 x)^2}\, dx$$

$$= \int \frac{d}{dx}\left(\frac{\tan x}{\sin^7 x}\right) dx$$

$$= \frac{\tan x}{\sin^7 x} + c$$

511 **(a)**

We have,

$$I=\int_{-1/2}^{1/2}\left|x\cos\frac{\pi}{2}x\right|dx$$

$$\Rightarrow I=-\int_{-1/2}^{0}x\cos\frac{\pi x}{2}dx+\int_{0}^{1/2}x\cos\frac{\pi x}{2}dx$$

$$=\frac{\pi\sqrt{2}+4\sqrt{2}-8}{\pi^2}$$

512 **(b)**

Let $I=\int\frac{\log_e(\tan x)}{\sin x\cos x}dx=\int\frac{\log_e(\tan x)}{\tan x}\sec^2 x\,dx$

Put $\tan x=t\ \Rightarrow\ \sec^2 x\,dx=dt$

$$\therefore I=\int\frac{\log_e t}{t}dt$$

Again, put $\log_e t=u\ \Rightarrow\ \frac{1}{t}dt=du$

$$\therefore\ I=\int u\,du=\frac{u^2}{2}+c$$

$$=\frac{(\log_e t)^2}{2}+c$$

$$=\frac{1}{2}[\log_e\tan x]^2+c$$

513 **(a)**

Let $I=\int\left(\frac{x+2}{x+4}\right)^2e^x dx=\int e^x\left[\frac{x^2+4+4x}{(x+4)^2}\right]dx$

$$\Rightarrow I=\int e^x\left[\frac{x(x+4)}{(x+4)^2}+\frac{4}{(x+4)^2}\right]dx$$

$$\Rightarrow I=\int\frac{e^x\,x}{x+4}\,dx+\int\frac{4e^x}{(x+4)^2}\,dx$$

$$\Rightarrow I=e^x\left(\frac{x}{x+4}\right)-\int\frac{4e^x}{(x+4)^2}\,dx$$

$$+\int\frac{4e^x}{(x+4)^2}\,dx$$

$\Rightarrow I=\frac{xe^x}{(x+4)}+c$

514 **(b)**

We have,

$$I=\int\frac{1}{\sin(x-a)\cos(x-b)}dx$$

$$\Rightarrow I=\frac{1}{\cos(a-b)}\int\frac{\cos\{(x-a)-(x-b)\}}{\sin(x-a)\cos(x-b)}dx$$

$$\Rightarrow I=\frac{1}{\cos(a-b)}\int\{\cot(x-a)+\tan(x-b)\}$$

$$+dx$$

$$\Rightarrow I=\frac{1}{\cos(a-b)}\{\log\sin(x-a)-\log\cos(x-b)\}$$

$$+C$$

$$\Rightarrow I=\frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$$

515 **(a)**

We have,

$$I=\int_{-\pi/2}^{\pi/2}\frac{\cos x}{1+e^x}dx$$

$\Rightarrow I=\int_{-\pi/2}^{0}\frac{\cos x}{1+e^x}dx+\int_{0}^{\pi/2}\frac{\cos x}{1+e^x}dx$ ...(i)

Putting $x=-t$ in $\int_{-\pi/2}^{0}\frac{\cos x}{1+e^x}dx$, we get

$$\int_{-\pi/2}^{0}\frac{\cos x}{1+e^x}dx=\int_{0}^{\pi/2}\frac{e^x\cos x}{1+e^x}dx$$

$$\therefore I=\int_{0}^{\pi/2}\frac{e^x\cos x}{1+e^x}dx+\int_{0}^{\pi/2}\frac{\cos x}{1+e^x}dx$$

$$\Rightarrow I=\int_{0}^{\pi/2}\frac{(1+e^x)\cos x}{(1+e^x)}dx$$

$$\Rightarrow I=\int_{0}^{\pi/2}\cos x\,dx=[\sin x]_0^{\pi/2}=1$$

516 **(a)**

$$\int_{-1}^{3}\left[\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right]dx$$

$$=\int_{-1}^{3}\left[\tan^{-1}\left(\frac{x}{x^2+1}\right)\right.$$

$$\left.+\cot^{-1}\left(\frac{x}{x^2+1}\right)\right]dx\quad\left(\because\tan^{-1}=\cot^{-1}\frac{1}{x}\right)$$

$$=\int_{-1}^{3}\left(\frac{\pi}{2}\right)dx=\left[\frac{\pi x}{2}\right]_{-1}^{3}=\frac{\pi}{2}[3+1]$$

$$=2\pi\quad\left(\because\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right)$$

517 **(c)**

We have, $\int_n^{n+1}f(x)dx=n^2$

Putting $n-2,-1,0,1,2,3$, we get

$$\int_{-2}^{-1}f(x)\,dx=4,\int_{-1}^{0}f(x)\,dx=1,\int_{0}^{1}f(x)\,dx$$

$$=0,\int_{1}^{2}f(x)\,dx=1,$$

$$\int_{2}^{3}f(x)\,dx=4\text{ and}\int_{3}^{4}f(x)\,dx=9$$

$$\therefore\int_{-2}^{4}f(x)\,dx=4+1+0+1+4+9=19$$

518 **(a)**

We have , $\int_0^{2\pi}(\sin x+|\sin x|)dx$

$$= \int_0^{\pi} (\sin x + \sin x) dx + \int_{\pi}^{2\pi} (\sin x - \sin x)\ dx$$

$$= \int_0^{\pi} 2\sin x\, dx + 0 = 2[-\cos x]_0^{\pi}$$

$$= -2(\cos \pi - \cos 0) = 4$$

519 **(c)**

$$f(x) = \lim_{y\to x} \frac{\sin^2 y - \sin^2 x}{y^2 - x^2}\ \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{y\to x} \frac{2\sin y \cos y - 0}{2y - 0}$$

$$= \frac{\sin 2x}{2x}$$

$$\therefore \int 4x\ f(x) dx = \int 4x\ \left(\frac{\sin 2x}{2x}\right) dx$$

$$= 2\int \sin 2x\ \ dx$$

$$= -\cos 2x + c$$

520 **(a)**

We have,

$$I = \int \frac{x^4 + 1}{(1 - x^4)^{3/2}} dx = \int \frac{x^2\left(\frac{1}{x^2} + x^2\right)}{x^3\left(\frac{1}{x^2} - x^2\right)^{3/2}} dx$$

$$\Rightarrow I = \int \frac{x + \frac{1}{x^3}}{\left(\frac{1}{x^2} - x^2\right)^{3/2}} dx = -\frac{1}{2}\int \frac{-2x - \frac{2}{x^3}}{\left(\frac{1}{x^2} - x^2\right)^{3/2}} dx$$

$$\Rightarrow I = -\frac{1}{2}\int \left(\frac{1}{x^2} - x^2\right)^{-3/2} d\left(-x^2 + \frac{1}{x}\right)$$

$$\Rightarrow I = -\frac{1}{2}\left\{\frac{\left(\frac{1}{x^2} - x^2\right)^{-1/2}}{-1/2}\right\} + C$$

$$\Rightarrow I = \frac{1}{\sqrt{\frac{1}{x^2} - x^2}} + C = \frac{x}{\sqrt{1 - x^4}} + C$$

521 **(a)**

We have,

$$I = e^x \frac{x}{(x+1)^2} dx = \int e^x \frac{x + 1 - 1}{(x+1)^2} dx$$

$$\Rightarrow I = \int e^x \left\{\frac{1}{x+1} +* \frac{-1}{(x+1)^2}\right\} dx = e^x \frac{1}{x+1} + C$$

522 **(c)**

Let $I = \int_{-\pi/2}^{\pi/2} \sin|x|\, dx = 2\int_0^{\pi/2} \sin x\ dx$

$$= 2[-\cos x]_0^{\pi/2} = 2$$

523 **(a)**

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

$$= \int \frac{\sec^2 x\ dx}{a^2 \tan^2 x + b^2}$$

$$= \frac{1}{a^2}\int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

Where $t = \tan x$

$$= \frac{1}{a^2} \cdot \frac{a}{b} \tan^{-1} \frac{t}{b/a} + c$$

$$= \frac{1}{ab} \tan^{-1} \frac{a(\tan x)}{b} + c$$

524 **(a)**

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\ \cos^2 x}\ dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(1 - 2\sin^2 x\ \cos^2 x)}\ dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(1 - 2\sin^2 x\ \cos^2 x)} \times [(\sin^2 x\ \cos^2 x)^2 - 2\sin^2 x\ \cos^2 x]\, dx$$

$$= -\int \frac{\cos 2x(1 - 2\sin^2 x \cos^2 x)}{(1 - 2\sin^2 x \cos^2 x)}\ dx$$

$$= -\int \cos 2x\ \ dx$$

$$\Rightarrow A\sin 2x + B = -\frac{1}{2}\sin 2x + B \quad \text{(given)}$$

$$\therefore\ A = -\frac{1}{2}$$

525 **(a)**

Let $I = \int \frac{4^{x+1} - 7^{x-1}}{28^x}\ dx$

$$= 4\int \frac{1}{7^x}\ dx - \frac{1}{7}\int \frac{1}{4^x}\ dx$$

$$= -\frac{4.7^{-x}}{\log_e 7} + \frac{1}{7\ \log_e 4} 4^{-x} + c$$

526 **(b)**

We have,

$$I = \int \frac{1}{5 + 4\cos x} dx$$

$$\Rightarrow I = \int \frac{1 + \tan^2 x/2}{5(1 + \tan^2 x/2) + 4(1 - \tan^2 x/2)} dx$$

$$\Rightarrow I = 2\int \frac{1}{3^2 + \left(\tan\frac{x}{2}\right)^2} d\left(\tan\frac{x}{2}\right)$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{\tan x/2}{3}\right) + C$$

Hence, $A = 2/3$ and $B = 1/3$

527 **(b)**

Since, $f''(x) = \sec^4 x + 4$

$f''(x) = (1 + \tan^2 x)\sec^2 x + 4$

$$\Rightarrow f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x + c$$

$$\Rightarrow f'(0) = 0$$

$\Rightarrow 0 = c$

Then, $f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$

$= \tan x + \frac{1}{3}\tan x(\sec^2 x - 1) + 4x$

$\Rightarrow f'(x) = \frac{2}{3}\tan x + \frac{1}{3}\tan x \sec^2 x + 4x$

$\therefore f(x) = \frac{2}{3}\log|\sec x| + \frac{\tan^2 x}{6} 2x^2 + d$

But $f(0) = 0$

$\Rightarrow d = 0$

Then, $f(x) = \frac{2}{3}\log|\sec x| + \frac{1}{6}\tan^2 x + 2x^2$

528 **(b)**

Let $I = \int 32x^3 (\log x)^2\, dx$

$= 32\left\{(\log x)^2 \frac{x^4}{4} - \int 2\log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx\right\}$

$= 8x^4(\log x)^2 - 16\left\{\log x. \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx\right\}$

$= 8x^4(\log x)^2 - 4x^4 \log x + x^4 + c$

$= x^4[8(\log x)^2 - 4\log x + 1] + c$

529 **(b)**

We have,

$$I = \int_1^{\frac{4\sqrt{3}}{5}-1} \frac{x+2}{\sqrt{x^2+2x-3}} dx$$

$$\Rightarrow I = \frac{1}{2}\int_1^{\frac{4\sqrt{3}}{5}-1} \frac{(2x+2)}{\sqrt{x^2+2x-3}} dx + \int_1^{\frac{4\sqrt{3}}{5}-1} \frac{1}{\sqrt{(x+1)^2 - 2^2}}$$

$$\Rightarrow I = \left[\sqrt{x^2+2x-3}\right]_1^{\frac{4\sqrt{3}}{5}-1} + \left[\log(x+1) + \sqrt{x^2+2x-3}\right]_1^{\frac{4\sqrt{3}}{5}-1}$$

$$\Rightarrow I = \frac{2\sqrt{3}}{3} + \frac{1}{2}\log 3$$

530 **(d)**

Let $I = \int_{-2}^{2} |[x]| dx$

$$= \int_{-2}^{-1} |[x]| dx + \int_{-1}^{0} |[x]| dx + \int_{0}^{1} |[x]| dx + \int_{1}^{2} |[x]| dx$$

$$= \int_{-2}^{-1} 2dx + \int_{-1}^{0} 1dx + \int_{0}^{1} 0dx + \int_{1}^{2} 1dx$$

$= 2[x]_{-2}^{-1} + [x]_{-1}^{0} + 0 + [x]_1^2 = 4$

531 **(a)**

$F = (e) = f(e) + f\left(\frac{1}{e}\right)$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$$

On putting $t = \frac{1}{t}$ in second integration, we get

$$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{\log t}{t} dt = \left[\frac{(\log t)^2}{2}\right]_1^e$$

$$= \frac{1}{2}[(\log e)^2 - (\log 1)^2] = \frac{1}{2}$$

532 **(a)**

Let $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^3 x} dx$

$= \int_{\pi/6}^{\pi/3} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$ ...(i)

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

533 **(c)**

Let $f^{-1}(x) = u \Rightarrow x = f(u)$

$\Rightarrow dx = f'(u)du$

$\therefore I = \int f^{-1}(x)\, dx = \int u f'(u) du$

$\Rightarrow I = uf(u) - \int f(u)\, du$

$= uf(u) - g(u) + c$ [$\because \int f(x)\, dx = g(x) + c$, given]

$\therefore$ On putting $u = f^{-1}(x)$ and $f(u) = x$

We get, $I = xf^{-1}(x) - g\{f^{-1}(x)\} + c$

534 **(a)**

$$\int \underset{\text{II}}{x} \underset{\text{I}}{\log x}\, dx = \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\frac{x^2}{2} + c$$

$$= \frac{x^2}{4}(2\log x - 1) + c$$

535 **(a)**

We have,

$$I = \int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4} dx$$

$$\Rightarrow I=\int_{1/3}^{1}\left(\frac{1}{x^2}-1\right)^{1/3}\frac{1}{x^3}\,dx$$

Putting $\frac{1}{x^2}-1=t$ and $-\frac{2}{x^3}dx=dt$, we get

$$I=\int_8^0 t^{1/3}\times-\frac{1}{2}dt=\frac{1}{2}\int_0^8 t^{1/3}\,dt=\frac{1}{2}\times\frac{3}{4}\left[t^{4/3}\right]_0^8$$

$$\Rightarrow I=\frac{3}{8}\times 2^4=6$$

536 **(c)**

Let $I=\int_{1/3}^{3}\frac{1}{x}\sin\left(\frac{1}{x}-x\right)dx$. Then,

$$\Rightarrow I=\int_{3}^{1/3} t\sin\left(t-\frac{1}{t}\right)\times-\frac{1}{t^2}dt, \text{where } x=\frac{1}{t}$$

$$\Rightarrow I=\int_{1/3}^{3}\frac{1}{t}\sin\left(t-\frac{1}{t}\right)dt$$

$$\Rightarrow I=-\int_{1/3}^{3}\frac{1}{t}\sin\left(\frac{1}{t}-t\right)dt$$

$\Rightarrow I=-I$

$\Rightarrow 2I=0\Rightarrow I=0$

537 **(b)**

Let $I_1=\int_0^1\frac{x\,e^{x^2}}{x^2+1}dx$

Putting $x^2=t$, we get

$$I_1=\frac{1}{2}\int_0^1\frac{e^t}{t+1}dt=\frac{1}{2}I$$

538 **(b)**

$$\int(\sin^4 x-\cos^4 x)dx$$

$$=\int(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)dx$$

$$=\int(\sin^2 x-\cos^2 x)dx=-\int(\cos^2 x-\sin^2 x)dx$$

$$=-\int\cos 2x\,dx=\frac{-\sin 2x}{2}+c$$

539 **(b)**

We know that

$$\int_0^{\pi/2}\sin^m x\cos^n x\,dx=\frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$$

$$\int_0^{\pi/2}\sin^6 x\,dx=\frac{\Gamma\frac{7}{2}\Gamma\frac{1}{2}}{2\sqrt{4}}=\frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\,\Gamma\frac{1}{2}\Gamma\frac{1}{2}}{2(3\,!)}=\frac{5\,\pi}{32}$$

540 **(a)**

We know that

$$\int_0^{2a}f(x)\,dx=\int_0^{a}\{f(x)+f(2a-x)\}\,dx$$

$$\Rightarrow\int_0^{2a}f(x)\,dx=\int_0^{a}f(x)\,dx+\int_0^{a}f(2a-x)\,dx$$

$$=\lambda+\mu$$

541 **(a)**

Let $I=\int\frac{dx}{7+5\cos x}=\int\frac{dx}{7+5\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}$

$$=\int\frac{\sec^2\left(\frac{x}{2}\right)dx}{7+7\tan^2\left(\frac{x}{2}\right)+5-5\tan^2\left(\frac{x}{2}\right)}$$

$$=\int\frac{\sec^2\left(\frac{x}{2}\right)dx}{12+2\tan^2\left(\frac{x}{2}\right)}=\int\frac{\frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx}{6+\tan^2\left(\frac{x}{2}\right)}$$

Put $\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

$$\therefore I=\int\frac{dt}{t^2+\left(\sqrt{6}\right)^2}=\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{t}{\sqrt{6}}\right)+c$$

$$=\frac{1}{\sqrt{6}}\tan^{-1}\left|\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{6}}\right|+c$$

542 **(d)**

Let $I=\int_0^{\pi/3}\frac{\cos x+\sin x}{\sqrt{1+2\sin x\cos x}}dx$

$$=\int_0^{\pi/3}\frac{\cos x+\sin x}{\sqrt{\cos^2 x+\sin^2 x+2\sin x\cos x}}dx$$

$$=\int_0^{\pi/3}\frac{\cos x+\sin x}{\sqrt{(\cos x+\sin x)^2}}dx$$

$$=\int_0^{\pi/3}\frac{\cos x+\sin x}{\cos x+\sin x}dx$$

$$=\int_0^{\pi/3}1\,dx=\frac{\pi}{3}$$

543 **(d)**

$$\because\int_0^n[x]dx=\int_0^n(x-\{x\})dx$$

$$=\int_0^n x\,dx-\int_0^n\{x\}dx$$

$$=\int_0^n x\,dx-n\int_0^1\{x\}dx$$

$$=\int_0^n x\,dx-n\int_0^1 x\,dx=\left(\frac{n^2}{2}-\frac{n}{2}\right)$$

$$\therefore\frac{\int_0^n[x]dx}{\int_0^n\{x\}\,dx}=\frac{\frac{n^2}{2}-\frac{n}{2}}{\frac{n}{2}}=n-1$$

544 **(c)**

Let $I=\int_{\pi/4}^{\pi/2}e^x(\log\sin x+\cot x)dx$

$$\Rightarrow\ I=\int_{\pi/4}^{\pi/2}e^x\log\sin x dx+\int_{\pi/4}^{\pi/2}e^x\cot x dx$$

$= \int_{\pi/4}^{\pi/2} e^x \log \sin x dx + [e^x \log \sin x]_{\pi/4}^{\pi/2}$

$- \int_{\pi/4}^{\pi/2} e^x \log \sin x \, dx$

$= e^{\pi/2} \log \sin \frac{\pi}{2} - e^{\pi/4} \log \sin \frac{\pi}{4}$

$= -e^{\pi/4} \log \left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} e^{\pi/4} \log 2$

545 **(b)**

Let

$I = \int_0^1 \cot^{-1}(1 - x + x^2) dx$

$I = \int_0^1 \tan^{-1}\left\{\frac{1}{1 - x(1-x)}\right\} dx$

$= \int_0^1 \tan^{-1}\left\{\frac{x + (1-x)}{1 - x(1-x)}\right\} dx$

$I = \int_0^1 \{\tan^{-1} x + \tan^{-1}(1-x)\} dx$

$I = \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(1-x) \, dx$

$I = \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}\big(1 - (1-x)\big)$

$= 2 \int_0^1 \tan^{-1} x \, dx$

$\therefore k = 2$

546 **(a)**

Let $I = \int_0^{\pi/2} \frac{(\sin^2 x - \cos^2 x) dx}{(\sin x + \cos x)(\sin^2 + \cos^2 x - \sin x \cos x)}$

$I = \int_0^{\pi/2} \frac{(\sin x - \cos x) dx}{(1 - \sin x \cos x)}$

$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 - \sin x \cos x} dx \quad \left(\because \int_0^a f(x) = \int_0^a f(a-x)\right)$

$\Rightarrow I = 0$

548 **(c)**

$\int \sqrt{1 + \sin \frac{x}{2}} \, dx$

$= \int \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2 \sin \frac{x}{4} \cos \frac{x}{4}} \, dx$

$= \int \left[\sin \frac{x}{4} + \cos \frac{x}{4}\right] dx$

$= 4 \left(\sin \frac{x}{4} - \cos \frac{x}{4}\right) + c$

549 **(c)**

$\int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x dx}{(x+4)^2}$

$= \int \left(\frac{x+4}{(x+4)^2} - \frac{1}{(x+4)^2}\right) e^x dx$

$= \int e^x \frac{1}{x+4} dx - \int e^x \frac{1}{(x+4)^2} dx$

$= \frac{e^x}{x+4} + \int e^x \frac{1}{(x+4)^2} dx - \int \frac{e^x}{(x+4)^2} dx + c$

$= \frac{e^x}{x+4} + c$

550 **(d)**

Let $I = \int_0^{\pi/2} x \sin^2 x cos^2 x \, dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \left(\frac{\pi}{2} - x\right) \sin^2 x \cos^2 x \, dx$ ... (ii)

On adding Eqs. (i) and (ii), we get

$2I = \frac{\pi}{2} \int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

$= \frac{\pi}{8} \int_0^{\pi/2} \sin^2 2x \, dx$

$= \frac{\pi}{8} \int_0^{\pi/2} \left(\frac{1 - \cos 4x}{2}\right) dx$

$\Rightarrow 2I = \frac{\pi}{16} \left[x - \frac{\sin 4x}{4}\right]_0^{\pi/2}$

$\Rightarrow 2I = \frac{\pi}{16} \left[\frac{\pi}{2}\right] \Rightarrow I = \frac{\pi^2}{64}$

551 **(a)**

Now, $\int uv'' \, dx = uv' - \int u'v' \, dx$

$= uv' - \left[vu' - \int vu'' \, dx\right]$

$= uv' - vu' + \int u''v \, dx$

$\therefore a = \int u'' v \, dx$

552 **(c)**

Let $I = \int (\sqrt[3]{x}) \left(\sqrt[5]{1 + \sqrt[3]{x^4}}\right) dx$

Put $\sqrt[3]{x^4} = t$

$\Rightarrow \frac{4}{3} . \sqrt[3]{x} \, dx = dt$

$\therefore I = \frac{3}{4} \int (\sqrt[5]{1+t}) \, dt$

$I = \frac{3}{4} \left[\frac{(1+t)^{\frac{1}{5}+1}}{\frac{1}{5}+1}\right] + c$

$$= \frac{5}{8}[\left(1+\sqrt[3]{x^4}\right)^{6/5}] + c$$

553 **(a)**

Let

$$I = \int \frac{1}{\cos x - \sin x} dx$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \frac{1}{\left(\frac{1}{\sqrt{2}}\cos x \frac{1}{\sqrt{2}}\sin x\right)} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\cos\left(x+\frac{\pi}{4}\right)} dx$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \int \sec\left(x+\frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2}+\frac{\pi}{8}+\frac{\pi}{4}\right)\right| + C$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$$

554 **(d)**

If $f(x)$ is a continuous function defined on $[a, b]$ then

$$m(b-a) \le \int_a^b f(x)\, dx \le M(b-a)$$

Where m and M are respectively minimum and maximum values of $f(x)$ in $[a, b]$

Clearly, $f(x) = 1 + e^{-x^2}$ is continuous on [0, 1]

Also,

$0 < x < 1 \Rightarrow x^2 < x \Rightarrow e^{x^2} < e^x \Rightarrow e^{-x^2} > e^{-x}$

and,

$0 < x < 1 \Rightarrow x^2 > 0 \Rightarrow e^{x^2} > e^0 \Rightarrow e^{-x^2} < 1$

$\therefore e^{-x} < e^{-x^2} < 1$ for all $x \in [0, 1]$

$\Rightarrow 1 + e^{-x} < 1 + e^{-x^2} < 2$ for all $x \in [0, 1]$

$$\Rightarrow \int_0^1 1 + e^{-x}\, dx < \int_0^1 1 + e^{-x^2} dx < \int_0^1 2.\, dx$$

$$\Rightarrow 2 - \frac{1}{e} < \int_0^1 1 + e^{-x^2} dx < 2$$

555 **(c)**

Let $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}}\, dx$

$$= \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}\, dx$$

$$= \int_0^{\pi/2} (\sin x + \cos x)\, dx$$

$$= [-\cos x + \sin x]_0^{\pi/2} = -0 + 1 + 1 - 0 = 2$$

556 **(b)**

Let $I = \int \sqrt{x} e^{\sqrt{x}}\, dx$

Put $x = t^2$

$\Rightarrow dx = 2t\, dt$

$$\therefore I = 2\int t^2 e^t dt$$

$$= 2[t^2 e^t - (2t)e^t + 2e^t] + c$$

$$= \left(2x - 4\sqrt{x} + 4\right)e^{\sqrt{x}} + c$$

557 **(c)**

Let

$$A = \lim_{n\to\infty} \left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \log\left(1+\frac{r^2}{n^2}\right)$$

$$\Rightarrow \log A = \int_0^1 \log(1+x^2)\, dx$$

$$= [x\log(1+x^2)]_0^1 - 2\int_0^1 \frac{x^2}{1+x^2} dx$$

$$\Rightarrow \log A = \log 2 - 2[x - \tan^{-1} x]_0^1$$

$$\Rightarrow \log A = \log 2 - 2\left[1-\frac{\pi}{4}\right] = \log 2 + \frac{\pi}{2} - 2$$

$$\Rightarrow A = e^{\log 2 + \frac{\pi}{2} - 2} = \frac{2}{e^2} e^{\pi/2}$$

558 **(d)**

We have,

$$\int_{\sqrt{2}}^{x} \frac{1}{x\sqrt{x^2-1}} dx = \frac{\pi}{12}$$

$$\Rightarrow [\sec^{-1} x]_{\sqrt{2}}^{x} = \frac{\pi}{12}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{12}$$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{12} \Rightarrow \sec^{-1} x = \frac{\pi}{3} \Rightarrow x = \sec\frac{\pi}{3} = 2$$

559 **(b)**

We have,

$$I = \int_0^{na} f(x) dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{(r-1)a}^{ra}$$

$\Rightarrow I = \sum_{r=1}^{n} \int_0^a f(y + (r-1)a) dy$, where $x = y + (r-1)a$

$$\Rightarrow I = \sum_{r=1}^{n} \int_0^a f(y) dy\ [\because f(y+a) = f(y)$$

$$\therefore f\{y + (r-1)a\} = f(y)]$$

$$\Rightarrow I = n\int_0^a f(x)\,dx$$

560 **(b)**

We have,

$$I = \int_{-\pi/3}^{\pi/3} \frac{x\sin x}{\cos^2 x}dx$$

$\Rightarrow I = 2\int_0^{\pi/3} \frac{x\sin x}{\cos^2 x}dx$ [∵integrand is an even function]

$$\Rightarrow I = 2\int_0^{\pi/3} \underset{I}{x} \underset{II}{\tan x \sec x}\ dx$$

$$\Rightarrow I = 2[x\sec x - \log(\sec x + \tan x)]_0^{\pi/3}$$

$$\Rightarrow I = 2[2\pi/3 - \log(2+\sqrt{3})]$$

$$\Rightarrow I = 4\pi/3 - 2\log\tan 5\pi/12 = 2(2\pi/3 - \log\tan 5\pi/12)$$

561 **(a)**

Let $I = \int \frac{a^{\frac{x}{2}}}{\sqrt{a^{-x}-a^x}}dx = \int \frac{a^x}{\sqrt{1-a^{2x}}}dx$

Put $a^x = t \quad \Rightarrow \quad a^x \log a\, dx = dt$

$$\therefore\ I = \frac{1}{\log a}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{\log a}\sin^{-1} t + c$$

$$= \frac{1}{\log a}\sin^{-1}(a^x) + c$$

562 **(a)**

Let $I = \int \frac{x+\sin x}{1+\cos x}dx = \int \left(\frac{x}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right)dx$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx$$

$$= x\tan\frac{x}{2} + c$$

563 **(c)**

Let $I = \int_{-1}^{1} \sin^3 x\cos^2 x\, dx$

Again, let $f(x) = \sin^3 x\cos^2 x$

$f(-x) = \sin^3(-x)\cos^2(-x)$

$= -\sin^3 x\cos^2 x$

$\therefore f(x)$ is an odd function.

Hence, $\int_{-1}^{1} \sin^3 x\cos^2 x\, dx = 0$

564 **(c)**

Let $I = \int x\{f(x^2)g''(x^2) - f''(x^2)g(x^2)\}dx$

Put $x^2 = t \Rightarrow 2x\, dx = dt$

$$\therefore I = \frac{1}{2}[\{f(t)g''(t)\}dt - \int f''(t^2)g(t^2)dt]$$

$$= \frac{1}{2}[f(t)g'(t) - \int f'(t)g(t)dt - g(t)f'(t) + \int g'(t)f'(t)dt] + c$$

$$= \frac{1}{2}[f(t)g'(t) - g(t)f'(t)] + c$$

$$= \frac{1}{2}[f(x^2)g'(x^2) - g(x^2)f'(x^2)] + c$$

565 **(c)**

We have,

$$I = \int_{-2}^{2} |1-x^4|\,dx$$

$$\Rightarrow I = 2\int_0^2 |1-x^4|\,dx\ [\because f(x) = 1-x^4 \text{ is an even function}]$$

$$\Rightarrow I = 2\left\{\int_0^1 (1-x^4)dx + \int_1^2 (x^4-1)dx\right\}$$

$$\Rightarrow I = 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^5\right\} = 12$$

566 **(c)**

$$I = \int_{-1}^{0} [x + [x+[x]]]dx + \int_0^1 [x+[x+[x]]]\,dx$$

$\Rightarrow I = I_1 + I_2$

For $I_1 \to [x] = -1$ and $-1 < x < 0$

$-2 < x + [x] < -1 \Rightarrow [x+[x]] = -2$

and now, $-3 < x + [x+[x]] < -2$

$\Rightarrow\ [x+[x+[x]]] = -3$

$$I_1 = \int_{-1}^{0} (-3)dx + -3[x]_{-1}^{0} = -3$$

For $I_2 \to [x] = 0$ and $0 < x < 1$

$\Rightarrow 0 < x + [x] < 1 \Rightarrow [x+[x]] = 0$

Again, $0 \le x + [x+[x]] < 1$

$\Rightarrow\ [x+[x+[x]]] = 0$

So, $I_2 = 0$

$\therefore\ I_1 + I_2 = -3$

567 **(d)**

Let $I = \int_0^{\pi} \sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x\ dx$

$$= \int_0^{\pi} a_0\cos^3 x\, dx + \int_0^{\pi} a_1 \cos^2 x \sin x dx + \int_0^{\pi} a_2 \cos x \sin^2 x\, dx + \int_0^{\pi} a_3 \sin^3 x\, dx$$

Since, $\int_0^{2a} f(x)dx$

$$= \begin{cases} 2\int_0^a f(x)dx & ,\text{if } f(2a-x) = f(x) \\ 0 & ,\text{if } f(2a-x) = -f(x) \end{cases}$$

∴ Integral Ist and IIIrd become zero.

∴The given integral is depend upon a_1 and a_3.

568 **(a)**

Let

$$\int \frac{x \sin x^2 \ e^{\sec x^2}}{\cos^2 x^2} dx$$
$$= \frac{1}{2}\int e^{\sec x^2} \tan x^2 \sec x^2 \, 2x \, dx$$
$$\Rightarrow I = \frac{1}{2}\int e^{\sec x^2} d\,(\sec x^2) = \frac{1}{2} e^{\sec x^2} + C$$

569 **(b)**

We have,

$$\int \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1+x}} dx = \int \sqrt{1+x}\, dx$$
$$= \frac{2}{3}(1+x)^{3/2} + C$$

570 **(a)**

Putting $x = \tan\theta$, we get

$$I = \int_0^\infty \frac{dx}{\left[x + \sqrt{x^2+1}\right]^3} = \int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{(\tan\theta + \sec\theta)^3}$$
$$I = \int_0^{\pi/2} \frac{\cos\theta}{(1+\sec\theta)^3} d\theta = \left[-\frac{1}{2(1+\sin\theta)^2}\right]_0^{\pi/2}$$
$$= -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$$

571 **(b)**

$$F(t) = \int_0^1 f(t-y) g(y) dy$$
$$= \int_0^t e^{t-y}. y dy = e^t \int_0^t e^{-y}\, y dy$$
$$= e^t[(-ye^{-y})_0^t - (e^{-y})_0^t]$$
$$= e^t[-te^{-t} - e^{-t} + 1] = e^t - (1+t)$$

573 **(a)**

Let $I = \int_1^{e^2} \frac{dx}{x(1+\log x)\text{^}2}$

Put $1 + \log x = t \ \Rightarrow \frac{1}{x} dx = dt$

$$\therefore I = \int_1^3 \frac{1}{t^2} dt = \left[-\frac{1}{t}\right]_1^3 = \frac{2}{3}$$

574 **(c)**

Let $I = \int \frac{dx}{(x+1)\sqrt{4x+3}}$

Put $4x + 3 = t^2 \Rightarrow 4\, dx = 2t\, dt$

$$\therefore I = \frac{1}{2}\int \frac{t\, dt}{\left(\frac{t^2-3}{4} + 1\right) t} = 2\int \frac{dt}{1+t^2}$$
$$= 2\tan^{-1} t + c = 2 \tan^{-1}\sqrt{4x+3} + c$$

575 **(b)**

Let $I = \int_a^b x\, f(x) dx$ ... (i)

$$\Rightarrow I = \int_a^b (a+b-x) f(x) dx$$

[since, $f(a+b-x) = f(x)$,given]

$$\Rightarrow I = (a+b)\int_a^b f(x) - I \text{ [from Eq. (i)]}$$
$$\Rightarrow 2I = (a+b)\int_a^b f(x) dx$$
$$\Rightarrow I = \left(\frac{a+b}{2}\right)\int_a^b f(x) dx$$

576 **(a)**

Here, $\int_0^{t^2} \{x\ f(x)\} dx = \frac{2}{5} t^5$

(Using Newton Leibnitz formula): differentiating both sides, we get

$$t^2\{f(t^2)\}.\left\{\frac{d}{dt}(t^2)\right\} - 0. f(0)\left\{\frac{d}{dt}(0)\right\} = 2t^4$$
$$\Rightarrow t^2 f(t^2). 2t = 2t^4$$
$$\Rightarrow f(t^2) = t$$
$$\therefore f\left(\frac{4}{25}\right) = \pm\frac{2}{5} \quad \left[\text{putting } t = \pm\frac{2}{5}\right]$$
$$\Rightarrow f\left(\frac{4}{25}\right) = \frac{2}{5} \quad \text{[neglecting negative]}$$

577 **(c)**

Given, $F(x^2) = x^2(1+x)$

$$\Rightarrow \int_0^{x^2} f(t) dt = x^2(1+x)$$

On differentiating both sides w.r.t., we get

$2xf(x^2) = 2x + 3x^2$

$$\Rightarrow f(x^2) = 1 + \frac{3x}{2}$$
$$\Rightarrow f(2^2) = 1 + \frac{3}{2}(2) = 4 \ \Rightarrow \ f(4) = 4$$

578 **(c)**

We have,

$$\int_0^{\sqrt{2}} [x^2]\, dx = \int_0^1 [x^2]\, dx + \int_0^{\sqrt{2}} [x^2]\, dx$$
$$\Rightarrow \int_0^{\sqrt{2}} [x^2]\, dx = \int_0^1 0\, dx + \int_0^{\sqrt{2}} 1. dx = \sqrt{2} - 1$$

579 **(a)**

Let $I = \int \frac{e^x}{(2+e^x)(e^x+1)} dx$

Put $e^x = t \ \Rightarrow \ e^x dx = dt$

$$\therefore I = \int \frac{dt}{(2+t)(t+1)}$$
$$= \int \left[\frac{1}{(1+t)} - \frac{1}{(2+t)}\right] dt$$
$$= \log(1+t) - \log(2+t) + c$$
$$= \log\left(\frac{1+t}{2+t}\right) + c$$
$$= \log\left(\frac{1+e^x}{2+e^x}\right) + c$$

580 **(a)**

Let $I = \int_0^{\pi/2} \frac{\sin^3 x \cos x}{\sin^4 x + \cos^4 x} dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 x \sin x}{\cos^4 x + \sin^4 x} dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin x \cos x}{\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{2 \sin x \cos x}{1 + \cos^2 2x} dx$$

$$\Rightarrow 2I = -\frac{1}{2}\int_1^{-1} \frac{1}{1+t^2} dt, \text{where } t = \cos 2x$$

$$\Rightarrow 2I = -\frac{1}{2}[\tan^{-1} t]_1^{-1} = -\frac{1}{2}\left(-\frac{\pi}{4} - \frac{\pi}{4}\right)$$

$$\Rightarrow I = \frac{\pi}{8}$$

581 **(c)**

We have,

$$\int_0^1 e^{x^2}(x-\alpha)dx = 0 \Rightarrow \int_0^1 e^{x^2} x\, dx = \int_0^1 e^{x^2} \alpha dx$$

$$\Rightarrow \frac{1}{2}\int_0^1 e^t dt = \alpha \int_0^1 e^{x^2}, \text{where } t = x^2$$

$\Rightarrow \frac{1}{2}(e-1) = \alpha \int_0^1 e^{x^2} dx$ ...(i)

Since e^{x^2} is an increasing function for $0 \le x \le 1$

$\therefore 1 \le e^{x^2} \le e$ when $0 \le x \le 1$

$\Rightarrow 1(1-0) \le \int_0^1 e^{x^2} dx \le e(1-0) \Rightarrow 1 \le \int_0^1 e^{x^2} dx \le e$ ...(ii)

From (i) and (ii) we find that LHS of (i) is positive and $\int_0^1 e^{x^2} dx$ lies between 1 and e. Therefore, α is a positive real number. Also, from (i), we have

$$\therefore \alpha = \frac{\frac{1}{2}(e-1)}{\int_0^1 e^{x^2} dx} \quad \text{... (iii)}$$

The denominator of (iii) is greater than unity and the numerator lies between 0 and 1. Therefore, $0 < \alpha < 1$

582 **(b)**

For $x \in [0, \pi/4]$, we have $x - [x] = x$

$$\therefore \int_0^{\pi/4} \sin x\, d(x - [x]) = \int_0^{\pi/4} \sin x\, dx = [-\cos x]_0^{\pi/4}$$

$$= 1 - \frac{1}{\sqrt{2}}$$

583 **(b)**

We know that

$$\int_a^b \frac{|x|}{x} dx = |b| - |a|$$

$$\therefore \int_{-1}^2 \frac{|x|}{x} dx = |2| - |-1| = 1$$

584 **(b)**

We have,

$$I = \int \frac{1+x^2}{x\sqrt{1+x^4}} dx = \int \frac{1 + \frac{1}{x^2}}{\sqrt{x^2 + \frac{1}{x^2}}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(x - \frac{1}{x}\right)^2 + 2}} d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow I = \log\left|\left(x - \frac{1}{x}\right) + \sqrt{\left(x - \frac{1}{x}\right)^2 + 2}\right| + C$$

585 **(d)**

$$\lim_{x\to 0} \frac{2x \tan|x|}{3x^2} = \frac{2}{3}\lim_{x\to 0}\frac{\tan|x|}{x}$$

RHL is $\frac{2}{3}$ and LHL is $-\frac{2}{3}$

So, limit does not exist

586 **(b)**

If $f(t)$ is an odd function, then $\int_0^x f(t)\, dt$ is an even function.

587 **(b)**

We have,

$$I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1}\left\{\frac{x + x - 1}{1 - x(x-1)}\right\} dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x + \tan^{-1}(x-1)\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1}(x-1)\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1}(1-x-1)\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1}(-x)\, dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1} x\, dx = 0$$

588 **(a)**

Let $I=\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx$ ...(i)

Then,

$$I=\int_{\pi/4}^{3\pi/4}\frac{x-\pi}{1+\sin(\pi-x)}dx$$

$I=\int_{\pi/4}^{3\pi/4}\frac{\pi-x}{1+\sin x}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_{\pi/4}^{3\pi/4}\frac{\pi}{1+\sin x}dx$$

$$\Rightarrow 2I=\pi\int_{\pi/4}^{3\pi/4}\frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I=\pi\int_{\pi/4}^{3\pi/4}(\sec^2 x-\tan x\sec x)\,dx$$

$$\Rightarrow 2I=\pi[\tan x-\sec x]_{\pi/4}^{3\pi/4}$$

$$\Rightarrow 2I=\pi[(\tan 3\pi/4-\sec 3\pi/4)-(\tan\pi/4-\sec\pi/4)]$$

$$\Rightarrow 2I=\pi\left[\left(-1+\sqrt{2}\right)-\left(1-\sqrt{2}\right)\right]$$

$$\Rightarrow 2I=2\pi\left(\sqrt{2}-1\right)$$

$$\Rightarrow I=\pi\left(\sqrt{2}-1\right)$$

589 **(a)**

We have,

$$\lim_{x\to1}\int_2^{f(x)}\frac{2\,tdt}{x-1}=4$$

$$\Rightarrow \lim_{x\to1}\frac{\int_2^{f(x)}2t\,dt}{x-1}=4$$

$\Rightarrow \lim_{x\to1}2f'(x)f(x)=4$ [Using De' L' Hospital's rule]

$\Rightarrow 2f'(1)f(1)=4\Rightarrow f'(1)=1$

590 **(d)**

$$f(x)+f(-x)=\frac{\sin^2\pi x}{1+\pi^x}+\frac{\sin^2(-\pi x)}{1+\pi^{-x}}$$

$$=\frac{\sin^2\pi x}{1+\pi^x}+\frac{(\sin^2\pi x)\pi^x}{\pi^x+1}$$

$$=\sin^2\pi x=\frac{1-\cos 2\pi x}{2}$$

$$\therefore \int[f(x)+f(-x)]dx=\int\left[\frac{1-\cos 2\pi x}{2}\right]dx$$

$$=\frac{1}{2}x-\frac{\sin 2\pi x}{4\pi}+c$$

591 **(b)**

$$\int_0^{2\pi}(\sin x+|\sin x)dx$$

$$=\int_0^{\pi}(\sin x+\sin x)dx+\int_{\pi}^{2\pi}(\sin x-\sin x)\,dx$$

$$=\int_0^{\pi}2\sin x\,dx+\int_{\pi}^{2\pi}0\,\,dx=2[-\cos x]_0^{\pi}+0$$

$$=-2(\cos\pi-\cos 0)=-2(-1-1)=4$$

592 **(b)**

We have,

$$f(x+2\pi)=\int_0^{x+2\pi}\sin^4 t\,dt$$

$$=\int_0^{x}\sin^4 t\,dt+\int_x^{x+2\pi}\sin^4 t\,dt$$

$$\Rightarrow f(x+2\pi)=f(x)+\int_0^{2\pi}\sin^4 t\,dt$$

$\left[\because \sin^4 t\right.$ is periodic with period $\left.\frac{\pi}{2}\right]$

$$\Rightarrow f(x+2\pi)=f(x)+f(2\pi)$$

593 **(d)**

$$\frac{d}{dx}\int_2^{x^2}(t-1)dt=\frac{d}{dx}\left\{\frac{x^4}{2}-x^2\right\}=2x(x^2-1)$$

594 **(d)**

We have,

$$\int_0^2\left|\cos\frac{\pi}{2}x\right|dx$$

$$=\int_0^1\cos\frac{\pi}{2}x\,dx+\int_1^2-\cos\frac{\pi}{2}x\,dx$$

$$=\frac{2}{\pi}\left[\sin\frac{\pi x}{2}\right]_0^1-\frac{2}{\pi}\left[\sin\frac{\pi x}{2}\right]_1^2=\left(\frac{2}{\pi}-0\right)-\left(0-\frac{2}{\pi}\right)$$

$$=\frac{4}{\pi}$$

595 **(c)**

We have, $I=\int_{-1}^{1}|1-x|dx$

Here, $-1\le x\le 1\ \Rightarrow\ 1-x\ge 0$

$$\therefore\ I=\int_{-1}^{1}(1-x)dx$$

$$=\left[x-\frac{x^2}{2}\right]_{-1}^{1}=1-\frac{1}{2}+1+\frac{1}{2}=2$$

596 **(d)**

Let $I=2\int_0^{\pi}\frac{\sin^4 x}{\sin^4 x+\cos^4 x\,dx}$

$=4\int_0^{\pi/2}\frac{\sin^4 x}{\sin^4 x+\cos^4 x}$ ...(i)

$\Rightarrow I=4\int_0^{\pi/2}\frac{\cos^4 x}{\sin^4+\cos^4 x}dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = 4\int_0^{\pi/2} 1.\,dx = 2\pi$$

$\Rightarrow\ I = \pi$

597 **(d)**

Let $f(x) = \sin^3 x \cos^2 x$

$f(-x) = -\sin^3 x \cos^2 x = -f(x)$

$\Rightarrow f(x)$is an odd function

$$\therefore \int_{-\pi}^{\pi} \sin^3 x \cos^2 x\, dx = 0$$

598 **(c)**

Let $I = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1} dx = \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1} dx$

Put $x + \frac{1}{x} = t \ \Rightarrow\ \left(1 - \frac{1}{x^2}\right) dx = dt$

$$\therefore\ I = \int \frac{dt}{t^2 - 1^2} = \frac{1}{2\times 1}\log\frac{t-1}{t+1} + c$$

$$= \frac{1}{2}\log\frac{x^2+1-x}{x^2+1+x} + c$$

599 **(c)**

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{2(\sin^2 x + \cos^2 x) - 1}{\cos^2 x} dx$$

$$= \int \sec^2 x\, dx = \tan x + c$$

600 **(b)**

Let $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

Putting $\sin x - \cos x = t$ and $(\sin x + \cos x)dx = dt$, we get

$$I = \int_{-1}^{0} \frac{dt}{3 + 1 - t^2}$$

$$I = \int_{-1}^{0} \frac{dt}{2^2 - t^2} = \frac{1}{4}\left[\log\left(\frac{2+t}{2-t}\right)\right]_{-1}^{0} = -\frac{1}{4}\log\frac{1}{3}$$

$$= \frac{1}{4}\log 3$$

601 **(b)**

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$= [\sin^{-1} x]_0^1 + \int_1^0 \frac{t}{t} dt$ (where, $t^2 = 1 - x^2$

$\Rightarrow t\, dt = -x\, dx$)

$= (\sin^{-1} 1 - \sin^{-1} 0) + [t]_1^0$

$= \frac{\pi}{2} - 1$

602 **(b)**

We have,

$p'(x) = p'(1-x), \forall x \in [0,1], p(0) = 1, p(1) = 41$

$\Rightarrow\ p(x) = -p(1-x) + C$

At $x = 0, p(0) = -p(1) + C$

$\Longrightarrow\ \ 1 = -41 + C$

$\Rightarrow\ C = 42$

$\therefore p(x) + p(1-x) = 42$

Now, $I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$

$$\Rightarrow\ 2I = \int_0^1 \big(p(x) + p(1-x)\big)dx = \int_4^1 42\, dx = 42$$

$\Rightarrow\ I = 21$

603 **(a)**

We have,

$$I = \int \frac{x^2 - 1}{x\sqrt{(x^2 + \alpha x + 1)(x^2 + \beta x + 1)}} dx$$

$$\Rightarrow I \int \frac{1 - \frac{1}{x^2}}{\sqrt{\left(x + \frac{1}{x} + \alpha\right)\left(x + \frac{1}{x} + \beta\right)}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(x+\frac{1}{x}\right)^2 + (\alpha+\beta)\left(x+\frac{1}{x}\right) + \alpha\beta}} d\left(x + \frac{1}{x}\right)$$

$$\Rightarrow I = \int \frac{1}{\sqrt{t^2 + (\alpha+\beta)t + \alpha\beta}} dt, \text{ where } t = x + \frac{1}{x}$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(t + \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha-\beta}{2}\right)^2}} dt$$

$$\Rightarrow I = \log\left|\left(t + \frac{\alpha+\beta}{2}\right) + \sqrt{\left(t + \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha-\beta}{2}\right)^2}\right| + C$$

$$\Rightarrow I = \log\left|t + \frac{\alpha+\beta}{2} + \sqrt{t^2 + (\alpha+\beta)t + \alpha\beta}\right| + C$$

$$\Rightarrow I = \log\left|(t+\alpha) + (t+\beta) + 2\sqrt{(t+\alpha)(t+\beta)}\right| + C - \log 2$$

$$\Rightarrow I = \log\left\{\sqrt{t+\alpha} + \sqrt{t+\beta}\right\}^2 + C_1$$

$$\Rightarrow I = 2\log\left\{\sqrt{t+\alpha} + \sqrt{t+\beta}\right\} + C_1$$

$$\Rightarrow I = 2\log\left\{\frac{\sqrt{x^2 + \alpha x + 1} + \sqrt{x^2 + \beta x + 1}}{\sqrt{x}}\right\} + C_1$$

604 **(a)**

$f(a) = \frac{e^a}{1+e^a}$ ...(i)

and $f(-a) = \frac{e^{-a}}{1+e^{-a}} = \frac{1}{1+e^a}$ ...(ii)

On adding Eqs.(i) and (ii), we get

$f(a) + f(-a) = 1$

$\Rightarrow f(a) = 1 - f(-a)$

Let $f(-a) = t$, then $f(a) = 1 - t$

Now, $I_1 = \int_t^{1-t} x g\,[x(1-x)]dx$ ... (i)

$\Rightarrow I_1 = \int_t^{1-t} (1-x) g[x(1-x)]dx$ ... (ii)

On adding Eqs. (i) and (ii). We get

$2I_1 = \int_t^{1-t} g[x(1-x)]dx = I_2$

$\Rightarrow \frac{I_2}{I_1} = 2$

605 **(d)**

We have,

$\int_2^5 f(x)\,dx = \int_{-2}^5 f(x)\,dx$

$\Rightarrow \int_2^5 f(x)\,dx - \int_{-2}^5 f(x)\,dx = 0$

$\Rightarrow \int_2^5 f(x)\,dx + \int_5^{-2} f(x)\,dx = 0$

$\Rightarrow \int_2^{-2} f(x)\,dx = 0$

$\Rightarrow \int_{-2}^2 f(x)\,dx = 0 \Rightarrow \int_0^2 \{f(x) + f(-x)\}dx = 0$

Thus, $f(x)$ may be an odd function

In general, nothing can be said about $f(x)$ defined on R

606 **(a)**

We have,

$f(x) = \int_0^x 2|t|\,dt \Rightarrow f'(x) = 2|x|$

Now,

$f'(x) = 1 \Rightarrow 2|x| = 1 \Rightarrow x = \pm\frac{1}{2}$

For $x = \frac{1}{2}$, we have $y = f(x) = \int_0^{1/2} 2t\,dt = \frac{1}{4}$

and, for $x = -\frac{1}{2}$, we have $y = \int_0^{-1/2} -2t\,dt = -\frac{1}{4}$

The equations of the tangents at $(1/2, 1/4)$ and $(-1/2, -1/4)$ are $y = x - \frac{1}{4}$ and $y = x + \frac{1}{4}$ respectively

607 **(c)**

Let $I = \int \cos\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]dx$

$= \int \cos\left[2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right]dx$

$= \int \cos[\cos^{-1}(-x)]dx \quad \left[\because 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

$= -\frac{x^2}{2} + c$

608 **(b)**

Let $I = \int_2^3 \frac{x+1}{x^2(x-1)}dx$

$= \int_2^3 \left(\frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1}\right)dx$

$= \left[-2\log x + \frac{1}{x} + 2\log(x-1)\right]_2^3$

$= \left[2\log\left(\frac{x-1}{x}\right) + \frac{1}{x}\right]_2^3$

$= \left[2\left(\log\frac{2}{3} - \log\frac{1}{2}\right) + \frac{1}{3} - \frac{1}{2}\right]$

$= 2\log\frac{4}{3} - \frac{1}{6} = \log\frac{16}{9} - \frac{1}{6}$

609 **(a)**

Let $I = \int x\,(x^x)^x(2\log x + 1)dx$

Let $(x^x)^x = t, x^2\log x = \log t$

$\Rightarrow \left(x^2.\frac{1}{x} + 2x\log x\right)dx = \frac{1}{t}dt$

$\Rightarrow x(1 + 2\log x)dx = \frac{1}{t}\,dt$

$\therefore I = \int t.\frac{1}{t}\,dt = t + c = (x^x)^x + c$

610 **(a)**

Let $I = \int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

$= \int \frac{\sec^2 x\,dx}{(\tan x - 2)(2\tan x + 1)}$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$\therefore I = \int \frac{dt}{(t-2)(2t+1)}$

$= \frac{1}{5}\int \frac{t}{t-2}dt - \frac{2}{5}\int \frac{1}{2t+1}dt$

$= \frac{1}{5}\log(t-2) - \frac{1}{5}\log(2t+1) + c$

$= \log \sqrt[5]{\frac{\tan x - 2}{2\tan x + 1}} + c$

611 **(b)**

$$\int_0^\infty \frac{dx}{(a^2+x^2)} = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^\infty = \frac{1}{a}\left(\frac{\pi}{2} - 0\right)$$
$$= \frac{\pi}{2a}$$

612 **(b)**

We have,

$$\int_1^e (\log x)^3 dx = [x(\log x)^3]_1^e - 3\int_1^e (\log x)^2 dx$$
$$\Rightarrow \int_1^e (\log x)^3 dx = e - 3\left[\{x(\log x)^2\}_1^e - 2\int_1^e (\log x) dx\right]$$
$$\Rightarrow \int_1^e (\log x)^3 dx = e - 3[e - 2[x\log x - x]_1^e]$$
$$= 6 - 2e$$

613 **(c)**

We have,

$$I = \int_1^{\sqrt[7]{2}} \frac{1}{x(2x^7+1)} dx$$
$$\Rightarrow I = \frac{1}{7}\int_1^2 \frac{1}{y(2y+1)} dy, \text{where } y = x^7$$
$$\Rightarrow I = \frac{1}{7}\left[\log\left(\frac{y}{2y+1}\right)\right]_1^2 = \frac{1}{7}\left[\log\frac{2}{5} - \log\frac{1}{3}\right]$$
$$= \frac{1}{7}\log\left(\frac{6}{5}\right)$$

614 **(a)**

We have, $\int f(x)dx = F(x)$ ...(i)

Let $I = \int x^3 f(x^2) dx$

Put $x^2 = t \Rightarrow 2x\ dx = dt$

$$\therefore I = \frac{1}{2}\int t\, f(t) dt$$
$$= \frac{1}{2}\{ t\int f(t)dt - \int (\int f(t)dt)\, dt \}$$
$$= \frac{1}{2}\{ tF(t) - \int F(t)dt \} \quad \text{[From Eq. (i)]}$$
$$= \frac{1}{2}\{ x^2 F(x^2) - \int F(x^2) dx^2 \}$$

615 **(b)**

We have,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$$
$$\therefore \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

$\Rightarrow \int_a^{2a} f(x)dx = \int_0^a f(x)dx$ ...(i)

Putting $2a - x = t$ in the integral on LHS, we get

$$\int_a^{2a} f(x)dx = \int_a^0 f(2a-t)(-dt) = \int_0^a f(2a-t)\,dt$$
$$\Rightarrow \int_a^{2a} f(x)dx = \int_0^a f(2a-x)\,dx$$
$$\therefore \int_0^a f(2a-x)\,dx = \int_0^a f(x)\,dx \Rightarrow f(2a-x)$$
$$= f(x)$$

616 **(c)**

Let $I = \int_0^\pi x\log\sin x\, dx$. Then,

$$I = \int_0^\pi (\pi - x)\log\sin(\pi - x)\,dx$$
$$\Rightarrow I = \int_0^\pi (\pi - x)\log\sin x\,dx$$
$$\Rightarrow I = \pi\int_0^\pi \log\sin x\,dx - \int_0^\pi x\log\sin x\,dx$$
$$\Rightarrow I = 2\pi\int_0^{\pi/2} \log\sin x\,dx - I$$
$$\Rightarrow I = \pi\int_0^{\pi/2} \log\sin x\,dx = \pi\left(\frac{-\pi}{2}\log 2\right)$$
$$= -\frac{\pi^2}{2}\log 2$$

617 **(c)**

Let $I = \int_1^2 [f\{(g(x)\}]^{-1} f'\{g(x)\}g'(x)dx$

Put $f(g(x)) = t \Rightarrow f'(g(x))g'(x)dx = dt$

$$\therefore I = \int_1^2 t^{-1}\,dt = [\log f\{g(x)\}]_1^2$$

$= \log f\{g(2)\} - \log f\{g(1)\} = 0$

$[\because g(2) = g(1)]$

618 **(b)**

We have,

$$\phi(x) = \int_a^x f(t)\ dt,$$
$$\Rightarrow \phi(-x) = \int_a^{-x} f(t)\,dt$$

$$\Rightarrow \phi(-x) = \int_{-a}^{x} f(-u)\,(-du), \text{where } t = -u$$

$$\Rightarrow \phi(-x) = \int_{-a}^{x} f(u)\,du \quad [\because f(-u) = -f(u)]$$

$$\Rightarrow \phi(-x) = \int_{-a}^{a} f(u)\,du + \int_{a}^{x} f(u)\,du$$

$$= 0 + \int_{a}^{x} f(u)\,du = \phi(x)$$

Hence, $\phi(x)$ is an odd function

619 **(b)**

We have,

$$I = \int \frac{x^5}{\sqrt{1+x^3}}dx = \frac{1}{3}\int \frac{x^3}{\sqrt{1+x^3}}3x^2\,dx$$

$$\Rightarrow I = \frac{1}{3}\int \frac{t}{\sqrt{1+t}}dt, \text{where } t = x^3$$

$$\Rightarrow I = \frac{1}{3}\int \frac{(u^2-1)2u\,du}{\sqrt{u^2}}, \text{where } 1+t = u^2$$

$$\Rightarrow I = \frac{2}{3}\int (u^2-1)\,du = \frac{2}{3}\left(\frac{u^3}{3} - u\right) + C$$

$$\Rightarrow I = \frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + C$$

620 **(b)**

Let $I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x}\ |\sin x| dx$$

$$= 2\int_{0}^{\pi/2} \sqrt{\cos x}\sin x dx$$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$$\therefore I = -2\int_{1}^{0} \sqrt{t}dt = -\frac{4}{3}[t^{3/2}]_1^0 = \frac{4}{3}$$

622 **(d)**

$$\int_{-3}^{3} (ax^5 + bx^3 + cx + k)\,dx$$

$$= \left[\frac{ax^6}{6} + \frac{bx^4}{4} + \frac{cx^2}{2} + kx\right]_{-3}^{3}$$

$$= \frac{a.3^6}{6} + \frac{b.3^4}{4} + \frac{c.3^2}{3} + k(3) - \frac{a.3^6}{6} - \frac{b.3^4}{4} - \frac{c.3^2}{3} + k(3)$$

$= 6k$

ie, Integral depends upon k

623 **(b)**

We have,

$$\lim_{n\to\infty} \frac{1}{n^2}\sum_{r=1}^{n} r\,e^{r/n} = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \left(\frac{r}{n}\right) e^{r/n}$$

$$= \int_0^1 x\,e^x dx = [e^x(x-1)]_0^1 = 1$$

624 **(a)**

Given, $I_n = \int x^n . e^{cx}\,dx$

$$= \frac{e^{cx}}{c}.x^n - \int \frac{e^{cx}}{c}.nx^{n-1}dx$$

$$\Rightarrow I_n = \frac{e^{cx}.x^n}{c} - \frac{n}{c} I_{n-1}$$

$$\Rightarrow cI_n + nI_{n-1} = e^{cx}.x^n$$

625 **(c)**

$$\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2}dx$$

$$= \int_0^{\pi/2} \frac{d}{dx}\left(\frac{\sin x}{2+\cos x}\right)dx$$

$$= \left[\frac{\sin x}{2+\cos x}\right]_0^{\pi/2} = \frac{1}{2}$$

626 **(a)**

Let $\int_0^{\pi/4} (\pi\,x - 4x^2)\log(1+\tan x)\,dx$ ...(i)

Then,

$$I = \int_0^{\pi/4} \left\{\pi\left(\frac{\pi}{4} - x\right) - 4\left(\frac{\pi}{4} - x\right)^2\right\}\log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right)dx$$

$\Rightarrow I = \int_0^{\pi/4} (\pi x - 4x^2)\log\frac{1}{1+\tan x}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/4} (\pi x - 4x^2)\log 2\,dx$$

$$\Rightarrow I = \frac{1}{2}\log_e 2\left[\frac{\pi x^2}{2} - \frac{4x^3}{3}\right]_0^{\pi/4}$$

$$\Rightarrow I = \frac{\pi^3}{192}\log_e 2$$

627 **(a)**

Let $I = \int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)}dx$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$

$$\therefore I = \int_0^1 \frac{dt}{(1+t)(2+t)}$$

$$= \int_0^1 \frac{1}{(1+t)}dt - \int_0^1 \frac{1}{(2+1)}dt$$

$= [\log(1+t) - \log(2+t)]_0^1$

$= (\log 2 - \log 3) - (0 - \log 2)$

$= \log\frac{4}{3}$

628 **(b)**

We have,

$$I = \int \frac{x^2}{(a+bx^2)^{5/2}}dx$$

$$\Rightarrow I = \int \frac{1}{x^3\left(\frac{a}{x^2}+b\right)^{5/2}}dx$$

$$\Rightarrow I = -\frac{1}{2a}\int \left(\frac{a}{x^2}+b\right)^{-5/2} \times \frac{-2a}{x^3}dx$$

$$\Rightarrow I = -\frac{1}{2a}\int \left(\frac{a}{x^2}+b\right)^{-5/2} d\left(\frac{a}{x^2}+b\right)$$

$$\Rightarrow I = -\frac{1}{2a} \times -\frac{2}{3} \times \left(\frac{a}{x^2}+b\right)^{-3/2} + C$$

$$= \frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2} + C$$

629 **(c)**

This is a standard result and its proof is beyond the scope of this book

630 **(a)**

Let $I = \int \frac{3^x}{\sqrt{9^x-1}}\,dx = \frac{3^x dx}{\sqrt{(3^x)^2-1}}$

Put $3^x = t \Rightarrow 3^x \log 3 dx = dt$

$$\Rightarrow 3^x dx = \frac{dt}{\log 3}$$

$$\therefore I = \frac{1}{\log 3}\int \frac{dt}{\sqrt{t^2-1}}$$

$$= \frac{1}{\log 3}\log\left[\, t + \sqrt{t^2-1}\,\right] + c$$

$$= \frac{1}{\log 3}\log\left[\, 3^x + \sqrt{9^x-1}\,\right] + c$$

631 **(c)**

Let $I = \int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2}\,dx$

$$= \int \{1 + 2\tan^2 x + 2\tan x \sec x\}^{1/2}\,dx$$

$$= \int \{\sec^2 x + \tan^2 x + 2\tan x \sec x\}^{1/2}\,dx$$

$$= \int (\sec x + \tan x)\,dx$$

$$= \log(\sec x + \tan x) + \log\sec x + c$$

$$= \log\sec x(\sec x + \tan x) + c$$

632 **(b)**

Given, $f(x) = \int_{-1}^{x} |t|dt$

$$= \int_{-1}^{0} |t|dt + \int_{0}^{x} |t|dt$$

$$= \int_{-1}^{0} -t\,dt + \int_{0}^{x} t\,dt$$

$$= -\left[\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_{0}^{x}$$

$$= -\left[0 - \frac{1}{2}\right] + \left[\frac{x^2}{2} - 0\right] = \frac{1}{2}(1+x^2)$$

633 **(b)**

If $0 \le a < b$, then $|x| = x$

$$\therefore \int_a^b \frac{|x|}{x}dx = \int_a^b 1\,dx = b - a = |b| - |a|$$

If $a < b < 0$, then $|x| = -x$

$$\therefore \int_a^b \frac{|x|}{x}dx = -\int_a^b 1\,dx = a - b = |b| - |a|$$

If $a < 0 < b$, then

$$\int_a^b \frac{|x|}{x}dx = \int_a^0 \frac{|x|}{x}dx + \int_0^b \frac{|x|}{x}dx$$

$$\Rightarrow \int_0^b = -\int_a^0 1.\,dx + \int_0^b 1\,dx = b + a = |b| - |a|$$

Hence, $\int_a^b \frac{|x|}{x}dx = |b| - |a|$

634 **(d)**

Let $I = \int \frac{1+\tan^2 x}{1-\tan^2 x}\,dx = \int \frac{\sec^2 x}{1-\tan^2 x}dx$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore I = \int \frac{dt}{1-t^2}$$

$$= \frac{1}{2\times 1}\log\left[\frac{1+t}{1-t}\right] + c = \frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$$

636 **(d)**

$$\int_2^3 \{x\}dx = \int_2^3 (x - [x])dx = \int_2^3 (x-2)dx$$

$$= \left[\frac{x^2}{2} - 2x\right]_2^3 = \frac{1}{2}$$

637 **(d)**

We have,

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}dx,$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right) + \sin^3\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\sin^3 x + \cos^3 x}dx = \frac{\pi}{2} - 0 \Rightarrow \frac{\pi}{4}$$

639 **(b)**

We have,

$$f(x) = \int_{-1/2}^{x} |t| dt$$

$\Rightarrow f'(x) = |x| > 0$ for all $x \in [-1/2, 1/2]$

$\Rightarrow f(x)$ is increasing on $[-1/2, 1/2]$

$\Rightarrow f\left(\frac{1}{2}\right)$ is the greatest value of $f(x)$

Now, $f\left(\frac{1}{2}\right) = \int_{-1/2}^{1/2} |t|\, dt = 2\int_0^{1/2} t\, dt = \frac{1}{4}$

640 **(b)**

Consider $f(x) = \int_0^x (1 + \cos^8 x)(ax^2 + bx + c)dx$

Obviously, $f(x)$ is continuous and differentiable on $[1, 2]$ and $(1, 2)$ respectively.

Also, $f(1) = f(2)$ (given)

$\therefore$ By Rolle's theorem there exist at least one point $k \in (1, 2)$ such that $f'(k) = 0$

Now, $f'(x) = (1 + \cos^8 x)(ax^2 + bx + c)$

$f'(k) = 0$

$\Rightarrow (1 + \cos^8 k)(ak^2 + bk + c) = 0$

$\Rightarrow ak^2 + bk + c = 0$ [as $(1 + \cos^8 k) \neq 0$]

$\therefore x = k$ is root of $ax^2 + bx + c = 0$,

Where $k \in (1, 2)$

Hence, at least one root $\in (1, 2)$

641 **(d)**

We have, $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right) = \sqrt{2}$

and $\int_0^1 f(x)dx = \frac{2A}{\pi}$

$$f'(x) = \frac{A\pi}{2}\cos\frac{\pi x}{2}$$

$$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{A\pi}{2}\cos\frac{\pi}{4} = \frac{A\pi}{2\sqrt{2}}$$

But $f'\left(\frac{1}{2}\right) = \sqrt{2}$

Therefore, $\frac{A\pi}{2\sqrt{2}} = \sqrt{2} \Rightarrow A = \frac{4}{\pi}$

Now, $\int_0^1 f(x)dx = \frac{2A}{\pi}$

$$\Rightarrow \int_0^1 \left\{A\sin\left(\frac{\pi x}{2}\right) + B\right\}dx = \frac{2A}{\pi}$$

$$\Rightarrow \left[-\frac{2A}{\pi}\cos\frac{\pi x}{2} + Bx\right]_0^1 = \frac{2A}{\pi}$$

$$\Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi} \Rightarrow B = 0$$

642 **(a)**

We have,

$1 + x^4 < (1 + x^2)^2$

$\Rightarrow \sqrt{1 + x^4} < 1 + x^2$

$$\Rightarrow \frac{1}{\sqrt{1 + x^4}} > \frac{1}{1 + x^2}$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}}dx > \int_0^1 \frac{1}{1 + x^2}dx$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}}dx > [\tan^{-1} x]_0^1$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}}dx > \frac{\pi}{4} > 0.78$$

Also, $\frac{1}{\sqrt{1+x^4}} \leq 1$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} \leq \int_0^1 1.dx = 1$$

Hence, $I > 0.78$ and $I \leq 1$

643 **(d)**

$$\because I_k = \int_{-2k\pi}^{0} |\sin x|[\sin x]\, dx + \int_0^{2k\pi} |\sin x|[\sin x]\, dx$$

$$= -\int_{2k\pi}^{0} |\sin x|[-\sin x]\, dx + \int_0^{2k\pi} |\sin x|[\sin x]\, dx$$

$$= \int_0^{2k\pi} |\sin x|([\sin x] + [-\sin x])\, dx$$

$$= 2k\int_0^{\pi} |\sin x|([\sin x] + [-\sin x])\, dx$$

$$= 2k\int_0^{\pi} \sin x(0 - 1)dx$$

$$= -2k[-\cos x]_0^{\pi} \quad \left(\begin{array}{l}\because \sin x > 0, x \in (0, \pi)\\ \therefore -\sin x < 0, x \in (0, \pi)\end{array}\right)$$

$= -4k$

$$\therefore \sum_{k=1}^{10} I_k = -4\sum_{k=1}^{10} k = -4 \cdot \frac{10.11}{2} = -220$$

644 **(a)**

Let $I = \int_0^8 |x - 5|dx$

$$= \int_0^5 -(x - 5)dx + \int_5^8 (x - 5)dx$$

$$= \left[-\frac{x^2}{2} + 5x\right]_0^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= \left(\frac{25}{2}\right) + \left(-\frac{16}{2} + \frac{25}{2}\right) = 17$$

645 **(b)**

Let

$$I = \int \frac{e^x}{(2 + e^x)(e^x + 1)}dx$$

$$\Rightarrow I = \int \frac{1}{(t + 2)(t + 1)}dt, \text{where } t = e^x$$

$$\Rightarrow I=\int\left(\frac{1}{t+1}-\frac{1}{t+2}\right)dt=\log\left(\frac{t+1}{t+2}\right)+C$$

$$=\log\left(\frac{e^x+1}{e^x+2}\right)+C$$

646 **(b)**

$$\int_{-1}^{2}|x|^3dx=\int_{-1}^{0}-x^3dx+\int_{0}^{2}x^3dx$$

$$=-\left[\frac{x^4}{4}\right]_{-1}^{0}+\left[\frac{x^4}{4}\right]_{0}^{2}$$

$$=-\frac{1}{4}[0-1]+\frac{1}{4}[2^4]$$

$$=\frac{1}{4}+4=\frac{17}{4}$$

647 **(c)**

Since $f(x)=x|x|$ is an odd function

$$\therefore \int_{-1}^{1} f|x|\,dx=0$$

648 **(a)**

$$\int\frac{(x+1)}{x(1+xe^x)^2}dx=\int\frac{e^x+xe^x}{xe^x(1+xe^x)^2}dx$$

Put $1+xe^x=t \Rightarrow e^x+xe^x\,dx=dt$

$$\therefore \int\frac{dt}{(t-1)t^2}=\int\left[\frac{1}{t-1}-\frac{(t+1)}{t^2}\right]dt$$

$$=\log(t-1)-\log t+\frac{1}{t}+c$$

$$=\log\left(\frac{t-1}{t}\right)+\frac{1}{t}+c$$

$$=\log\left(\frac{xe^x}{xe^x+1}\right)+\frac{1}{1+xe^x}+c$$

649 **(c)**

We have,

$$F(x)=\int_{1}^{x}|t|\,dt$$

$\Rightarrow F'(x)=|x|$ for all $x\in[-1/2,1/2]$

$\Rightarrow F'(x)>0$ for all $x\in[-1/2,1/2]$

$\Rightarrow F(x)$ is increasing on $[-1/2,1/2]$

$\Rightarrow F(-1/2)$ and $F\ 1/2$ are minimum and maximum values of $F(x)$

$\therefore$ Required value $=F\left(\frac{1}{2}\right)$

$$=\int_{1}^{1/2}|t|\,dt=\left(\frac{t^2}{2}\right)_1^{1/2}=\frac{1}{2}\left(\frac{1}{4}-1\right)=-\frac{3}{8}$$

650 **(a)**

We have,

$I=\int_0^{\pi/2}\frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)}dx \quad$ ...(i)

$\Rightarrow I=\int_0^{\pi/2}\frac{f\left(\frac{\pi}{2}-x\right)}{f\left(\frac{\pi}{2}-x\right)+f(x)}dx \quad$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_{0}^{\pi/2}1\cdot dx=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

651 **(c)**

Let $I=\int\frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$

On dividing Nr and Dr by x^5, we get

$$I=\int\frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$$

Put $2-\frac{2}{x^2}+\frac{1}{x^4}=t \quad\Rightarrow\quad \left(\frac{4}{x^3}-\frac{4}{x^5}\right)dx=dt$

$$\therefore I=\frac{1}{4}\int\frac{dt}{\sqrt{t}}=\frac{1}{2}\sqrt{t}+c=\frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$$

652 **(c)**

$$I=\int_0^1 x(1-x)^n dx$$

Put $1-x=z\Rightarrow -dx=dz$

$$\therefore I=\int_1^0(1-z)z^n(-dz)$$

$$=\int_0^1(z^n-z^{n+1})dz$$

$$=\left[\frac{z^{n=1}}{n+1}-\frac{z^{n+2}}{n+2}\right]_0^1=\frac{1}{n+1}-\frac{1}{n+2}$$

Alternate Given integral can be rewritten as

$$I=\int_0^1 x^{2-1}(1-x)^{(n+1)-1}dx$$

It is the form of beta function

$$\therefore\ I=B(2,n+1)=\frac{\sqrt{2}\sqrt{n+1}}{\sqrt{(n+3}}$$

$$=\frac{1}{(n+1)(n+2)}=\frac{1}{n+1}-\frac{1}{n+2}$$

653 **(b)**

Let $I=\frac{x^2}{(ax+b)}dx$

Put $ax+b=t \quad\Rightarrow\quad dx=\frac{1}{a}dt$ and $x=\left(\frac{t-b}{a}\right)$

$$\therefore\ I=\frac{1}{a^3}\int\frac{(t-b)^2}{t^2}dt=\frac{1}{a^3}\int\left(1+\frac{b^2}{t^2}-\frac{2b}{t}\right)dt$$

$$=\frac{1}{a^3}\left(t-\frac{b^2}{t}-2b\log t\right)+c$$

$$=\frac{1}{a^3}\left[ax+b-\frac{b^2}{ax+b}-2b\log(ax+b)\right]+c$$

$$= \frac{1}{a^3}\left[\frac{a^2x^2 + b^2 + 2axb - b^2}{(ax+b)} - 2b\log(ax+b)\right] + c$$

$$= \frac{1}{a^3}\left[\frac{2ax(ax+b)}{ax+b} - \frac{a^2x^2}{ax+b} - 2b\log(ax+b)\right] + c$$

$$= \frac{1}{a^3}\left[2ax - \frac{a^2x^2}{ax+b} - 2b\log(ax+b)\right] + c$$

$$= \frac{2}{a^2}\left[x - \frac{b}{a}\log(ax+b)\right] - \frac{x^2}{a(ax+b)} + c$$

654 **(d)**

$$I = \int_{-1}^{1} \frac{1}{\sqrt{1-2ax+a^2}}dx$$

$$\Rightarrow I = \int_{-1}^{1} (1-2ax+a^2)^{1/2}dx$$

$$\Rightarrow \left[-\frac{1}{a}(1-2ax+a^2)^{1/2}\right]_{-1}^{1}$$

$$\Rightarrow I = -\frac{1}{a}\sqrt{(1-a)^2} + \frac{1}{a}\sqrt{(1+a)^2}$$

$$\Rightarrow I = -\frac{1}{a}|1-a| + \frac{1}{a}(1+a)$$

$$\Rightarrow I = 2$$

656 **(a)**

Let $I = \int \frac{\sin x + \cos x}{3+\sin 2x}dx = \int \frac{\sin x+\cos x}{4+\sin 2x - 1}dx$

$$\Rightarrow \quad I = \int \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2}dx$$

Put $\sin x - \cos x = t \;\; \Rightarrow \;\; (\cos x + \sin x)dx = dt$

$$\therefore \; I = \int \frac{dt}{4-t^2} = \frac{1}{4}\log\frac{2-t}{2+t} + c$$

$$= \frac{1}{4}\log\left(\frac{2-\sin x+\cos x}{2+\sin x - \cos x}\right) + c$$

657 **(d)**

Let $I = \int_{e^{-1}}^{e} \frac{dt}{t(t+1)}$

$$= \int_{e^{-1}}^{e} \frac{1}{t}dt - \int_{e^{-1}}^{e} \frac{1}{(t+1)}dt$$

$$= [\log t - \log(t+1)]_{e^{-1}}^{e}$$

$$= [\log e - \log(e+1)] - [\log e^{-1} - \log(e^{-1}+1)]$$

$$= \log\left(\frac{e}{e+1}\right) - [-\log e - \log(e^{-1}+1)]$$

$$= \log\left(\frac{e}{e+1}\right) + \left[\log e\left(\frac{1}{e}+1\right)\right]$$

$$= \log_e e = 1$$

658 **(b)**

Let

$$I = \int_0^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1}dx = \int_0^{\pi/2} \frac{\cos 3x - \cos 3\pi/3}{2(\cos x - \cos\pi/3)}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(4\cos^3 x - 3\cos x) - (4\cos^3\pi/3 - 3\cos\pi/3)}{2(\cos x - \cos\pi/3)}$$

$$\Rightarrow I = 2\int_0^{\pi/2}\left(\frac{\cos^3 x - \cos^3\pi/3}{\cos x - \cos\pi/3}\right)dx - \frac{3}{2}\int_0^{\pi/2}\left(\frac{\cos x - \cos\pi/3}{\cos x - \cos\pi/3}\right)dx$$

$$\Rightarrow I = 2\int_0^{\pi/2} (\cos^2 x + \cos^2\pi/3 + \cos\pi/3)\,dx - \frac{3}{2}\int_0^{\pi/2} 1 \,.\, dx$$

$$\Rightarrow I = \int_0^{\pi/2}\left(1 + \cos 2x + \frac{1}{2} + \cos x\right)dx - \frac{3\pi}{4}$$

$$= \frac{3\pi}{4} + 1 - \frac{3\pi}{4} = 1$$

659 **(a)**

Putting $x = \frac{1}{1+y}, dx = -\frac{1}{(1+y)^2}dy$, we have

$$I(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$$

$$\Rightarrow I(m,n) = \int_{\infty}^{0} \frac{1}{(1+y)^{m-1}}\left(1 - \frac{1}{1+y}\right)^{n-1} = \frac{(-1)}{(1+y)^2}dy$$

$$\Rightarrow I(m,n) = \int_{\infty}^{0} \frac{y^{n-1}}{(1+y)^{m-n}}dy = \int_0^{\infty}\frac{x^{n-1}}{(1+x)^{m+n}}dx$$

But $I(m,n) = I(n,m)$

$$\therefore I(m,n) = \int_0^{\infty}\frac{x^{m-1}}{(1+x)^{m+n}}dx = \int_0^{\infty}\frac{x^{n-1}}{(1+x)^{m+n}}dx$$

660 **(b)**

$\int_{-1}^{15} \text{sgn}(\{x\})dx = \int_0^{16}\text{sgn}(\{x-1\})dx$ (by property)

$$= \int_0^{16}\text{sgn}(\{x\})dx$$

$$= 16\int_0^1 \text{sgn}(\{x\})dx = 16\int_0^1 \text{sgn}(x)dx$$
$$= 16\int_0^1 1\cdot dx = 16$$

661 **(b)**

Let $I = \int_5^{10}\frac{1}{(x-1)(x-2)}dx$

$$\int_5^{10}\left[\frac{-1}{x-1}+\frac{1}{x-2}\right]dx$$
$$= [-\log(x-1)+\log(x-2)]_5^{10}$$
$$= -\log 9 + \log 8 + \log 4 - \log 3$$
$$= -2\log 3 + 3\log 2 + 2\log 2 - \log 3$$
$$= -3\log 3 + 5\log 2$$
$$= -\log 27 + \log 32$$
$$= \log\frac{32}{27}$$

662 **(d)**

Let $f(x) = \left(e^{x^3}+e^{-x^3}\right)(e^x - e^{-x})$

$\Rightarrow f(-x) = \left(e^{-x^3}+e^{x^3}\right)(e^{-x}-e^{x})$

$= -\left(e^{x^3}+e^{-x^3}\right)(e^x - e^{-x})$

$= -f(x)$

$\Rightarrow f(x)$is an odd function.

$\therefore \int_{-1}^1 f(x)dx = 0$

663 **(b)**

$$\int_{b-1}^{b}\frac{e^{-t}}{t-b-1}dt = \int_{-1}^{0}\frac{e^{-(t+b)}}{(t+b)-b-1}dt$$
$$= e^{-b}\int_{-1}^{0}\frac{e^{-t}}{t-1}dt$$
$$= e^{-b}\int_0^1\frac{e^{-(t-1)}}{t-2}dt$$

Put $t-1=-s$

$\Rightarrow\ dt = -ds$

$$= -e^{-b}\int_1^0\frac{e^s}{-(s+1)}ds$$
$$= e^{-b}\int_1^0\frac{e^s}{s+1}ds$$
$$= -e^{-b}\int_0^1\frac{e^t}{t+1}dt$$
$$= -ae^{-b}$$

664 **(c)**

Let $x = \alpha\cos^2\theta + \beta\sin^2\theta$. Then,

$x-\alpha = (\beta-\alpha)\sin^2\theta,\ \beta - x = (\beta-\alpha)\cos^2\theta$

and, $dx = 2(\beta-\alpha)\sin\theta\cos\theta\, d\theta$

Also, $x=\alpha \Rightarrow \theta = 0$ and, $x = \beta \Rightarrow \theta = \pi/2$

$$\therefore \int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$$
$$= \int_0^{\pi/2}\frac{2(\beta-\alpha)\sin\theta\cos\theta}{(\beta-\alpha)\sin\theta\cos\theta}d\theta$$
$$= 2\int_0^{\pi/2} d\theta = \pi$$

665 **(a)**

We have,

$$I = \int\frac{1}{\{(x-1)^3(x+2)^5\}^{1/4}}dx$$
$$= \int\frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$$
$$\Rightarrow I = \frac{1}{3}\int\left(\frac{x-1}{x+2}\right)^{-3/4} d\left(\frac{x-1}{x+2}\right)$$
$$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$$

666 **(b)**

Let $I =$

$\int_{-\pi/3}^{\pi/3} x\sec x\tan x\, dx$ [$x\sec x\tan x$ is an even]

$$= 2\int_0^{\pi/3} x\sec x\tan x\, dx$$
$$= 2\left[x\sec x - \log\left\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}\right]_0^{\pi/3}$$
$$= \frac{4\pi}{3} - 2\log\tan\frac{5\pi}{12}$$

667 **(a)**

We have,

$$I = \int\frac{(x^4-x)^{1/4}}{x^5}dx = \int\frac{1}{x^4}\left(1-\frac{1}{x^3}\right)^{1/4}dx$$
$$\Rightarrow I = \frac{1}{3}\int\left(1-\frac{1}{x^3}\right)^{1/4} d\left(1-\frac{1}{x^3}\right)$$
$$= \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4} + C$$

668 **(c)**

$f(x)$

$$= \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$$

[Applying $C_1 \to C_1 - (C_2 + C_3)$]

$$= \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4\sin x \\ 0 & \sin x & 1 \end{vmatrix}$$

$= \sin x(3 - 4\sin^2 x)$

$= \sin 3x$

Then, $\int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} \sin 3x \, dx$

$= -\left[\frac{\cos 3x}{3}\right]_0^{\pi/2}$

$= \left(-\frac{1}{3}\right)\left[\cos\left(3 \times \frac{\pi}{2}\right) - \cos 0\right]$

$= \left(-\frac{1}{3}\right)(0 - 1)$

$= \frac{1}{3}$

669 **(a)**

Let $I = \int \frac{(x^3+3x^2+3x+1)}{(x+1)^5} \, dx$

$= \int \frac{(x+1)^3}{(x+1)^5} dx = \int \frac{1}{(x+1)^2} \, dx$

$= -\frac{1}{(x+1)} + c$

670 **(c)**

Put $x = a\,(\sin\theta)^{2/3}$

$\Rightarrow dx = \frac{2}{3}\, a(\sin\theta)^{-1/3} \cos\theta \; d\theta$

$\therefore \int \sqrt{\frac{x}{a^3 - x^3}} \, dx$

$= \int \frac{a^{1/2}(\sin\theta)^{1/3} . \frac{2}{3}\, a\,(\sin\theta)^{-1/3} \cos\theta}{\sqrt{a^3 - a^3 \sin^2\theta}} d\theta$

$= \frac{2}{3}\, a^{3/2} \int \frac{\cos\theta}{a^{3/2}\sqrt{1 - \sin^2\theta}} \, d\theta$

$= \frac{2}{3} \sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c = g(x) + c$

$\therefore g(x) = \frac{2}{3} \sin^{-1} \sqrt{\frac{x^3}{a^3}}$

671 **(c)**

We know that, if

$I_n = \int \sin^n x \;\, dx$, then

$I_n = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n+1}{n}\; I_{n-2}$

where n is a positive integer

$\Rightarrow\; nI_n - (n-1)I_{n+2} = -\sin^{n-1} x \cos x$

672 **(b)**

Let $I = \int_0^{\pi} e^{\sin^2 x} \cos^3 x \, dx$ ...(i)

$\Rightarrow\; I = \int_0^{\pi} e^{\sin^2(\pi - x)} \cos^3(\pi - x) \, dx$

$\Rightarrow\; I = -\int_0^{\pi} e^{\sin^2 x} \cos^3 x \, dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = 0 \;\Rightarrow\; I = 0$

673 **(b)**

$\lim_{x \to \frac{\pi}{2}} \frac{\int_{\pi/2}^{x} (2^{-\cos t} - 1) dt}{\int_{\pi^2/4}^{x^2} \left(\sqrt{t} - \frac{\pi}{2}\right) d\,t} = \lim_{x \to \frac{\pi}{2}} \left(\frac{2^{-\cos x} - 1}{\left(x - \frac{\pi}{2}\right) \cdot 2x}\right)$

Applying L' Hospital's rule

$= \lim_{n \to \frac{\pi}{2}} \frac{2^{-\cos x}(\log_e 2)(\sin x)}{2x + 2\left(x - \frac{\pi}{2}\right)} = \frac{\log_e 2}{\pi}$

674 **(c)**

$\int f(x) g''(x) dx - \int f''(x) g(x) dx$

$= [f(x)g'(x) - \int f'(x)g'(x)dx] - [g(x)f'(x)$

$- \int g'(x)f'(x)dx]$

$= f(x)g'(x) - f'(x)g(x)$

675 **(d)**

We have,

$f(x) = A\sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right)$

$= \sqrt{2}$ and, $\int_0^1 f(x)\, dx = \frac{2A}{\pi}$

$\therefore f'(x) = \frac{A\pi}{2} \cos\left(\frac{\pi x}{2}\right)$

$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{A\pi}{2} \cos\frac{\pi}{4} = \frac{A\pi}{2\sqrt{2}} \Rightarrow \frac{A\pi}{2\sqrt{2}} = \sqrt{2} \Rightarrow A = \frac{4}{\pi}$

Now,

$\int_0^1 f(x)\, dx = \frac{2A}{\pi}$

$\Rightarrow \int_0^1 \left\{A\sin\left(\frac{\pi x}{2}\right) + B\right\} dx = \frac{2A}{\pi}$

$\Rightarrow \left[-\frac{2A}{\pi}\cos\left(\frac{\pi x}{2}\right) + Bx\right]_0^1 = \frac{2A}{\pi} \Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi}$

$\Rightarrow B = 0$

676 **(d)**

$I = \int \frac{dx}{\cos x - \sin x}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\cos\left(x + \frac{\pi}{4}\right)}$

$= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx$

$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right)\right| + c$

$= \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + c$

677 **(b)**

Let $I = \int \frac{(x+1)^2}{x(x^2+1)}\, dx = \int \frac{x^2+1+2x}{x(x^2+1)}\, dx$

$= \int \frac{1}{x} dx + \int \frac{2}{x^2+1} dx$

$\Rightarrow\ I = \log_e x + 2\tan^{-1} x + c$

678 **(a)**

Let $I = \int \sqrt{2}\sqrt{1+\sin x}\, dx = \sqrt{2} \int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$

$= 2\int \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = -4\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$

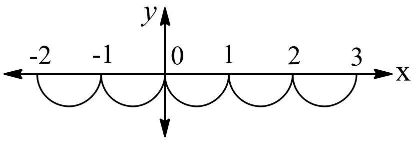
But $I = -4\cos(ax+b) + c$ (given)

On comparing, we get

$a = \frac{1}{2}, b = \frac{\pi}{4}$

679 **(b)**

Graph of $y = f(x)$ will be of the following form.
Form the graph we can observe that period of $f(x)$ is 1 unit.



$\therefore \left|\int_2^4 f(x)dx\right| = 2\left|\int_0^1 x(x-1)dx\right| = \frac{1}{3}$ sq unit

680 **(a)**

Let $I = \int_0^{\pi/2} \frac{\sin 8x \log \cot x}{\cos 2x} dx$ ...(i)

Then,

$I = \int_0^{\pi/2} \frac{\sin 8\left(\frac{\pi}{2} - x\right) \log\cot\left(\frac{\pi}{2} - x\right)}{\cos 2\left(\frac{\pi}{2} - x\right)} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin 8x \log \tan x}{\cos 2x} dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{\sin 8x}{\cos 2x} \log 1\, dx = 0$

$\Rightarrow I = 0$

681 **(a)**

We have,

$f(x) = \begin{cases} 3[x] - 5, & x > 0 \\ 2, & x = 0 \\ 3[x] + 5, & x < 0 \end{cases}$

$\therefore I = \int_{-3/2}^{2} f(x)dx\, dx$

$\Rightarrow I = \int_{-\frac{3}{2}}^{-1} (3[x] + 5)\, dx + \int_{-1}^{0} (3[x] + 5)$

$+ \int_0^1 (3[x] - 5)\, dx + \int_1^2 (3[x] - 5)\, dx$

$\Rightarrow I = \int_{-\frac{3}{2}}^{-1} (-6 + 5)\, dx + \int_{-1}^{0} (-3 + 5)$

$+ \int_0^1 (-5)\, dx + \int_1^2 (3 - 5)\, dx$

$\Rightarrow I = -\left(-1 + \frac{3}{2}\right) + 2(0+1) - 5(1-0) + (-2)(2-1)$

$\Rightarrow I = -\frac{1}{2} + 2 - 5 - 2 = \frac{-11}{2}$

682 **(a)**

Let $I = \int \frac{(1+x^4)}{(1-x^4)^{3/2}}\, dx$

$= \int \frac{x^2\left(\frac{1}{x^2} + x^2\right)}{x^3\left(\frac{1}{x^2} - x^2\right)} dx = \int \frac{\left(\frac{1}{x^3} + x\right)}{\left(\frac{1}{x^2} - x^2\right)} dx$

Put $\frac{1}{x^2} - x^2 = z$

$\Rightarrow \left(\frac{1}{x^3} + x\right) dx = -\frac{dz}{z}$

$\therefore I = -\frac{1}{2}\int \frac{dz}{z^{3/2}}$

$= z^{-1/2} + c = \frac{x}{\sqrt{1-x^4}} + c$

683 **(a)**

We have,

$I = \int \frac{\sin\theta - \cos\theta}{(\sin\theta + \cos\theta)\sqrt{\sin\theta\cos\theta + \sin^2\theta\cos^2\theta}} d\theta$

$\Rightarrow I = \int \frac{\sin^2\theta - \cos^2\theta}{(\sin\theta + \cos\theta)^2\sqrt{\left(\sin\theta\cos\theta + \frac{1}{2}\right)^2 - \frac{1}{4}}} d\theta$

$\Rightarrow I = -\int \frac{2\cos 2\theta}{(1+\sin 2\theta)\sqrt{(1+\sin 2\theta)^2 - 1}} d\theta$

$\Rightarrow I = -\int \frac{1}{(1+\sin 2\theta)\sqrt{(1+\sin 2\theta)^2 - 1}} d(1 + \sin 2\theta)$

$\Rightarrow I = \operatorname{cosec}^{-1}(1 + \sin 2\theta)$

Hence, $f(\theta) = 1 + \sin 2\theta$

684 **(d)**

$\because f(x) = \cos x - \cos^2 x + \cos^3 x - \ldots \infty$

$$= \frac{\cos x}{1+\cos x}$$

$$\therefore \int f(x)\ dx = \int \frac{1+\cos x}{1+\cos x}dx - \int \frac{1}{1+\cos x}\ dx$$

$$= \int 1\ dx - \frac{1}{2}\int \sec^2\frac{x}{2}\ dx$$

$$= x - \frac{1}{2}\tan\frac{x}{2}.2 + c$$

$$= x - \tan\frac{x}{2} + c$$

685 **(c)**

We have,

$$F(x) = \frac{1}{x^2}\int_4^x [4t^2 - 2F'(t)]dt$$

$$\Rightarrow F'(x) = \frac{1}{x^2}[4x^2 - 2F'(x)] - \frac{2}{x^3}\int_4^x \{4t^2 - 2F'(t)\}\,dt$$

$$\Rightarrow F'(4) = \frac{1}{16}[64 - 2F'(4)] - 0$$

$$\Rightarrow 18F'(4) = 64 \Rightarrow F'(4) = \frac{32}{9}$$

686 **(d)**

We have, $f'(x) = f(x)$

$$\therefore \frac{f'(x)}{f(x)} = 1$$

$\Rightarrow \log f(x) = x\log C$

$\Rightarrow f(x) = Ce^x$

$\Rightarrow 1 = Ce^0 \Rightarrow C = 1 \quad [\because f(0) = 1]$

$\therefore f(x) = e^x$ and $g(x) = x^2 - e^x$

$$\therefore \int_0^1 f(x)\,g(x)dx = \int_0^1 e^x\,(x^2 - e^x)dx$$

$$= e - \frac{1}{2}e^2 - \frac{3}{2}$$

687 **(a)**

We have,

$$\int_0^\pi \frac{1}{a+b\cos x}dx = \frac{\pi}{\sqrt{a^2-b^2}}$$

$$\Rightarrow \frac{d}{da}\int_0^\pi \frac{1}{a+b\cos x}dx = -\frac{\pi}{2(a^2-b^2)^{3/2}} \times 2a$$

$$\Rightarrow \int_0^\pi -\frac{1}{(a+b\cos x)^2}dx = -\frac{\pi a}{(a^2-b^2)^{3/2}}$$

$$\Rightarrow \int_0^\pi \frac{1}{(a+b\cos x)^2}dx = \frac{\pi a}{(a^2-b^2)^{3/2}}$$

688 **(c)**

$$\int \frac{e^x(1+\sin x)}{(1+\cos x)}dx$$

$$= \int e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right]dx$$

$$= \frac{1}{2}\int e^x\sec^2\frac{x}{2}dx + \int e^x\tan\frac{x}{2}dx$$

$$= e^x\tan\frac{x}{2} - \int e^x\tan\frac{x}{2}dx + \int e^x\tan\frac{x}{2}dx$$

$$= e^x\tan\frac{x}{2} + c$$

But $I = e^x f(x) + c$ (given)

$$\therefore f(x) = \tan\frac{x}{2}$$

689 **(c)**

Let $I = \int \frac{dx}{(e^x+e^{-x})^2} = \int \frac{e^{2x}\ dx}{(e^{2x}+1)^2}$

Put $e^{2x} + 1 = z$

$\Rightarrow 2e^{2x}dx = dz$

$$\therefore I = \frac{1}{2}\int \frac{dz}{z^2}$$

$$= -\frac{1}{2z} + c$$

$$= -\frac{1}{2(e^{2x}+1)} + c$$

$$= -\frac{1}{2}(e^{2x}+1)^{-1} + c$$

690 **(a)**

We have,

$$I = \int \frac{x\cos x + 1}{\sqrt{2x^3\,e^{\sin x} + x^2}}dx$$

$$\Rightarrow I = \int \frac{1}{2x\,e^{\sin x}} \times \frac{(2x\cos x + 2)\,e^{\sin x}}{\sqrt{2x\,e^{\sin x+1}}}dx$$

$$\Rightarrow I = 2\int \frac{1}{t^2-1}dt, \text{where } 2\,xe^{\sin x} + 1 = t^2$$

$$\Rightarrow I = \log\left|\frac{t-1}{t+1}\right| + C$$

$$= \log\left|\frac{\sqrt{2x\,e^{\sin x}+1} - 1}{\sqrt{2x\,e^{\sin x}+1} + 1}\right| + C$$

691 **(a)**

We have,

$$I = \int_1^3 \frac{3\log_e|\sin x^3|}{x}dx$$

$$\Rightarrow I = \int_1^{27} \frac{3x^2 \log_e|\sin x^3|}{x^3} dx$$

$$\Rightarrow I = \int_1^{3} \frac{\log_e|\sin x^3|}{x^3} d(x^3), \text{where } t = x^3$$

$$\Rightarrow I = \int_1^{27} \frac{\log_e|\sin t|}{t} dt$$

$$\Rightarrow I = \int_1^{27} \phi'(t)\, dt = [\phi(t)]_1^{27} = \phi(27) - \phi(1)$$

Hence, $k = 27$

692 **(c)**

$$\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}}\, dx$$

$$= \int \frac{e^{\log_e x^5} - e^{\log_e x^4}}{e^{\log_e x^3} - e^{\log_e x^2}}\, dx$$

$$= \int \frac{x^5 - x^4}{x^3 - x^2} dx = \int \frac{x^4(x-1)}{x^2(x-1)}\, dx$$

$$= \int x^2\, dx = \frac{x^3}{3} + c$$

693 **(c)**

Let $I = \int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$. Then,

$$I = \int_0^{2a} \frac{f(2a-x)}{f(2a-x)+f(x)} dx$$

$$\therefore 2I = \int_0^{2a} \frac{f(x)+f(2a-x)}{f(x)+f(2a-x)} dx = \int_0^{2a} 1 \cdot dx = 2a$$

$$\Rightarrow I = a$$

694 **(c)**

$$|x| < \sqrt{1+x^2} \;\; \Rightarrow \frac{1}{|x|} > \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \int_0^1 \frac{1}{|x|} dx > \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$$

$$\Rightarrow I_1 > I_2$$

695 **(d)**

$\because$ Integrand is discontinuous at $\frac{\pi}{2}$, then

$$\int_0^{\pi/2} 0.dx + \int_{\pi/2}^{3\pi/2} 0.dx = 0$$

$\because \; 0 < x < \frac{\pi}{2}, |\tan^{-1}\tan x| = |\sin^{-1}\sin x|$

and $\frac{\pi}{2} < x < \frac{3\pi}{2}, |\tan^{-1}\tan x| = |\sin^{-1}\sin x|$

696 **(c)**

We have,

$$I = \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx$$

$$= \int \sqrt{\frac{\cos x}{1 - \cos^3 x}} \sin x\; dx$$

$$\Rightarrow I = -\frac{2}{3}\int \frac{1}{1^2 - (\cos^{3/2} x)^2} d\,(\cos^{3/2} x)$$

$$\Rightarrow I = \frac{2}{3}\cos^{-1}\left(\cos^{3/2} x\right) + C$$

697 **(b)**

We have,

$$\int x \sqrt{\frac{2\sin(x^2+1) - \sin 2(x^2+1)}{2\sin(x^2+1) + \sin 2(x^2+1)}} dx$$

$$= \int x \sqrt{\frac{2\sin(x^2+1) - 2\sin(x^2+1)\cos(x^2+1)}{2\sin(x^2+1) + 2\sin(x^2+1)\cos(x^2+1)}}$$

$$= \int x \sqrt{\frac{1 - \cos(x^2+1)}{1 + \cos(x^2+1)}} dx$$

$$= \int x \tan\left(\frac{x^2+1}{2}\right) dx$$

$$= \int \tan\left(\frac{x^2+1}{2}\right) d\left(\frac{x^2+1}{2}\right)$$

$$= \log\left|\sec\left(\frac{x^2+1}{2}\right)\right| + C$$

698 **(a)**

Let $I = \int_0^{\pi} \frac{x\, dx}{1+\cos\alpha \sin x} dx$. Then,

$$I = \int_0^{\pi} \frac{\pi - x}{1 + \cos\alpha \sin(\pi - x)} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{\pi}{1 + \cos\alpha \sin x} dx - I$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sec^2 x/2}{1 + \tan^2(x/2) + 2\tan(x/2)\cos\alpha} dx$$

$$\Rightarrow 2I = 2\pi \int_0^{\infty} \frac{dt}{t^2 + 2t\cos\alpha + 1}, \text{where } t = \tan\frac{x}{2}$$

$$\Rightarrow 2I = 2\pi \int_0^{\infty} \frac{dt}{(t+\cos\alpha)^2 + \sin^2\alpha}$$

$$\Rightarrow I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$$

$$\Rightarrow I = \frac{\pi}{\sin\alpha}\left[\frac{\pi}{2} - \tan^{-1}(\cot\alpha)\right]$$

$$\Rightarrow I = \frac{\pi}{\sin\alpha}\left[\frac{\pi}{2} - \tan^{-1}\tan\left(\frac{\pi}{2} - \alpha\right)\right] \Rightarrow I = \frac{\pi\alpha}{\sin\alpha}$$

699 **(d)**

Let, $I = \int \tan(\sin^{-1} x)\, dx$

$$= \int \tan\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right)dx = \int \frac{x}{\sqrt{1-x^2}}dx$$

Put, $1-x^2 = t^2$

$\Rightarrow -2x\,dx = 2t\,dt$

$$\therefore\ I = -\int \frac{t\,dt}{t} = -t + c = -\sqrt{1-x^2} + c$$

700 **(b)**

Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}}dx$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}}dx - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$$

$$= [\sin^{-1}x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}dt$$

Put $t^2 = 1-x^2 \Rightarrow tdt = -xdx$

$$\therefore\ I = [\sin^{-1}1 - \sin^{-1}0] + \int_1^0 \frac{t}{t}dt$$

$$= \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1$$

Alternate

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta\, d\theta$

$$\therefore I = \int_{\pi/4}^{0} -\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}.2\sin 2\theta\, d\,\theta$$

$$= 2\int_0^{\pi/4}\frac{\sin\theta}{\cos\theta}.2\sin\theta\cos\theta d\theta$$

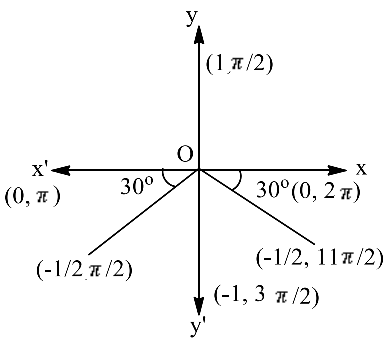
$$= 2\int_0^{\pi/4}(1+\cos 2\theta)d\theta = 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= \frac{\pi}{2} - 1$$

701 **(b)**

We know that $\sin\frac{\pi}{6} = \frac{1}{2}$

$$\sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6} = -\frac{1}{2}$$



$$\sin\frac{11\pi}{6} = \sin\left(2\pi - \frac{\pi}{6}\right)$$

$$= -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\sin\frac{9\pi}{6} = \sin\frac{3\pi}{6} = -1$$

Hence, we divide the interval π to 2π as

$$\left(\pi, \frac{7\pi}{6}\right), \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$$

$$\sin x = \left(0, -\frac{1}{2}\right), \left(-\frac{1}{2}, -1\right), \left(-\frac{1}{2}, 0\right)$$

$2\sin x = (0,-1), (-1,-2), (-1,0)$

$$\therefore \int_{\pi}^{2\pi}[2\sin x]dx$$

$$= \int_{\pi}^{7\pi/6} -1dx + \int_{7\pi/6}^{11\pi/6} -2\,dx$$

$$+ \int_{11\pi/6}^{2\pi} -1dx$$

$$= -\frac{5\pi}{3}$$

702 **(d)**

We have,

$$I = \int \frac{1}{(\sin x + 4)(\sin x - 1)}dx$$

$$\Rightarrow I = \frac{1}{5}\int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)}dx$$

$$\Rightarrow I = \frac{1}{5}\int \frac{1}{\sin x - 1}dx - \frac{1}{5}\int \frac{1}{\sin x + 4}dx$$

$$\Rightarrow I = \frac{1}{5}\int \frac{2dt}{2t - 1 - t^2}$$

$$- \frac{1}{5}\int \frac{2dt}{2t + 4(1+t^2)}, \text{where } t = \tan\frac{x}{2}$$

$$\Rightarrow I = -\frac{2}{5}\int \frac{dt}{t^2 - 2t + 1} - \frac{1}{10}\int \frac{dt}{t^2 + \frac{1}{2}t + 1}$$

$$\Rightarrow I = -\frac{2}{5}\int \frac{1}{(1-t)^2}dt$$

$$- \frac{1}{10}\int \frac{1}{\left(t + \frac{1}{4}\right)^2 + \left(\sqrt{15}/4\right)^2}dt$$

$$\Rightarrow I = \frac{2}{5}\frac{1}{(t-1)} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right) + C$$

$$\Rightarrow I = \frac{2}{5}\cdot\frac{1}{\tan\frac{x}{2} - 1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2} + 1}{\sqrt{15}}\right) + C$$

Hence, $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4\tan\frac{x}{2} + 1}{\sqrt{15}}$

703 **(d)**

Given, $\int e^x(1+x).\sec^2(xe^x)\,dx = f(x) +$ constant

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt$ in LHS, we get

LHS $= \int \sec^2 t \ dt$

$= \tan t + \text{constant}$

$\Rightarrow \tan(xe^x) + \text{constant} = f(x) + \text{constant}$

$\Rightarrow f(x) = \tan(xe^x)$

704 **(c)**

Since $e^{x-[x]}$ is a periodic function with period 1

$$\therefore \int_0^{1000} e^{x-[x]}dx = 1000\int_0^1 e^{x-[x]}\,dx$$

$$= 1000\int_0^1 e^x\,dx = 1000(e-1)$$

706 **(b)**

$$\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}}\,dx = \int \frac{2e^{2x}+3}{3e^{2x}+4}dx$$

Let $2e^{2x} + 3 = A(3e^{2x} + 4) + B(6e^{2x})$

$\Rightarrow\ 2e^{2x} + 3 = (3A + 6B)e^{-2x} + 4A$

On comparing both sides, we get

$2 = 3A + 6B$ ...(i)

and $3 = 4A$ ...(ii)

From Eqs. (i) and (ii), we get

$$2 = \frac{9}{4} + 6B$$

$$\Rightarrow 6B = 2 - \frac{9}{4} = -\frac{1}{4}$$

$$\Rightarrow B = -\frac{1}{24}$$

$$\therefore \int \frac{2e^{2x}+3}{3e^{2x}+4}dx$$

$$= \int \frac{3(3e^{2x}+4)}{4(3e^{2x}+4)}dx - \frac{1}{24}\int \frac{6e^{2x}}{3e^{2x}+4}dx$$

$$= \frac{3}{4}x - \frac{1}{24}\log(3e^{2x}+4) + c$$

707 **(b)**

Let $I = \int \frac{x^2}{1+(x^3)^2}dx$

Put $x^3 = t\ \Rightarrow 3x^2dx = dt$

$$\therefore I = \frac{1}{3}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{3}\tan^{-1} t + c = \frac{1}{3}\tan^{-1}(x^3) + c$$

708 **(b)**

We have,

$$I = \int_0^\pi x \sin x \cos^4 x\,dx$$

$$\Rightarrow I = \int_0^\pi (\pi - x)\sin(\pi - x)\cos^4(\pi - x)\,dx$$

$$\Rightarrow I = \int_0^\pi (\pi - x)\sin x \cos^4 x\,dx$$

$$\Rightarrow I = \pi\int_0^\pi \sin x \cos^4 x\,dx - I$$

$$\Rightarrow 2I = \pi\left[\frac{-\cos^5 x}{5}\right]_0^\pi = \frac{\pi}{5}(1+1) = \frac{2\pi}{5}$$

$$\Rightarrow I = \frac{\pi}{5}$$

709 **(b)**

Let $I = \int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}\,dx$

$$= \int \frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

Put $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\Rightarrow \left(-\frac{2}{x^3} - \frac{5}{x^6}\right)dx = dt$$

Then, $I = -\int \frac{dt}{t^3} = \frac{1}{2t^2} + c$

$$= \frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2} + c$$

$$= \frac{x^{10}}{2(x^5+x^3+1)^2} + c$$

710 **(c)**

Let $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\cos x}2^{\sin x}}dx$ ....(ii)

$\left[\text{put } x = \left(\frac{\pi}{2} - x\right)\right]$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1dx = \frac{\pi}{2}$$

$$\Rightarrow\ I = \frac{\pi}{4}$$

711 **(b)**

$$\int_0^\pi \sqrt{\frac{\cos 2x + 1}{2}}dx$$

$$= \int_0^\pi \sqrt{\cos^2 x}\,dx = \int_0^\pi |\cos x|\,dx$$

$$= \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^\pi \cos x\,dx$$

$= 2$

712 **(d)**

Let $I = \int e^{x\log a}.e^x\ dx = \int a^x.e^x\,dx$

$$= \int (ae)^x\, dx = \frac{(ae)^x}{\log_e ae} + c$$

713 **(a)**

Let $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$ and Replacing x by $-x$, we get

$$f(-x) = \log\left(\sqrt{1 + x^2} - x\right)$$

$$= \log\left(\sqrt{1 + x^2} - x\right)\frac{\left(\sqrt{1 + x^2} + x\right)}{\left(\sqrt{1 + x^2} + x\right)}$$

$$= \log\frac{[(1 + x^2) - x^2]}{\left(\sqrt{1 + x^2} + x\right)}$$

$$= \log 1 - \log\left(\sqrt{1 + x^2} + x\right)$$

$$= -\log\left(\sqrt{1 + x^2} + x\right)$$

$$\Rightarrow f(-x) = -f(x)$$

Hence, $f(x)$ is an odd function.

$$\therefore \int_{-1}^{1} \log\left(x + \sqrt{1 + x^2}\right) dx = 0$$

714 **(b)**

$$I_n = \int (\log x)^n\, dx$$

$$= (\log x)^n . x - \int x . n(\log x)^{n-1}\frac{1}{x}dx$$

$$\Rightarrow\ I_n = x(\log x)^n - nI_{n-1}I$$

$$\Rightarrow\ I_n + nI_{n-1} = x(\log x)^n$$

715 **(a)**

$$\because x^2 < x^{\frac{\pi}{2}} < x$$

$$\Rightarrow 1 + x^2 < 1 + x^{\frac{\pi}{2}} < 1 + x$$

$$\Rightarrow \frac{1}{1 + x} < \frac{1}{1 + x^{\frac{\pi}{2}}} < \frac{1}{1 + x^2}$$

$$\Rightarrow \int_0^1 \frac{dx}{1 + x} < \int_0^1 \frac{1}{1 + x^{\frac{\pi}{2}}}dx < \int_0^1 \frac{dx}{1 + x^2}$$

$$\Rightarrow \{\log(1 + x)\}_0^1 < I < (\tan^{-1} x\,)_0^1$$

$$\Rightarrow \log 2 < I < \frac{\pi}{4}$$

716 **(b)**

Let $I = \int_a^b \frac{x}{|x|} dx = \int_a^b -1 dx$

$$\Rightarrow\ I = -[x]_a^b = [-b + a]$$

$$= |b| - |a| \quad [\because a < b < 0]$$

717 **(d)**

We have,

$$f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$$

$$\Rightarrow f(x) = \begin{vmatrix} 0 & \sin x - x^2 & 2 - \cos x \\ x^2 - \sin x & 0 & 2x - 1 \\ \cos x - 2 & 1 - 2x & 0 \end{vmatrix}$$

[Interchanging rows and columns]

$$\Rightarrow f(x)$$

$$= (-1)^3 \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 2x - 1 \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$$

[Taking (-1) common from each column]

$$\Rightarrow f(x) = -f(x) \Rightarrow f(x) = 0 \Rightarrow \int f(x)\, dx = 0$$

718 **(d)**

$$f(x) = \lim_{n\to\infty} n^2\, x^{1/(n+1)}\left[x^{\frac{1}{n}-\frac{1}{n+1}} - 1\right]$$

$$= \lim_{n\to\infty} \frac{x^{1/(n+1)}\left(\frac{1}{x^{n(n+1)}} - 1\right)}{\frac{1}{n(n+1)} \times \frac{n(n+1)}{n^2}} \log x = \log x$$

Hence, $\int x\, f(x)dx = \int x \log x\, dx$

$$= \frac{x^2}{2}\log x - \frac{1}{4}x^2 + c$$

719 **(b)**

We have,

$$I = \int \frac{1}{x^{1/2}(1 + x^2)^{5/4}}dx = \int \frac{1}{x^3(1 + x^2)^{5/4}}dx$$

$$\Rightarrow I = -\frac{1}{2}\int \left(1 + \frac{1}{x^2}\right)^{-5/4} d\left(1 + \frac{1}{x^2}\right)$$

$$\Rightarrow I = -\frac{1}{2}\left\{\frac{\left(1 + \frac{1}{x^2}\right)^{-1/4}}{-\frac{1}{4}}\right\} + C = \frac{2\sqrt{x}}{(x^2 + 1)^{1/4}} + C$$

720 **(c)**

$\{x\} = x$ when $0 < x < 1$ and $[x + 1] = 1$

$$\therefore \int_0^1 (\{x\}[x + 1])dx = \int_0^1 x\, dx = \frac{1}{2}$$

721 **(a)**

We have,

$$\int_0^2 (x - \log_2 a)\, dx = 2\log_2\left(\frac{2}{a}\right)$$

$$\Rightarrow 2 - 2\log_2 a = 2\log_a\left(\frac{2}{a}\right)$$

$$\Rightarrow 1 = \log_2 a + \log_a 2 - \log_a a$$

$$\Rightarrow 2 = \log_2 a + \log_a 2$$

$$\Rightarrow 2 = \log_2 a + \frac{1}{\log_2 a} \Rightarrow \log_2 a = 1 \Rightarrow a = 2$$

722 **(c)**

By definition $\text{sgn}\,(x - [x]) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

$$\therefore \int_{-2}^{10} \text{sgn}\,(x - [x])dx$$

$$= -\int_{-2}^{0} 1\, dx + \int_0^{10} 1\, dx$$

$$= -[x]_{-2}^{0} + [x]_0^{10} = 8$$

723 **(c)**

We have,

$$I = \int_0^{\infty} \frac{1}{1+x^4} dx$$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2\theta}{\cos^4\theta+\sin^4\theta} d\theta$, where $x = \tan\theta$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^2\theta}{\cos^4\theta+\sin^4\theta} d\theta$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{1}{\cos^4\theta + \sin^4\theta} d\theta$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sec^4\theta}{1+\tan^4\theta} d\theta$$

$$\Rightarrow 2I = \int_0^{\infty} \frac{t^2+1}{t^4+1} dt$$

$$= \int_0^{\infty} \frac{1}{\left(t-\frac{1}{t}\right)^2 + \left(\sqrt{2}\right)^2} d\left(t - \frac{1}{2}\right)$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t-1/t}{\sqrt{2}}\right)\right]_0^{\infty} = \frac{1}{\sqrt{2}}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{\pi}{\sqrt{2}}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{2}}$$

724 **(a)**

$$\int \frac{x^2}{x^2+4} dx = \int \frac{x^2+4-4}{(x^2+4)} dx$$

$$= \int \left(\frac{x^2+4}{x^2+4} - \frac{4}{x^2+4}\right) dx$$

$$= \int \left[1 - \frac{4}{x^2+4}\right] dx = \int dx - \int \frac{4}{x^2+4} dx$$

$$= x - 4\int \frac{dx}{x^2+(2)^2} = x - \frac{4}{2}\tan^{-1}\frac{x}{2} + c$$

$$= x - 2\tan^{-1}\frac{x}{2} + c$$

725 **(c)**

Let $I = \int \frac{1+x}{x+e^{-x}} dx = \int \frac{e^x(x+1)}{xe^x+1} dx$

Put $xe^x + 1 = t$

$\Rightarrow (xe^x + e^x)dx = dt$

$\Rightarrow (x+1)e^x \, dx = dt$

$\therefore \; I = \int \frac{dt}{t} = \log t + c$

$= \log|(xe^x+1)| + c$

726 **(c)**

Let $I = \int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$

Put $\frac{x+1}{x+2} = t \;\Rightarrow\; \frac{1}{(x+2)^2} dx = dt$

$$\therefore I = \int \frac{dt}{t^{5/4}} \Rightarrow I = \frac{t^{-1/4}}{\left(-\frac{1}{4}\right)} + c$$

$$\Rightarrow I = -4\left(\frac{x+2}{x+1}\right)^{1/4} + c$$

727 **(a)**

$\frac{x+1}{e^x} < 1$ ...(i)

$\Rightarrow \frac{[x+1]}{e^x} < 1$ ...(ii)

(as $[x+1] < x+1$)

$\therefore \left[\frac{[x+1]}{e^x}\right] = 0$ [from Eqs.(i) and (ii)]

728 **(b)**

Let $\tan^{-1} x = t$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\therefore \int \frac{(\tan^{-1}x)^3}{1+x^2} dx = \int t^3 dt$$

$$= \frac{t^4}{4} + c = \frac{(\tan^{-1}x)^4}{4} + c$$

729 **(c)**

Put $x^x = t$

$\Rightarrow x^x(1+\log_e|x|)dx = dt$

$\therefore \int x^x(1+\log_e|x|)dx = t + c = x^x + c$

730 **(a)**

$$\int \left\{\frac{(\log x - 1)}{1+(\log x)^2}\right\}^2 dx$$

$$= \int \frac{(\log x)^2 + 1 - 2\log x}{[(\log x)^2+1]^2} dx$$

$$= \int \frac{(\log x)^2 + 1 - 2x\left(\log x . \frac{1}{x}\right)}{[(\log x)^2+1]^2} dx$$

$$= \int \frac{d}{dx}\left(\frac{x}{(\log x)^2+1}\right) dx = \frac{x}{(\log x)^2+1} + c$$

731 **(d)**

$$\lim_{n\to\infty} \frac{1+2^4+3^4+\ldots+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+\ldots+n^3}{n^5}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^4 - \lim_{n\to\infty}\frac{1}{n} \times \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^3$$

$$= \int_0^1 x^4 dx - \lim_{n\to\infty}\frac{1}{n} \times \int_0^1 x^3 dx$$

$$= \left[\frac{x^5}{5}\right]_0^1 - 0 = \frac{1}{5}$$

732 **(c)**

Let $I = \int \frac{dx}{\sqrt{1-e^{2x}}} = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}} dx$

Put $e^{-x} = t \Rightarrow e^{-x}dx = -dt$

$\therefore\ I = -\int \frac{dt}{\sqrt{t^2-1}} = -\log\left|t+\sqrt{t^2-1}\right| + c$

$= -\log\left|e^{-x}+\sqrt{e^{-2x}-1}\right| + c$

733 **(d)**

Since, $\cos(\sin x) > \cos x > \sin(\cos x)$, for $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \int_0^{\pi/2} \cos(\sin x)dx > \int_0^{\pi/2} \cos x\, dx$

$> \int_0^{\pi/2} \sin(\cos x)dx$

$\Rightarrow\ I > K > J$

734 **(c)**

Let $I = \int_{-1}^{1} \frac{17x^5+29x^3-31x}{x^2+1}dx - \int_{-1}^{1}\frac{x^4-1}{x^2+1}dx$

$= 0 - 2\int_0^1 \frac{(x^2-1)(x^2+1)}{(x^2+1)}dx$

$= -2\left[\left(\frac{x^3}{3}-x\right)\right]_0^1 = \frac{4}{3}$

735 **(b)**

Let $I = \int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx$

Put $x = \cos 2\theta$,

then $\sin\left[2\tan^{-1}\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\right]$

$= \sin[2\tan^{-1}(\cot\theta)]$

$= \sin\left[2\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)\right]$

$= \sin\left[2\left(\frac{\pi}{2}-\theta\right)\right] = \sin(\pi-2\theta) = \sin 2\theta$

$= \sqrt{1-\cos^2 2\theta} = \sqrt{1-x^2}$

Now, $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx = \int_0^1 \sqrt{1-x^2}\,dx$

$= \left[\frac{1}{2}x\cdot\sqrt{1-x^2}\right]_0^1 + \frac{1}{2}[\sin^{-1}x]_0^1$

$= \frac{1}{2}[1\sqrt{1-1}-0] + \frac{1}{2}[\sin^{-1}(1)-0]$

$= \frac{1}{2}[0] + \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

736 **(a)**

Let $I = 2\int e^{2x}\sin 3x\,dx + 3\int e^{2x}\cos 3x\,dx$

$= 2\left[\frac{e^{2x}}{2}\sin 3x - \int\frac{e^{2x}}{2}3\cos 3x\,dx\right]$

$+ 3\int e^{2x}\cos 3x\,dx$

$= e^{2x}\sin 3x + c$

737 **(b)**

$\int_0^2 |x^2-1|dx = \int_0^1 -(x^2-1)dx$

$+ \int_1^2 (x^2-1)dx$

$= \left[-\frac{x^3}{3}+x\right]_0^1 + \left[\frac{x^3}{3}-x\right]_1^2$

$= -\frac{1}{3}+1+\frac{8}{3}-2-\frac{1}{3}+1$

$=2$

738 **(a)**

For $0 < x < 1$, we get

$x^2 > x^3$

For, $1 < x < 2$, we have $x^3 + x^2$

$\therefore 2^{x^2} > 2^{x^3}$ for $0 < x < 1$ and $2^{x^2} < 2^{x^3}$ for $1 < x < 2$

$\Rightarrow \int_0^1 2^{x^2}dx > \int_0^1 2^{x^3}dx$ and $\int_0^1 2^{x^2}dx < \int_0^1 2^{x^3}dx$

$\Rightarrow I_1 > I_2$ and $I_3 > I_4$

739 **(b)**

$\int(\sin 2x - \cos 2x)dx = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow \sqrt{2}\int\left(\frac{1}{\sqrt{2}}\sin 2x - \frac{1}{\sqrt{2}}\cos 2x\right)dx$

$= \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow -\sqrt{2}\int\left(\frac{1}{2}\cos 2x - \frac{1}{\sqrt{2}}\sin 2x\right)dx$

$= \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow\ -\sqrt{2}\int\left(2x+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow\ -\frac{\sqrt{2}}{2}\sin\left(2x+\frac{\pi}{4}\right)+c = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right)+c = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\therefore a = -\frac{5\pi}{4}, b \in R$

740 **(d)**

Let $I = \int e^{x+x^{-1}}.1\,dx + \int\left(x-\frac{1}{x}\right)e^{x+x^{-1}}dx$

$= xe^{x+x^{-1}} - \int\left(1-\frac{1}{x^2}\right)xe^{x+x^{-1}}dx$

$+ \int\left(x-\frac{1}{x}\right)e^{x+x^{-1}}dx$

$= xe^{x+x^{-1}} + c$

741 **(c)**

We have,

$$I = \int_0^1 f(k-1+x)\,dx$$

$$\Rightarrow I = \int_{k-1}^{k} f(t)\,dt, \text{where } t = k-1+x$$

$$\Rightarrow I = \int_{k-1}^{k} f(x)\,dx$$

$$\therefore I = \sum_{k=1}^{n} \int_{k-1}^{k} f(x)\,dx$$

$$\Rightarrow I = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \cdots + \int_{n-1}^{n} f(x)dx$$

$$= \int_0^n f(x)\,dx$$

742 **(c)**

Let $I = \int_0^\pi \frac{1}{1+3^{\cos x}} dx$ ...(i)

Then,

$$I = \int_0^\pi \frac{1}{1+3^{\cos(\pi-x)}} dx$$

$$\Rightarrow I = \int_0^\pi \frac{1}{3+e^{-\cos x}} dx$$

$\Rightarrow I = \int_0^\pi \frac{3^{\cos x}}{3^{\cos x}+1}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^\pi 1\,.\,dx = \pi \Rightarrow I = \frac{\pi}{2}$$

ALITER We have,

$$I = \int_0^\pi \frac{1}{1+3^{\cos x}} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left\{\frac{1}{1+3^{\cos x}} + \frac{1}{3^{\cos(\pi-x)}}\right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left\{\frac{1}{1+3^{\cos x}} + \frac{1}{1+3^{-\cos x}}\right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

743 **(c)**

We have,

$$I = \int_0^1 \frac{1}{x^2+2x\cos\alpha+1} dx$$

$$\Rightarrow I = \int_0^1 \frac{1}{(x+\cos\alpha)^2 \sin^2\alpha} dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left\{\tan^{-1}\left(\frac{x+\cos\alpha}{\sin\alpha}\right)\right\}_0^1$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\tan^{-1}\frac{1+\cos\alpha}{\sin\alpha} - \tan^{-1}\frac{\cos\alpha}{\sin\alpha}\right]$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\cot\frac{\alpha}{2}\right) - \tan^{-1}(\cot\alpha)\right]$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - \left(\frac{\pi}{2}-\alpha\right)\right] = \frac{\alpha}{2\sin\alpha}$$

744 **(c)**

Let $I \int_0^{\pi/4} \frac{\sin\theta+\cos\theta}{9+16\sin 2\theta} d\theta$

Putting $\sin\theta - \cos\theta = t$ and $(\sin\theta+\cos\theta)d\theta = dt$, we get

$$I = \int_{-1}^{0} \frac{1}{9+16(1-t^2)} dt = \int_{-1}^{0} \frac{1}{25-16t^2} dt$$

$$\Rightarrow I = \frac{1}{40}\left[\log\left(\frac{5+4t}{5-4t}\right)\right]_{-1}^{0} = -\frac{1}{40}\log\left(\frac{1}{9}\right)$$

$$= \frac{1}{20}\log 3$$

745 **(a)**

Let $I = \int_0^\infty \frac{dx}{\left(x+\sqrt{x^2+1}\right)^3}$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\sec^2\theta d\theta}{\left(\tan\theta + \sqrt{\tan^2\theta+1}\right)^3}$$

$$= \int_0^{\pi/2} \frac{\sec^2\theta d\theta}{(\tan\theta + \sec\theta)^3}$$

$$= \int_0^{\pi/2} \frac{1/\cos^2\theta}{\left(\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)^3} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{(\sin\theta+1)^3} d\theta$$

$$= \left[-\frac{1}{2(\sin\theta+1)^2}\right]_0^{\pi/2} = \frac{-1}{8} + \frac{1}{2} = \frac{3}{8}$$

746 **(b)**

$$\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{x^2+4x+4+9}$$

$$= \int \frac{dx}{(x+2)^2+3^2} = \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + c$$

747 **(c)**

Since, $J = \int \frac{e^{3x}}{1+e^{2x}+e^{4x}} dx$

$$\therefore\ J - I = \int \frac{(e^{3x}-e^x)}{1+e^{2x}+e^{4x}}\,dx$$

$= \int \frac{(u^2-1)}{1+u^2+u^4} du \quad [u = e^x]$

$$= \int \frac{\left(1-\frac{1}{u^2}\right)}{1+\frac{1}{u^2}+u^2} du = \int \frac{\left(1-\frac{1}{u^2}\right)}{\left(u+\frac{1}{u}\right)^2-1} du$$

$= \int \frac{dt}{t^2-1}$ $\left[\text{put } u+\frac{1}{u}=t \Rightarrow \left(1-\frac{1}{u^2}\right)du=dt\right]$

$$= \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| + c = \frac{1}{2}\log\left|\frac{u^2-u+1}{u^2+u+1}\right| + c$$

$$= \frac{1}{2}\log\left|\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right| + c$$

748 **(b)**

$$\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}} = \int_a^b \frac{1}{\sqrt{-x^2+(a+b)x-ab}} dx$$

$$= \int_a^b \frac{1}{\sqrt{\left(\frac{b-a}{2}\right)^2-\left(x-\frac{a+b}{2}\right)^2}} dx$$

$$= \left[\sin^{-1}\left(\frac{x-\frac{a+b}{2}}{\frac{b-a}{2}}\right)\right]_a^b$$

$$= \sin^{-1}1 - \sin^{-1}(-1)$$

$$= \frac{\pi}{2}+\frac{\pi}{2} = \pi$$

749 **(d)**

Let $I = \int_{-\pi}^{\pi}(\sin^6 x+\cos^6 x)\,dx =$

$2\int_0^{\pi}(\sin^6 x+\cos^6 x)\,dx$

$$= 4\int_0^{\pi/2}\sin^6 x\,dx + 4\int_0^{\pi/2}\cos^6\left(\frac{\pi}{2}-x\right)dx$$

$$= 8\int_0^{\pi/2}\sin^6 x\,dx = 8\times\frac{5.3.1}{6.4.2}\times\frac{\pi}{2} = \frac{5\pi}{4}$$

750 **(b)**

We have,

$$I_m = \int_1^x (\log x)^m dx$$

$$\Rightarrow I_m = [(\log x)^m x]_1^x - \int_1^x m(\log x)^{m-1}\times\frac{1}{x}\times x\,dx$$

$\Rightarrow I_m = (\log x)^m x - mI_{m-1}$

$\Rightarrow k - lI_{m-1} = x(\log x)^m$

$- m\,I_{m-1}$ $[\because I_m = k - lI_{m-1}]$

$\Rightarrow k = x(\log x)^m, l = m$

751 **(b)**

We have,

$$I = \int x^{-2/3}\left(1+x^{1/2}\right)^{-5/3}dx$$

$$= \int x^{3/2}\left(1+x^{-1/2}\right)^{-5/3}dx$$

$$\Rightarrow I = -2\int\left(1+x^{-1/2}\right)^{-5/3}d\left(1+x^{-1/2}\right)$$

$$\Rightarrow I = -2\times\frac{\left(1+x^{-1/2}\right)^{-2/3}}{-2/3}+C$$

$$= 3\left(1+x^{-1/2}\right)^{-2/3}+C$$

752 **(a)**

$$\int(1-\cos x)\,\text{cosec}^2 dx = \int\frac{1-\cos x}{1-\cos^2 x}dx$$

$$= \int\frac{1}{1+\cos x}dx$$

$$= \frac{1}{2}\int\sec^2\frac{x}{2}dx = \tan\frac{x}{2}+c$$

753 **(d)**

$$\because \int_0^{2a} f(x)dx = \int_0^a \{f(2a-x)+f(x)\}dx$$

$$= \int_0^a f(2a-x)dx + \int_0^a f(x)dx$$

$= m+n$

754 **(a)**

$I_n = \int(\log x)^n\,dx$ ....(i)

$\therefore I_{n-1} = \int(\log x)^{n-1}\,dx$ ....(ii)

Now, $I_n = \int(\log x)^n\cdot 1dx$

$$= (\log x)^n x - n\int(\log x)^{n-1}\frac{1}{x}x\,dx$$

$$= x(\log x)^n - n\int(\log x)^{n-1}dx$$

$\Rightarrow I_n = x(\log x)^n - nI_{n-1}$

$\therefore I_n + n\,I_{n-1} = x(\log x)^n$

755 **(a)**

Let $f(x) = \int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$. Then,

$$f'(x) = 2x\frac{(x^4-5x^2+4)}{(2+e^{x^2})}$$

For points of extremum, we must have

$f'(x) = 0 \Rightarrow x = 0, x = \pm1, \pm2$

756 **(c)**

Let $I = \int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2}\frac{2^{\sin x(\pi/2-x)}}{2^{\sin(\pi/2-x)}+2^{\cos x(\pi/2-x)}}dx$$

$\Rightarrow I = \int_0^{\pi/2}\frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}}dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2}\frac{2^{\sin x}+2^{\cos x}}{2^{\sin x}+2^{\cos x}}dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

757 **(b)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n} = \int_0^1 e^x dx = [e^x]_0^1$$

$= e-1$

758 **(d)**

The integrand is an odd function. So the value of the integral is zero

759 **(a)**

We have,

$$I=\int \frac{2x-3}{(x^2+x+1)^2}dx=\int \frac{(2x+1)-4}{(x^2+x+1)^2}dx$$

$$\Rightarrow I=\int \frac{2x+1}{(x^2+x+1)^2}dx - 4\int \frac{1}{\left[(x+1/2)^2+\left(\sqrt{3}/2\right)^2\right]^2}dx$$

$$\Rightarrow I=-\frac{1}{x^2+x+1}-4I_1, \text{where } I_1 = \int \frac{1}{\left[(x+1/2)^2+\left(\sqrt{3}/2\right)^2\right]^2}dx$$

Putting $x+1/2=(\sqrt{3}/2)\tan\theta$ in I_1, we get

$$I_1=\int \frac{\sqrt{3}/2\sec^2\theta\ d\theta}{\left[\left(\sqrt{3}/2\tan\theta\right)^2+\left(\sqrt{3}/2\right)^2\right]^2}$$

$$\Rightarrow I_1=\frac{8}{3\sqrt{3}}\int \cos^2\theta\ d\theta=\frac{8}{3\sqrt{3}}\int \frac{1+\cos 2\theta}{2}d\theta$$

$$\Rightarrow I_1=\frac{4}{3\sqrt{3}}\{\theta+\sin 2\theta\}+C$$

$$\Rightarrow I_1=\frac{4}{3\sqrt{3}}\left\{\tan^{-1}\frac{2x+1}{\sqrt{3}}+\frac{\sqrt{3}}{4}\left(\frac{2x+1}{x^2+x+1}\right)\right\}+C$$

$$\therefore I=-\frac{1}{x^2+x+1}-\frac{16}{3\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}} -\frac{4}{3}\left(\frac{2x+1}{x^2+x+1}\right)+C$$

760 **(d)**

Let $I=\int \frac{\sqrt{x}\sqrt{x}}{\sqrt{x}(x+1)}\ dx$

$$=\int \frac{x+1}{\sqrt{x}(x+1)}\ dx-\int \frac{1}{\sqrt{x}(x+1)}\ dx$$

$$=\int \frac{1}{\sqrt{x}}dx-\int \frac{1}{\sqrt{x}(x+1)}dx$$

$$=2\sqrt{x}-2\tan^{-1}\sqrt{x}+c$$

$$=2(\sqrt{x}-\tan^{-1}\sqrt{x})+c$$

761 **(b)**

We have,

$$I=\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$$

$$\Rightarrow I=\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx+2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I=0+2\times 2\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I=4\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}$$

$$\Rightarrow I=4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

$$\Rightarrow I=4\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx-4\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I=-4\pi[\tan^{-1}(\cos x)]_0^{\pi}-I$$

$$\Rightarrow 2I=-4\pi[-\pi/4-\pi/4]\Rightarrow I=\pi^2$$

762 **(a)**

Let

$$A=\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)...\left(1+\frac{n}{n}\right)\right\}^{1/n}$$

$$\Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log A=\lim_{n\to\infty}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log A=\int_0^1 \log(1+x)\,dx$$

$$\Rightarrow \log A=[n\log(1+x)-x+\log(x+1)]_0^1$$

$$\Rightarrow \log A=2\log 2-1\Rightarrow A=\frac{4}{e}$$

763 **(d)**

$$\int_{-2}^{2}|x|dx=-\int_{-2}^{0}x\ dx+\int_0^2 x\ dx$$

$$=-\left[\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}\right]_0^2$$

$$=-(0-2)+(2-0)=4$$

764 **(b)**

$\because \sin^{-1}(\cos x)+\cos^{-1}(\sin x)$ is periodic with period 2π, then

$$\int_{a+2\pi}^{a+5\pi/2}(\sin^{-1}(\cos x)+\cos^{-1}(\sin x))\,dx$$

$$=\int_0^{\pi/2}\left(\sin^{-1}\left\{\cos\left(\frac{\pi}{2}-x\right)\right\}+\cos^{-1}\left\{\sin\left(\frac{\pi}{2}-x\right)\right\}\right)dx$$

$$=\int_0^{\pi/2}(\sin^{-1}\sin x+\cos^{-1}\cos x)dx$$

$$=2\int_0^{\pi/2}x\ dx=2\left\{\frac{x^2}{2}\right\}_0^{\frac{\pi}{2}}=\frac{\pi^2}{4}$$

766 **(d)**

Put $x+\sqrt{1+x^2}=t$

$\Rightarrow \left(1+\frac{1}{2\sqrt{1+x^2}}.2x\right)dx=dt$

$\Rightarrow \frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\ dx=dt$

$\Rightarrow \frac{dx}{\sqrt{1+x^2}}=\frac{dt}{t}$

$\therefore \int \frac{\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\ dx=\int \frac{\log t}{t}\ dt$

$=\frac{(\log t)^2}{2}+c$

$=\frac{1}{2}[\log(x+\sqrt{1+x^2})]^2+c$

767 **(c)**

Putting $e^x-1=t^2$ and $e^x dx=2t\,dt$, we have

$I=\int_{\log 2}^{x}\frac{dx}{\sqrt{e^x-1}}dx$

$\Rightarrow I=\int_{1}^{t}\frac{dt}{t^2+1}=2[\tan^{-1}t]_1^t$

$\Rightarrow I=2[\tan^{-1}t-\tan^{-1}1]=2\tan^{-1}t=-\frac{\pi}{2}$

$\therefore \int_{\log 2}^{x}\frac{dx}{\sqrt{e^x-1}}dx=\frac{\pi}{6}$

$\Rightarrow 2\tan^{-1}t-\frac{\pi}{2}=\frac{\pi}{6}$

$\Rightarrow \tan^{-1}t=\frac{\pi}{3}$

$\Rightarrow t=\sqrt{3}=t^2=3\Rightarrow e^x-1=3\Rightarrow e^x=4\Rightarrow x=\log_e 4$

768 **(d)**

$\int \frac{\cos x+x\sin x}{x^2+x\cos x}\ dx$

$=\int \frac{x+\cos x-x+x\sin x}{x^2+x\cos x}\ dx$

$=\int \frac{dx}{x}-\int \frac{x(1-\sin x)}{x(x+\cos x)}dx$

$=\log x-\log(x+\cos x)+c$

$=\log\left|\frac{x}{x+\cos x}\right|+c$

769 **(c)**

Let $I=\int \frac{dx}{x\sqrt{x^6-16}}$

$=\frac{1}{3}\int \frac{3x^2}{x^3\sqrt{(x^3)^2-4^2}}dx$

Put $x^3=t\ \Rightarrow 3x^2dx=dt$

$\therefore\ I=\frac{1}{3}\int \frac{dt}{t\sqrt{t^2-4^2}}$

$=\frac{1}{3\times 4}\sec^{-1}\left(\frac{t}{4}\right)+c$

$=\frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right)+c$

770 **(a)**

$F(x)=\int_{x^2}^{x^3}\log t\ dt$

Applying Leibnitz theorem, we get

$F'(x)=\log x^3\cdot\frac{d}{dx}x^3-\log x^2\cdot\frac{d}{dx}x^2$

$=3\log x\cdot 3x^2-2\log x\cdot 2x$

$=(9x^2-4x)\log x$

771 **(b)**

We have,

$\int_0^{\infty}\frac{\log(1+x^2)}{1+x^2}dx=k\int_0^1\frac{\log(1+x)}{1+x^2}dx$

$\Rightarrow \int_0^{\pi/2}\ \log\sec^2\theta\ d\theta$

$=k\int_0^{\pi/4}\ \log(1+\tan\theta)\ d\theta$, where $x=\tan\theta$

$\Rightarrow -2\int_0^{\pi/2}\ \log\cos\theta\ d\theta=k\int_0^{\pi/4}\ \log(1+\tan\theta)\ d\theta$

$\Rightarrow -2\times -\frac{\pi}{2}\log_e 2=k\left(\frac{\pi}{8}\log 2\right)$

$\Rightarrow k=8$

772 **(b)**

$\int_0^4|x-1|dx=\int_0^1(1-x)dx+\int_1^4(x-1)dx$

$=\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^4$

$=1-\frac{1}{2}+\left[8-4-\left(\frac{1}{2}-1\right)\right]$

$=\frac{1}{2}+4+\frac{1}{2}=5$

773 **(b)**

Given, $f(x)$ is a continuous function.

Let us consider $f(x)=x$

$\therefore \int_{-3}^{5}2x\ \ dx=[x^2]_{-3}^5=16$

and $\int_{-6}^{10}(x-1)\,dx=\left[\frac{x^2}{2}-x\right]_{-6}^{10}=16$

$$\therefore \int_{-3}^{5} 2f(x)dx = \int_{-6}^{10} f(x-1)dx$$

774 **(c)**

$$\int [\sin(\log x) + \cos(\log x)]dx$$

$$= \int \frac{d}{dx}[x\sin(\log x)\} dx$$

$$= x\sin(\log x) + c$$

775 **(b)**

Let $I = \int_0^a \frac{1}{1+e^{f(x)}} dx$ ...(i)

Then,

$$\Rightarrow I = \int_0^a \frac{1}{1+e^{f(a-x)}} dx$$

$$\Rightarrow I = \int_0^a \frac{1}{1+e^{-f(x)}} dx \quad [\because f(a-x) + f(x) = 0]$$

$\Rightarrow I = \int_0^a \frac{e^{f(x)}}{e^{f(x)}+1} dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^a \frac{e^{f(x)}+1}{e^{f(x)}+1} dx = \int_0^a 1\, dx = a \Rightarrow I = \frac{a}{2}$$

776 **(c)**

Let $I = \int_{-1}^{1} |1-x| dx = \int_{-1}^{1} (1-x)dx$

$$= \left[x - \frac{x^2}{2}\right]_{-1}^{1} = 2$$

777 **(d)**

Let $I = \int_{-1}^{3} \left[\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right] dx$

Now, $\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}$

$$= \tan^{-1}\left[\frac{\frac{x}{x^2+1} + \frac{x^2+1}{x}}{1 - \frac{x}{x^2+1}\cdot\frac{x^2+1}{x}}\right]$$

$$= \tan^{-1}\left[\frac{x^2+(x^2+1)^2}{x(x^2+1).0}\right] = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Hence, $I = \int_{-1}^{3} \frac{\pi}{2}\, dx = 2\pi$

778 **(b)**

Let $I = \int \frac{x^2}{1+x^6} dx$

Put $x^3 = t \Rightarrow x^2 dx = \frac{1}{3} dt$

$$\therefore I = \frac{1}{3}\int \frac{1}{1+t^2} dt = \frac{1}{3}\tan^{-1}(x^3) + c$$

780 **(b)**

$$\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$= \int \frac{\sqrt{(1+x)^2}+\sqrt{x}\sqrt{x+1}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$= \int \sqrt{1+x}\; dx = \frac{2}{3}(1+x)^{3/2} + c$$

781 **(b)**

We have,

$f(x) = (2x+1)|\sin x|, \pi < x < 2\pi$

$\Rightarrow f(x) = -(2x+1)\sin x$

Hence, required primitive is given by

$$-\int (2x+1)\sin x\; dx$$

$$= -[-(2x+1)\cos x + 2\sin x] + C$$

$I \qquad II$

$= (2x+1)\cos x - 2\sin x + C$

782 **(a)**

Let $I = k\int_0^1 xf(3x)dx$

Let $3x = t \Rightarrow dx = \frac{dt}{3}$

$$\therefore I = k\int_0^3 \frac{t}{3}.f(t).\frac{dt}{3} = \frac{k}{9}\int_0^3 t\,.f(t)dt$$

Now, $\frac{k}{9}\int_0^3 t\,.f(t)dt = \int_0^3 t\,.f(t)dt$ [given]

$$\Rightarrow \frac{k}{9} = 1 \Longrightarrow k = 9$$

783 **(c)**

Put $x^e + e^x = t \Rightarrow e(x^{e-1} + e^{x-1})dx = dt$

$$\therefore \int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx = \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}\log t + c$$

$$= \frac{1}{e}\log(x^e + e^x) + c$$

784 **(a)**

Let $I = \int \operatorname{cosec}(x-a)\operatorname{cosec} x\, dx$

$$I = \int \frac{\sin a}{\sin a \sin(x-a)\sin x} dx$$

$$= -\frac{1}{\sin a}\int \left[\frac{\sin(x-a)\cos x - \cos(x-a)\sin x}{\sin(x-a)\sin x}\right]$$

$$= -\frac{1}{\sin a}\int [\cot x - \cot(x-a)]\, dx$$

$$= -\frac{1}{\sin a}[\log|\sin x| - \log|\sin(x-a)|] + c$$

$$= -\frac{1}{\sin a}[\log|\sin x\ \operatorname{cosec}(x-a)|] + c$$

785 **(b)**

Let $I = \int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}} d\theta$

Put $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$\therefore\ I = \int_0^1 \frac{dt}{\sqrt{4-t^2}}$

$= \left[\sin^{-1}\frac{t}{2}\right]_0^1$

$= \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

786 **(a)**

We have,

$I = \int_0^{\pi} \frac{1}{a^2 - 2a\cos x + 1} dx$

$\Rightarrow I$

$= \int_0^{\pi} \frac{1 + \tan^2 x/2}{(a^2+1)(1+\tan^2 x/2) - 2a(1-\tan^2 x/2)} dx$

$\Rightarrow I$

$= \int_0^{\infty} \frac{2\,dt}{(a^2+1)(1+t^2) - 2a(1-t^2)}$, where $\tan\frac{x}{2}$

$= t,$

$\Rightarrow I = 2\int_0^{\infty} \frac{dt}{t^2(a+1)^2 + (a-1)^2}$

$\Rightarrow I = \frac{2}{(a+1)^2}\int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1-a}{1+a}\right)^2}$

$\Rightarrow I = \frac{2}{(a+1)^2}\left(\frac{1+a}{1-a}\right)\left[\tan^{-1}\frac{t(1+a)}{1-a}\right]_0^{\infty}$

$\Rightarrow I = \frac{2}{1-a^2}(\tan^{-1}\infty - \tan^{-1}0) = \frac{\pi}{1-a^2}$

787 **(b)**

Let $I = \int_{-1}^{1} \frac{|x+2|}{x+2} dx$

For $-1 \le x \le 1, |x+2| = 2+x$

$\therefore I = \int_{-1}^{1} \frac{x+2}{x+2}\, dx = \int_{-1}^{1} 1\, dx$

$= [x]_{-1}^{1} = 1 - (-1)$

$= 2$

788 **(c)**

$\because \sqrt{x + 2\sqrt{(x-1)}}$

$= \sqrt{\left(\sqrt{(x-1)}\right)^2 + 1^2 + 2\sqrt{(x-1)}}$

$= \sqrt{(x-1)} + 1$

and $\sqrt{x - 2\sqrt{(x-1)}} +$

$\sqrt{\left(\sqrt{(x-1)}\right)^2 + 1^2 - 2\sqrt{(x-1)}}$

$= |\sqrt{(x-1)} - 1|$

Then, $\int_1^5 \left(\sqrt{x + 2\sqrt{(x-1)}} + \sqrt{x - 2\sqrt{(x-1)}}\right) dx$

$= \int_1^5 (\sqrt{(x-1)} + 1)dx + \int_1^5 |\sqrt{(x-1)} - 1| dx$

$= \int_1^5 (\sqrt{(x-1)} + 1)dx + \int_1^2 (1 - \sqrt{(x-1)})\, dx$

$+ \int_1^5 (\sqrt{(x-1)} - 1)dx$

$= \int_0^4 (\sqrt{x} + 1)dx$

$+ \int_0^1 (1 - \sqrt{x})dx + \int_1^4 (\sqrt{x} - 1)dx$

$= \left[\frac{2}{3}(x^{3/2}) + x\right]_0^4 + \left[x - \frac{2}{3}x^{3/2}\right]_0^1 + \left[\frac{2}{3}x^{3/2} - x\right]_1^4$

$= \left(\frac{16}{3} + 4\right) + \left(1 - \frac{2}{3}\right) + \left(\frac{16}{3} - 4\right) + \left(\frac{2}{3} - 1\right)$

$= \frac{32}{3}$

789 **(d)**

Let $I = \int \frac{1}{(\sin x + 4)(\sin x - 1)} dx$

$= \frac{1}{5}\int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)} dx$

$= \frac{1}{5}\int \frac{1}{(\sin x - 1)} dx - \frac{1}{5}\int \frac{1}{(\sin x + 4)} dx$

$= \frac{1}{5}\int \frac{\sec^2 x/2}{2\tan x/2 - 1 - \tan^2 x/2} dx$

$- \frac{1}{5}\int \frac{\sec^2 x/2}{2\tan x/2 + 4 + 4\tan^2 x/2} dx$

Put $\tan\frac{x}{2} = t$

$\Rightarrow \sec^2\frac{x}{2} dx = 2t\, dt$

$\therefore I = \frac{1}{5}\int \frac{2dt}{2t - 1 - t^2} - \frac{1}{5}\int \frac{2dt}{[2t + 4(1+t^2)]}$

$\therefore\ I = -\frac{2}{5}\int \frac{dt}{2t - 1 - t^2} - \frac{1}{10}\int \frac{dt}{t^2 - \frac{1}{2}t + 1}$

$= \frac{-2}{5}\int \frac{1}{(t-1)^2} dt - \frac{1}{10}\int \frac{1}{\left(t + \frac{1}{4}\right)^2 \left(\frac{\sqrt{15}}{4}\right)^2}$

$= \frac{2}{5}\frac{1}{(t-1)} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right) + c$

$= \frac{2}{5}\frac{1}{(\tan\frac{x}{2} - 1)} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2} + 1}{\sqrt{15}}\right)$

$+ c$

$\therefore A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4\tan\frac{x}{2} + 1}{\sqrt{15}}$

790 **(a)**

$$\int(\sqrt{\tan x}+\sqrt{\cot x})dx$$
$$=\int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$
$$=\int\left(\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}\right)dx$$
$$=\int\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{2\sin x\cos x}}dx$$
$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}$$
Put $\sin x-\cos x=t$
$\Rightarrow(\cos x+\sin x)dx=dt$
$$=\sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}$$
$=\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

791 **(b)**

Let $I=\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx+2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

$\Rightarrow I=0+4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$ ...(i)

$$\left[\because\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx\text{ is an odd function}\right]$$

$\Rightarrow I=4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$...(ii)

On adding Eqs. (i) and (ii), we get
$$2I=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$
Put $\cos x=t\ \Rightarrow\ \sin x\,dx=-dt$
$$\therefore\ I=2\pi\int_1^{-1}\frac{-dt}{1+t^2}$$
$$=2\pi[\tan^{-1}t]_{-1}^{1}=2\pi\left[\frac{\pi}{4}+\frac{\pi}{4}\right]$$
$$=\pi^2$$

792 **(c)**

We have,
$$I=\int\frac{\sin x}{\sin(x-\alpha)}dx=\int\frac{\sin\{(x-\alpha)+\alpha\}}{\sin(x-\alpha)}$$
$$\Rightarrow I=\int\{\cos\alpha+\sin\alpha\cot(x-\alpha)\}dx$$
$\Rightarrow I=x\cos\alpha+\sin\alpha\log_e\sin(x-\alpha)+C$

$\therefore A=\cos\alpha,B=\sin\alpha$

793 **(b)**

Since $x-[x]$ is a periodic function with period 1. Therefore, $\sin(x-[x])\,\pi$ is a also a periodic function with period 1
$$\therefore I=\int_0^{100}\sin(x-[x])\pi dx$$
$$=100\int_0^1\sin(x-[x])\,\pi dx$$
$$\Rightarrow I=100\int_0^1\sin\pi x dx=-\frac{100}{\pi}[\cos\pi x]_0^1=\frac{200}{\pi}$$

794 **(b)**

$\because f(x)=\cos x-\cos^2 x+\cos^3 x-\ldots\infty$
$$=\frac{\cos x}{1+\cos x}$$
$$\therefore\int f(x)dx=\int\frac{1+\cos x}{1+\cos x}dx-\int\frac{1}{1+\cos x}dx$$
$$=x-\frac{1}{2}\int\sec^2\frac{x}{2}dx$$
$$=x-\frac{1}{2}\tan\frac{x}{2}\cdot2+c=x-\tan\frac{x}{2}+c$$

795 **(b)**

We have, $\int_0^a f(2a-x)dx=\mu$

and $\int_0^a f(x)dx=\lambda$

Now, using properties of definite integral
$$\int_0^{2a}f(x)dx=\int_0^a f(x)dx+\int_0^a f(2a-x)dx$$
$$\Rightarrow\int_0^{2a}f(x)dx=\lambda+\mu$$

796 **(c)**

Let
$$I=\int\frac{f'(x)}{f(x)\log\{f(x)\}}dx$$
$$\Rightarrow I=\int\frac{1}{\log\{f(x)\}}d[\log\{f(x)\}]$$
$$=\log[\log\{f(x)\}]+C$$

797 **(b)**
$$\int_1^4|x-3|dx=\int_1^3(3-x)dx+\int_3^4(x-3)dx$$
$$=\left[3x-\frac{x^2}{2}\right]_1^3+\left[\frac{x^2}{2}-3x\right]_3^4$$
$$=\left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right)+\left(\frac{16}{2}-12\right)-\left(\frac{9}{2}-9\right)$$
$$=\frac{5}{2}$$

798 **(a)**
$$\int e^x\left(\log x+\frac{1}{x}\right)dx=\int e^x\log x\,dx+\int e^x.\frac{1}{x}dx$$
$$=[e^x\log x]-\int e^x.\frac{1}{x}dx+\int e^x.\frac{1}{x}dx$$
$$=e^x\log x+c$$

799 **(b)**

Let $I=\int_{-3\pi/2}^{-\pi/2}[(x+\pi)^3+\cos^2 x]dx$ ...(i)

$$\Rightarrow I=\int_{-3\pi/2}^{-\pi/2}\left[\left(-\frac{\pi}{2}-\frac{3\pi}{2}-x+\pi\right)^3\right.$$

$$\left.=\cos^2\left(-\frac{\pi}{2}-\frac{3\pi}{2}-x\right)\right]dx$$

$$\Rightarrow I=\int_{-3\pi/2}^{-\pi/2}[-(x+\pi)^3+\cos^2]dx \ldots \text{(ii)}$$

On adding Eqs. (i)and (ii),we get

$$2I=\int_{-3\pi/2}^{-\pi/2}2\cos^2 x\,dx$$

$$=\int_{-3\pi/2}^{-\pi/2}(1+\cos 2x)dx$$

$$=\left[x+\frac{\sin 2x}{2}\right]_{-3\pi/2}^{-\pi/2}$$

$$=\left[-\frac{\pi}{2}+\frac{\sin(-\pi)}{2}-\left(-\frac{3\pi}{2}+\frac{\sin(-3\pi)}{2}\right)\right]=\pi$$

$$\Rightarrow I=\frac{\pi}{2}$$

800 **(a)**

Let $I=\int_1^x\frac{\log(x^2)}{x}dx=2\int_1^x\log x.\frac{1}{x}dx$

Put $\log x=t \Rightarrow \frac{1}{x}dx=dt$

$$\therefore I=2\int_0^{\log x}t\,dt=2\left[\frac{t^2}{2}\right]_0^{\log x}=(\log x)^2$$

801 **(b)**

Given, $f'(x)=x+\frac{1}{x}$

On integrating both sides, we get

$$f(x)=\frac{x^2}{2}+\log x+c$$

802 **(d)**

Let $I=\int e^{-\log x}\,dx=\int\frac{1}{x}\,dx=\log|x|+c$

803 **(b)**

$$\int\frac{dx}{x(x+1)}=\int\frac{1}{x}dx-\int\frac{1}{x+1}dx$$

$$=\log x-\log(x+1)+c$$

$$=\log\left|\frac{x}{x+1}\right|+c$$

804 **(c)**

We have,

$$I=\int_0^{\pi^2/4}\sin\sqrt{x}\,dx=2\int_0^{\pi/2}\sin t\,dt\text{, where } t^2=x$$

$$\Rightarrow I=2[-t\cos t+\sin t]_0^{-\pi/2}=2$$

805 **(b)**

Let $f(x)=\int_0^x\left[\frac{1}{\sqrt{1+t^2}}-\frac{1}{1+t}\right]dt$

$$=\int_0^x\frac{1}{\sqrt{1+t^2}}dt-\int_0^x\frac{1}{1+t}dt$$

Let $I_1=\int_0^x\frac{1}{\sqrt{1+t^2}}dt$

Put $t=\tan\theta\Rightarrow dt=\sec^2\theta\,d\theta$

$$\therefore I_1=\int_0^{\tan^{-1}x}\frac{\sec^{-1}\theta}{\sec\theta}d\theta$$

$$=[\log(\sec\theta+\tan\theta)]_0^{\tan^{-1}x}$$

$$=\log(\sec\tan^{-1}x+x)$$

$$\therefore f(x)=\log(\sec\tan^{-1}x+x)-\log(1+x)$$

$$\therefore \lim_{x\to\infty}f(x)=\lim_{x\to\infty}\log\left(\frac{\sec\tan^{-1}x+x}{1+x}\right)$$

$$=\lim_{x\to\infty}\log\left(\frac{\sqrt{1+x^2}+x}{1+x}\right)$$

$$=\log\lim_{x\to\infty}\left(\frac{\sqrt{\frac{1}{x^2}+1}+1}{\frac{1}{x}+1}\right)$$

$$=\log_e 2$$

806 **(c)**

Given, $I_1=\int\sin^{-1}x\,dx$

and $I_2=\int\sin^{-1}\sqrt{1-x^2}\,dx$

$$\Rightarrow I_2=\int\cos^{-1}x\,dx$$

Now, $I_1+I_2=\int(\sin^{-1}x+\cos^{-1}x)dx$

$$=\int\frac{\pi}{2}dx=\frac{\pi}{2}x$$

$$\therefore I_1+I_2=\frac{\pi}{2}x$$

807 **(b)**

Let $I=\int\cos^3 x\,.\,e^{\log\sin x}dx=\int\cos^3 x\sin x\,dx$

Put $\cos x=t \Rightarrow -\sin x\,dx=dt$

$$\therefore\quad I=-\int t^3\,dt=-\frac{t^4}{4}+c=-\frac{\cos^4 x}{4}+c$$

808 **(c)**

We have,

$$\sqrt{\frac{1+\cos 2x}{2}}=|\cos x|=\begin{cases}\cos x, 0\le x\le\pi/2\\-\cos x,\pi/2\le x\le\pi\end{cases}$$

$$\therefore I=\int_0^{\pi}\sqrt{\frac{1+\cos 2x}{2}}dx=\int_0^{\pi}|\cos x|dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos x \; dx + \int_{\pi/2}^{\pi} -\cos x \; dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} = 2$$

809 **(c)**

Let $\int_0^{\pi/2} \sin 2x \log \tan x \, dx$ ...(i)

$$I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right) \log \tan\left(\frac{\pi}{2} - x\right) dx \quad \left[\because \int_0^a f(x)dx \int_0^a f(a - x)dx\right]$$

$I = \int_0^{\pi/2} \sin 2x \log \cot x \, dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \sin 2x \log(\tan x \cot x) dx$$

$$= \int_0^{\pi/2} \sin 2x \log 1 \, dx = \int_0^{\pi/2} 0 \, dx$$

$\Rightarrow I = 0$

810 **(a)**

Since, the given function is an odd function.

$$\therefore \; I = \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx = 0$$

811 **(a)**

$$\text{Let } A = \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n}\sec^2\left(\frac{1}{n}\right)^2 + \frac{2}{n}\sec^2\left(\frac{2}{n}\right)^2 + \ldots + \frac{n}{n}\sec^2\left(\frac{n}{n}\right)^2\right]$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right) \sec^2\left(\frac{r}{n}\right)^2$$

$$\Rightarrow \; A = \int_0^1 x \sec^2(x)^2 dx$$

Put $x^2 = t \Longrightarrow x \, dx = \frac{dt}{2}$

$$\therefore A = \frac{1}{2}\int_0^1 \sec^2 t \, dt = \frac{1}{2}[\tan t\,]_0^1 = \tan 1$$

812 **(b)**

$$\int \frac{1}{1 + \cos ax} dx = \int \frac{1}{2\cos^2(ax/2)} dx$$

$$= \frac{1}{2}\int \sec^2\frac{ax}{2} dx = \frac{1}{2}.\frac{\tan \quad ax/2}{a/2}$$

$$= \frac{1}{a}\tan\frac{ax}{2} + c$$

813 **(b)**

$$\int \sqrt{1 + \cos x} \; dx$$

$$= \int \sqrt{2\cos^2\frac{x}{2}} \; dx$$

$$= \sqrt{2}\int \cos\frac{x}{2} \; dx = 2\sqrt{2}.\sin\frac{x}{2} + c$$

814 **(c)**

$$\int_{-2}^{3} f(x)dx = \int_{-2}^{2} f(x)dx + \int_{2}^{3} f(x)dx$$

$$= \int_{-2}^{2} e^{\cos x} \sin x \, dx + \int_2^3 2 \, dx$$

Function of Ist integral is an odd function, therefore the value of Ist integral is zero.

$$\therefore \int_{-2}^{3} f(x)dx = 0 + 2[x]_2^3$$

$= 0 + 2(3 - 2) = 2$

815 **(d)**

Let $I = \int x \frac{e^x}{\sqrt{1+e^x}} dx$

Putting $1 + e^x = t^2$ i.e. $x = \log(t^2 - 1)$, we get

$$I = 2\int \log(t^2 - 1) \, dt$$

$$\Rightarrow I = 2\left\{t\log(t^2 - 1) - 2\int \frac{t^2}{t^2 - 1} dt\right\}$$

$$\Rightarrow I = 2\left\{t\log(t^2 - 1) - 2\int 1 + \left(\frac{1}{t^2 - 1}\right) dt\right\}$$

$$\Rightarrow I = 2\left\{t\log(t^2 - 1) - 2t - \log\left(\frac{t - 1}{t + 1}\right)\right\} + C$$

$$\Rightarrow I = 2x\sqrt{1 + e^x} - 4\sqrt{1 + e^x} - 2\log\left\{\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right\} + C$$

Hence, $f(x) = 2x - 4 = 2(x - 2)$ and $g(x) = \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

816 **(c)**

$\because v(t) = \cos \pi t,$

$$\Rightarrow \frac{ds}{dt} = \cos \pi t$$

$$\Rightarrow s = \frac{\sin \pi t}{\pi} + c, \text{at } t = 0, s = 4, c = 4$$

$$\Rightarrow s = \frac{\sin \pi t}{\pi} + 4$$

817 **(a)**

Let $I = \int \cos(\log_e x)dx = \int \cos(\log_e x) \cdot 1dx$

$$= \cos(\log_e x)x - \int \frac{-\sin \log_e x)}{x} \cdot x \, dx$$

$$= x\cos(\log_e x) + \int \sin(\log_e x)dx$$

$$= x\cos(\log_e x) + \int \sin(\log_e x) \cdot 1 \, dx$$

$= x\cos(\log_e x) - \sin(\log_e x)x$

$$- \int \frac{\cos(\log_e x)}{x} x\, dx$$

$= x\cos(\log_e x) + x\sin(\log_e x) - I$

$\Rightarrow 2I = x[\cos(\log_e x) + \sin(\log_e x)]$

$\Rightarrow I = \frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)]$

818 **(d)**

Let $I = \int \frac{(e^x - e^{-x})}{(e^x + e^{-x})\log(\cosh x)} dx$

Put $\log(\cosh x) = t \Rightarrow \frac{1}{\cosh x}.\sinh x\, dx = dt$

$\Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = dt$

$[\because \sinh x = \frac{e^x - e^{-x}}{2}$

and $\cosh x = \frac{e^x + e^{-x}}{2}]$

$\therefore I = \int \frac{1}{t} dt = \log t + c$

$= \log(\log\cosh x) + c$

819 **(b)**

Given, $u = -f''(\theta)\sin\theta + f'(\theta)\cos\theta$

and $v = f''(\theta)\cos\theta + f'(\theta)\sin\theta$

On differentiating w. r. t θ respectively, we get

$\frac{du}{d\theta} = -f'''(\theta)\sin\theta$

$$- f''(\theta)\cos\theta$$
$$+ f''(\theta)\cos\theta - f'(\theta)\sin\theta$$

$= -f'''(\theta)\sin\theta - f'(\theta)\sin\theta$

and $\frac{dv}{d\theta} = f'''(\theta)\cos\theta - f''(\theta)\sin\theta + f''(\theta)\sin\theta + f'(\theta)\cos\theta$

$= -f'''(\theta)\cos\theta + f'(\theta)\cos\theta$

$\therefore \left(\frac{du}{d\theta}\right)^2 + \left(\frac{dv}{d\theta}\right)^2$

$$= [f'''(\theta)]^2 + [f'(\theta)]^2 + 2f'(\theta)f'''(\theta)$$

$= [f'''(\theta) + f'(\theta)]^2$

Now, $\int \left[\left(\frac{du}{d\theta}\right)^2 + \left(\frac{dv}{d\theta}\right)^2\right]^{1/2} d\theta = \int [f'''(\theta) + f'(\theta)]\, d\theta$

$= f''(\theta) + f(\theta) + c$

820 **(d)**

We have,

$$I = \int_0^\pi \frac{\sin k\, x}{\sin x} dx = \int_0^\pi \frac{\sin k(\pi - x)}{\sin(\pi - x)} dx$$

$$\Rightarrow I = -\int_0^\pi \frac{\sin kx}{\sin x} dx = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

821 **(d)**

Let $I = \int \frac{\sin x}{\sin(x-\alpha)} dx$

Put $x - \alpha = t \quad \Rightarrow \quad dx = dt$

$\therefore \quad I = \frac{\sin(\alpha + t)}{\sin t} dt$

$= \int \frac{\sin\alpha\cot t + \cos\alpha\sin t}{\sin t} dt$

$= \sin\alpha \int \cot t\, dt + \cos\alpha \int dt$

$= \sin\alpha\log\sin t + + \cos\alpha\, .\, t + c_1$

$= \sin\alpha\log\sin(x - \alpha) + \cos\alpha\, .\, (x - \alpha) + c_1$

$= \sin\alpha\log\sin(x - \alpha) + x\cos\alpha + c$

[let $c = -\alpha\cos\alpha + c_1$]

823 **(b)**

$\int x^3 \log x\, dx$

$= \frac{x^4}{4}\log x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$= \frac{x^4}{4}\log x - \int \frac{x^3}{4} dx = \frac{x^4}{4}\log x - \frac{x^4}{16} + c$

$= \frac{1}{16}[4x^4\log x - x^4] + c$

824 **(a)**

We have,

$$I = \int \frac{\sin^3 x}{(1 + \cos^2 x)\sqrt{1 + \cos^2 x + \cos^4 x}} dx$$

$\Rightarrow I$

$$= -\int \frac{(1 - \cos^2 x)}{(1 + \cos^2 x)\sqrt{1 + \cos^2 x + \cos^4 x}} d(\cos x)$$

$\Rightarrow I = -\int \frac{1-t^2}{(1+t^2)\sqrt{1+t^2+t^4}} dt$, where $t = \cos x$

$$\Rightarrow I = \int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right)\sqrt{\left(t + \frac{1}{t}\right)^2 - 1}} dt$$

$$\Rightarrow I = \int \frac{1}{\left(1 + \frac{1}{t}\right)\sqrt{\left(t + \frac{1}{t}\right)^2 - 1^2}} d\left(t + \frac{1}{t}\right)$$

$\Rightarrow I = \sec^{-1}\left(t + \frac{1}{t}\right) + C$

$$= \sec^{-1}(\cos x + \sec x) + C$$

825 **(a)**

We have,

$x = \int_2^{\sin t} \sin^{-1}\theta\, d\theta$ and $y = \int_n^{\sqrt{t}} \frac{\sin\theta^2}{\theta} d\,\theta$

$\Rightarrow \frac{dx}{dt} = \cos t\sin^{-1}(\sin t)$ and $\frac{dy}{dt} = \frac{1}{2\sqrt{t}} \times \frac{\sin t}{\sqrt{t}}$

$\Rightarrow \frac{dx}{dt} = t\cos t$ and $\frac{dy}{dt} = \frac{\sin t}{2t}$

$\Rightarrow \frac{dy}{dx} = \frac{\sin t}{2t} \times \frac{1}{t\cos t} = \frac{\tan t}{2t^2}$

826 **(c)**

Let $I = 100\int_0^{\pi/2} \frac{(\sin x+\cos x)+\sin x}{\sin x+\cos x} dx$

$= 100\left[\int_0^{\pi/2} 1dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx\right]$

Let $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}$ ...(i)

$\Rightarrow\ I_1 = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx$ ...(ii)

$\left[\text{put } x = \frac{\pi}{2} - x\right]$

On adding Eqs. (i) and (ii), we get

$2I_1 = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

$\Rightarrow\ I_1 = \frac{\pi}{4}$

$\therefore\ I = 100\left[\frac{\pi}{2} + \frac{\pi}{4}\right] = 100 \times \frac{3\pi}{4} = 75\pi$

827 **(c)**

We have, $\frac{d}{dx}(f(x)) = g(x) \Rightarrow df(x) = g(x)dx$

Now,

$I = \int_a^b f(x)d(f(x))$

$\Rightarrow I = \frac{1}{2}[\{f(x)\}^2]_a^b \Rightarrow I = \frac{1}{2}\{[f(b)]^2 - [f(a)]^2\}$

828 **(b)**

$\because \sin^4 t + \cos^4 t$ is periodic with period $\frac{\pi}{2}$

Now, $f(x + \pi) = \int_0^{\pi+x}(\sin^4 t + \cos^4 t)dt$

$= \int_0^{x}(\sin^4 t + \cos^4 t)dt + \int_0^{x+\pi}(\sin^4 t + \cos^4 t)dt$

$= f(x) + \int_0^{\pi}(\cos^4 t + \sin^4 t)dt = f(x) + f(\pi)$

or

$= f(x) + 2\int_0^{\pi/2}(\cos^4 t + \sin^4 t)dt$

$= f(x) + 2f\left(\frac{\pi}{2}\right)$

829 **(b)**

Let $I = \int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx = \int_0^1 \frac{x^6 x\, dx}{\sqrt{1-x^4}}$

Put $x^2 = \sin\theta \ \Rightarrow\ 2x\, dx = \cos\theta\, d\theta$

$\therefore\ I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^3\theta\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}}$

$\Rightarrow\ I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^3\theta\cos\theta\, d\theta}{\cos\theta} = \frac{1}{2}\int_0^{\pi/2} \sin^3\theta\, d\theta$

$\Rightarrow\ I = \frac{1}{2}\frac{\sqrt{2}\sqrt{\left(\frac{1}{2}\right)}}{\sqrt{2}.\sqrt{\left(\frac{5}{2}\right)}} = \frac{\sqrt{\left(\frac{1}{2}\right)}}{4\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\left(\frac{1}{2}\right)}} = \frac{1}{3}$

830 **(a)**

Let $\frac{x-1}{x+2} = t \ \Rightarrow\ \frac{1}{(x+2)^2}dx = \frac{1}{3}\ dt$

$\therefore\ \int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\ dx = \frac{1}{3}\int t^{-3/4}\, dt$

$= \frac{4}{3}\ t^{1/4} + c$

$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$

831 **(c)**

Let $I = \int \frac{dx}{x+\sqrt{x-1}}$

Put $x = t^2 + 1 \ \Rightarrow\ dx = 2t\, dt$

$\therefore\ I = \int \frac{2t}{t^2+t+1}dt$

$= \int \frac{2t+1}{t^2+t+1}dt - \int \frac{1}{t^2+t+1}dt$

$= \log(t^2+t+1) - \int \frac{1}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dt$

$= \log(t^2+t+1) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right) + c$

$= \log\left(x+\sqrt{x-1}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$

832 **(b)**

Given, $2f(x) - 3f\left(\frac{1}{x}\right) = x$ ...(i)

Put $x = \frac{1}{x}$ in Eq.(i), we get

$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$f(x) = -\frac{3+2x^2}{5x}$

Now, $\int_1^2 f(x)dx = -\int_1^2 \frac{3+2x^2}{5x}dx$

$= -\int_1^2\left(\frac{3}{5x} + \frac{2x}{5}\right)dx$

$= -\frac{1}{5}\left[3\log x + \frac{2x^2}{2}\right]_1^2$

$= -\frac{1}{5}(3\log 2 + 4 - 3\log 1 - 1)$

$= -\frac{1}{5}(3\log 2 + 4 - 0 - 1)$

$= -\frac{3}{5}(1 + \log 2)$

833 **(c)**

$\int e^x\left(\frac{2}{x} - \frac{2}{x^2}\right)dx$

$= \frac{2e^x}{x} + c\ [\because f(x) = \frac{2}{x}$ and $f'(x) = -\frac{2}{x^2}$

834 **(b)**

We have,

$$I=\int_2^4 \frac{\sqrt{x^2-4}}{x^4}dx=\int_2^4 \frac{1}{x^3}\sqrt{1-\frac{4}{x^2}}dx$$

Let, $1-\frac{4}{x^2}=t \Rightarrow \frac{8}{x^3}dx=dt$

$$\therefore I=\frac{1}{8}\int_0^{3/4}\sqrt{t}\,dt=\frac{1}{8}\times\frac{2}{3}\left[t^{3/2}\right]_0^{3/4}=\frac{1}{12}\times\frac{3}{4}\sqrt{\frac{3}{4}}$$

$$=\frac{\sqrt{3}}{32}$$

835 **(b)**

We have,

$$\int e^{3\log x}(x^4+1)^{-1}dx=\int e^{\log x^3}\frac{1}{x^4+1}dx$$

$$\Rightarrow I=\int\frac{x^3}{x^4+1}dx=\frac{1}{4}\log(x^4+1)+C$$

836 **(d)**

Given, $\int_0^p \frac{dx}{1+4x^2}=\frac{\pi}{8}$

LHS= $\int_0^p \frac{dx}{1+4x^2}=\frac{1}{4}\int_0^p\frac{dx}{\frac{1}{4}+x^2}$

$$=\frac{1}{4}\times\frac{1}{\frac{1}{2}}\left[\tan^{-1}\left(\frac{x}{1/2}\right)\right]_0^p$$

$$\Rightarrow \frac{1}{2}\left[\tan^{-1}2p\right]=\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}2p=\frac{\pi}{4}$$

$$\Rightarrow 2p=1$$

$$\Rightarrow p=\frac{1}{2}$$

837 **(c)**

Let, $I=\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$ ...(i)

Then,

$$I=\int_0^{\pi/2}\frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$\Rightarrow I=\int_0^{\pi/2}\frac{\cos^2 x}{\sin x+\cos x}dx$ ...(ii)

Adding (i) and (ii), we get

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{\sin^2 x+\cos^2 x}{\sin x+\cos x}dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{1}{\sin x+\cos x}dx$$

$$\Rightarrow 2I=\frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$

$$\Rightarrow 2I=\frac{1}{\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$

$$\Rightarrow 2I=\frac{1}{\sqrt{2}}\left[\log\left(\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right)\right]_0^{\pi/2}$$

$$\Rightarrow 2I=\frac{1}{\sqrt{2}}\left[\log(\sqrt{2}+1)-\log(\sqrt{2}-1)\right]$$

$$\Rightarrow 2I=\frac{1}{\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)=\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)^2$$

$$=\sqrt{2}\log(\sqrt{2}+1)$$

$$\therefore I=\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$$

838 **(b)**

Let $I=\int\frac{\sqrt{x^2+1}\,[\log(x^2+1)-2\log x]}{x^4}dx$ · Then,

$$I=\int\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)\cdot\frac{1}{x^3}dx$$

Putting $1+\frac{1}{x^2}=t$ and $-\frac{2}{x^3}dx=dt$, we get

$$I=-\frac{1}{2}\int\sqrt{t}\log t\,dt$$

$$\Rightarrow I=-\frac{1}{2}\left\{\log t\left(\frac{t^{3/2}}{3/2}\right)-\frac{2}{3}\int\frac{1}{t}t^{3/2}dt\right\}$$

$$\Rightarrow I=-\frac{1}{2}\left\{\frac{2}{3}(\log t)t^{3/2}-\frac{2}{3}\left(\frac{2}{3}t^{3/2}\right)\right\}+C$$

$$\Rightarrow I=-\frac{1}{3}t^{\frac{3}{2}}\log t+\frac{2}{9}t^{3/2}+C$$

$$\Rightarrow I=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left\{\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right\}+C$$

839 **(b)**

We have,

$$I=\int_3^5\frac{x^2}{x^2-4}dx=\int_3^5\left(\frac{4}{x^2-4}\right)dx$$

$$\Rightarrow I=\left[x+\frac{4}{2\times 2}\log\left(\frac{x-2}{x+2}\right)\right]_3^5$$

$$=2+\left(\log\frac{3}{7}-\log\frac{1}{5}\right)=2+\log\frac{15}{7}$$

840 **(d)**

Since $|\sin x|$ is a periodic function with period π

$$\therefore \int_\pi^{10\pi}|\sin x|dx=9\int_0^\pi|\sin x|dx=9\int_0^\pi\sin x\,dx$$

$$=18$$

841 **(b)**

We have,

$$\Delta(y)=\begin{vmatrix}a-c & b-d & a-c+b-d\\ b-c & c-d & b-d-1\\ y+c & y+d & y-b+d\end{vmatrix}$$

Applying $R_1\to R_1-R_3$ and , $R_2\to R_2-R_3$

$$\therefore \int_0^2 \Delta(y)\,dy$$

$$= \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ \int_0^2 (y+c)\,dy & \int_0^2 (y+d)\,dy & \int_0^2 (y-b)\,dy \end{vmatrix}$$

$$\Rightarrow \int_0^2 \Delta(y)\,dy = \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ 2+2c & 2+2d & 2-2(b-d) \end{vmatrix}$$

Let x be the common difference of the A.P. a, b, c, d. The,

$$\int_0^2 \Delta(y)\,dy = \begin{vmatrix} -2x & -2x & -4x \\ -x & -x & -2x-1 \\ 2x+2c & 2+2d & 2+2x \end{vmatrix}$$

$$\Rightarrow \int_0^2 \Delta(y)\,dy$$

$$= \begin{vmatrix} 0 & 0 & 2 \\ -x & -x & -2x-1 \\ 2+2c & 2+2d & 2+2x \end{vmatrix} \quad \text{Applying } R_1 \to R_1 - 2R_2$$

$$\Rightarrow \int_0^2 \Delta(y)\,dy = 2\begin{vmatrix} -x & -x \\ 2+2c & 2+2d \end{vmatrix}$$

$$\Rightarrow \int_0^2 \Delta(y)\,dy = -4x\begin{vmatrix} 1 & 1 \\ 1+c & 1+d \end{vmatrix} = -4x(d-c)$$

$$= -4x^2$$

$$\therefore \int_0^2 \Delta(y)\,dy = -16 \Rightarrow -4x^2 = -16 \Rightarrow x = \pm 2$$

842 **(c)**

$$\int \operatorname{cosec}^4 x\,dx = \int \operatorname{cosec}^2 x.\operatorname{cosec}^2 x\ dx$$

$$= \int \operatorname{cosec}^2 x(1+\cot^2 x)\,dx$$

$$= \int \operatorname{cosec}^2 x\,dx + \int \cot^2 x.\operatorname{cosec}^2 x\ dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

843 **(d)**

If $f(x)$ is a continuous function defined on $[a, b]$, then

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

Where M and m are maximum and minimum values respectively of $f(x)$ in $[a, b]$

Here, $f(x) = 1 + e^{-x^2}$ is continuous on $[0, 1]$.

Now, $0 < x < 1 \Rightarrow x^2 < x$

$\Rightarrow e^{x^2} < e^x \Rightarrow e^{-x^2} > e^{-x}$

Again, $0 < x < 1 \Rightarrow x^2 > 0$

$\Rightarrow e^{x^2} < e^0 \Rightarrow e^{-x^2} > 1$

$\therefore e^{-x} < e^{-x^2} < 1$ for all $x \in [0, 1]$

$\Rightarrow 1 + e^{-x} < 1 + e^{-x^2} < 2$ for all $x \in [0, 1]$

$$\Rightarrow \int_0^1 (1+e^{-x})dx < \int_0^1 (1+e^{-x^2})dx < \int_0^1 2\,dx$$

$$\Rightarrow 2 - \frac{1}{e} < \int_0^1 (1+e^{-x^2})dx < 2$$

844 **(c)**

We have,

$$I = \int e^{\sec x}(\tan^2 x + \sec x + \sec^2 x + \tan x \sec^2 x + \tan x \sec x)dx$$

$$\Rightarrow I = \int e^{\sec x}[\sec x \tan x(\sec x + \tan x) + (\sec x \tan x + \sec^2 x)]dx$$

$$\Rightarrow I = \int e^{\sec x}\sec x \tan x \cdot (\sec x + \tan x)dx + \int e^{\sec x} \cdot (\sec x \tan x + \sec^2 x)dx$$

$$\Rightarrow I = (\sec x + \tan x)e^{\sec x} - \int (\sec x + \tan x + \sec^2 x)e^{\sec x}\,dx + \int e^{\sec x}(\sec x \tan x + \sec^2 x)dx$$

$$\Rightarrow I = e^{\sec x}(\sec x + \tan x) + C$$

845 **(a)**

Let $I = \int \frac{\log\sqrt{x}}{3x}\,dx$

Put $\sqrt{x} = t$

$$\frac{1}{\sqrt{x}}dx = 2\,dt$$

$$\therefore\ I = \int \frac{2\log t}{3t}dt$$

$$= \frac{2}{3}\int \frac{\log t}{t}\,dt$$

$$= \frac{2}{3} \cdot \frac{(\log t)^2}{2} + c$$

$$= \frac{1}{3}\left(\log\sqrt{x}\right)^2 + c$$

846 **(b)**

We have,

$$I = \int_0^{2\pi} \sin^2 x\,dx$$

$$\Rightarrow I = 2\int_0^{\pi} \sin^2 x\ dx\ [\because \sin^2(2\pi - x) = \sin^2 x]$$

$$\Rightarrow I=4\int_0^{\pi/2}\sin^2 x\,dx\ \ [\because \sin^2(\pi-x)=\sin^2 x]$$

847 **(a)**

We have,

$0\le x\le 1$

$\Rightarrow 1\le 1+x^4\le 2$

$\Rightarrow 1\le\sqrt{1+x^4}\le\sqrt{2}$

$$\Rightarrow \frac{1}{\sqrt{2}}\le\frac{1}{\sqrt{1+x^4}}\le 1$$

$$\Rightarrow \frac{1}{\sqrt{2}}(1-0)\le\int_0^1\frac{1}{\sqrt{1+x^4}}dx\le 1(1-0)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\le\int_0^1\frac{1}{\sqrt{1+x^4}}dx\le 1$$

$$\Rightarrow \int_0^1\frac{1}{\sqrt{1+x^4}}dx\in\left[\frac{1}{\sqrt{2}},1\right]$$

848 **(c)**

Let $I=\int\frac{3x+2}{(x-2)^2(x-3)}\,dx$ ...(i)

Again, let

$\frac{3x+2}{(x-2)^2(x-3)}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x-3)}$ ...(ii)

$\Rightarrow\ 3x+2=A(x-2)(x-3)+B(x-3)+C(x-2)^2$...(iii)

On putting the values of $x=2,3$ respectively, we get

$3\times 2+2=B(2-3)\quad\Rightarrow\quad B=-8$

and $3\times 3+2=C(3-2)^2\quad\Rightarrow\quad C=11$

On equation the coefficient of x^2in Eq. (iii), we get

$0=A+C\quad\Rightarrow\quad A=-11$

On putting the values of A,B and C in Eq. (ii), we get

$$\frac{3x+2}{(x-2)^2(x-3)}=-\frac{11}{(x-2)}-\frac{8}{(x-2)^2}+\frac{11}{(x-3)}$$

$$\therefore\ I=\int\left[-\frac{11}{(x-2)}-\frac{8}{(x-2)^2}+\frac{11}{(x-3)}\right]dx$$

$$=-11\log(x-2)+\frac{8}{(x-2)}+11\log(x-3)+c$$

$$=11\ \log\left(\frac{x-3}{x-2}\right)+\frac{8}{(x-2)}+c$$

849 **(a)**

Let $I=\int\frac{\cos x}{1+\sin x}.e^x dx-\int\frac{1}{\sin x+1}.e^x dx$

$$=\frac{(\cos x)e^x}{1+\sin x}-\int\left[\frac{-(1+\sin x)\sin x-\cos^2 x}{(1+\sin x)^2}\right].e^x dx-\int\frac{e^x}{\sin x+1}dx$$

$$=\frac{e^x\cos x}{1+\sin x}+\int\frac{1}{1+\sin x}.e^x dx-\int\frac{e^x dx}{1+\sin x}$$

$$=\frac{e^x\cos x}{1+\sin x}+c$$

850 **(c)**

Let $I=\int_0^{2\pi}\sin^6 x\cos^5 x dx$

$=2\int_0^{\pi}\sin^6 x\cos^5 x dx\ \ [\because f(2\pi-x)=f(x)]$

Let $f(x)=\sin^6 x\cos^5 x$

$f(\pi-x)=\sin^6(\pi-x)\cos^5(\pi-x)$

$=-\sin^6 x.\cos\ ^5x=-f(x)$

$\therefore\ I=0$

851 **(c)**

$$\int\cos^{-3/7}x\sin^{-11/7}x\,dx$$

$$=\int\frac{\sin^{-11/7}x}{\cos^{-11/7}x}.\sec^2 x\,dx$$

$$=\int\tan^{-11/7}x\sec^2 x\,dx$$

Put $\tan x=t\ \Rightarrow\ \sec^2 x\,dx=dt$

$$\therefore\ I=\int t^{-11/7}\,dt=-\frac{7}{4}\tan^{-4/7}x+c$$

852 **(a)**

We have,

$$\lim_{n\to\infty}\left\{\frac{1}{na}+\frac{1}{na+1}+\cdots+\frac{1}{na+n(b-a)}\right\}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n(b-a)}\frac{1}{na+r}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n(b-a)}\frac{1}{a+(r/n)}\times\frac{1}{n}=\int_0^{b-a}\frac{1}{a+x}dx$$

$$=\log\left(\frac{b}{a}\right)$$

853 **(b)**

Since, $\frac{xe^{x^2}}{1+x^2}$is an odd function

$$\therefore\int_{-a}^{a}\frac{xe^{x^2}}{1+x^2}dx=0$$

854 **(c)**

Let $I=\int\frac{\sin\theta+\cos\theta}{\sqrt{1+\sin 2\theta-1}}d\theta$

$$=\int\frac{\sin\theta+\cos\theta}{\sqrt{1-(\sin\theta-\cos\theta)^2}}d\theta$$

Put $\sin\theta-\cos\theta=t\ \Rightarrow(\cos\theta+\sin\theta)d\theta=dt$

$\therefore\ I = \int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t + c$

$= \sin^{-1}(\sin\theta - \cos\theta) + c$

855 **(c)**

$\int \frac{e^x - 1}{e^x + 1}\, dx = \int \left(\frac{2e^x}{e^x+1} - 1\right) dx$

$= 2\log(e^x + 1) - x + c = f(x) + c$ [given]

$\therefore\ f(x) = 2\log(e^x + 1) - x$

856 **(b)**

Let $I = \int e^{3\log x}(x^4+1)^{-1}\, dx$

$= \int e^{\log x^3}(x^4+1)^{-1} dx$

$= \int x^3 (x^4+1)^{-1} dx = \frac{1}{4}\int \frac{4x^3}{(x^4+1)} dx$

Let $x^4 + 1 = t \Rightarrow 4x^3 dx = dt$

$\therefore I = \frac{1}{4}\int \frac{1}{t} dt = \frac{1}{4}\log t + c$

$\Rightarrow I = \frac{1}{4}\log(x^4+1) + c$

857 **(b)**

$y = \int \frac{dx}{(3 + 5\sin x + 3\cos x)}$

$= \int \frac{\sec^2(x/2)}{10\tan(x/2) + 6} dx$

$= \frac{1}{5}\log|5\tan(x/2) + 3| + c$

$= \frac{1}{5}\log\left|\frac{5}{3}\tan(x/2) + 1\right| + c$

This curve passes through (0, 0).

$\therefore c = 0$

Then, $y = \frac{1}{5}\log\left|1 + \frac{5}{3}\tan\left(\frac{x}{2}\right)\right|$

858 **(d)**

We have,

$f'(x) = f(x)$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

$\Rightarrow \log f(x) = x + \log C$

$\Rightarrow f(x) = C\, e^x \Rightarrow f(0) = C\, e^0 \Rightarrow 1 = C$

$\therefore f(x) = e^x$

Now,

$\int_0^1 f(x)\ g(x)\, dx$

$= \int_0^1 e^x(x^2 - e^x)\, dx$ [$\because f(x)$ $+ g(x) = x^2$]

$\Rightarrow \int_0^1 f(x)\ g(x)\, dx = \int_0^1 x^2 e^x dx - \int_0^1 e^{2x}\, dx$

$\Rightarrow \int_0^1 f(x)\ g(x)\, dx$

$= [(x^2 - 2x + 2)e^x]_0^1 - \frac{1}{2}[e^{2x}]_0^1$

$= e - \frac{e^2}{2} - \frac{3}{2}$

859 **(c)**

We have,

$f(x) = \int_0^{x^2} \sqrt{1+t^2} dt,$

$\Rightarrow f'(x) = \frac{d}{dx}\left\{\int_0^{x^2} \sqrt{1+t^2} dt\right\}$

$\Rightarrow f'(x) = \int_0^{x^2} 0 dt + 2x\sqrt{1+x^4} - 0$

$= 2x\sqrt{1+x^4}$

860 **(a)**

For $x > e$, we know that

$1 < \log x < \frac{x}{e}$

$\Rightarrow 1 > \frac{1}{\sqrt[2]{\log x}} > \left(\frac{e}{x}\right)^{1/3}$

$\Rightarrow \int_3^4 dx > I > \int_3^4 e^{1/3}x^{-1/3}\, dx$

$\Rightarrow 1 > I > \frac{3}{2}e^{1/3}\left(4^{2/3} - 3^{2/3}\right)$

$\Rightarrow 1 > I > \frac{3}{2}e^{1/3}\left(16^{1/3} - 9^{1/3}\right)$

$\Rightarrow 1 > I > 0.92$

861 **(d)**

We have,

$f(y) = e^y$ and $g(y) = y$

$\therefore F(t) = \int_0^t f(t-y)\, g(y) dy$

$\Rightarrow F(t) = \int_0^t e^{t-y}.\, y\, dy$

$\Rightarrow F(t) = [-y\, e^{t-y} - e^{t-y}]_0^t$

$\Rightarrow F(t) = (-t-1) - (0 - e^t) = e^t - t - 1$

862 **(a)**

Putting $x^2 - 1 = t^2$ and $x\, dx = t\, dt$, we get

$I = \int_0^\infty \frac{t^2 - 1}{(t^2+1)^2} dt$

$$\Rightarrow I=\int_0^1 \frac{t^2-1}{(t^2+1)^2}dt+\int_1^\infty \frac{t^2-1}{(t^2+1)^2}dt$$

$\Rightarrow I=I_1+I_2$,

Where $I_1=\int_0^1 \frac{t^2-1}{(t^2+1)^2}dt$ and $I_2=\int_1^\infty \frac{t^2-1}{(t^2+1)^2}dt$

Putting $t=\frac{1}{u}$ in I_2, we get,

$$I_2=-\int_0^1 \frac{u^2-1}{(u^2+1)^2}du=-I_1$$

Hence, $I=I_1+I_2=I_1-I_1=0$

863 **(b)**

We have,

$$I=\int(\sin 2x-\cos 2x)\,dx$$

$$\Rightarrow I=\sqrt{2}\int\left(\frac{1}{\sqrt{2}}\sin 2x-\frac{1}{\sqrt{2}}\cos 2x\right)dx$$

$$\Rightarrow I=\sqrt{2}\int\cos\left(2x+\frac{\pi}{4}\right)+\text{constant}$$

$$I=\frac{1}{\sqrt{2}}\sin\left(\pi+2x+\frac{\pi}{4}\right)+\text{constant}$$

$$I=\frac{1}{\sqrt{2}}\sin\left(\frac{5\pi}{4}+2x\right)+\text{constant}$$

But, $\int(\sin 2x-\cos 2x)\,dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\therefore a=-\frac{5\pi}{4}$ and $b=$ any constant

864 **(a)**

Let $I=\int\frac{\sec x}{\sec x+\tan x}\,dx$

$$=\int\frac{\sec x(\sec x-\tan x)}{\sec^2 x-\tan^2 x}dx$$

$$=\int(\sec^2 x-\sec x\tan x)\,dx$$

$=\tan x-\sec x+c$

865 **(b)**

$\int 1\cos^{-1}x\;dx=x\cos^{-1}x+\int x.\frac{1}{\sqrt{1-x^2}}\,dx$

$$=x\cos^{-1}x-\frac{1}{2}\int\frac{(-2x)}{\sqrt{1-x^2}}\,dx$$

$$=x\cos^{-1}x-\sqrt{1-x^2}+c$$

866 **(a)**

We have,

$$\int_0^{36}\frac{1}{2x+9}dx=\log k$$

$$\Rightarrow\frac{1}{2}[\log(2x+9)]_0^{36}=\log k\Rightarrow\log\left(\frac{81}{9}\right)=\log k^2$$

$$\Rightarrow k=3$$

867 **(d)**

Let $I=\int 1.\log 2x\,dx$

$$\Rightarrow I=x\log 2x-\int\frac{1}{2x}.2.x\,dx$$

$$\Rightarrow I=x\log 2x-x+c$$

868 **(d)**

Given, $f''(x)=\tan^2 x=(1-\sec^2 x)$

On integrating both sides, we get

$f'(x)=x-\tan x+c_1$

$f'(0)=0-0+c_1\Rightarrow c_1=0$

$\Rightarrow f'(x)=x-\tan x$

Again integrating both sides, we get

$$\Rightarrow f(x)=\frac{x^2}{2}-\log\sec x+c_2$$

$$\Rightarrow f(0)=0-\log 1+c_2\Rightarrow c_2=0$$

$$\therefore f(x)=\frac{x^2}{2}-\log\sec x$$

869 **(c)**

Let $I=\int\frac{dx}{x(x^5+1)}$

$$=\int\frac{x^4dx}{x^5(x^5+1)}$$

Put $x^5=t$ and $x^4dx=\frac{1}{5}\,dt$

$$\therefore I=\frac{1}{5}\int\frac{dt}{t(t+1)}$$

$$=\frac{1}{5}\int\left[\frac{1}{t}-\frac{1}{t+1}\right]dt$$

$$=\frac{1}{5}[\log t-\log(t+1)]+c$$

$$=\frac{1}{5}\log\frac{t}{t+1}+c$$

$$=\frac{1}{5}\log\frac{x^5}{x^5+1}+c$$

870 **(a)**

We have,

$$I=\int\frac{1}{1+\sin x}dx=\int\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\frac{1}{2}\int\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx=-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)+b$$

$\Rightarrow I=\tan\left(-\frac{\pi}{4}+\frac{x}{2}\right)+b$, where $b=$ count

Hence, $a=-\frac{\pi}{4}$ and $b\in R$

871 **(a)**

Let $I=\int\sqrt{e^x-1}\,dx=\int\frac{(\sqrt{e^x-1})e^x}{1+(\sqrt{e^x-1})}dx$

Put $e^x-1=t^2\;\Rightarrow\;e^xdx=2t\,dt$

$$\therefore I=2\int\frac{t^2dt}{1+t^2}$$

$$=2\int\left(\frac{1+t^2}{1+t^2}\right)dt-2\int\frac{1}{1+t^2}dt$$

$= 2[t - \tan^{-1} t] + c$

$= 2[\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

872 **(c)**

$\int \frac{dx}{x^4 + x^3} = \int \frac{(x+1) - x}{x^3(x+1)} dx$

$= \int \left(\frac{1}{x^3} - \frac{1}{x^2(x+1)}\right) dx$

$= \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x(x+1)}\right) dx$

$= \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1}\right) dx$

$= -\frac{1}{2x^2} + \frac{1}{x} + \log|x| - \log|x+1| + c$

$= -\frac{1}{2x^2} + \frac{1}{x} + \log\left|\frac{x}{x+1}\right| + c$

$\therefore\ A = -\frac{1}{2}$

and $B = 1$

873 **(c)**

We have,

$I = \int_\alpha^\beta x\,|x| dx$

$\Rightarrow I = \int_\alpha^0 x|x|\,dx + \int_0^\beta x|x|\,dx$

$\Rightarrow I = -\int_\alpha^0 x^2\,dx + \int_0^\beta x^2\,dx$

$\Rightarrow I = -\left[\frac{x^3}{3}\right]_\alpha^0 + \left[\frac{x^3}{3}\right]_0^\beta = -\left[0 - \frac{\alpha^3}{3}\right] + \left[\frac{\beta^3}{3} - 0\right]$

$= \frac{\alpha^3 + \beta^3}{3}$

874 **(c)**

We have,

$\sin\alpha + \int_\alpha^{2\alpha} \cos 2x\,dx = 0$

$\Rightarrow \sin\alpha + \frac{1}{2}[\sin 2x]_\alpha^{2\alpha} = 0$

$\Rightarrow \sin\alpha + \frac{1}{2}(\sin 4\alpha - \sin 2\alpha) = 0$

$\Rightarrow \sin\alpha + \sin\alpha\cos 3\alpha = 0$

$\Rightarrow \sin\alpha = 0$ or $(1 + \cos 3\alpha) = 0$

$\Rightarrow \alpha = 0, \alpha = -\pi, \alpha = -\pi/3 \Rightarrow \alpha$

$= -\frac{\pi}{3}\ \ [\because \alpha \in (-\pi, 0)]$

875 **(a)**

$I_1 = \int_0^{\pi/2} x\sin x\,dx$

$= -[x\cos x]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x) dx$

$= [0 - 0] + [\sin x]_0^{\pi/2} = 1$ ...(i)

And $I_2 = \int_0^{\pi/2} x\cos x\,dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} \sin x\,dx$

$= \left[\frac{\pi}{2} - 0\right] + [\cos x]_0^{\pi/2}$

$= \frac{\pi}{2} - 1$ ...(ii)

From Eqs. (i)and (ii), we get

$I_1 + I_2 = \frac{\pi}{2}$

876 **(b)**

We have,

$I = \int_{1/2}^{2} |\log_{10} x|\,dx$

$\Rightarrow I = \int_{1/2}^{2} |\log_e x \,.\, \log_{10} e|\,dx$

$\Rightarrow I = |\log_{10} e|\left[\int_{1/2}^{1} -\log_e x\ dx + \int_1^2 \log_e x\ dx\right]$

$\Rightarrow I = \log_{10} e\left[-\{x\log x - x\}_{1/2}^1 + \{x\log x - x\}_1^2\right]$

$\Rightarrow I = \log_{10} e\left[1 + \left(-\frac{1}{2}\log 2 - \frac{1}{2}\right) + 2\log 2 - 2 + 1\right]$

$\Rightarrow I = \log_{10} e\left[-\frac{1}{2} + \frac{3}{2}\log 2\right]$

$\Rightarrow I = \frac{1}{2}(\log_{10} e)(\log 8 - 1)$

$\Rightarrow I = \frac{1}{2}(\log_{10} e)\log_e\left(\frac{8}{e}\right) = \frac{1}{2}\log_{10}\left(\frac{8}{e}\right)$

877 **(d)**

We have,

$I = \int_0^1 |\sin 2\pi\, x|\,dx$

$\Rightarrow I = \int_0^{1/2} \sin 2\pi\ x\,dx + \int_{1/2}^{1} -(\sin 2\pi x)\,dx$

$\Rightarrow I = -\frac{1}{2\pi}[\cos 2\pi x]_0^{1/2} + \frac{1}{2\pi}[\cos 2\,\pi x]_{1/2}^1 = \frac{2}{\pi}$

878 **(c)**

Given, $I = \int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$ ...(i)

$$\Rightarrow I = \int_{-\pi}^{\pi} \frac{e^{\sin(2\pi - x)}}{e^{\sin(2\pi-x)} + e^{-\sin(2\pi-x)}} dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx \quad \text{...(ii)}$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_{-\pi}^{\pi} \frac{e^{\sin x} + e^{-\sin x}}{e^{\sin x} + e^{-\sin x}} dx$$

$$= \int_{-\pi}^{\pi} 1 dx = [x]_{-\pi}^{\pi}$$

$I = \pi$

879 **(a)**

Let $I = \int \frac{1}{1+\cos x + \sin x} dx$

$$= \int \frac{1}{1 + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\left(1 + \tan\frac{x}{2}\right)} dx$$

Put $\tan\frac{x}{2} = t$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}\, dx = dt$$

$$\therefore\ I = \int \frac{dt}{1+t} = \log(1+t) + c$$

$$= \log\left|1 + \tan\frac{x}{2}\right| + c$$

880 **(d)**

$$\int \frac{x^4 - 1}{x^2(x^4+x^2+1)^{1/2}} dx$$

$$= \int \frac{2x^4 + x^2 - (x^4+x^2+1)}{x^2\sqrt{x^4+x^2+1}} dx$$

$$= \int \frac{\frac{x(4x^3+2x)}{2\sqrt{x^4+x^2+1}} - \sqrt{x^4+x^2+1}}{x^2} dx$$

$$= \int \frac{d}{dx}\left(\frac{\sqrt{x^4+x^2+1}}{x}\right) = \frac{\sqrt{x^4+x^2+1}}{x} + c$$

881 **(c)**

Let $I = \int_0^1 \frac{x^3}{1+x^8} dx$

Putting $x^4 = t, 4x^3 dx = dt$, we get

$$I = \frac{1}{4}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{4}[\tan^{-1} t]_0^1 = \frac{\pi}{16}$$

882 **(a)**

Let $I = \int \frac{\sin x + 8\cos x}{4\sin x + 6\cos x} dx$

We can write

$$\sin x + 8\cos x = A(4\sin x + 6\cos x) + B\frac{d}{dx}(4\sin x + 6\cos x)$$

$$\Rightarrow \sin x + 8\cos x = A(4\sin x + 6\cos x) + B(4\cos x - 6\sin x)$$

On equating the coefficients of $\sin x$ and $\cos x$, we get

$1 = 4A - 6B,\ \ 8 = 6A + 4B$

$\Rightarrow\ A = 1,\ \ B = \frac{1}{2}$

$\therefore\ I$

$$= \int \frac{(4\sin x + 6\cos x) + \frac{1}{2}(4\cos x - 6\sin x)}{4\sin x + 6\cos x} dx$$

$$= \int \left(1 + \frac{1}{2}.\frac{4\cos x - 6\sin x}{4\sin x + 6\cos x}\right) dx$$

$$= x + \frac{1}{2}\log(4\sin x + 6\cos x) + c$$

883 **(b)**

$$I = \int \frac{x^2}{(ax+b)^2} dx$$

Put $ax + b = t \Rightarrow dx = \frac{1}{a}dt$

and $x = \left(\frac{t-b}{a}\right)$

$$\therefore I = \frac{1}{a^3}\int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3}\int\left(1 + \frac{b^2}{t^2} - \frac{2b}{t}\right) dt$$

$$= \frac{1}{a^3}\left(t - \frac{b^2}{t} - 2b\log t\right) + c$$

$$= \frac{1}{a^3}\left(ax + b - \frac{b^2}{ax+b} - 2b\log(ax+b)\right) + c$$

$$= \frac{2}{a^2}\left(x - \frac{b}{a}\log(ax+b)\right) - \frac{x^2}{a(ax+b)} + c$$

884 **(b)**

Let $I = \int_1^4 e^{\sqrt{x}} dx$

Put $x = t^2 \Rightarrow dx = 2t\, dt$

$$\therefore I = \int_1^2 2e^t t\, dt = 2[te^t - e^t]_1^2$$

$$= 2[(2e^2 - e) - e^2 + e] = 2e^2$$

885 **(a)**

Let $I = \int \frac{x^4}{x+x^5} dx = \int \frac{x^4+1-1}{x+x^5} dx$

$$\Rightarrow\ I = \int \frac{1}{x} dx - \int \frac{1}{x^5+x} dx$$

$\Rightarrow\ I = \log x - [f(x) + c]$ [given]

$\Rightarrow\ I = \log x - f(x) + c_1$ [where $c_1 = -c$]

886 **(c)**

$$\int e^x(1 - \cot x + \cot^2 x) dx$$

$$= \int e^x(-\cot x + \operatorname{cosec}^2 x)dx$$

$$= -\int e^x \cot x \, dx + \int e^x \operatorname{cosec}^2 x \ \ dx$$

$$= -e^x \cot x - \int e^x \operatorname{cosec}^2 x \ \ dx \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x(-\cot x) + c = -e^x \cot x + c$$

887 **(c)**

Given, $f(x) = 4x^2 - 3x + 1, g(x) = \frac{f(-x)-f(x)}{x^2+3}$

$$\therefore g(x) = \frac{(4x^2 + 3x + 1) - (4x^2 - 3x + 1)}{x^2 + 3}$$

$$= \frac{6x}{x^2 + 3}$$

Now, $g(-x) = -\frac{6x}{x^2+3} = -g(x)$

Which is an odd function

$$\therefore \int_{-2}^{2} g(x)dx = 0$$

888 **(a)**

$$\int \sqrt{1 + \sin\left(\frac{x}{4}\right)} \ dx$$

$$= \int \sqrt{\left(\sin^2\frac{x}{8} + \cos^2\frac{x}{8}\right) + \left(2\sin\frac{x}{8}\cos\frac{x}{8}\right)} \ dx$$

$$= \int \sqrt{\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right)^2} \ dx = \int \left(\sin\frac{x}{8} + \cos\frac{x}{8}\right) dx$$

$$= \frac{-\cos\frac{x}{8}}{\left(\frac{1}{8}\right)} + \frac{\sin\frac{x}{8}}{\left(\frac{1}{8}\right)} + c = \left(8\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$$

889 **(a)**

If we put $h(x) = t$, then the integral reduces to

$$\int_{h(a)}^{h(b)} \left[f\left(g\left(h(t)\right)\right)\right]^{-1} f'(g(t))g'(t)dt$$

$$= 0 \quad [\because h(a) = h(b)]$$

890 **(a)**

Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x cos^2 x} dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \quad \text{... (ii)}$$

On adding Eqs. (i) and(ii), we get

$$2I = \int_0^{\pi/2} 1 \, dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

891 **(b)**

Putting $\frac{\pi x}{2} = t$ and $dx = \frac{2}{\pi} dt$, we get

$$\int_0^1 \log\sin\left(\frac{\pi x}{2}\right) dx = \frac{2}{\pi} \int_0^{\pi/2} \log\sin t \, dt$$

$$= \frac{2}{\pi}\left(-\frac{\pi}{2}\log 2\right) = -\log 2$$

892 **(d)**

Given, $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}, x > 0$

On integrating both sides, we get

$F(x) = \int \frac{e^{\sin x}}{x} dx$ ...(i)

Also $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} . e^{\sin x^3} dx = F(k) - F(1)$

Let $x^3 = z \Rightarrow 3x^2 dx = dz$

$$\therefore \int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$$

$\Rightarrow [F(z)]_1^{64} = F(k) - F(1)$ [from Eq.(i)]

$\Rightarrow F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64$

893 **(b)**

$$\int_0^{10} |x(x-1)(x-2)|dx$$

$$= \int_0^1 (x^3 - 3x^2 + 2x)dx + \int_1^2 (-x^3 + 3x^2 - 2x)dx$$

$$= \int_2^{10} (x^3 - 3x^2 + 2x)dx$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[-\frac{x^4}{4} + x^3 - x^2\right]_1^2 + \left[\frac{x^4}{4} - x^3 + x^2\right]_2^{10}$$

$= 1600.5$

894 **(d)**

Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}\left(\sin x \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{8}\right)}$$
$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2}+\frac{\pi}{8}\right) dx$$
$$= \frac{1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\frac{1}{2}} + c$$
$$= -\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2}+\frac{\pi}{8}\right) + c$$

895 **(a)**

We have,
$$I = \int_{-\pi/4}^{\pi/4} \sin^{-4} x \; dx$$
$$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{1}{\sin^4 x} \; dx$$
$$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \operatorname{cosec}^4 x \, dx$$
$\Rightarrow I = \int_{-1}^{1} (1+t^2)dt$, where $\cot x = t$
$$\Rightarrow I = -2\int_0^1 (1+t^2)\,dt = -2\left[t + \frac{t^3}{3}\right]_0^1 = -\frac{8}{3}$$

896 **(a)**
$$I_8 + I_6 = \int_0^{\pi/4} \tan^8\theta \, d\theta + \int_0^{\pi/4} \tan^6\theta \, d\theta$$
$$= \int_0^{\pi/4} \sec^2\theta \tan^6\theta \, d\theta$$
$$= \left[\frac{\tan^7\theta}{7}\right]_0^{\pi/4} = \frac{1}{7}$$

897 **(b)**

We have,
$$\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx = 2\int \frac{a^{\sqrt{x}}}{2\sqrt{x}} dx = 2\int a^{\sqrt{x}} \, d(\sqrt{x})$$
$$= \frac{2a^{\sqrt{x}}}{\log a} + C$$

898 **(a)**

Let $I = \int_a^b \sqrt{(x-a)(b-x)}dx$

Put $x = a\cos^2\theta + b sin^2\theta$

$\Rightarrow dx = -2a\cos\theta\sin\theta + 2b\sin\theta\cos\theta$

$\Rightarrow dx = \sin 2\theta(b-a)$
$$\therefore \; I = \int_0^{\pi/2} \sqrt{(b-a)\sin^2\theta(b-a)\cos^2\theta} \; (b-a)\sin 2\theta d\theta$$
$$= \int_0^{\pi/2} \frac{(b-a)^2}{2} \sin^2 2\theta d\theta$$
$$= \frac{(b-a)^2}{2} \int_0^{\pi/2} \left[\frac{1-\cos 4\theta}{2}\right] d\theta$$
$$= \frac{(b-a)^2}{4} \left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$
$$= \frac{(b-a)^2}{4} \left[\frac{\pi}{2} - 0\right]$$
$$= \frac{\pi(b-a)^2}{8}$$

899 **(b)**
$$\int_0^{x^2} f(t) \; dt = x\cos\pi x$$
On differentiating both sides, we get
$$2x\, f(x^2) = \frac{-x\sin\pi x}{\pi} + \cos\pi x$$
$$\Rightarrow \; f(x^2) = -\frac{x\sin\,\pi x}{2\pi x} + \frac{\cos\pi x}{2x}$$
$$\therefore \; f(4) = f(2^2) = \frac{1}{4}$$

900 **(c)**
$$f'(x) = \frac{dx}{(1+x^2)^{3/2}}$$
On integrating both sides,
$$f(x) = \int \frac{dx}{(1+x^2)^{3/2}} + c$$
Put $x = \tan\theta \Rightarrow dx = \sec^2\theta \; d\theta$
$$\therefore \; f(x) = \int \frac{\sec^2\theta}{\sec^3\theta} \, d\theta + c = \int \cos\theta \; d\theta + c$$
$$\Rightarrow \; f(x) = \sin\theta + c$$
$$\Rightarrow f(x) = \frac{x}{\sqrt{1+x^2}} + c$$
$$\Rightarrow f(0) = 0 + c \Rightarrow c = 0$$
$$\therefore f(x) = \frac{x}{\sqrt{1+x^2}} \Rightarrow f(1) = \frac{1}{\sqrt{2}}$$

901 **(c)**

We have,
$$I = \int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$$
$$\Rightarrow I = \int_0^{\pi/2} \frac{1}{2} x \sec^2\frac{x}{2} + \int_0^{\pi/2} \tan\frac{x}{2} dx$$
$$\Rightarrow I = \left[x\tan\frac{x}{2}\right]_0^{\pi/2} - \int_0^{\pi/2} \tan\frac{x}{2} dx + \int_0^{\pi/2} \tan\frac{x}{2} dx$$
$$= \pi/2$$

902 **(a)**

We have,

$$\int f(x)\sin x\cos x\ dx = \frac{1}{2(b^2-a^2)}\log\{f(x)\} + C$$

$$\Rightarrow f(x)\sin x\cos x = \frac{1}{2(b^2-a^2)}\cdot\frac{1}{f(x)}f'(x)$$

$$\Rightarrow 2(b^2-a^2)\sin x\cos x = \frac{f'(x)}{\{f(x)\}^2}$$

$$\Rightarrow \int (2b^2\sin x\cos x - 2a^2\sin x\cos x)\,dx$$

$$= \int \frac{f'(x)}{\{f(x)\}^2}dx$$

$$\Rightarrow -b^2\cos^2 x - a^2\sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{1}{(a^2\sin^2 x + b^2\cos^2 x)}$$

903 **(c)**

$$\int \sqrt{\frac{1+x}{1-x}}dx = \int \frac{1+x}{\sqrt{1-x^2}}dx$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}}dx$$

$$= \sin^{-1}x - \sqrt{1-x^2} + c$$

904 **(b)**

Let $I = \int_1^2 \frac{dx}{x(1+x^4)} = \int_1^2 \frac{x^3}{x^4(1+x^4)}dx$

Put $x^4 = t \Rightarrow 4x^3dx = dt$

$$\therefore\quad I = \frac{1}{4}\int_1^{16}\frac{dt}{t(1+t)} = \frac{1}{4}\int_1^{16}\left(\frac{1}{t} - \frac{1}{1+t}\right)dt$$

$$= \frac{1}{4}\left(\log\frac{t}{1+t}\right)_1^{16} = \frac{1}{4}\left(\log\frac{16}{17} - \log\frac{1}{2}\right)$$

$$= \frac{1}{4}\log\frac{32}{17}$$

905 **(c)**

We have,

$$I = \int \frac{1}{(x+1)^2\sqrt{x^2+2x+2}}dx$$

$$= \int \frac{1}{(x+1)^2\sqrt{(x+1)^2+1}}dx$$

$$\Rightarrow I = \int \frac{\sec^2\theta\, d\theta}{\tan^2\theta\sec\theta}, \text{where } x+1 = \tan\theta$$

$$\Rightarrow I = \int \frac{\cos\theta}{\sin^2\theta}d\theta = \int (\sin\theta)^{-2}d(\sin\theta)$$

$$\Rightarrow I = -\frac{1}{\sin\theta} + C = -\frac{\sqrt{x^2+2x+2}}{x+1} + C$$

906 **(c)**

We have,

$$I = \int_{-1}^{1}\left\{[x^2] + \log\left(\frac{2+x}{2-x}\right)\right\}dx$$

$$= \int_{-1}^{1}[x^2]\,dx + \int_{-1}^{1}\log\left(\frac{2+x}{2-x}\right)dx$$

$\Rightarrow I$

$$= \int_{-1}^{1} 0\ dx$$

$+ 0 \quad \left[\because \log\left(\frac{2+x}{2-x}\right) \text{ is an odd function}\right]$

$\Rightarrow I = 0$

907 **(b)**

$$\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = \int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx$$

$$= \int e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx$$

$$\frac{1}{2}\int e^x\operatorname{cosec}^2\frac{x}{2}dx - \int e^x\cot\frac{x}{2}dx$$

$$= \frac{1}{2}\int e^x\operatorname{cosec}^2\frac{x}{2}dx - e^x\cot\frac{x}{2}\int e^x\operatorname{cosec}^2\frac{x}{2}dx$$

$$= e^x(-\cot\frac{x}{2}) + c$$

$$= -e^x\cot\frac{x}{2} + c$$

908 **(d)**

Let $I = \int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}\ dx$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$$

Put $x + \frac{1}{x} = t$

$\Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$ and $\left(x+\frac{1}{x}\right)^2 = t^2$

$$\Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$\therefore I = \int \frac{1}{(t^2+1)\tan^{-1}t}dt$$

$$= \log(\tan^{-1}t) + c$$

$$= \log\left[\tan^{-1}\left(x+\frac{1}{x}\right)\right] + c$$

909 **(b)**

We have,

$$\int_0^1 x(1-x)^{-3/4}dx$$

$$= \left[-4x + (1-x)^{1/4}\right]_0^1$$

$$+ 4\int_0^1 (1-x)^{1/4} = \frac{16}{5}$$

910 **(b)**

$f'(x) = 1 - \cos x$ always positive, so $f(x)$ is an increasing function so least value of $f(x)$ on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is $f\left(\frac{\pi}{2}\right)$

$$\therefore\ f\left(\frac{\pi}{2}\right) = \int_0^{\pi/2} (1 - \cos t)dt$$

$$= [t - \sin t]_0^{\pi/2} = \left(\frac{\pi}{2} - 1\right)$$

911 **(d)**

$$\int \frac{x^2 - 2}{x^3 - \sqrt{x^2 - 1}}\ dx$$

$$= \int \frac{dx}{x\sqrt{x^2 - 1}} - 2\int \frac{dx}{x^3\sqrt{x^2 - 1}}$$

$$= \sec^{-1} x - 2\int \frac{\sec\theta \tan\theta}{\sec^3\theta \tan\theta} d\theta$$

[Putting $x = \sec\theta \Rightarrow dx = \sec\theta \tan\theta\ d\theta$]

$$= \sec^{-1} x - 2\int \cos^2\theta\, d\theta$$

$$= \sec^{-1} x - \int (1 + \cos 2\theta)\, d\theta$$

$$= \sec^{-1}x - \left(\theta + \frac{\sin 2\theta}{2}\right) + c$$

$$= \sec^{-1} x - \sec^{-1} x - \frac{\sqrt{x^2 - 1}}{x^2} + c$$

$$= -\frac{\sqrt{x^2 - 1}}{x^2} + c$$

912 **(b)**

We have,

$$I = \int \frac{(x - x^3)^{1/3}}{x^4} dx = \int \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3} dx$$

$$\Rightarrow = -\frac{1}{2}\int \left(\frac{1}{x^2} - 1\right)^{1/3} d\left(\frac{1}{x^2} - 1\right)$$

$$= -\frac{3}{8}\left(\frac{1}{x^2} - 1\right)^{4/3} + C$$

913 **(a)**

Let $I = \int_0^\infty \log\left(x + \frac{1}{x}\right)\frac{1}{1+x^2} dx$

Put $x = \tan\theta \ \Rightarrow\ dx = \sec^2\theta\, d\theta$

$$\therefore\ I = \int_0^{\pi/2} \log\left(\tan\theta + \frac{1}{\tan\theta}\right)\frac{\sec^2\theta}{\sec^2\theta} d\theta$$

$$\Rightarrow\ I = \int_0^{\pi/2} \log\frac{(1 + \tan^2\theta)}{\tan\theta} d\theta$$

$$\Rightarrow\ I = 2\int_0^{\pi/2} \log\sec\theta\, d\theta - \int_0^{\pi/2} \log\tan\theta\, d\theta$$

$$\Rightarrow\ I = 2\int_0^{\pi/2} \log\sec\theta\, d\theta \quad \left(\because \int_0^{\pi/2} \log\tan\theta\, d\theta = 0\right)$$

$$\Rightarrow\ I = -2\int_0^{\pi/2} \log\cos\theta\, d\theta$$

$$= -2 \times \frac{-\pi}{2}\log 2 \quad \left(\because \int_0^{\pi/2} \log\cos\theta\, d\theta = -\frac{\pi}{2}\log 2\right)$$

$$\therefore\ I = \pi\log 2$$

914 **(a)**

Given, $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx = x[f(x) - g(x)] + c$

$$\text{LHS} = \int \left[\underset{\text{II}}{1}.\log \underset{\text{I}}{(\log x)}\, dx + \int \frac{1}{(\log x)^2}\right] dx$$

$$= x\log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2}\ dx$$

$$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx + c$$

$$= x\left[\log(\log x) - \frac{1}{\log x}\right] + c$$

$$\therefore f(x) = \log(\log x), g(x) = \frac{1}{\log x}$$

915 **(c)**

Required integral I is given by

$$I = \int \frac{1}{\sqrt{x^2 + 9}} d(x^2 + 1)$$

$$\Rightarrow I = \int \frac{1}{\sqrt{x^2 + 9}} d(x^2 + 9) \quad [\because d(x^2 + 1) = d(x^2 + 9)]$$

$$\Rightarrow I = 2\sqrt{x^2 + 9} + C$$

916 **(b)**

We have,

$$I = \int \frac{\sqrt{x}}{1 + x^{3/4}} dx = \int \frac{t^2 \times 4t^3}{1 + t^3} dt\text{, where } x = t^4$$

$$\Rightarrow I = \frac{4}{3}\int \frac{t^3}{1 + t^3} \times 3t^2 dt = \frac{4}{3}\int \frac{u - 1}{u} du\text{, where } u = 1 + t^3$$

$$\Rightarrow I = \frac{4}{3}|u - \log u| + C$$

$$= \frac{4}{3}[1 + x^{3/4} - \log(1 + x^{3/4})] + C$$

917 **(b)**

We have,

$$\int e^{x\log a}\, e^x\, dx = \int e^{\log a^x}\cdot e^x\, dx = \int a^x\; e^x\, dx$$

$$\Rightarrow \int e^{x\log a}\, e^x\, dx = \int (ae)^x\, dx = \frac{(ae)^x}{\log(ae)} + C$$

918 **(b)**

Given, $\int f(x)dx = g(x)$

$$\therefore \int f(x)g(x)\,dx$$

$$= g(x)\int f(x)\,dx - \int [g'(x)\int f(x)\;dx]\,dx$$

$$= g(x)g(x) - \int g'(x)\,g(x)dx - [g(x)]^2 - \int f(x)\,g(x)dx$$

$$\Rightarrow \int f(x)\,g(x)dx = \frac{1}{2}[g(x)]^2$$

919 **(b)**

Given, $\int \frac{\cos 4x+1}{\cos x-\tan x}\,dx = A\cos 4x + B$

Let $I = \int \frac{\cos 4x+1}{\cos x-\tan x}dx$

$$= \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}dx = \int \frac{2\cos^2 x}{\frac{\cos 2x}{\sin x\cos x}}dx$$

$$= \int \sin 2x\cos 2x\,dx$$

$$= \frac{1}{2}\int 2\sin 2x\cos 2x\,dx$$

$$= \frac{1}{2}\int \sin 4x\,dx = \frac{1}{2}\left[-\frac{\cos 4x}{4}\right] + B$$

$$= -\frac{1}{8}\cos 4x + B$$

But $I = A\cos 4x + B$ (given)

$$\therefore A = -\frac{1}{8}$$

920 **(d)**

Let $I = \int_0^{\pi}(\pi\, x - x^2)^{100}\sin 2x\,dx$. Then,

$$\Rightarrow I = \int_0^{\pi}\{\pi(\pi-x)-(\pi-x)^2\}\sin 2\,(\pi-x)dx$$

$$\Rightarrow I = -\int_0^{\pi}(\pi x - x^2)\sin 2x\,dx$$

$$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

921 **(a)**

We have,

$$I = \int_{\pi}^{2\pi}[2\sin x]\,dx$$

$$\Rightarrow I = \int_{\pi}^{2\pi/6}[2\sin x]\,dx + \int_{7\pi/6}^{11\pi/6}[2\sin x]\,dx + \int_{11\pi/6}^{2\pi}[2\sin x]\,dx$$

$$\Rightarrow I = \int_{\pi}^{7\pi/6} -1\,dx + \int_{7\pi/6}^{11\pi/6} -2\,dx + \int_{11\pi/6}^{2\pi} -1\,dx$$

$$= -\frac{5\pi}{3}$$

922 **(a)**

We have,

$$I = \int \frac{1}{\sqrt{\sin^3 x\cos x}}dx = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}}dx$$

$$\Rightarrow I = \int (\tan x)^{-3/2}d(\tan x) = \frac{-2}{\sqrt{\tan x}} + C$$

923 **(c)**

Let $\int_0^{1.5}[x^2]dx = \int_0^1[x^2]dx + \int_1^{\sqrt{2}}[x^2]dx + \int_{\sqrt{2}}^{1.5}[x^2]dx$

$$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{1.5} 2\,dx = 2 - \sqrt{2}$$

924 **(d)**

$$\int_0^{\pi}|\cos x|dx = \int_0^{\pi/2}\cos x\,dx - \int_{\pi/2}^{\pi}\cos x\,dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$$

$$= \sin\frac{\pi}{2} - \sin 0 - \sin\pi + \sin\frac{\pi}{2}$$

$$= 2\sin\frac{\pi}{2} = 2$$

925 **(a)**

We have,

$$\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$$

$$\Rightarrow I = \int \frac{(t^6+t^4+t)}{t^6(1+t^2)}6t^2\,dt\text{, where } x = t^6$$

$$\Rightarrow I = 6\int \frac{t^5+t^3+1}{t^2+1}dt = 6\int \frac{t^3(t^2+1)+1}{t^2+1}dt$$

$$\Rightarrow I = 6\int \left(t^3 + \frac{1}{t^2+1}\right)dt$$

$$\Rightarrow I = 6\left\{\frac{t^4}{4} + \tan^{-1}t\right\} + C$$

$$= \frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + C$$

926 **(c)**

Given, $\int_0^{\pi} e^{\cos^2 x}\cdot\cos^3(2n+1)x\,dx$

Let $f(x) = e^{\cos^2 x}\cdot\cos^3(2n+1)x$

Then, $f(\pi - x) = e^{\cos^2(\pi - x)} \cdot \cos^3[(2n+1)\pi - (2n+1)x]$

$= -e^{\cos^2 x} \cdot \cos^3(2n+1)x$

$\Rightarrow f(\pi - x) = -f(x)$

Then, $f(x)$ is an odd function

$\therefore \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3(2n+1)x\, dx = 0$

927 **(b)**

Let $I = \int_0^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1}\, dx$

$= \int_0^{\pi/2} \frac{\cos 3x - \cos\frac{3\pi}{3}}{2\left(\cos x - \cos\frac{\pi}{3}\right)}\, dx$

$= \int_0^{\pi/2} \frac{\left(4\cos^3 x - 3\cos x\right) - \left(4\cos^3\frac{\pi}{3} - 3\cos\frac{\pi}{3}\right)}{2\left(\cos x - \cos\frac{\pi}{3}\right)} dx$

$= 2\int_0^{\pi/2} \left(\frac{\cos^3 x - \cos^3\frac{\pi}{3}}{\cos x - \cos\frac{\pi}{3}}\right) dx - \frac{3}{2}\int_0^{\pi/2} \left(\frac{\cos x - \cos\frac{\pi}{3}}{\cos x - \cos\frac{\pi}{3}}\right) dx$

$= 2\int_0^{\pi/2} \left(\cos^2 x + \cos^2\frac{\pi}{3} + \cos x \cos\frac{\pi}{3}\right) dx - \frac{3}{2}\int_0^{\pi/2} 1\, dx$

$= \int_0^{\pi/2} \left(1 + \cos 2x + \frac{1}{2} + \cos x\right) dx - \frac{3\pi}{4}$

$= \frac{3\pi}{4} + 1 - \frac{3\pi}{4} = 1$

928 **(c)**

$\int_1^3 \frac{\cos(\log x)}{x} dx = [\sin(\log x)]_1^3$

$= \sin(\log 3) - \sin(\log 1) = \sin(\log 3)$

929 **(d)**

Let $I = \int (e^x + e^{-x})^2 . (e^x - e^{-x}) dx$

Put $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})dx = dt$

$\therefore I = \int t^2 dt = \frac{t^3}{3} + c$

$= \frac{(e^x + e^{-x})^3}{3} + c$

930 **(b)**

Let $I = \int \frac{dx}{\sin x \cos x} = 2\int \frac{dx}{\sin 2x}$

$= 2\int \operatorname{cosec} 2x\, dx$

$= 2\log|\tan\left(\frac{2x}{2}\right)| \times \frac{1}{2} + c$

$= \log|\tan x| + c$

931 **(c)**

Using Walli's formula, we get

$\int_0^{\pi/2} \sin^n x\, dx$

$= \frac{n-1}{n} . \frac{n-3}{n-2} . \frac{n-5}{n-4} \ldots\ldots \frac{2}{3}$ if n is odd

$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^5 x\, dx = \frac{5-1}{5} . \frac{5-3}{5-2}$

$= \frac{4}{5} . \frac{2}{3}$

$= \frac{8}{15}$

932 **(b)**

Let

$I = \int_{-\pi/2}^{\pi/2} \log_e \left\{\left(\frac{ax^2 + bx + c}{ax^2 - bx + c}\right)(a+b)|\sin x|\right\} dx$

$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log_e \left(\frac{ax^2 + bx + c}{ax^2 - bx + c}\right) dx +$

$\int_{-\pi/2}^{\pi/2} \log_e(a+b)\, dx + \int_{-\pi/2}^{\pi/2} \log_e|\sin x|\, dx$

$\Rightarrow I = 0 + \pi \log_e(a+b) + 2\int_0^{\pi/2} \log_e \sin x\, dx$

$\Rightarrow I = \pi \log_e(a+b) + 2 \times -\frac{\pi}{2}\log 2$

$= \pi \log_e\left(\frac{a+b}{2}\right)$

933 **(c)**

Let $I = \int_{1/e}^{e} \frac{|\log x|}{x^2} dx$. Then,

$I = -\int_e^{1/e} |-\log t|\, dt$, where $x = \frac{1}{t}$

$\Rightarrow I = \int_{1/e}^{e} |\log t|\, dt$

$\Rightarrow I = \left\{\int_{1/e}^{1} -\log t\, dt + \int_1^e \log t\, dt\right\}$

$\Rightarrow I = \{-[(t\log t - t)]_{1/e}^{1} + [t\log t - t]_1^e\}$

$\Rightarrow I = \left(1 - \frac{2}{e} + 1\right) = 2\left(1 - \frac{1}{e}\right)$

934 **(c)**

Let $f(x) = x|x|$

$f(-x) = -x|x|$

$= -f(x)$

$\therefore f(x)$ is an odd function

Hence, $I = 0$

935 **(c)**

We have,

$$I=\int\frac{1}{x^3+x^4}dx=\int\frac{1}{x^3(x+1)}dx$$
$$=\int\frac{1}{x^2\{x(x+1)\}}dx$$
$$\Rightarrow I=\int\frac{1}{x^2}\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$$
$$=\int\frac{1}{x}\left\{\frac{1}{x^2}-\frac{1}{x(x+1)}\right\}dx$$
$$\Rightarrow I=\int\frac{1}{x}\left\{\frac{1}{x^2}-\left(\frac{1}{x}-\frac{1}{x+1}\right)\right\}dx$$
$$=\int\frac{1}{x^3}-\frac{1}{x^2}+\frac{1}{x(x+1)}dx$$
$$\Rightarrow I=\int\left(\frac{1}{x^3}-\frac{1}{x^2}+\frac{1}{x}-\frac{1}{x+1}\right)dx$$
$$=-\frac{1}{2x^2}+\frac{1}{x}+\log\left|\frac{x}{x+1}\right|+C$$
$\therefore A=-\frac{1}{2}$ and $B=1$

936 **(a)**

We have,
$$I=\int\frac{1}{(x^2+1)(x^2+4)}dx$$
$$=\frac{1}{3}\left(\frac{1}{x^2+1}-\frac{1}{x^2+4}\right)dx$$
$$\Rightarrow I=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+C$$
Hence, $A=\frac{1}{3}$ and $B=-\frac{1}{6}$

937 **(c)**

We have,
$$I=\int_{-1/2}^{1/2}\left\{\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right\}^{1/2}dx$$
$$\Rightarrow I=\int_{-1/2}^{1/2}\left\{\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)^2\right\}^{1/2}dx$$
$$\Rightarrow I=\int_{-1/2}^{1/2}\left|\frac{4x}{x^2-1}\right|dx$$
$$\Rightarrow I=\int_{-1/2}^{0}\left|\frac{4x}{1-x^2}\right|dx+\int_{0}^{1/2}\left|\frac{4x}{1-x^2}\right|dx$$
$$\Rightarrow I=-4\int_{-1/2}^{0}\frac{x}{1-x^2}dx+4\int_{0}^{1/2}\frac{x}{1-x^2}dx$$
$$\Rightarrow I=2[\log(1-x^2)]_{-1/2}^{0}-2[\log(1-x^2)]_0^{1/2}$$
$$\Rightarrow I=-2\log\left(1-\frac{1}{4}\right)-2\log\left(1-\frac{1}{4}\right)$$
$$\Rightarrow I=-4\log\frac{3}{4}=4\log\frac{4}{3}$$

938 **(b)**

$$\int\operatorname{cosec}x\,dx=\log|\operatorname{cosec}x-\cot x|+c$$
$$=\log\left|\frac{(1-\cos x)}{\sin x}\right|+c$$
$$=\log\left|\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}}\right|+c$$
$$=\log\left|\tan\frac{x}{2}\right|+c=f(x)+\text{constant}$$
$$\therefore\ f(x)=\log\left|\tan\frac{x}{2}\right|$$

939 **(a)**

Let $I=\int\frac{dx}{x(x^n+1)}$

Put $x^n+1=t\ \Rightarrow\ nx^{n-1}dx=dt$
$$\therefore\ I=\frac{1}{n}\int\frac{dt}{t(t-1)}=\frac{1}{n}\int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt$$
$$=\frac{1}{n}[\log(t-1)-\log t]+c$$
$$=\frac{1}{n}\log\left(\frac{t-1}{t}\right)+c=\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$$

940 **(c)**

$$\int\frac{ax^3+bx^2+c}{x^4}dx$$
$$=\int\left[\frac{a}{x}+\frac{b}{x^2}+\frac{c}{x^4}\right]dx$$
$$=a\int\frac{1}{x}dx+b\int x^{-2}dx+c\int x^{-4}dx$$
$$=a\log x-bx^{-1}-\frac{c}{3}x^{-3}+k$$
$$=a\log x-\frac{b}{x}-\frac{c}{3x^3}+k$$

941 **(b)**

Let $I=\int\frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$

Putting $x+1=t^2$ and $dx=2t\,dt$, we get
$$I=2\int\frac{t^2+1}{t^4+t^2+1}dt$$
$$\Rightarrow I=2\int\frac{1+(1/t)^2}{\left(t-\frac{1}{t}\right)^2+3}dt$$
$$=2\int\frac{1}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{3}\right)^2}d\left(t-\frac{1}{t}\right)$$
$$\Rightarrow t=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right)+C$$
$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+C$$

942 **(d)**

Let $I=\int_0^{\pi/2}\frac{dx}{1+\cot x}=\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}dx$ ...(i)

$$\Rightarrow I=\int_0^{\pi/2}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx$$

$\Rightarrow I=\int_0^{\pi/2}\frac{\sin x}{\cos x+\sin x}dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I=\int_0^{\pi/2}1\,dx=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

943 **(d)**

Let $I=\int\frac{2\sin x\cos x}{1+\cos^2 x}dx$

Put $1+\cos^2 x=t \Rightarrow -2\cos x\sin x\,dx=dt$

$$\therefore I=-\int\frac{dt}{t}=-\log t+c$$

$$=-\log(1+\cos^2 x)+c$$

944 **(c)**

We have,

$$I=\int_0^{\pi}e^{\sin^2 x}\cos^3(2n+1)\,x\,dx,$$

$$\Rightarrow I=\int_0^{\pi}e^{\sin^2(\pi-x)}\cos^3\{(2x+1)(\pi-x)\}\,dx$$

$$\Rightarrow I=\int_0^{\pi}e^{\sin^2 x}\cos^3\{(2n+1)\pi-(2n+1)x\}dx$$

$$\Rightarrow I=-\int_0^{\pi}e^{\sin^2 x}\cos^3(2n+1)\,x\,dx=-I$$

$\Rightarrow 2I=0\Rightarrow I=0$

945 **(c)**

On putting $\log(x+\sqrt{1+x^2})=t$

$$\Rightarrow \frac{1}{x+\sqrt{1+x^2}}\times\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx=dt$$

$$\Rightarrow \frac{dx}{\sqrt{1+x^2}}=dt$$

$$\therefore \int\frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx=\int t\,dt=\frac{1}{2}t^2+c$$

$$=\frac{1}{2}\log(x+\sqrt{x^2+1})^2+c$$

Thus, $f(x)=\log(x+\sqrt{x^2+1})$

and $g(x)=\frac{x^2}{2}$

946 **(d)**

Let $I=\int\frac{1+\tan^2 x}{1-\tan^2 x}\,dx=\int\frac{\sec^2 x}{1-\tan^2 x}\,dx$

Put $\tan x=t \Rightarrow \sec^2 x\,dx=dt$

$$\therefore I=\int\frac{dt}{1-t^2}=\frac{1}{2\times 1}\log\left(\frac{1+t}{1-t}\right)+c$$

$$=\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$$

947 **(a)**

$(x+1)(x+3)=(x+2-1)(x+2+1)$

$=(x+2)^2-1$

$$\therefore \int(x+1)(x+2)^7(x+3)\,dx$$

$$=\int\{(x+2)^9-(x+2)^7\}\,dx$$

$$=\frac{(x+2)^{10}}{10}-\frac{(x+2)^8}{8}+c$$

948 **(b)**

Let $I=\int_1^2\frac{dx}{(x+1)\sqrt{x^2-1}}$

Put $x=\sec\theta \Rightarrow dx=\sec\theta\tan\theta d\theta$ ... (i)

$$\therefore I=\int_0^{\pi/3}\frac{\sec\theta\tan\theta}{(\sec\theta+1)\tan\theta}\,d\theta$$

$$=\int_0^{\pi/3}\frac{1}{1+\cos\theta}d\theta=\int_0^{\pi/3}\frac{\sec^2\frac{\theta}{2}}{2}d\theta$$

$$=\frac{1}{2}\left[\frac{\tan\frac{\theta}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{3}}=\frac{1}{\sqrt{3}}$$

949 **(a)**

Let $I=\int_0^{\pi}\cos^3 x\,dx$ ...(i)

$\Rightarrow I=\int_0^{\pi}\cos^3(\pi-x)dx=-\int_0^{\pi}\cos^3 x\,dx$

...(ii)

On adding Eqs. (i) and(ii), we get

$2I=0\Rightarrow I=0$

950 **(d)**

$\int_0^a\sqrt{a^2-x^2}\,dx=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$

$$=\left[0+\frac{a^2}{2}\sin^{-1}(1)-0-\frac{a^2}{2}\sin^{-1}(0)\right]$$

$$=\frac{a^2}{2}.\frac{\pi}{2}-0=\frac{a^2\pi}{4}$$

951 **(b)**

We have,

$$I=\int\frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$$

$$\Rightarrow I=\int\frac{x^2-1}{(x+1)^2\sqrt{x^3+x^2+x}}dx$$

$$\Rightarrow I=\int\frac{1-\frac{1}{x^2}}{(x+\frac{1}{x}+2)\sqrt{x+\frac{1}{x}+1}}dx$$

$$\Rightarrow I=\int \frac{1}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}\,d\left(x+\frac{1}{x}+1\right)$$

$$\Rightarrow I=2\int \frac{1}{t^2+1}\,dt, \text{where } x+\frac{1}{x}+1=t^2$$

$$\Rightarrow I=2\tan^{-1}\left(x+\frac{1}{x}+1\right)+C$$

$$=2\tan^{-1}\left(\frac{x^2+x+1}{x}\right)+C$$

952 **(d)**

Let $A=\lim_{n\to\infty}\left\{\frac{n!}{(k\,n)^n}\right\}^{1/n}$

$$\Rightarrow A=\lim_{n\to\infty}\left\{\frac{1}{kn}\cdot\frac{2}{kn}\cdots\frac{n}{kn}\right\}^{1/n}$$

$$\Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\left\{\log\frac{1}{kn}+\log\frac{2}{kn}+\cdots+\log\frac{n}{kn}\right\}$$

$$\Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{kn}\right)$$

$$\Rightarrow \log A=\int_0^1 \log\left(\frac{x}{k}\right)dx$$

$$\Rightarrow \log A=\left[x\log\frac{x}{k}-x\right]_0^1$$

$$\Rightarrow \log A=\log\left(\frac{1}{k}\right)-\log e \Rightarrow A=\frac{1}{ke}$$

953 **(d)**

Put $10^x+x^{10}=t$

$\therefore (10^x\log_e 10+10x^9)dx=dt$

$$\therefore \int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}\,dx=\int\frac{dt}{t}$$

$=\log_e t+c$

$=\log_e(10^x+x^{10})+c$

954 **(c)**

Let $I=\int e^{\sin\theta}(\log\sin\theta)\cos\theta\, d\theta+$
$\int e^{\sin\theta}\operatorname{cosec}^2\theta\cos\theta\;d\theta$

Put $\sin\theta=t \;\Rightarrow \cos\theta\;d\theta=dt$

$$\therefore\; I=\int e^t\log t\;dt+\int e^t\,t^{-2}\,dt$$

$$=\log t\;e^t-\int\frac{e^t}{t}\,dt+\frac{e^t t^{-1}}{-1}-\int\frac{e^t t^{-1}}{-1}\,dt$$

$$=e^t\left(\log t-\frac{1}{t}\right)+c$$

$$=e^{\sin\theta}(\log\sin\theta-\operatorname{cosec}\theta)+c$$

955 **(b)**

We have,

$$f(x)=\int\frac{1}{(1+x^2)^{3/2}}\,dx$$

$$\Rightarrow f(x)=\int\cos\theta\;d\theta\,, \text{where } x=\tan\theta$$

$$\Rightarrow f(x)=\sin\theta+C$$

$$\Rightarrow f(x)=\frac{x}{\sqrt{x^2+1}}+C$$

Now, $f(0)=0\Rightarrow C=0$

$$\therefore f(x)=\frac{x}{\sqrt{x^2+1}}\Rightarrow f(1)=\frac{1}{\sqrt{2}}$$