

MATHS (QUESTION BANK)**INVERSE TRIGONOMETRICE FUNCTIONS**

Single Correct Answer Type

- If $[\sin^{-1} \cos^{-1} \sin^{-1} x] = 1$, where $[.]$ denotes the greatest integer function, then x belongs to the interval
 - $[\tan \sin \cos 1, \tan \sin \cos \sin 1]$
 - $(\tan \sin \cos 1, \tan \sin \cos \sin 1)$
 - $[-1, 1]$
 - $[\sin \cos \tan 1, \sin \cos \sin \tan 1]$
- $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$ is equal to
 - 1
 - 5
 - 10
 - 15
- If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then the value of x is
 - $\frac{3\pi}{4}$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{3}$
 - None of these
- If $\tan^{-1} \frac{x-1}{x+2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then x is equal to
 - $\frac{1}{\sqrt{2}}$
 - $-\frac{1}{\sqrt{2}}$
 - $\pm \sqrt{\frac{5}{2}}$
 - $\pm \frac{1}{2}$
- $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$ is equal to
 - $2 \sin^{-1} \frac{x}{a}$
 - $\sin^{-1} \frac{2x}{a}$
 - $\sin^{-1} \frac{x}{a}$
 - $\cos^{-1} \frac{x}{a}$
- The sum of the infinite series

$$\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) + \sin^{-1} \left(\frac{\sqrt{2}-1}{\sqrt{6}} \right) + \sin^{-1} \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}} \right) + \dots$$

$$+ \dots + \sin^{-1} \left(\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{\{n(n+1)\}}} \right) + \dots$$
 is
 - $\frac{\pi}{8}$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{2}$
 - π
- If $\theta_1 = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{1}{3}$ and $\theta_2 = \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{1}{3}$, then
 - $\theta_1 > \theta_2$
 - $\theta_1 = \theta_2$
 - $\theta_1 < \theta_2$
 - None of these
- If $\cos^{-1} x > \sin^{-1} x$, then
 - $x < 0$
 - $-1 < x < 0$
 - $0 \leq x < \frac{1}{\sqrt{2}}$
 - $-1 \leq x < \frac{1}{\sqrt{2}}$
- If $e^{[\sin^2 \alpha + \sin^4 \alpha + \sin^6 \alpha + \dots] \log_e 2}$ is a root of equation $x^2 - 9x + 8 = 0$, where $0 < \alpha < \frac{\pi}{2}$, then the principle value of $\sin^{-1} \sin \left(\frac{2\pi}{3} \right)$ is
 - α
 - 2α
 - $-\alpha$
 - -2α
- If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z + \cos^{-1} t = 4\pi$, then the value of $x^2 + y^2 + z^2 + t^2$ is
 - $xy + zy + zt$
 - $1 - 2xyzt$
 - 4
 - 6
- Sum of infinite terms of the series

$$\cot^{-1} \left(1^2 + \frac{3}{4} \right) + \cot^{-1} \left(2^2 + \frac{3}{4} \right) + \cot^{-1} \left(3^2 + \frac{3}{4} \right) + \dots$$
 is
 - $\frac{\pi}{4}$
 - $\tan^{-1}(2)$
 - $\tan^{-1} 3$
 - None of these
- If $\sin^{-1} \alpha + \sin^{-1} \beta + \sin^{-1} \gamma = \frac{3\pi}{2}$, then $\alpha\beta + \alpha\gamma + \beta\gamma$ is equal to
 - 1
 - 0
 - 3
 - 3

13. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to
 a) π b) $\frac{5\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these
14. If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = 3\pi$, then $p^2 + q^2 + r^2 + 2pqr$ is equal to
 a) 3 b) 1 c) 2 d) -1
15. If $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x \geq 0$, then the smallest interval in which θ lies, is given by
 a) $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ b) $-\frac{\pi}{4} \leq \theta \leq 0$ c) $0 \leq \theta \leq \frac{\pi}{4}$ d) $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$
16. $\cot\left\{\cos^{-1}\left(\frac{7}{25}\right)\right\} =$
 a) $\frac{25}{24}$ b) $\frac{25}{7}$ c) $\frac{24}{25}$ d) None of these
17. If $x \in (1, \infty)$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
 a) $2 \tan^{-1} x$ b) $\pi - 2 \tan^{-1} x$ c) $-\pi - 2 \tan^{-1} x$ d) None of these
18. $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is equal to
 a) $\sqrt{\frac{x^2+2}{x^2+3}}$ b) $\sqrt{\frac{x^2+2}{x^2+1}}$ c) $\sqrt{\frac{x^2+1}{x^2+2}}$ d) None of these
19. The value of $\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$ is
 a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$
20. The solution set of the equation $\tan^{-1} x - \cot^{-1} x = \cos^{-1}(2-x)$ is
 a) $[0,1]$ b) $[-1,1]$ c) $[1,3]$ d) None of these
21. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to
 a) π b) $\frac{5\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these
22. If a, b are positive quantities and if $a_1 = \frac{a+b}{2}, b_1 = \sqrt{a_1 b}, a_2 = \frac{a_1+b_1}{2}, b_2 = \sqrt{a_2 b_1}$ and so on, then
 a) $a_\infty = \frac{\sqrt{b^2 - a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$ b) $b_\infty = \frac{\sqrt{b^2 - a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$ c) $b_\infty = \frac{\sqrt{a^2 + b^2}}{\cos^{-1}\left(\frac{b}{a}\right)}$ d) None of these
23. The value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is
 a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $-\frac{1}{2}$
24. The value of $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}]$, is
 a) $\sqrt{\frac{x^2+2}{x^2+1}}$ b) $\sqrt{\frac{x^2+1}{x^2+2}}$ c) $\frac{x}{\sqrt{x^2+2}}$ d) $\frac{1}{\sqrt{x^2+2}}$
25. $\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \dots$ upto ∞ is equal to
 a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{5}$
26. If $x \in (1, \infty)$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals
 a) $2 \tan^{-1} x$ b) $-\pi + 2 \tan^{-1} x$ c) $\pi + 2 \tan^{-1} x$ d) None of these
27. The value of $\cos(2 \cos^{-1} x + \sin^{-1} x)$ at $x = \frac{1}{5}$ is
 a) 1 b) 3 c) 0 d) $-\frac{2\sqrt{6}}{5}$
28. The value of $\cot^{-1} \frac{xy+1}{x-y} + \cot^{-1} \frac{yz+1}{y-z} + \cot^{-1} \frac{zx+1}{z-x}$ is
 a) 0 b) 1 c) $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z$ d) None of the above

29. If we consider only the principle value of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is
- a) $\sqrt{\frac{29}{3}}$ b) $\frac{29}{3}$ c) $\sqrt{\frac{3}{29}}$ d) $\frac{3}{29}$
30. The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$, is
- a) 1 b) 0 c) $\frac{7}{17}$ d) $-\frac{7}{17}$
31. If in a ΔABC , $\angle A = \tan^{-1} 2$ and $\angle B = \tan^{-1} 3$, then angle C is equal to
- a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) None of these
32. $\cos^{-1}\left\{\frac{1}{2}x^2 + \sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$ holds for
- a) $|x| \leq 1$ b) $x \in R$ c) $0 \leq x \leq 1$ d) $-1 \leq x \leq 0$
33. If θ and ϕ are the roots of the equation $8x^2 + 22x + 5 = 0$, then
- a) Both $\sin^{-1}\theta$ and $\sin^{-1}\phi$ are equal b) Both $\sec^{-1}\theta$ and $\sec^{-1}\phi$ are equal
c) Both $\tan^{-1}\theta$ and $\tan^{-1}\phi$ are equal d) None of the above
34. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x is equal to
- a) 0 b) 2 c) 1 d) -1
35. $\sum_{m=1}^n \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to
- a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ d) None of these
36. If we consider only the principle value of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is
- a) $\sqrt{\frac{29}{3}}$ b) $\frac{29}{3}$ c) $\sqrt{\frac{3}{29}}$ d) $\frac{3}{29}$
37. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is
- a) 0 b) 1 c) 2 d) 3
38. If $y = \cos^{-1}(\cos 10)$, then y is equal to
- a) 10 b) $4\pi - 10$ c) $2\pi + 10$ d) $2\pi - 10$
39. If $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$, then value of x is
- a) a b) b c) $\frac{a+b}{1-ab}$ d) $\frac{a-b}{1+ab}$
40. The value of $\sum_{r=0}^{\infty} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$ is equal to
- a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) None of these
41. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$ is equal to
- a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$
42. $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$, is
- a) $\pi/4$ b) $\pi/2$ c) π d) 0
43. If $\sec^{-1}\sqrt{1+x^2} + \operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y} + \cot^{-1}\frac{1}{z} = \pi$, then $x + y + z$ is equal to
- a) xyz b) $2xyz$ c) xyz^2 d) x^2yz
44. If $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$, then x is

a) $\left(0, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$

b) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, 0\right)$

c) $(-2, 2)$

d) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$

62. If x takes negative permissible value, then $\sin^{-1} x$ is equal to

a) $-\cos^{-1} \sqrt{1 - x^2}$

b) $\cos^{-1} \sqrt{x^2 - 1}$

c) $\pi - \cos^{-1} \sqrt{1 - x^2}$

d) $\cos^{-1} \sqrt{1 - x^2}$

63. The value of $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{7}{8}$ is

a) $\tan^{-1} \frac{7}{8}$

b) $\cot^{-1} 15$

c) $\tan^{-1} 15$

d) $\tan^{-1} \frac{25}{24}$

64. The smallest and the largest values of $\tan^{-1} \left(\frac{1-x}{1+x}\right)$, $0 \leq x \leq 1$ are

a) $0, \pi$

b) $0, \frac{\pi}{4}$

c) $-\frac{\pi}{4}, \frac{\pi}{4}$

d) $\frac{\pi}{4}, \frac{\pi}{2}$

65. If $\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = \tan^{-1} \frac{2x}{1-x^2}$, then the value of x is

a) a

b) b

c) $\frac{a+b}{1-ab}$

d) $\frac{a-b}{1+ab}$

66. Number of solutions of the equation

$\tan^{-1} \left(\frac{1}{2x+1}\right) + \tan^{-1} \left(\frac{1}{4x+1}\right) = \tan^{-1} \left(\frac{2}{x^2}\right)$ is

a) 1

b) 2

c) 3

d) 4

67. If $-\frac{1}{2} \leq x \leq \frac{1}{2}$, then $\sin^{-1}(3x - 4x^3)$ equals

a) $3 \sin^{-1} x$

b) $\pi - 3 \sin^{-1} x$

c) $-\pi - 3 \sin^{-1} x$

d) None of these

68. If $x \in (-\infty, -1)$, then $\sin^{-1} \left(\frac{2x}{1+x^2}\right)$ equals

a) $2 \tan^{-1} x$

b) $\pi - 2 \tan^{-1} x$

c) $-\pi - 2 \tan^{-1} x$

d) None of these

69. The sum of the infinite series

$\sin^{-1} \left(\frac{1}{\sqrt{2}}\right) + \sin^{-1} \left(\frac{\sqrt{2}-1}{\sqrt{6}}\right) + \sin^{-1} \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}\right) + \dots$

$+ \dots + \sin^{-1} \left(\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{n(n+1)}}\right) + \dots$ is

a) $\frac{\pi}{8}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) π

70. If $\cos^{-1} x = \alpha$, ($0 < x < 1$) and

$\sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1} \left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$, then $\tan^{-1}(2x)$ equals

a) $\pi/6$

b) $\pi/4$

c) $\pi/3$

d) $\pi/2$

71. If $\alpha = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{1}{3}$ and $\beta = \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{1}{3}$, then

a) $\alpha < \beta$

b) $\alpha = \beta$

c) $\alpha > \beta$

d) None of these

72. If $\theta = \tan^{-1} a$, $\phi = \tan^{-1} b$ and $ab = -1$, then $(\theta - \phi)$ is equal to

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) None of these

73. If $\tan \theta + \tan \left(\frac{\pi}{3} + \theta\right) + \tan \left(-\frac{\pi}{3} + \theta\right) = a \tan 3\theta$, then a is equal to

a) $1/3$

b) 1

c) 3

d) None of these

74. If the $(\cos^{-1} x) = \sin \left(\cot^{-1} \frac{1}{2}\right)$, then x is equal to

a) $\pm \frac{5}{3}$

b) $\pm \frac{\sqrt{5}}{3}$

c) $\pm \frac{5}{\sqrt{3}}$

d) None of these

75. If $\sin^{-1} = \frac{\pi}{5}$, for some $x \in (-1, 1)$, then the value of $\cos^{-1} x$ is

a) $\frac{3\pi}{10}$

b) $\frac{5\pi}{10}$

c) $\frac{7\pi}{10}$

d) $\frac{9\pi}{10}$

76. If $\frac{1}{2} \leq x \leq 1$, then $\sin^{-1}(3x - 4x^3)$ equals
 a) $3 \sin^{-1} x$ b) $\pi - 3 \sin^{-1} x$ c) $-\pi - 3 \sin^{-1} x$ d) None of these
77. $\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} =$
 a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) 0
78. If x, y, z are in AP and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in AP, then
 a) $x = y = z$ b) $x = y = -z$ c) $x = 1, y = 2, z = 3$ d) $x = 2, y = 4, z = 6$
79. If $a_1, a_2, a_3, \dots, a_n$ are in AP with common difference 5 and if $a_i a_j \neq -1$ for $i, j = 1, 2, \dots, n$ then
 $\tan^{-1} \left(\frac{5}{1+a_1 a_2} \right) + \tan^{-1} \left(\frac{5}{1+a_2 a_3} \right) + \dots + \tan^{-1} \left(\frac{5}{1+a_{n-1} a_n} \right)$ is equal to
 a) $\tan^{-1} \left(\frac{5}{1+a_n a_{n-1}} \right)$ b) $\tan^{-1} \left(\frac{5a_1}{1+a_n a_1} \right)$ c) $\tan^{-1} \left(\frac{5n-5}{1+a_n a_1} \right)$ d) $\tan^{-1} \left(\frac{5n-5}{1+a_1 a_{n+1}} \right)$
80. The relation $\tan^{-1} \left(\frac{1+x}{1-x} \right) = \frac{\pi}{4} + \tan^{-1} x$ holds true for all
 a) $x \in R$ b) $x \in (-\infty, 1)$ c) $x \in (-1, \infty)$ d) $x \in (-\infty, -1)$
81. If $A = \tan^{-1} \left(\frac{x\sqrt{3}}{2k-x} \right)$ and $B = \tan^{-1} \left(\frac{2x-k}{k\sqrt{3}} \right)$, then the value of $A - B$ is
 a) 10° b) 45° c) 60° d) 30°
82. If $0 < x < 1$, then
 $\sqrt{1+x^2} [\{ x \cos(\cot^{-1} x) + \sin(\cot^{-1} x) \}^2 - 1]^{1/2}$ is equal to
 a) $\frac{x}{\sqrt{1+x^2}}$ b) x c) $x\sqrt{1+x^2}$ d) $\sqrt{1+x^2}$
83. If $\sin^{-1} \left(\frac{x}{5} \right) + \operatorname{cosec}^{-1} \left(\frac{5}{4} \right) = \frac{\pi}{2}$, then value of x is
 a) 1 b) 3 c) 4 d) 5
84. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ and $f(1) = 2$,
 $f(p+q) = f(p) \cdot f(q), \forall p, q \in R$, then
 $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{(x+y+z)}{x^{f(1)+y^{f(2)}+z^{f(3)}}$ is equal to
 a) 0 b) 1 c) 2 d) 3
85. If $A = 2 \tan^{-1}(2\sqrt{2} - 1)$ and $B = 3 \sin^{-1} \frac{1}{3} + \sin^{-1} \frac{3}{5}$, then
 a) $A = B$ b) $A < B$ c) $A > B$ d) None of these
86. If $\tan^{-1}(x+2) + \tan^{-1}(x-2) - \tan^{-1} \left(\frac{1}{2} \right) = 0$, then one of the values of x is equal to
 a) -1 b) 5 c) $\frac{1}{2}$ d) 1
87. $\cos \left[\cos^{-1} \left(-\frac{1}{7} \right) + \sin^{-1} \left(-\frac{1}{7} \right) \right]$ is equal to
 a) $-\frac{1}{3}$ b) 0 c) $\frac{1}{3}$ d) $\frac{4}{9}$
88. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is
 a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) None of these
89. The value of $\sec \left[\tan^{-1} \left(\frac{b+a}{b-a} \right) - \tan^{-1} \left(\frac{a}{b} \right) \right]$ is
 a) 2 b) $\sqrt{2}$ c) 4 d) 1
90. Which one of following is true?
 a) $\sin(\cos^{-1} x) = \cos(\sin^{-1} x)$ b) $\sec(\tan^{-1} x) = \tan(\sec^{-1} x)$
 c) $\cos(\tan^{-1} x) = \tan(\cos^{-1} x)$ d) $\tan(\sin^{-1} x) = \sin(\tan^{-1} x)$
91. If $a > b > 0$, then the value of $\tan^{-1} \left(\frac{a}{b} \right) + \tan^{-1} \left(\frac{a+b}{a-b} \right)$ depends on
 a) Both a and b b) b and not a c) a and not b d) Neither a nor b

92. If $x \geq 1$, then $2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right)$ is equal to
 a) $4 \tan^{-1} x$ b) 0 c) $\pi/2$ d) π
93. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then
 a) $x^2 + y^2 = z^2$ b) $x^2 + y^2 + z^2 = 0$
 c) $x^2 + y^2 + z^2 = 1 - 2xyz$ d) None of the above
94. If $x > \frac{1}{\sqrt{3}}$, then $\tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$ equals
 a) $3 \tan^{-1} x$ b) $-\pi + 3 \tan^{-1} x$ c) $\pi + 3 \tan^{-1} x$ d) None of these
95. If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then the value of x is
 a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these
96. The value of $\cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13}$ is
 a) $\sin^{-1} \frac{63}{65}$ b) $\sin^{-1} \frac{12}{13}$ c) $\sin^{-1} \frac{65}{68}$ d) $\sin^{-1} \frac{5}{12}$
97. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$ and $\tan^{-1} x - \tan^{-1} y = 0$, then $x^2 + xy + y^2$ is equal to
 a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{3}{2}$ d) $\frac{1}{8}$
98. The number of real solution of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{2}$ is
 a) 0 b) 1 c) 2 d) ∞
99. If $x + y + z = xyz$, then $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z =$
 a) 0 b) $\pi/2$ c) π d) None of these
100. The number of positive integral solutions of the equation $\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$ is
 a) One b) Two c) Zero d) None of these
101. $\sin^{-1} \left(\frac{3}{5} \right) + \tan^{-1} \left(\frac{1}{7} \right) =$
 a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\cos^{-1} \left(\frac{4}{5} \right)$ d) π
102. If $xy + yz + zx = 1$, then $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z =$
 a) π b) $\pi/2$ c) π d) none of these
103. If $x^2 + y^2 + z^2 = r^2$, then
 $\tan^{-1} \left(\frac{xy}{zr} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) + \tan^{-1} \left(\frac{xz}{yr} \right)$ is equal to
 a) π b) $\frac{\pi}{2}$ c) 0 d) None of these
104. If $f(x) = \sin^{-1} \left\{ \frac{\sqrt{3}}{2} x - \frac{1}{2} \sqrt{1-x^2} \right\}$, $-\frac{1}{2} \leq x \leq 1$, then $f(x)$ is equal to
 a) $\sin^{-1} \frac{1}{2} - \sin^{-1} x$ b) $\sin^{-1} x - \frac{\pi}{6}$ c) $\sin^{-1} x + \frac{\pi}{6}$ d) None of these
105. $\cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right)$ is equal to
 a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$
106. The solution of $\tan^{-1} 2\theta + \tan^{-1} 3\theta = \frac{\pi}{4}$ is
 a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{\sqrt{6}}$
107. The value of $\cos^{-1} \left(-\frac{1}{2} \right)$ among the following, is
 a) $\frac{9\pi}{3}$ b) $\frac{8\pi}{3}$ c) $\frac{5\pi}{3}$ d) $\frac{11\pi}{3}$
108. If $\tan \theta + \tan \left(\frac{\pi}{3} + \theta \right) + \tan \left(-\frac{\pi}{3} + \theta \right) = a \tan 3\theta$, then a is equal to
 a) $1/3$ b) 1 c) 3 d) None of these

109. The value of $\cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13}$ is
 a) $\sin^{-1} \frac{63}{65}$ b) $\sin^{-1} \frac{12}{13}$ c) $\sin^{-1} \frac{65}{68}$ d) $\sin^{-1} \frac{5}{12}$
110. The value of x for which $\cos^{-1}(\cos 4) > 3x^2 - 4x$ is
 a) $\left(0, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$ b) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, 0\right)$
 c) $(-2, 2)$ d) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$
111. If $x \in (-\infty, 1)$, then $\tan^{-1} \left(\frac{2x}{1-x^2}\right)$ equals
 a) $2 \tan^{-1} x$ b) $-\pi + 2 \tan^{-1} x$ c) $\pi + 2 \tan^{-1} x$ d) None of these
112. If $\frac{1}{\sqrt{2}} \leq x \leq 1$, then $\sin^{-1}(2x\sqrt{1-x^2})$ equals
 a) $2 \sin^{-1} x$ b) $\pi - 2 \sin^{-1} x$ c) $-\pi - 2 \sin^{-1} x$ d) None of these
113. $\frac{\alpha^3}{2} \operatorname{cosec}^2 \left(\frac{1}{2} \tan^{-1} \frac{\alpha}{\beta}\right) + \frac{\beta^3}{2} \sec^2 \left(\frac{1}{2} \tan^{-1} \left(\frac{\beta}{\alpha}\right)\right)$ is
 a) $(\alpha - \beta)(\alpha^2 + \beta^2)$ b) $(\alpha + \beta)(\alpha^2 - \beta^2)$
 c) $(\alpha + \beta)(\alpha^2 + \beta^2)$ d) None of these
114. If $\sum_{i=1}^{20} \sin^{-1} x_i = 10\pi$, then $\sum_{i=1}^{20} x_i$ is equal to
 a) 20 b) 10 c) 0 d) None of these
115. Which one of the following is correct?
 a) $\tan 1 > \tan^{-1} 1$ b) $\tan 1 < \tan^{-1} 1$ c) $\tan 1 = \tan^{-1} 1$ d) None of these
116. If $\alpha = \sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{3}$ and $\beta = \cos^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{1}{3}$, then
 a) $\alpha > \beta$ b) $\alpha = \beta$ c) $\alpha < \beta$ d) $\alpha + \beta = 2\pi$
117. $2 \tan^{-1} \left(\frac{1}{3}\right) + \tan^{-1} \left(\frac{1}{7}\right)$ is equal to
 a) $\left(\frac{49}{29}\right)$ b) $\frac{\pi}{2}$ c) $-\left(\frac{49}{29}\right)$ d) $\frac{\pi}{4}$
118. $\tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2a}{1+a^2}\right) + \frac{1}{2} \cos^{-1} \left(\frac{1-a^2}{1+a^2}\right)\right]$ is equal to
 a) $\frac{2a}{1+a^2}$ b) $\frac{1-a^2}{1+a^2}$ c) $\frac{2a}{1-a^2}$ d) None of these
119. The sum of the infinite series $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$ is
 a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) None of these
120. If $\tan^{-1} \left(\frac{a}{x}\right) + \tan^{-1} \left(\frac{b}{x}\right) = \frac{\pi}{2}$, then x is equal to
 a) \sqrt{ab} b) $\sqrt{2ab}$ c) $2ab$ d) ab
121. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta - x \cos \beta - \sin \beta = 0$, then $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$ is equal to
 a) β b) $\frac{\pi}{2} - \beta$ c) $\pi - \beta$ d) $-\beta$
122. If $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the value of $\tan^{-1} \left(\frac{\tan x}{4}\right) + \tan^{-1} \left(\frac{3 \sin 2x}{5+3 \cos 2x}\right)$ is
 a) $\frac{x}{2}$ b) $2x$ c) $3x$ d) x
123. $\frac{\alpha^3}{2} \operatorname{cosec}^2 \left(\frac{1}{2} \tan^{-1} \frac{\alpha}{\beta}\right) + \frac{\beta^3}{2} \sec^2 \left(\frac{1}{2} \tan^{-1} \left(\frac{\beta}{\alpha}\right)\right)$ is
 a) $(\alpha - \beta)(\alpha^2 + \beta^2)$ b) $(\alpha + \beta)(\alpha^2 - \beta^2)$ c) $(\alpha + \beta)(\alpha^2 + \beta^2)$ d) None of these
124. If $-1 \leq x \leq 0$, then $\cos^{-1}(2x^2 - 1)$ equals
 a) $2 \cos^{-1} x$ b) $\pi - 2 \cos^{-1} x$ c) $2\pi - 2 \cos^{-1} x$ d) $-2 \cos^{-1} x$

- a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
175. If θ and ϕ are the roots of the equation $8x^2 + 22x + 5 = 0$, then
a) Both $\sin^{-1} \theta$ and $\sin^{-1} \phi$ are equal b) Both $\sec^{-1} \theta$ and $\sec^{-1} \phi$ are equal
c) Both $\tan^{-1} \theta$ and $\tan^{-1} \phi$ are equal d) None of the above
176. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta - x \cos \beta - \sin \beta = 0$, then $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$ is equal to
a) β b) $\frac{\pi}{2} - \beta$ c) $\pi - \beta$ d) $-\beta$
177. The value of $\cos(2 \cos^{-1} x + \sin^{-1} x)$ at $x = \frac{1}{5}$ is
a) 1 b) 3 c) 0 d) $-\frac{2\sqrt{6}}{5}$
178. Solution of the equation $\cot^{-1} x + \sin^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{4}$ is
a) $x = 3$ b) $x = \frac{1}{\sqrt{5}}$ c) $x = 0$ d) None of these
179. Let $\cos(2 \tan^{-1} x) = \frac{1}{2}$, then the value of x is
a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) $1 - \sqrt{3}$ d) $1 - \frac{1}{\sqrt{3}}$
180. If $\sin^{-1} \frac{1}{3} + \sin^{-1} \frac{2}{3} = \sin^{-1} x$, then the value of x is
a) 0 b) $\frac{(\sqrt{5} - 4\sqrt{2})}{9}$ c) $\frac{(\sqrt{5} + 4\sqrt{2})}{9}$ d) $\frac{\pi}{2}$
181. The solution set of the equation $\tan^{-1} x - \cot^{-1} x = \cos^{-1}(2 - x)$ is
a) $[0, 1]$ b) $[-1, 1]$ c) $[1, 3]$ d) None of these
182. $\cos^{-1} \left\{ \frac{1}{2}x^2 + \sqrt{1-x^2} \sqrt{1-\frac{x^2}{4}} \right\} = \cos^{-1} \frac{x}{2} - \cos^{-1} x$ holds for
a) $|x| \leq 1$ b) $x \in R$ c) $0 \leq x \leq 1$ d) $-1 \leq x \leq 0$
183. The solutions of the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$ are
a) $-\frac{1}{4}, 8$ b) $\frac{1}{4}, -8$ c) $-4, \frac{1}{8}$ d) $4, -\frac{1}{8}$
184. If $3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1+x}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$, then value of x is
a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) None of these
185. If $x^2 + y^2 + z^2 = r^2$, then
 $\tan^{-1} \left(\frac{xy}{zr} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) + \tan^{-1} \left(\frac{xz}{yr} \right)$ is equal to
a) π b) $\frac{\pi}{2}$ c) 0 d) None of these
186. The greatest and the least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are respectively
a) $-\frac{\pi}{2}, \frac{\pi}{2}$ b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$ c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ d) None of these
187. For the principle value branch of the graph of the function $y = \sin^{-1} x, -1 \leq x \leq 1$, which among the following is a true statement?
a) Graph is symmetric about the x -axis b) Graph is symmetric about the y -axis
c) Graph is not continuous d) The line $x = 1$ is a tangent
188. If $-1 \leq x \leq -\frac{1}{\sqrt{2}}$, then $\sin^{-1}(2x\sqrt{1-x^2})$ equals
a) $2 \sin^{-1} x$ b) $\pi - 2 \sin^{-1} x$ c) $-\pi - 2 \sin^{-1} x$ d) None of these
189. If a, b, c be positive real number and the value of

206. $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$ is equal to
 a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$
207. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is
 a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-1}{2}$ d) None of these
208. If the mapping $f(x) = ax + b, a > 0$ maps $[-1, 1]$ onto $[0, 2]$ then $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$ is equal to
 a) $f(-1)$ b) $f(0)$ c) $f(1)$ d) $f(2)$
209. The value of $\sin^{-1} \left(\cos \frac{33\pi}{5} \right)$ is
 a) $\frac{3\pi}{5}$ b) $\frac{7\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$
210. For the equation $\cos^{-1} x + \cos^{-1} 2x + \pi = 0$, then the number of real solutions is
 a) 1 b) 2 c) 0 d) ∞
211. The value of $\tan \left\{ \frac{1}{2} \cos^{-1} \left(\frac{\sqrt{5}}{3} \right) \right\}$, is
 a) $\frac{3 + \sqrt{5}}{2}$ b) $3 + \sqrt{5}$ c) $\frac{1}{2}(3 - \sqrt{5})$ d) None of these
212. The value of $\sin \left[2 \cos^{-1} \frac{\sqrt{5}}{3} \right]$ is
 a) $\frac{\sqrt{5}}{3}$ b) $\frac{2\sqrt{5}}{3}$ c) $\frac{4\sqrt{5}}{9}$ d) $\frac{2\sqrt{5}}{9}$
213. If $x > -\frac{1}{\sqrt{3}}$, then $\tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$ equals
 a) $3 \tan^{-1} x$ b) $-\pi + 3 \tan^{-1} x$ c) $\pi + 3 \tan^{-1} x$ d) None of these
214. $\cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right)$ is equal to
 a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$
215. If $\sin \left(\sin^{-1} \frac{1}{5} + \cos^{-1} x \right) = 1$, then the value of x is
 a) -1 b) $\frac{2}{5}$ c) $\frac{1}{3}$ d) $\frac{1}{5}$
216. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx$ is equal to
 a) 0 b) 1 c) 3 d) -3
217. If $0 \leq x < \infty$, then $\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$ equals
 a) $2 \tan^{-1} x$ b) $-2 \tan^{-1} x$ c) $\pi - 2 \tan^{-1} x$ d) $\pi + 2 \tan^{-1} x$
218. The value of $\cos[2 \tan^{-1}(-7)]$ is
 a) $\frac{49}{50}$ b) $-\frac{49}{50}$ c) $\frac{24}{25}$ d) $-\frac{24}{25}$
219. The value of $\sin \left(4 \tan^{-1} \frac{1}{3} \right) - \cos \left(2 \tan^{-1} \frac{1}{7} \right)$ is
 a) $\frac{3}{7}$ b) $\frac{7}{8}$ c) $\frac{8}{21}$ d) None of these
220. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx$ is equal to
 a) 0 b) 1 c) 3 d) -3
221. If $a_1, a_2, a_3, \dots, a_n$ are in AP with common ratio d , then
 $\tan \left[\tan^{-1} \frac{d}{1+a_1 a_2} + \tan^{-1} \frac{d}{1+a_2 a_3} + \dots + \tan^{-1} \frac{d}{1+a_{n-1} a_n} \right]$ is equal to
 a) $\frac{(n-1)d}{a_1 + a_n}$ b) $\frac{(n-1)d}{1 + a_1 a_n}$ c) $\frac{nd}{1 + a_1 a_n}$ d) $\frac{a_n - a_1}{a_n + a_1}$
222. $\sin \left(2 \sin^{-1} \sqrt{\frac{63}{65}} \right)$ is equal to

- a) $\frac{2\sqrt{126}}{65}$ b) $\frac{4\sqrt{65}}{65}$ c) $\frac{8\sqrt{63}}{65}$ d) $\frac{\sqrt{63}}{65}$
223. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is
a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) None of these
224. If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 4 \tan^{-1} x$, then
a) $x \in -(-\infty, -1)$ b) $x \in (1, \infty)$ c) $x \in [0, 1]$ d) $x \in [-1, 0]$
225. $\tan^{-1} \frac{c_1 x - y}{c_1 y + x} + \tan^{-1} \frac{c_2 - c_1}{1 + c_2 c_1} + \tan^{-1} \frac{c_3 - c_2}{1 + c_3 c_2} + \dots + \tan^{-1} \frac{1}{c_n}$ is equal to
a) $\tan^{-1} \frac{y}{x}$ b) $\tan^{-1} yx$ c) $\tan^{-1} \frac{x}{y}$ d) $\tan^{-1}(x - y)$
226. If $\tan^{-1} a + \tan^{-1} b = \sin^{-1} 1 - \tan^{-1} c$, then
a) $a + b + c = abc$
b) $ab + bc + ca = abc$
c) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{1}{abc} = 0$
d) $ab + bc + ca = a + b + c$
227. The value of $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is
a) $\sqrt{\frac{x^2 + 1}{x^2 - 1}}$ b) $\sqrt{\frac{1 - x^2}{x^2 + 2}}$ c) $\sqrt{\frac{1 - x^2}{1 + x^2}}$ d) $\sqrt{\frac{x^2 + 1}{x^2 + 2}}$
228. If $[\cot^{-1} x] + [\cos^{-1} x] = 0$, where x is a non-negative real number and $[.]$ denotes the greatest integer function, then complete set of values of x is
a) $(\cos 1, 1]$ b) $(\cot 1, 1)$ c) $(\cos 1, \cot 1)$ d) None of these
229. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx$ is equal to
a) 1 b) 0 c) -3 d) 3
230. A solution of the equation $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$, is
a) $x = 1$ b) $x = -1$ c) $x = 0$ d) $x = \pi$
231. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x\right)$, $x \neq 0$ is equal to
a) x b) $2x$ c) $\frac{2}{x}$ d) None of these
232. The equation $2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ has
a) No solution b) Only one solution c) Two solutions d) Three solutions
233. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$, is
a) 0 b) 1 c) 2 d) 3
234. If $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$, then x is equal to
a) 1 b) 0 c) $\frac{4}{5}$ d) $\frac{1}{5}$
235. If the mapping $f(x) = ax + b$, $a > 0$ maps $[-1, 1]$ onto $[0, 2]$ then $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$ is equal to
a) $f(-1)$ b) $f(0)$ c) $f(1)$ d) $f(2)$
236. If $\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = \tan^{-1} \frac{2x}{1-x^2}$, then value of x is
a) a b) b c) $\frac{a+b}{1-ab}$ d) $\frac{a-b}{1+ab}$
237. The sum of the two angles $\cot^{-1} 3$ and $\operatorname{cosec}^{-1} \sqrt{5}$, is
a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
238. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$ is equal to

- a) $\frac{2a}{1+a^2}$ b) $\frac{1-a^2}{1+a^2}$ c) $\frac{2a}{1-a^2}$ d) None of these
239. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then
a) $x + y + xy = 1$ b) $x + y - xy = 1$
c) $x + y + xy + 1 = 0$ d) $x + y - xy + 1 = 0$
240. If $0 \leq x \leq 1$, then $\cos^{-1}(2x^2 - 1)$ equals
a) $2 \cos^{-1} x$ b) $\pi - 2 \cos^{-1} x$ c) $2\pi - 2 \cos^{-1} x$ d) None of these
241. If a, b, c be positive real number and the value of

$$\theta = \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1} \sqrt{\frac{c(a+b+c)}{ab}}$$
Then $\tan \theta$ is equal to
a) 0 b) 1 c) $\frac{a+b+c}{abc}$ d) None of these
242. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to
a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) π d) $\frac{3\pi}{4}$
243. $\tan^{-1} \frac{c_1 x - y}{c_1 y + x} + \tan^{-1} \frac{c_2 - c_1}{1 + c_2 c_1} + \tan^{-1} \frac{c_3 - c_2}{1 + c_3 c_2} + \dots + \tan^{-1} \frac{1}{c_n}$ is equal to
a) $\tan^{-1} \frac{y}{x}$ b) $\tan^{-1} yx$ c) $\tan^{-1} \frac{x}{y}$ d) $\tan^{-1}(x - y)$
244. The value of $\cos\{\tan^{-1}(\tan 2)\}$, is
a) $\frac{1}{\sqrt{5}}$ b) $-\frac{1}{\sqrt{5}}$ c) $\cos 2$ d) $-\cos 2$
245. If $\tan^{-1} \frac{x-1}{x+2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then x is equal to
a) $\frac{1}{\sqrt{2}}$ b) $-\frac{1}{\sqrt{2}}$ c) $\pm \sqrt{\frac{5}{2}}$ d) $\pm \frac{1}{2}$
246. The sum of series

$$\tan^{-1} \frac{1}{1+1+1^2} + \tan^{-1} \frac{1}{1+2+2^2} + \tan^{-1} \frac{1}{1+3+3^2} + \dots$$
 ∞ is equal to
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$
247. The value of 'a' for which $ax^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has a real solution, is
a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$
248. If $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$, then A is equal to
a) $x - y$ b) $x + y$ c) $\frac{x - y}{1 + xy}$ d) $\frac{x + y}{1 - xy}$
249. If $\tan^{-1} \left(\frac{a}{x}\right) + \tan^{-1} \left(\frac{b}{x}\right) = \frac{\pi}{2}$, then x is equal to
a) \sqrt{ab} b) $\sqrt{2ab}$ c) $2ab$ d) ab
250. $\cos^{-1} \frac{1}{2} + 2 \sin^{-1} \frac{1}{2}$ is equal to
a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$
251. $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$ is equal to
a) π b) $\pi/2$ c) $\pi/3$ d) $\pi/4$
252. If $2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, then x is equal to
a) $[-1, 1]$ b) $\left[-\frac{1}{\sqrt{2}}, 1\right]$ c) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ d) None of these

253. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$, is
- a) $\frac{4}{17}$ b) $\frac{5}{17}$ c) $\frac{6}{17}$ d) $\frac{3}{17}$
254. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. Then, the third angle is
- a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$
255. If $e^{[\sin^2 \alpha + \sin^4 \alpha + \sin^6 \alpha + \dots] \log_e 2}$ is a root of equation $x^2 - 9x + 8 = 0$, where $0 < \alpha < \frac{\pi}{2}$, then the principle value of $\sin^{-1} \sin\left(\frac{2\pi}{3}\right)$ is
- a) α b) 2α c) $-\alpha$ d) -2α
256. If $\frac{1}{2} \leq x \leq 1$, then $\cos^{-1}(4x^3 - 3x)$ equals
- a) $3 \cos^{-1} x$ b) $2\pi - 3 \cos^{-1} x$ c) $-2\pi - 3 \cos^{-1} x$ d) None of these
257. If $\sin^{-1}(2x\sqrt{1-x^2}) - 2 \sin^{-1} x = 0$, then x belongs to the interval
- a) $[-1, 1]$ b) $[-1/\sqrt{2}, 1/\sqrt{2}]$ c) $[-1, -1/\sqrt{2}]$ d) $[1/\sqrt{2}, 1]$
258. Solution set of $[\sin^{-1} x] > [\cos^{-1} x]$, where $[.]$ denote the greatest integer function, is
- a) $\left[\frac{1}{\sqrt{2}}, 1\right]$ b) $(\cos 1, \sin 1)$ c) $[\sin 1, 1]$ d) None of these
259. If $[\sin^{-1} \cos^{-1} \sin^{-1} x] = 1$, where $[.]$ denotes the greatest integer function, then x belongs to the interval
- a) $[\tan \sin \cos 1, \tan \sin \cos \sin 1]$ b) $(\tan \sin \cos 1, \tan \sin \cos \sin 1)$
c) $[-1, 1]$ d) $[\sin \cos \tan 1, \sin \cos \sin \tan 1]$
260. The solution of $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$ is
- a) $-\frac{1}{\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$ c) $-\sqrt{3}$ d) $\sqrt{3}$
261. If $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the value of $\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3 \sin 2x}{5+3 \cos 2x}\right)$ is
- a) $\frac{x}{2}$ b) $2x$ c) $3x$ d) x
262. If $-\frac{1}{2} \leq x \leq \frac{1}{2}$, then $\sin^{-1}(3x - 4x^3)$ equals
- a) $3 \sin^{-1} x$ b) $\pi - 3 \sin^{-1} x$ c) $-\pi - 3 \sin^{-1} x$ d) None of these
263. If $\tan \theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(\frac{-\pi}{3} + \theta\right) = K \tan 3\theta$, then the value of K is
- a) 1 b) $1/3$ c) 3 d) none of these
264. If $-1 \leq x \leq 0$, then $\cos^{-1}(2x^2 - 1)$ equals
- a) $2 \cos^{-1} x$ b) $\pi - 2 \cos^{-1} x$ c) $2\pi - 2 \cos^{-1} x$ d) $-2 \cos^{-1} x$
265. If $\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3}$, $\beta = \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{3}$, then
- a) $\alpha > \beta$ b) $\alpha = \beta$ c) $\alpha < \beta$ d) $\alpha + \beta = 2\pi$
266. If $x \in [-1, 1]$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
- a) $2 \tan^{-1} x$ b) $\pi - 2 \tan^{-1} x$ c) $-\pi - 2 \tan^{-1} x$ d) None of these
267. $\sin\left[3 \sin^{-1}\left(\frac{1}{5}\right)\right]$ is equal to
- a) $\frac{71}{125}$ b) $\frac{74}{125}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$
268. If $\sum_{i=1}^{20} \sin^{-1} x_i = 10\pi$, then $\sum_{i=1}^{20} x_i$ is equal to
- a) 20 b) 10 c) 0 d) None of these
269. The value of x for which $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$ is
- a) $\frac{1}{2}$ b) 1 c) 0 d) $-\frac{1}{2}$
270. $\tan\left[\frac{\pi}{2} + \frac{1}{2} \cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1}\left(\frac{a}{b}\right)\right]$ is equal to

- a) $\frac{2a}{b}$ b) $\frac{2b}{a}$ c) $\frac{a}{b}$ d) $\frac{b}{a}$
271. $\tan^{-1} \frac{x}{y} - \tan^{-1} \frac{x-y}{x+y}$ is equal to
(where $x < y > 0$)
a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) None of these
272. The value of 'a' for which $ax^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has a real solution, is
a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$
273. $\cos^{-1}\left(\frac{-1}{2}\right) - 2 \sin^{-1}\left(\frac{1}{2}\right) + 3 \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - 4 \tan^{-1}(-1)$ equals
a) $\frac{19\pi}{12}$ b) $\frac{35\pi}{12}$ c) $\frac{47\pi}{12}$ d) $\frac{43\pi}{12}$
274. If $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$, $1 \leq x < \infty$, then the smallest interval in which θ lies is
a) $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ b) $0 \leq \theta \leq \frac{\pi}{4}$ c) $-\frac{\pi}{4} \leq \theta \leq 0$ d) $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$
275. If $4 \sin^{-1} x + \cos^{-1} x = \pi$, then x is equal to
a) 0 b) $1/2$ c) $-1/2$ d) 1
276. The value of $\sin^{-1}\left(\cos \frac{33\pi}{5}\right)$ is
a) $\frac{3\pi}{5}$ b) $\frac{7\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$
277. If $a_1, a_2, a_3, \dots, a_n$ are in AP with common ratio d , then
 $\tan\left[\tan^{-1} \frac{d}{1+a_1a_2} + \tan^{-1} \frac{d}{1+a_2a_3} + \dots + \tan^{-1} \frac{d}{1+a_{n-1}a_n}\right]$ is equal to
a) $\frac{(n-1)d}{a_1 + a_n}$ b) $\frac{(n-1)d}{1 + a_1a_n}$ c) $\frac{nd}{1 + a_1a_n}$ d) $\frac{a_n - a_1}{a_n + a_1}$
278. If $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then x is equal to
a) \sqrt{ab} b) $\sqrt{2ab}$ c) $2ab$ d) ab
279. If $A = \tan^{-1} x$, $x \in R$, then the value of $\sin 2A$ is
a) $\frac{2x}{1-x^2}$ b) $\frac{2x}{\sqrt{1-x^2}}$ c) $\frac{2x}{1+x^2}$ d) $\frac{1-x^2}{1+x^2}$
280. The value of x , where $x > 0$ and $\tan\left\{\sec^{-1}\left(\frac{1}{x}\right)\right\} = \sin(\tan^{-1} 2)$ is
a) $\sqrt{5}$ b) $\frac{\sqrt{5}}{3}$ c) 1 d) $\frac{2}{3}$
281. If $a < \frac{1}{32}$, then the number of solutions of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = a\pi^3$, is
a) 0 b) 1 c) 2 d) Infinite
282. If $\sqrt{3} + i = (a + ib)(c + id)$, then $\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right)$ has the value
a) $\frac{\pi}{3} + 2n\pi, n \in I$ b) $n\pi + \frac{\pi}{6}, n \in I$ c) $n\pi - \frac{\pi}{3}, n \in I$ d) $2n\pi - \frac{\pi}{3}, n \in I$
283. If $\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x$, then the value of x is
a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) 2
284. The sum of the infinite series
 $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$ is
a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) None of these
285. If $y = \cos^{-1}(\cos 10)$, then y is equal to
a) 10 b) $4\pi - 10$ c) $2\pi + 10$ d) $2\pi - 10$
286. The principle value of $\sin^{-1} \tan\left(\frac{-5\pi}{4}\right)$ is

287. The value of $\sum_{r=0}^{\infty} \tan^{-1} \left(\frac{1}{1+r+r^2} \right)$ is equal to
- a) $\frac{\pi}{4}$ b) $-\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $-\frac{\pi}{2}$
288. If $-1 \leq x \leq -\frac{1}{2}$, then $\cos^{-1}(4x^3 - 3x)$ equals
- a) $3 \cos^{-1} x$ b) $2\pi - 3 \cos^{-1} x$ c) $-2\pi + 3 \cos^{-1} x$ d) None of these
289. If $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$, then A is equal to
- a) $x - y$ b) $x + y$ c) $\frac{x - y}{1 + xy}$ d) $\frac{x + y}{1 - xy}$
290. If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then the value of x is
- a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these
291. The number of real solution of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is
- a) 0 b) 1 c) 2 d) ∞
292. $\cos \left\{ \cos^{-1} \left(-\frac{1}{7} \right) + \sin^{-1} \left(-\frac{1}{7} \right) \right\} =$
- a) $-\frac{1}{3}$ b) 0 c) $\frac{1}{3}$ d) $\frac{4}{9}$
293. The number of triplets (x, y, z) satisfying $\sin^{-1} x + \cos^{-1} y + \sin^{-1} z = 2\pi$, is
- a) 0 b) 2 c) 1 d) Infinite
294. If $\sin^{-1}(1 - x) - 2 \sin^{-1} x = \frac{\pi}{2}$, then x equals
- a) $0, -\frac{1}{2}$ b) $0, \frac{1}{2}$ c) 0 d) None of these
295. $\cos^{-1} \left(\frac{15}{17} \right) + 2 \tan^{-1} \left(\frac{1}{5} \right) =$
- a) $\frac{\pi}{2}$ b) $\cos^{-1} \left(\frac{171}{221} \right)$ c) $\frac{\pi}{4}$ d) None of these
296. If $x < 0$, then $\tan^{-1} \left(\frac{1}{x} \right)$ equals
- a) $\cot^{-1} x$ b) $-\cot^{-1} x$ c) $-\pi + \cot^{-1} x$ d) $-\pi - \cot^{-1} x$
297. If $\cos^{-1} \sqrt{p} + \cos^{-1} \sqrt{1-p} + \cos^{-1} \sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is
- a) 1 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$
298. $\tan^{-1} \frac{m}{n} - \tan^{-1} \frac{m-n}{m+n}$ is equal to
- a) $\tan^{-1} \frac{n}{m}$ b) $\tan^{-1} \frac{m+n}{m-n}$ c) $\frac{\pi}{4}$ d) $\tan^{-1} \left(\frac{1}{2} \right)$
299. The value of $\sin \left[\frac{\pi}{2} - \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right]$ is
- a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $-\frac{1}{2}$
300. If $\cos^{-1} x > \sin^{-1} x$, then
- a) $x < 0$ b) $-1 < x < 0$ c) $0 \leq x < \frac{1}{\sqrt{2}}$ d) $-1 \leq x < \frac{1}{\sqrt{2}}$
301. The value of $\cot^{-1} \frac{xy+1}{x-y} + \cot^{-1} \frac{yz+1}{y-z} + \cot^{-1} \frac{zx+1}{z-x}$ is
- a) 0 b) 1 c) $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z$ d) None of the above
302. The value of $\tan \left\{ \cos^{-1} \left(-\frac{2}{7} \right) - \frac{\pi}{2} \right\}$ is
- a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

303. $\sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x}$ is equal to

- a) π b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) None of these

304. If $x = \sin(2 \tan^{-1} 2)$ and $y = \sin\left(\frac{1}{2} \tan^{-1} \frac{4}{3}\right)$, then

- a) $x = y^2$ b) $y^2 = 1 - x$ c) $x^2 = \frac{y}{2}$ d) $y^2 = 1 + x$

305. The simplified expression of $\sin(\tan^{-1} x)$, for any real number x is given by

- a) $\frac{1}{\sqrt{1+x^2}}$ b) $\frac{x}{\sqrt{1+x^2}}$ c) $-\frac{1}{\sqrt{1+x^2}}$ d) $-\frac{x}{\sqrt{1+x^2}}$

306. The number of real solutions of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{2}$ is

- a) 0 b) 1 c) 2 d) ∞

307. If $-\infty < x \leq 0$, then $\cos^{-1} \left(\frac{1-x^2}{1+x^2}\right)$ equals

- a) $2 \tan^{-1} x$ b) $-2 \tan^{-1} x$ c) $\pi - 2 \tan^{-1} x$ d) $\pi + 2 \tan^{-1} x$

308. If $-\frac{\pi}{2} < \sin^{-1} x < \frac{\pi}{2}$, then $\tan(\sin^{-1} x)$ is equal to

- a) $\frac{x}{1-x^2}$ b) $\frac{x}{1+x^2}$ c) $\frac{x}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$

MATHS (QUESTION BANK)

INVERSE TRIGONOMETRICE FUNCTIONS

: ANSWER KEY :

1)	a	2)	d	3)	b	4)	c	145)	a	146)	b	147)	a	148)	b
5)	c	6)	c	7)	c	8)	d	149)	d	150)	a	151)	b	152)	b
9)	a	10)	c	11)	b	12)	c	153)	b	154)	b	155)	b	156)	d
13)	a	14)	b	15)	d	16)	d	157)	c	158)	d	159)	c	160)	a
17)	b	18)	c	19)	a	20)	c	161)	c	162)	b	163)	b	164)	c
21)	a	22)	b	23)	c	24)	b	165)	d	166)	b	167)	a	168)	c
25)	a	26)	b	27)	d	28)	a	169)	c	170)	d	171)	d	172)	c
29)	d	30)	d	31)	c	32)	a	173)	b	174)	c	175)	c	176)	b
33)	c	34)	d	35)	a	36)	d	177)	d	178)	a	179)	b	180)	c
37)	a	38)	b	39)	d	40)	a	181)	c	182)	a	183)	b	184)	b
41)	d	42)	c	43)	a	44)	d	185)	b	186)	d	187)	d	188)	c
45)	a	46)	a	47)	a	48)	a	189)	a	190)	a	191)	c	192)	b
49)	a	50)	d	51)	a	52)	c	193)	c	194)	c	195)	b	196)	b
53)	b	54)	d	55)	b	56)	c	197)	b	198)	b	199)	b	200)	c
57)	b	58)	a	59)	c	60)	c	201)	d	202)	c	203)	d	204)	c
61)	d	62)	a	63)	c	64)	b	205)	a	206)	d	207)	b	208)	d
65)	d	66)	b	67)	d	68)	c	209)	d	210)	c	211)	c	212)	c
69)	c	70)	c	71)	a	72)	c	213)	c	214)	c	215)	d	216)	c
73)	c	74)	b	75)	a	76)	b	217)	a	218)	d	219)	d	220)	c
77)	c	78)	a	79)	c	80)	b	221)	b	222)	a	223)	b	224)	c
81)	d	82)	c	83)	b	84)	c	225)	c	226)	c	227)	d	228)	b
85)	c	86)	d	87)	b	88)	b	229)	d	230)	c	231)	c	232)	a
89)	b	90)	a	91)	d	92)	d	233)	a	234)	d	235)	d	236)	d
93)	c	94)	b	95)	b	96)	a	237)	c	238)	c	239)	a	240)	a
97)	c	98)	c	99)	a	100)	a	241)	a	242)	a	243)	c	244)	d
101)	a	102)	b	103)	b	104)	b	245)	c	246)	a	247)	c	248)	c
105)	c	106)	c	107)	b	108)	c	249)	a	250)	d	251)	d	252)	c
109)	a	110)	d	111)	c	112)	b	253)	c	254)	b	255)	a	256)	a
113)	c	114)	a	115)	a	116)	c	257)	b	258)	c	259)	a	260)	d
117)	d	118)	c	119)	c	120)	a	261)	d	262)	a	263)	c	264)	d
121)	b	122)	d	123)	c	124)	c	265)	c	266)	a	267)	a	268)	a
125)	b	126)	d	127)	d	128)	c	269)	d	270)	b	271)	b	272)	c
129)	d	130)	a	131)	a	132)	a	273)	d	274)	b	275)	b	276)	d
133)	b	134)	a	135)	c	136)	a	277)	b	278)	a	279)	c	280)	b
137)	c	138)	d	139)	c	140)	c	281)	a	282)	b	283)	b	284)	c
141)	d	142)	c	143)	a	144)	b	285)	b	286)	d	287)	a	288)	c

289) c	290) b	291) c	292) b
293) c	294) c	295) d	296) c
297) d	298) c	299) c	300) d
301) a	302) a	303) a	304) b
305) b	306) c	307) b	308) c

MATHS (QUESTION BANK)

INVERSE TRIGONOMETRIC FUNCTIONS

: HINTS AND SOLUTIONS :

1 (a)

We have, $1 \leq \sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x \leq \frac{\pi}{2}$

$$\Rightarrow \sin 1 \leq \cos^{-1} \sin^{-1} \tan^{-1} x \leq 1$$

$$\Rightarrow \cos \sin 1 \geq \sin^{-1} \tan^{-1} x \geq \cos 1$$

$$\Rightarrow \sin \cos \sin 1 \geq \tan^{-1} x \geq \sin \cos 1$$

$$\Rightarrow \tan \sin \cos \sin 1 \geq x \geq \tan \sin \cos 1$$

$$\therefore x \in [\tan \sin \cos 1, \tan \sin \cos \sin 1]$$

3 (b)

Given, $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\therefore \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\Rightarrow \frac{2 \cos x}{1 - \cos^2 x} = 2 \operatorname{cosec} x$$

$$\Rightarrow \frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x$$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

4 (c)

We have, $\tan^{-1} \frac{x-1}{x+2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \left[\frac{2x(x+2)}{x^2 + 4 + 4x - x^2 + 1} \right] = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$$

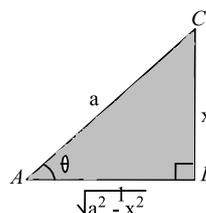
$$\Rightarrow 2x^2 + 4x = 4x + 5$$

$$\Rightarrow x = \pm \sqrt{\frac{5}{2}}$$

5 (c)

Let $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}} = \theta$

$$\Rightarrow \tan \theta = \frac{x}{\sqrt{a^2-x^2}}$$



$$\therefore \sin \theta = \frac{x}{a}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{x}{a} \right)$$

6

(c)

$$\therefore T_r = \sin^{-1} \left(\frac{\sqrt{r} - \sqrt{(r-1)}}{\sqrt{r(r+1)}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{r} - \sqrt{(r-1)}}{1 + \sqrt{r}\sqrt{(r-1)}} \right)$$

$$S_n = \sum_{r=1}^n \tan^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{1 + \sqrt{r}\sqrt{r-1}} \right)$$

$$= \sum_{r=1}^n \{ \tan^{-1} \sqrt{r} - \tan^{-1} \sqrt{(r-1)} \}$$

$$= \tan^{-1} \sqrt{n} - \tan^{-1} \sqrt{0}$$

$$= \tan^{-1} \sqrt{n} - 0$$

$$\therefore S_\infty = \tan^{-1} \infty = \frac{\pi}{2}$$

7

(c)

We have,

$$\theta_1 = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{1}{3}$$

$$\Rightarrow \theta_1 = \frac{\pi}{2} - \cos^{-1} \frac{4}{5} + \frac{\pi}{2} - \cos^{-1} \frac{1}{3}$$

$$\Rightarrow \theta_1 = \pi - \left(\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{1}{3} \right)$$

$$\Rightarrow \theta_1 = \pi - \theta_2 \Rightarrow \theta_2 = \pi - \theta_1$$

Also,

$$\theta_1 = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{1}{3}$$

$$\Rightarrow \theta_1 = \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \theta_1 = \tan^{-1} \left(\frac{\frac{4}{3} + \frac{1}{2\sqrt{2}}}{1 - \frac{4}{3} \times \frac{1}{2\sqrt{2}}} \right) = \tan^{-1} \left(\frac{8\sqrt{2} + 3}{6\sqrt{2} - 4} \right)$$

$$< \frac{\pi}{2}$$

$$\therefore \theta_2 = \pi - \theta_1 \Rightarrow \theta_2 > \frac{\pi}{2}$$

Hence, $\theta_1 < \theta_2$

8 **(d)**

$\cos^{-1} x, \sin^{-1} x$ are real, if $-1 \leq x \leq 1$

But $\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow 2 \cos^{-1} x > \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{4}$$

$$\therefore \cos(\cos^{-1} x) < \cos \frac{\pi}{4}$$

$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

The common value are $-1 \leq x < \frac{1}{\sqrt{2}}$

9 **(a)**

Roots of equation $x^2 - 9x + 8 = 0$ are 1 and 8

Let $y = [\sin^2 \alpha + \sin^4 \alpha + \sin^6 \alpha + \dots \infty] \log_e 2$

$$\Rightarrow y = \frac{\sin^2 \alpha}{1 - \sin^2 \alpha} \log_e 2 = \tan^2 \alpha \log_e 2$$

$$\Rightarrow y = \log_e 2^{\tan^2 \alpha}$$

$$\Rightarrow e^y = 2^{\tan^2 \alpha}$$

According to question,

$$2^{\tan^2 \alpha} = 8 = 2^3 \Rightarrow \tan^2 \alpha = 3$$

$$\Rightarrow \tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\therefore \sin^{-1} \left(\sin \frac{2\pi}{3} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} = \alpha$$

10 **(c)**

Given, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z + \cos^{-1} t = 4\pi$

Which is possible only when

$$\cos^{-1} x = \cos^{-1} y = \cos^{-1} z = \cos^{-1} t = \pi$$

[\because Domain of $\cos^{-1} x$ is $[0, \pi]$]

$$\Rightarrow x = y = z = t = \cos \pi = -1$$

$$\therefore x^2 + y^2 + z^2 + t^2$$

$$= (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2$$

$$= 4$$

11 **(b)**

$$\text{Here, } T_n = \cot^{-1} \left(n^2 + \frac{3}{4} \right)$$

$$= \tan^{-1} \left(\frac{4}{4n^2 + 3} \right)$$

$$= \tan^{-1} \left(\frac{1}{1 + (n + \frac{1}{2})(n - \frac{1}{2})} \right)$$

$$= \tan^{-1} \left[\frac{\left(n + \frac{1}{2} \right) - \left(n - \frac{1}{2} \right)}{1 + \left(n + \frac{1}{2} \right) \left(n - \frac{1}{2} \right)} \right]$$

$$= \tan^{-1} \left(n + \frac{1}{2} \right) - \tan^{-1} \left(n - \frac{1}{2} \right)$$

$$\therefore S_\infty = T_\infty^{-1} - \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{2} - \tan^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow S_\infty = \cot^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow S_\infty = \tan^{-1}(2)$$

12 **(c)**

Since, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$$\therefore \sin^{-1} \alpha = \frac{\pi}{2}, \sin^{-1} \beta = \frac{\pi}{2} \text{ and } \sin^{-1} \gamma = \frac{\pi}{2}$$

$$\therefore \alpha = \beta = \gamma = 1$$

Thus, $\alpha\beta + \alpha\gamma + \gamma\beta = 3$

13 **(a)**

$$\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$$

$$= \frac{\pi}{4} + \pi + \tan^{-1} \left(\frac{2+3}{1-2 \cdot 3} \right) \text{ (as } 2 \cdot 3 > 1)$$

$$= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

14 **(b)**

$$\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = 3\pi$$

We know that, if $y = \cos^{-1} x$, then $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$,

Hence, the given equation will hold only when each is π

$$\therefore p = q = r = \cos \pi = -1$$

$$\therefore p^2 + q^2 + r^2 + 2pqr$$

$$= (-1)^2 + (-1)^2 + (-1)^2 + 2(-1)(-1)(-1)$$

$$= 1 + 1 + 1 - 2$$

$$= 3 - 2 = 1$$

15 **(d)**

We have, $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$

$$= \frac{\pi}{2} - \tan^{-1} x = \cot^{-1} x$$

Since, $0 \leq x \leq 1$, therefore $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

16 **(d)**

We have,

$$\cot \left\{ \cos^{-1} \left(\frac{7}{25} \right) \right\} = \cot \left\{ \cot^{-1} \left(\frac{7}{24} \right) \right\} = \frac{7}{24}$$

17 **(b)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$x \in (1, \infty) \Rightarrow \tan \theta > 1 \Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 2\theta$$

$$< \pi$$

Now,

$$\begin{aligned} & \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\ &= \sin^{-1}(\sin 2\theta) \\ &= \sin^{-1}(\sin(\pi - 2\theta)) \\ &= \pi - 2\theta \quad \left[\because \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < \pi - 2\theta < 0\right] \\ &= \pi - 2 \tan^{-1} x \end{aligned}$$

19 (a)

$$\begin{aligned} & \tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\} \\ &= \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\} \\ &= \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\} = \tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\} \\ &= \tan\left\{\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)\right\} = \frac{2}{3\sqrt{5}} \end{aligned}$$

20 (c)

Since, $\tan^{-1} x$ and $\cot^{-1} x$ exists for all $x \in \mathbb{R}$ and $\cos^{-1}(2-x)$ exists, if $-1 \leq 2-x \leq 1$
 $\therefore \tan^{-1} x - \cot^{-1} x = \cos^{-1}(2-x)$
 Is possible only if $1 \leq x \leq 3$.
 Thus the solution of given equation is $[1, 3]$.

21 (a)

$$\begin{aligned} & \tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3) \\ &= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2\cdot 3}\right) \quad (\text{as } 2\cdot 3 > 1) \\ &= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi \end{aligned}$$

22 (b)

Let $a = b \cos \theta$. Then,

$$\begin{aligned} a_1 &= \frac{b \cos \theta + b}{2} = b \cos^2 \frac{\theta}{2} \\ \Rightarrow b_1 &= \sqrt{b \cos^2 \frac{\theta}{2} b} = b \cos \frac{\theta}{2} \end{aligned}$$

Now,

$$\begin{aligned} a_2 &= \frac{a_1 + b_1}{2} \\ \Rightarrow a_2 &= \frac{b \cos^2 \frac{\theta}{2} + b \cos \frac{\theta}{2}}{2} \\ \Rightarrow a_2 &= b \cos \frac{\theta}{2} \cos^2 \frac{\theta}{4} \\ \Rightarrow b_2 &= \sqrt{a_2 b_1} = \sqrt{b \cos \frac{\theta}{2} \cos^2 \frac{\theta}{4} b \cos \frac{\theta}{2}} \\ \Rightarrow b_2 &= b \cos \frac{\theta}{2} \cos \frac{\theta}{2^2} \end{aligned}$$

$$\text{Thus, } b_2 = b \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2^2}\right)$$

Similarly, we have

$$b_3 = b \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2^2}\right) \cos\left(\frac{\theta}{2^3}\right)$$

and, so on

$$b_n = b \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2^2}\right) \cos\left(\frac{\theta}{2^3}\right) \dots \cos\left(\frac{\theta}{2^n}\right)$$

Now,

$$\begin{aligned} b_\infty &= \lim_{n \rightarrow \infty} b_n \\ &= \lim_{n \rightarrow \infty} b \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2^2}\right) \cos\left(\frac{\theta}{2^3}\right) \dots \cos\left(\frac{\theta}{2^n}\right) \\ \Rightarrow b_\infty &= \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{b \sin \theta}{2^n \sin\left(\frac{\theta}{2^n}\right)} \\ \Rightarrow b_\infty &= \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{\left(\frac{\theta}{2^n}\right) b \sin \theta}{\sin\left(\frac{\theta}{2^n}\right) \theta} \\ \Rightarrow b_\infty &= \lim_{n \rightarrow \infty} b_n = \frac{b \sin \theta}{\theta} = \frac{b \sqrt{1 - \frac{a^2}{b^2}}}{\cos^{-1}\left(\frac{a}{b}\right)} = \frac{\sqrt{b^2 - a^2}}{\cos^{-1}\left(\frac{a}{b}\right)} \end{aligned}$$

23 (c)

$$\begin{aligned} & \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ &= \cos \cos^{-1} \sqrt{1 - \frac{3}{4}} \\ &= \cos \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \end{aligned}$$

24 (b)

We have,

$$\begin{aligned} & \sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] \\ &= \sin\left[\cot^{-1}\left\{\frac{1}{\sqrt{1 + \tan^2(\tan^{-1} x)}}\right\}\right] \\ &= -\sin\left\{\cot^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)\right\} \\ &= \frac{1}{\sqrt{1 + \cot^2\left\{\cot^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)\right\}}} = \frac{1}{\sqrt{1 + \frac{1}{1 + x^2}}} \\ &= \sqrt{\frac{1 + x^2}{2 + x^2}} \end{aligned}$$

25 (a)

$$\begin{aligned} & \cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \dots \infty \\ &= \sum_{r=1}^{\infty} \cot^{-1}(2.r^2) \\ &= \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right) \\ &= \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{(1+2r) + (1-2r)}{1 - (1+2r)(1-2r)}\right) \\ &= \sum_{r=1}^{\infty} [\tan^{-1}(1+2r) + \tan^{-1}(1-2r)] \end{aligned}$$

$$\begin{aligned}
&= \tan^{-1} 3 - \tan^{-1} 1 \\
&\quad + \tan^{-1} 5 \\
&\quad - \tan^{-1} 3 \\
&\quad + \tan^{-1} 7 - \tan^{-1} 5 + \dots + \tan^{-1} \infty \\
&= -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}
\end{aligned}$$

26 (b)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$\begin{aligned}
x \in (1, \infty) &\Rightarrow 1 < x < \infty \Rightarrow 1 < \tan \theta < \infty \Rightarrow \frac{\pi}{4} \\
&< \theta < \frac{\pi}{2}
\end{aligned}$$

Now,

$$\begin{aligned}
&\tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1}(\tan 2\theta) \\
&= \tan^{-1}(-\tan(\pi - 2\theta)) \\
&= \tan^{-1}(\tan(2\theta - \pi)) \\
&= 2\theta - \pi \left[\begin{array}{l} \because \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow < 2\theta < \pi \\ \Rightarrow -\frac{\pi}{2} - 2\theta - \pi < 0 \end{array} \right] \\
&= 2 \tan^{-1} x - \pi
\end{aligned}$$

27 (d)

$$\begin{aligned}
&\cos(2 \cos^{-1} x + \sin^{-1} x) \\
&= \cos[2(\cos^{-1} x + \sin^{-1} x) - \sin^{-1} x] \\
&= \cos(\pi - \sin^{-1} x) = -\cos(\sin^{-1} x) \\
&= -\cos \left[\sin^{-1} \left(-\frac{1}{5} \right) \right] \quad \left(\because x = \frac{1}{5} \right) \\
&= -\cos \left(\cos^{-1} \frac{2\sqrt{6}}{5} \right) \\
&= -\frac{2\sqrt{6}}{5}
\end{aligned}$$

28 (a)

$$\begin{aligned}
&\cot^{-1} \frac{xy+1}{x-y} + \cot^{-1} \frac{yz+1}{y-z} + \cot^{-1} \frac{zx+1}{z-x} \\
&= \cot^{-1} y - \cos^{-1} x \\
&\quad + \cot^{-1} z \\
&\quad - \cot^{-1} y + \cot^{-1} x - \cot^{-1} z \\
&= 0
\end{aligned}$$

29 (d)

$$\begin{aligned}
&\tan \left(\cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right) \\
&= \tan(\tan^{-1} 7 - \tan^{-1} 4) \\
&= \tan \left[\tan^{-1} \left(\frac{7-4}{1+28} \right) \right] = \frac{3}{29}
\end{aligned}$$

31 (c)

Given that, $\angle A = \tan^{-1} 2, \angle B = \tan^{-1} 3$

We know that, $\angle A + \angle B + \angle C = \pi$

$$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi$$

$$\Rightarrow \tan^{-1} \left(\frac{2+3}{1-2 \times 3} \right) + \angle C = \pi$$

$$\Rightarrow \tan^{-1}(-1) + \angle C = \pi$$

$$\Rightarrow \frac{3\pi}{4} + \angle C = \pi$$

$$\Rightarrow \angle C = \frac{\pi}{4}$$

32 (a)

$$\text{Since, } 0 \leq \cos^{-1} \left(\frac{x^2}{2} + \sqrt{1-x^2} \sqrt{1-\frac{x^2}{4}} \right) \leq \frac{\pi}{2}$$

Because $\cos^{-1} x$ is in first quadrant when x is positive

$$\text{And } \cos^{-1} \frac{x}{2} - \cos^{-1} x \geq 0$$

$$\text{So, } \cos^{-1} \frac{x}{2} \geq \cos^{-1} x$$

$$\text{Also, } \left| \frac{x}{2} \right| \leq 1, |x| \leq 1 \Rightarrow |x| \leq 1$$

33 (c)

$$8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, -\frac{5}{2}$$

$$\therefore -1 < -\frac{1}{4} < 1 \text{ and } -\frac{5}{2} < -1$$

$$\therefore \sin^{-1} \left(-\frac{1}{4} \right) \text{ exists but } \sin^{-1} \left(-\frac{5}{2} \right) \text{ does not exist.}$$

$$\sec^{-1} \left(-\frac{5}{2} \right) \text{ exists but } \sec^{-1} \left(-\frac{1}{4} \right) \text{ does not exist.}$$

$$\tan^{-1} \left(-\frac{1}{4} \right) \text{ and } \tan^{-1} \left(-\frac{5}{2} \right) \text{ both exist.}$$

34 (d)

$$\text{Given, } (\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\begin{aligned} \therefore (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) \\ = \frac{5\pi^2}{8} \end{aligned}$$

$$\Rightarrow \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} = \tan^{-1} x + 2 (\tan^{-1} x)^2 \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4}$$

$$\text{Now, we take } \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1$$

35 (a)

$$\text{We have, } \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4+m^2+2} \right)$$

$$= \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)} \right)$$

$$= \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2+m+1) - (m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)} \right)$$

$$= \sum_{m=1}^n [\tan^{-1}(m^2+m+1) - \tan^{-1}(m^2-m+1)]$$

$$= (\tan^{-1} 3$$

$$- \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + (\tan^{-1} 13 - \tan^{-1} 7$$

$$+ \dots + \tan^{-1}(n^2 - n + 1) - \tan^{-1}(n^2 - n + 1)]$$

$$= \tan^{-1} \frac{n^2 + n + 1 - 1}{1 + (n^2 + n + 1) \cdot 1}$$

$$= \tan^{-1} \left(\frac{n^2 + n}{2 + n^2 + n} \right)$$

36 (d)

$$\tan \left(\cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right)$$

$$= \tan(\tan^{-1} 7 - \tan^{-1} 4)$$

$$= \tan \left[\tan^{-1} \left(\frac{7-4}{1+28} \right) \right] = \frac{3}{29}$$

37 (a)

As we know that

$$|\sin^{-1} x| \leq \frac{\pi}{2}$$

∴ Given relation

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Is possible only when

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow x = y = z = 1$$

$$\therefore x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$$

$$= 1 + 1 + 1 - \frac{9}{1 + 1 + 1}$$

$$= 3 - \frac{9}{3} = 0$$

39 (d)

$$\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = \tan^{-1} \frac{2x}{1-x^2}$$

$$\Rightarrow 2 \tan^{-1} a - 2 \tan^{-1} b = 2 \tan^{-1} x$$

$$\Rightarrow \tan^{-1} \frac{a-b}{1+ab} = \tan^{-1} x$$

$$\Rightarrow x = \frac{a-b}{1+ab}$$

40 (a)

$$\therefore \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \tan^{-1} \left(\frac{r+1-r}{1+r(r+1)} \right)$$

$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$

$$\therefore \sum_{r=0}^n [\tan^{-1}(r+1) - \tan^{-1}(r)]$$

$$= \tan^{-1}(n+1) - \tan^{-1}(0)$$

$$= \tan^{-1}(n+1)$$

$$\Rightarrow \sum_{r=0}^{\infty} \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

41 (d)

$$4 \tan^{-1} \frac{1}{5} = 2 \left[2 \tan^{-1} \frac{1}{5} \right]$$

$$= 2 \tan^{-1} \frac{\frac{2}{5}}{1 - \frac{1}{25}} = 2 \tan^{-1} \frac{5}{12}$$

$$= \tan^{-1} \frac{\frac{10}{12}}{1 - \frac{25}{144}}$$

$$= \tan^{-1} \frac{120}{119}$$

$$\text{So, } 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239} = \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \times \frac{1}{239}}$$

$$= \tan^{-1} \frac{(120 \times 239) - 119}{(119 \times 239) + 120}$$

$$= \tan^{-1} \frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}$$

42 (c)

The given expression can be written as

$$\tan^{-1} \left\{ a \sqrt{\frac{a+b+c}{abc}} \right\} + \tan^{-1} \left\{ b \sqrt{\frac{a+b+c}{abc}} \right\}$$

$$+ \tan^{-1} \left\{ c \sqrt{\frac{a+b+c}{abc}} \right\}$$

$$= \tan^{-1}(ay) + \tan^{-1}(by) + \tan^{-1}(cy), \text{ where } y = \sqrt{\frac{a+b+c}{abc}}$$

$$= \tan^{-1} \left\{ \frac{ay + by + cy - abc y^3}{1 - ab y^2 - bc y^2 - ac y^2} \right\}$$

$$= \tan^{-1} \left\{ y \left(\frac{a+b+c - abc y^2}{1 - y^2(ab+bc+ca)} \right) \right\} = \tan^{-1} 0$$

$$= 0$$

43 (a)

$$\text{Given, } \sec^{-1} \sqrt{1+x^2} + \operatorname{cosec}^{-1} \frac{\sqrt{1+y^2}}{y} + \cot^{-1} \frac{1}{z} =$$

$$\pi$$

$$\therefore \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$$

$$\Rightarrow \tan^{-1} \left(\frac{x+y+z-xyz}{1-xy-yz-zx} \right) = \pi$$

$$\Rightarrow x+y+z = xyz$$

44 (d)

$$\text{Given, } \tan^{-1}(x-1) + \tan^{-1} x = \tan^{-1} 3x - \tan^{-1}(x+1)$$

$$\Rightarrow \tan^{-1} \left[\frac{(x-1)+x}{1-(x-1)x} \right] = \tan^{-1} \left[\frac{3x-(x+1)}{1+3x(x+1)} \right]$$

$$\Rightarrow (1+3x^2+3x)(2x-1) = (1-x^2+x)(2x-1)$$

$$\Rightarrow (2x-1)(4x^2+2x) = 0$$

$$\Rightarrow x = 0, \pm \frac{1}{2}$$

45 (a)

As we know that

$$|\sin^{-1} x| \leq \frac{\pi}{2}$$

∴ Given relation

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Is possible only when

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow x = y = z = 1$$

$$\therefore x^{100} + y^{100} + z^{100} = \frac{9}{x^{101} + y^{101} + z^{101}}$$

$$= 1 + 1 + 1 - \frac{9}{1 + 1 + 1}$$

$$= 3 - \frac{9}{3} = 0$$

46 (a)

$$\begin{aligned} & \sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x} \\ &= [\sin^{-1} x + \cos^{-1} x] + \left[\sin^{-1} \left(\frac{1}{x} \right) + \cos^{-1} \left(\frac{1}{x} \right) \right] \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \end{aligned}$$

47 (a)

$$\begin{aligned} & \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right) \\ &= \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\ &= \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) \\ & \quad - \tan^{-1}(m^2 - m + 1)] \\ &= \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1 \\ &= \tan^{-1} \left(\frac{n^2 + n}{2 + n^2 + n} \right) \end{aligned}$$

48 (a)

$$\text{Let } \sin^{-1} a = A, \sin^{-1} b = B, \sin^{-1} c = C$$

$$\therefore \sin A = a, \sin B = b, \sin C = c \dots \text{(i)}$$

$$\text{And } A + B + C = \pi$$

$$\text{Then } \sin 2A + \sin 2B + \sin 2C =$$

$$4 \sin A \sin B \sin C \dots \text{(ii)}$$

$$\Rightarrow \sin A \cos A + \sin B \cos B + \sin C \cos C$$

$$= 2 \sin A \sin B \sin C$$

$$\Rightarrow \sin A \sqrt{1 - \sin^2 A} + \sin B \sqrt{1 - \sin^2 B}$$

$$+ \sin C \sqrt{1 - \sin^2 C} = 2 \sin A \sin B \sin C \dots \text{(iii)}$$

$$\Rightarrow a\sqrt{1 - a^2} + b\sqrt{1 - b^2} + c\sqrt{1 - c^2}$$

$$= 2abc$$

49 (a)

$$1 \text{ rad} > 45^\circ$$

$$\Rightarrow \tan 1^\circ > \tan 45^\circ \Rightarrow \tan 1 > 1$$

$$\text{Also, } \tan^{-1} 1 = \frac{\pi}{4} < 1$$

$$\text{Hence, } \tan 1 > \tan^{-1} 1$$

50 (d)

$$\text{Since, } 2 \cos^{-1} x = \cos^{-1}(2x^2 - 1)$$

Therefore,

$$2 \cos^{-1} 0.8 = \cos^{-1}(2 \times 0.64 - 1) = \cos^{-1}(0.28)$$

$$\Rightarrow \cos(2 \cos^{-1} 0.8) = \cos(\cos^{-1} 0.28) = 0.28$$

51 (a)

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

$$\Rightarrow \frac{3x + 2x}{1 - 6x^2} = \frac{\pi}{4}$$

$$\Rightarrow 5x = 1 - 6x^2$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow x = -1, \frac{1}{6}$$

But when $x = -1$,

$$\tan^{-1} 2x = \tan^{-1}(-2) < 0$$

$$\text{And } \tan^{-1} 3x = \tan^{-1}(-3) < 0$$

This value will not satisfy the given equation

$$\text{Hence, } x = \frac{1}{6}$$

52 (c)

We have,

$$\cos \left[\frac{1}{2} \cos^{-1} \left\{ \cos \left(\sin^{-1} \frac{\sqrt{63}}{8} \right) \right\} \right]$$

$$= \cos \left[\frac{1}{2} \cos^{-1} \left\{ \cos \left(\cos^{-1} \frac{1}{8} \right) \right\} \right]$$

$$= \cos \left[\frac{1}{2} \cos^{-1} \left(\frac{1}{8} \right) \right] = \sqrt{\frac{1 + \cos \left(\cos^{-1} \frac{1}{8} \right)}{2}} = \frac{3}{4}$$

53 (b)

$$\text{Let } \cos^{-1} x = \theta. \text{ Then, } x = \cos \theta$$

$$\text{Also, } 0 \leq x \leq 1 \Rightarrow 0 \leq \cos \theta \leq 1 \Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$$

Now,

$$\cos^{-1}(2x^2 - 1)$$

$$= \cos^{-1}(2 \cos^2 \theta - 1)$$

$$= \cos^{-1}(\cos 2\theta) = 2\theta = 2 \cos^{-1} x \quad [\because 0 \leq 2\theta \leq \pi]$$

54 (d)

$$\text{Let } \cot^{-1} x = \theta \Rightarrow x = \cot \theta$$

$$\text{Now, } \operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + x^2}$$

$$\Rightarrow \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\sqrt{1 + x^2}}$$

$$\Rightarrow \theta = \sin^{-1} \frac{1}{\sqrt{1 + x^2}}$$

$$\therefore \sin(\cot^{-1} x) = \sin \left(\sin^{-1} \frac{1}{\sqrt{1 + x^2}} \right)$$

$$= \frac{1}{\sqrt{1 + x^2}} = (1 + x^2)^{-1/2}$$

55 (b)

$$\frac{\sin 2 - 1}{\cos 2} = -\frac{1 - \sin 2}{\cos 2}$$

$$\begin{aligned}
&= -\frac{(\cos 1 - \sin 1)^2}{(\cos 1 + \sin 1)(\cos 1 - \sin 1)} \\
&= -\frac{\cos 1 - \sin 1}{\cos 1 + \sin 1} \\
&= -\frac{1 - \tan 1}{1 + \tan 1} \\
&= -\tan\left(\frac{\pi}{4} - 1\right) \\
&= \tan\left(1 - \frac{\pi}{4}\right) \\
\Rightarrow \tan^{-1}\left(\frac{\sin 2 - 1}{\cos 2}\right) \\
&= \tan^{-1}\left[\tan\left(1 - \frac{\pi}{4}\right)\right] \\
&= 1 - \frac{\pi}{4}
\end{aligned}$$

56 (c)

$\therefore \Delta ABC$ is right angled at A .

$$\therefore a^2 = b^2 + c^2 \quad \dots(i)$$

$$\begin{aligned}
\text{Now, } \tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right) \\
&= \tan^{-1}\left[\frac{\frac{c}{a+b} + \frac{b}{a+c}}{1 - \left(\frac{c}{a+b}\right)\left(\frac{b}{a+c}\right)}\right] \\
&= \tan^{-1}\left[\frac{ac + c^2 + ab + b^2}{a^2 + ac + ab + bc - bc}\right] \\
&= \tan^{-1}\left[\frac{a^2 + ac + ab}{a^2 + ac + ab}\right] \\
&= \tan^{-1}(1) = \frac{\pi}{4} \quad [\text{using Eq. (i)}]
\end{aligned}$$

57 (b)

Let $\cos^{-1} x = \theta$. Then, $x = \cos \theta$

Also,

$$-\frac{1}{2} \leq x \leq \frac{1}{2} \Rightarrow -\frac{1}{2} \leq \cos \theta \leq \frac{1}{2} \Rightarrow \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$$

Now,

$$\begin{aligned}
\cos^{-1}(4x^3 - 3x) &= \cos^{-1}(\cos 3\theta) \\
&= \cos^{-1}(\cos(2\pi - 3\theta)) \\
&= 2\pi - 3\theta \quad \left[\because \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \Rightarrow 0 \leq 2\pi - 3\theta \leq \pi\right] \\
&= 2\pi - 3\cos^{-1} x
\end{aligned}$$

58 (a)

Let $\sin^{-1} a = A, \sin^{-1} b = B, \sin^{-1} c = C$

$$\therefore \sin A = a, \sin B = b, \sin C = c \dots (i)$$

And $A + B + C = \pi$

Then $\sin 2A + \sin 2B + \sin 2C =$

$$4 \sin A \sin B \sin C \dots (ii)$$

$$\Rightarrow \sin A \cos A + \sin B \cos B + \sin C \cos C$$

$$= 2 \sin A \sin B \sin C$$

$$\Rightarrow \sin A \sqrt{(1 - \sin^2 A)} + \sin B \sqrt{(1 - \sin^2 B)}$$

$$+ \sin C \sqrt{1 - \sin^2 C} = 2 \sin A \sin B \sin C \dots (iii)$$

$$\begin{aligned}
&\Rightarrow a\sqrt{(1 - a^2)} + b\sqrt{(1 - b^2)} + c\sqrt{(1 - c^2)} \\
&= 2abc
\end{aligned}$$

59 (c)

Given, $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$

$$\begin{aligned}
\Rightarrow \sin^{-1}(1 - x) &= \frac{\pi}{2} - \sin^{-1} x - \sin^{-1} x \\
&= \frac{\pi}{2} - 2\sin^{-1} x
\end{aligned}$$

$$\Rightarrow \sin^{-1}(1 - x) = \sin^{-1} 1 - \sin^{-1} 2x\sqrt{1 - x^2}$$

$$\Rightarrow \sin^{-1}(1 - x) = \sin^{-1}[1\sqrt{1 - 4x^2(1 - x^2)} - 0]$$

$$\Rightarrow (1 - x) = \sqrt{1 - 4x^2 + 4x^4}$$

$$\Rightarrow 1 - x = 1 - 2x^2$$

$$\Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x \in \left\{0, \frac{1}{2}\right\}$$

60 (c)

Given, $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \sin^{-1} 1$

$$\therefore \tan^{-1}\left(\frac{a + b + c - abc}{1 - ab - bc - ca}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{a + b + c - abc}{1 - ab - bc - ca} = \frac{1}{0} \Rightarrow ab + bc + ca - 1$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{1}{abc} = 0$$

61 (d)

Now, $\cos^{-1}(\cos 4) = \cos^{-1}\{\cos(2\pi - 4)\} = 2\pi - 4$

$$\Rightarrow 2\pi - 4 > 3x^2 - 4x$$

$$\Rightarrow 3x^2 - 4x - (2\pi - 4) < 0$$

$$\Rightarrow \frac{2 - \sqrt{6\pi - 8}}{3} < x < \frac{2 + \sqrt{6\pi - 8}}{3}$$

62 (a)

Let $x = -y, y > 0$

$$\therefore \sin^{-1} x = \sin^{-1}(-y)$$

$$= -\sin^{-1} y$$

$$= -\cos^{-1} \sqrt{1 - y^2}$$

$$= -\cos^{-1} \sqrt{1 - x^2}$$

63 (c)

$$\text{Now, } \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\frac{7}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right) + \tan^{-1}\frac{7}{8}$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) + \tan^{-1}\frac{7}{8}$$

$$= \tan^{-1}(1) + \tan^{-1}\frac{7}{8}$$

$$= \tan^{-1} \left(\frac{1 + \frac{7}{8}}{1 - \frac{7}{8}} \right)$$

$$= \tan^{-1}(15)$$

64 (b)

We have,

$$\tan^{-1} \left(\frac{1-x}{1+x} \right) = \tan^{-1} 1 - \tan^{-1} x = \frac{\pi}{4} - \tan^{-1} x$$

We have, $0 \leq x \leq 1$

$$\therefore 0 \leq -\tan^{-1} x \leq -\frac{\pi}{4}$$

$$\Rightarrow 0 \geq -\tan^{-1} x \geq -\frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} \geq \frac{\pi}{4} - \tan^{-1} x \geq 0 \Rightarrow \frac{\pi}{4} \geq \tan^{-1} \left(\frac{1-x}{1+x} \right) \geq 0$$

65 (d)

$$\text{Given, } \sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2}$$

$$= \tan^{-1} \frac{2x}{1-x^2}$$

$$\Rightarrow 2 \tan^{-1} a - 2 \tan^{-1} b = 2 \tan^{-1} x$$

$$\Rightarrow \tan^{-1} \left(\frac{a-b}{1+ab} \right) = \tan^{-1} x$$

$$\Rightarrow x = \frac{a-b}{1+ab}$$

66 (b)

Given,

$$\tan^{-1} \left(\frac{1}{2x+1} \right) + \tan^{-1} \left(\frac{1}{4x+1} \right)$$

$$= \tan^{-1} \left(\frac{2}{x^2} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{2x+1} \times \frac{1}{4x+1}} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{2}{x^2} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{6x+2}{8x^2+6x} \right) = \tan^{-1} \left(\frac{2}{x^2} \right)$$

$$\Rightarrow \frac{6x+2}{8x^2+6x} = \frac{2}{x^2}$$

$$\Rightarrow 6x^3 + 2x^2 = 16x^2 + 12x$$

$$\Rightarrow 6x^3 - 14x^2 - 12x = 0$$

$$\Rightarrow 2x(3x^2 - 7x - 6) = 0$$

$$\Rightarrow 2x(3x+2)(x-3) = 0$$

$$\Rightarrow x = 0, -\frac{2}{3}, 3$$

But $x = -\frac{2}{3}$ does not satisfy the given relation

67 (d)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$-\frac{1}{2} \leq x \leq \frac{1}{2} \Rightarrow -\frac{1}{2} \leq \sin \theta \leq \frac{1}{2} \Rightarrow -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$$

Now,

$$\sin^{-1}(3x - 4x^3) = \sin^{-1}(\sin 3\theta)$$

$$\Rightarrow \sin^{-1}(3x - 4x^3)$$

$$= 3\theta \left[\because -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} \right.$$

$$\left. \leq 3\theta \leq \frac{\pi}{2} \right]$$

$$\Rightarrow \sin^{-1}(3x - 4x^3) = 3 \sin^{-1} x$$

68 (c)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$-\infty < x < -1 \Rightarrow -\infty < \tan \theta < -1 \Rightarrow -\frac{\pi}{2} < \theta$$

$$< -\frac{\pi}{4}$$

Now,

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= \sin^{-1}(-\sin(\pi + 2\theta))$$

$$= \sin^{-1}(\sin(-\pi - 2\theta))$$

$$= -\pi - 2\theta \left[\because -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < -\pi - 2\theta < 0 \right]$$

$$= -\pi - 2 \tan^{-1} x$$

69 (c)

$$\therefore T_r = \sin^{-1} \left(\frac{\sqrt{r} - \sqrt{(r-1)}}{\sqrt{r(r+1)}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{r} - \sqrt{(r-1)}}{1 + \sqrt{r}\sqrt{(r-1)}} \right)$$

$$S_n = \sum_{r=1}^n \tan^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{1 + \sqrt{r}\sqrt{r-1}} \right)$$

$$= \sum_{r=1}^n \{ \tan^{-1} \sqrt{r} - \tan^{-1} \sqrt{(r-1)} \}$$

$$= \tan^{-1} \sqrt{n} - \tan^{-1} \sqrt{0}$$

$$= \tan^{-1} \sqrt{n} - 0$$

$$\therefore S_\infty = \tan^{-1} \infty = \frac{\pi}{2}$$

70 (c)

Given, $\cos^{-1} x = \alpha$

$$\Rightarrow x = \cos \alpha, \quad 0 < x < 1 \quad \dots(i)$$

$$\text{Also, } \sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1} \left(\frac{1}{2x^2-1} \right) = \frac{2\pi}{3}$$

$$\therefore \sin^{-1}(2 \cos \alpha \sqrt{1 - \cos^2 \alpha})$$

$$+ \sec^{-1} \left(\frac{1}{2 \cos^2 \alpha - 1} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}(\sin 2\alpha) + \sec^{-1}(\sec 2\alpha) = \frac{2\pi}{3}$$

$$\Rightarrow 2\alpha + 2\alpha = \frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{6}$$

$$\text{Now, } x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 2x = \sqrt{3}$$

$$\therefore \tan^{-1}(2x) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

71 (a)

$$\text{Since, } \alpha = \sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{16}{25}}\right)$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{8\sqrt{2}}{15} + \frac{3}{15}\right) = \sin^{-1}\left(\frac{8\sqrt{2}+3}{15}\right)$$

$$\text{Since, } \frac{8\sqrt{2}+3}{15} < 1$$

$$\therefore \alpha < \frac{\pi}{2}$$

$$\text{Now, } \beta = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{1}{3}\right)$$

$$\Rightarrow \beta = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right) + \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}\right)$$

$$= \pi - \alpha$$

$$\Rightarrow \beta > \alpha \quad (\because \alpha < \frac{\pi}{2})$$

72 (c)

Given that, $\theta = \tan^{-1} a$ and $\phi = \tan^{-1} b$

And $ab = -1$

$$\therefore \tan \theta \tan \phi = ab = -1$$

$$\Rightarrow \tan \theta = -\cot \phi$$

$$\Rightarrow \tan \theta = \tan\left(\frac{\pi}{2} + \phi\right)$$

$$\Rightarrow \theta - \phi = \frac{\pi}{2}$$

73 (c)

$$\tan \theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(-\frac{\pi}{3} + \theta\right) = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{\sqrt{3} + \tan \theta}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow \frac{3(3 \tan \theta - \tan^3 \theta)}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow 3 \tan 3\theta = a \tan 3\theta$$

$$\Rightarrow a = 3$$

74 (b)

$$\text{Let } \cot^{-1} \frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot \phi$$

$$\Rightarrow \sin \phi = \frac{1}{\sqrt{1 + \cot^2 \phi}} = \frac{2}{\sqrt{5}}$$

$$\text{Let } \cos^{-1} x = \theta \Rightarrow \sec \theta = \frac{1}{x}$$

$$\Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{x^2} - 1}$$

$$\Rightarrow \tan \theta = \frac{\sqrt{1-x^2}}{x}$$

$$\text{Now, } \tan(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$$

$$\Rightarrow \tan\left(\tan^{-1} \frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1} \frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \sqrt{(1-x^2)5} = 2x$$

On squaring both sides, we get

$$(1-x^2)5 = 4x^2$$

$$\Rightarrow 9x^2 = 5$$

$$\Rightarrow x = \pm \frac{\sqrt{5}}{3}$$

75 (a)

$$\text{We know, } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{\pi}{5}$$

$$\Rightarrow \cos^{-1} x = \frac{3\pi}{10}$$

76 (b)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$\frac{1}{2} \leq x \leq 1 \Rightarrow \frac{1}{2} \leq \sin \theta \leq 1 \Rightarrow \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \leq 3\theta \leq \frac{3\pi}{2}$$

Now,

$$\sin^{-1}(3x - 4x^3)$$

$$= \sin^{-1}(\sin 3\theta)$$

$$= \sin^{-1}(\sin(\pi - 3\theta))$$

$$= \pi - 3\theta \quad \left[\because \frac{\pi}{2} \leq 3\theta \leq \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} \leq 3\theta \leq \frac{\pi}{2} \right]$$

$$= \pi - 3 \sin^{-1} x$$

78 (a)

Since, x, y, z are in AP

$$\therefore y = \frac{x+z}{2} \quad \dots(i)$$

And $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in AP.

$$\therefore 2 \tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{2y}{1-y^2} = \frac{2y}{1-xz} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow y^2 = xz$$

$\Rightarrow x, y, z$ are in GP.

$$\therefore x = y = z$$

79 (c)

Since, $a_1, a_2, a_3, \dots, a_n$ are in AP with common difference 5

$$\Rightarrow a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = 5$$

$$\begin{aligned} \text{Now } T_1 &= \tan^{-1} \frac{5}{1+a_1 a_2} \\ &= \tan^{-1} \frac{a_2 - a_1}{1 + a_2 a_1} \\ &= \tan^{-1} a_2 - \tan^{-1} a_1 \end{aligned}$$

Similarly

$$\begin{aligned} T_2 &= \tan^{-1} a_3 - \tan^{-1} a_2 \\ T_3 &= \tan^{-1} a_4 - \tan^{-1} a_3 \\ T_{n-1} &= \tan^{-1} a_n - \tan^{-1} a_{n-1} \end{aligned}$$

On adding all, we get

$$\begin{aligned} \therefore \text{Required sum} &= \tan^{-1} a_n - \tan^{-1} a_1 \\ &= \tan^{-1} \frac{a_n - a_1}{1 + a_n a_1} \\ &= \tan^{-1} \frac{a_1 + 5(n-1) - a_1}{1 + a_n a_1} \\ &= \tan^{-1} \frac{5(n-1)}{1 + a_n a_1} \end{aligned}$$

80 (b)

$$\begin{aligned} \text{Given, } \tan^{-1} \left(\frac{1+x}{1-x} \right) &= \frac{\pi}{4} + \tan^{-1} x \\ \text{RHS} &= \frac{\pi}{4} + \tan^{-1} x = \tan^{-1} 1 + \tan^{-1} x \\ &= \tan^{-1} \left(\frac{1+x}{1-x} \right), \text{ if } x < 1 \\ \therefore x &\in (-\infty, 1) \end{aligned}$$

81 (d)

We know that,

$$\begin{aligned} \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ &= \frac{\frac{x\sqrt{3}}{2k-x} - \frac{2x-k}{k\sqrt{3}}}{1 + \frac{x\sqrt{3}}{2k-x} \cdot \frac{2x-k}{k\sqrt{3}}} = \frac{1}{\sqrt{3}} \\ \Rightarrow A-B &= 30^\circ \end{aligned}$$

82 (c)

$$\begin{aligned} &\sqrt{1+x^2} [\{x \cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2} \\ &= \sqrt{1+x^2} \left[\left\{ x \cos \left(\cos^{-1} \frac{x}{\sqrt{1+x^2}} \right) \right. \right. \\ &\quad \left. \left. + \sin \left(\sin^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right\}^2 - 1 \right]^{1/2} \\ &= \sqrt{1+x^2} \left[\left\{ x \cdot \frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} \right\}^2 - 1 \right]^{1/2} \\ &= \sqrt{1+x^2} [1+x^2-1]^{1/2} \\ &= x\sqrt{1+x^2} \end{aligned}$$

83 (b)

$$\begin{aligned} \text{Since, } \sin^{-1} \left(\frac{x}{5} \right) + \operatorname{cosec}^{-1} \left(\frac{5}{4} \right) &= \frac{\pi}{2} \\ \Rightarrow \sin^{-1} \left(\frac{x}{5} \right) + \sin^{-1} \left(\frac{4}{5} \right) &= \frac{\pi}{2} \end{aligned}$$

$$\Rightarrow \sin^{-1} \left(\frac{x}{5} \right) = \cos^{-1} \left(\frac{4}{5} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{x}{5} \right) = \sin^{-1} \left(\frac{3}{5} \right)$$

$$\Rightarrow x = 3$$

84 (c)

$$\because -\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}, -\frac{\pi}{2} \leq \sin^{-1} y \leq \frac{\pi}{2}$$

$$\text{And } -\frac{\pi}{2} \leq \sin^{-1} z \leq \frac{\pi}{2}$$

$$\text{Given that, } \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Which is possible only when

$$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\text{Or } x = y = z = 1$$

$$\text{Put } p = q = 1$$

$$\text{Then } f(2) = f(1)f(1) = 2 \cdot 2 = 4$$

$$\text{And put } p = 1, q = 2$$

$$\text{Then, } f(3) = f(1)f(2) = 2 \cdot 2^2 = 8$$

$$\therefore x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$

$$= 1 + 1 + 1 - \frac{3}{1+1+1}$$

$$= 3 - 1 = 2$$

86 (d)

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{x+2+x-2}{1-(x+2)(x-2)} = \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \frac{2x}{1-x^2+4} = \frac{1}{2}$$

$$\Rightarrow 4x = 5 - x^2$$

$$\Rightarrow x^2 + 4x - 5 = 0$$

$$\Rightarrow (x-1)(x+5) = 0$$

$$\Rightarrow x = 1, -5$$

87 (b)

$$\begin{aligned} \cos \left[\cos^{-1} \left(-\frac{1}{7} \right) + \sin^{-1} \left(-\frac{1}{7} \right) \right] &= \cos \frac{\pi}{2} \\ &= 0 \end{aligned}$$

88 (b)

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x \right) - \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$$

89 (b)

$$\sec \left[\tan^{-1} \left(\frac{b+a}{b-a} \right) - \tan^{-1} \left(\frac{a}{b} \right) \right]$$

$$\begin{aligned}
&= \sec \left[\tan^{-1} \left\{ \frac{\frac{b+a}{b-a} - \frac{a}{b}}{1 + \left(\frac{b+a}{b-a} \right) \left(\frac{a}{b} \right)} \right\} \right] \\
&= \sec[\tan^{-1}(1)] \\
&= \sec \frac{\pi}{4} = \sqrt{2}
\end{aligned}$$

90 (a)

$$\begin{aligned}
&\sin(\cos^{-1} x) = \cos(\sin^{-1} x) \\
&\Rightarrow \sin\left(\frac{\pi}{2} - \sin^{-1} x\right) = \cos(\sin^{-1} x) \\
&\Rightarrow \sin^{-1} x = \sin^{-1} x
\end{aligned}$$

91 (d)

$$\begin{aligned}
&\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right) \\
&= \tan^{-1}\left\{ \frac{\frac{a}{b} + \frac{a+b}{a-b}}{1 - \frac{a}{b} \left(\frac{a+b}{a-b} \right)} \right\} \\
&= \tan^{-1}\left(-\frac{a^2 + b^2}{a^2 + b^2}\right) \\
&= \tan^{-1}(-1)
\end{aligned}$$

\therefore The value is neither depends on a nor b

92 (d)

We have,

$$\begin{aligned}
&\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2 \tan^{-1} x \text{ for } x \geq 1 \\
&\therefore 2 \tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\
&= 2 \tan^{-1} x + \pi - 2 \tan^{-1} x = \pi
\end{aligned}$$

93 (c)

$$\begin{aligned}
&\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi \\
&\Rightarrow \cos^{-1}(xy - \sqrt{1-y^2} \sqrt{1-x^2}) = \pi - \cos^{-1} z \\
&\Rightarrow xy - \sqrt{1-x^2} \sqrt{1-y^2} = \cos(\pi - \cos^{-1} z) \\
&\Rightarrow xy - \sqrt{1-x^2} \sqrt{1-y^2} = -z \\
&\Rightarrow xy + z = \sqrt{1-x^2} \sqrt{1-y^2} \\
&\text{On squaring both sides, we get} \\
&x^2 y^2 + z^2 + 2xyz - 1 - x^2 - y^2 + x^2 y^2 \\
&\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz
\end{aligned}$$

94 (b)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$\begin{aligned}
x > \frac{1}{\sqrt{3}} \Rightarrow \tan \theta > \frac{1}{\sqrt{3}} \Rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 3\theta \\
&< \frac{3\pi}{2}
\end{aligned}$$

Now,

$$\begin{aligned}
&\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan 3\theta) \\
&\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan(\pi - 3\theta))
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan(3\theta - \pi)) \\
&\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)
\end{aligned}$$

$$= 3\theta - \pi \left[\begin{array}{l} \because \frac{\pi}{6} < \theta < \frac{\pi}{2} \\ \Rightarrow -\frac{\pi}{2} < 3\theta - \pi < \frac{\pi}{2} \end{array} \right]$$

$$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3 \tan^{-1} x - \pi$$

95 (b)

We have, $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\begin{aligned}
&\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x) \\
&\Rightarrow \frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x \\
&\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}
\end{aligned}$$

96 (a)

$$\begin{aligned}
&\text{Let } \cot^{-1} \frac{3}{4} = \theta \Rightarrow \cot \theta = \frac{3}{4} \\
&\text{And } \sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sqrt{1 + \left(\frac{9}{16}\right)}} = \frac{4}{5} \\
&\therefore \cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13} = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} \\
&= \sin^{-1} \left[\frac{4}{5} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{16}{25}} \right] \\
&= \sin^{-1} \left[\frac{4}{5} \cdot \frac{12}{13} + \frac{5}{13} \cdot \frac{3}{5} \right] \\
&= \sin^{-1} \left[\frac{48 + 15}{65} \right] = \sin^{-1} \frac{63}{65}
\end{aligned}$$

97 (c)

$$\begin{aligned}
&\therefore \tan^{-1} x - \tan^{-1} y = 0 \Rightarrow x = y \\
&\text{Also, } \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} \\
&\Rightarrow \cos^{-1} x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2} \\
&\text{Hence, } x^2 + xy + y^2 = 3x^2 = \frac{3}{2}
\end{aligned}$$

98 (c)

$$\begin{aligned}
&\text{Clearly, } x(x+1) \geq 0 \text{ and } x^2 + x + 1 \leq 1 \\
&\Rightarrow x(x+1) = 0 \\
&\Rightarrow x = 0, -1 \\
&\text{When } x = 0, \\
&\text{LHS} = \tan^{-1} 0 + \sin^{-1} 1 = \frac{\pi}{2} \\
&\text{When } x = -1, \\
&\text{LHS} = \tan^{-1} 0 + \sin^{-1} \sqrt{1-1+1} \\
&= 0 + \sin^{-1}(1) = \frac{\pi}{2}
\end{aligned}$$

Thus, the number of solution is 2

99 (a)

We have,

$$\begin{aligned} & \tan^{-1} x + \tan^{-1} y + \tan^{-1} z \\ &= \tan^{-1} \left\{ \frac{x + y + z - xyz}{1 - (xy + yz + zx)} \right\} = \tan^{-1} 0 = 0 \end{aligned}$$

100 (a)

$$\text{Given, } \tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{1}{y} = \sin^{-1} \frac{3}{\sqrt{10}}$$

$$\Rightarrow \tan^{-1} \left(\frac{x + \frac{1}{y}}{1 - \frac{x}{y}} \right) = \tan^{-1} 3$$

$$\Rightarrow x + \frac{1}{y} = 3 \left(1 - \frac{x}{y} \right)$$

$$\Rightarrow x = 1, y = 2$$

\therefore The number of solutions of given equation is 1.

101 (a)

We have,

$$\begin{aligned} & \sin^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \left\{ \frac{3/4 + 1/7}{1 - 3/4 \times 1/7} \right\} = \tan^{-1} \left(\frac{25}{25} \right) = \tan^{-1} 1 \\ &= \frac{\pi}{4} \end{aligned}$$

103 (b)

$$\text{Given that, } x^2 + y^2 + z^2 = r^2$$

$$\text{Now, } \tan^{-1} \left(\frac{xy}{zr} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) + \tan^{-1} \left(\frac{xz}{yr} \right)$$

$$\begin{aligned} &= \tan^{-1} \left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \left(\frac{x^2 + y^2 + z^2}{r^2} \right)} \right] \\ &= \tan^{-1} \left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \frac{r^2}{r^2}} \right] \\ &= \tan^{-2} \infty = \frac{\pi}{2} \end{aligned}$$

104 (b)

Put $x = \sin \theta$, we get

$$f(x) = \sin^{-1} \left\{ \sin \left(\theta - \frac{\pi}{6} \right) \right\}$$

$$\text{For, } -\frac{1}{2} \leq x \leq 1$$

$$\Rightarrow -\frac{1}{2} \leq \sin \theta \leq 1$$

$$\Rightarrow -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

is in the fourth on the first quadrant

$$\therefore f(x) = \theta - \frac{\pi}{6} = \sin^{-1} x - \frac{\pi}{6}$$

105 (c)

$$\begin{aligned} & \cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right) \\ &= \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$

106 (c)

$$\text{Given, } \tan^{-1} 2\theta + \tan^{-1} 3\theta = \frac{\pi}{4}$$

$$\therefore \tan^{-1} \left(\frac{2\theta + 3\theta}{1 - 2\theta \times 3\theta} \right) = \tan^{-1} 1$$

$$\Rightarrow 6\theta^2 + 5\theta - 1 = 0$$

$$\Rightarrow \theta = \frac{-5 \pm \sqrt{25 + 24}}{2 \times 6}$$

$$= \frac{-5 \pm 7}{12} = -1, \frac{1}{6}$$

$$\Rightarrow \theta = \frac{1}{6}$$

107 (b)

$$\text{Let } \theta = \cos^{-1} \left(-\frac{1}{2} \right)$$

$$\Rightarrow \cos \theta = -\frac{1}{2} = -\cos \left(\frac{\pi}{3} \right)$$

$$= \cos \left(\pi - \frac{\pi}{3} \right) = \cos \left(\frac{2\pi}{3} \right)$$

$$\Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \dots$$

108 (c)

$$\tan \theta + \tan \left(\frac{\pi}{3} + \theta \right) + \tan \left(-\frac{\pi}{3} + \theta \right) = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{\sqrt{3} + \tan \theta}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow \frac{3(3 \tan \theta - \tan^3 \theta)}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow 3 \tan 3\theta = a \tan 3\theta$$

$$\Rightarrow a = 3$$

109 (a)

$$\text{Let } \cot^{-1} \frac{3}{4} = \theta \Rightarrow \cot \theta = \frac{3}{4}$$

$$\text{And } \sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sqrt{1 + \left(\frac{9}{16} \right)}} = \frac{4}{5}$$

$$\therefore \cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13} = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13}$$

$$= \sin^{-1} \left[\frac{4}{5} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{16}{25}} \right]$$

$$= \sin^{-1} \left[\frac{4}{5} \cdot \frac{12}{13} + \frac{5}{13} \cdot \frac{3}{5} \right]$$

$$= \sin^{-1} \left[\frac{48 + 15}{65} \right] = \sin^{-1} \frac{63}{65}$$

110 (d)

$$\begin{aligned} \text{Now, } \cos^{-1}(\cos 4) &= \cos^{-1}\{\cos(2\pi - 4)\} = 2\pi - 4 \\ \Rightarrow 2\pi - 4 &> 3x^2 - 4x \\ \Rightarrow 3x^2 - 4x - (2\pi - 4) &< 0 \\ \Rightarrow \frac{2 - \sqrt{6\pi - 8}}{3} < x < \frac{2 + \sqrt{6\pi - 8}}{3} \end{aligned}$$

111 (c)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$x \in (-\infty, -1)$$

$$\begin{aligned} \Rightarrow -\infty < x < -1 &\Rightarrow -\infty < \tan \theta < -1 \Rightarrow -\frac{\pi}{2} < \theta \\ &< -\frac{\pi}{4} \end{aligned}$$

Now,

$$\begin{aligned} \tan^{-1}\left(\frac{2x}{1-x^2}\right) &= \tan^{-1}(\tan 2\theta) \\ &= \tan^{-1}(\tan(\pi + 2\theta)) \\ &= \pi + 2\theta \quad \left[\because -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \Rightarrow 0 < \pi + 2\theta < \frac{\pi}{2}\right] \\ &= \pi + 2 \tan^{-1} x \end{aligned}$$

112 (b)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$ and $\sqrt{1-x^2} = \cos \theta$

Also,

$$\frac{1}{\sqrt{2}} \leq x \leq 1 \Rightarrow \frac{1}{\sqrt{2}} \leq \sin \theta \leq 1 \Rightarrow \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$$

$$\begin{aligned} \therefore \sin^{-1}(2x\sqrt{1-x^2}) \\ &= \sin^{-1}(\sin 2\theta) \\ &= \sin^{-1}(\sin(\pi - 2\theta)) \\ &= \pi - 2\theta \quad \left[\because \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 \leq \pi - 2\theta \leq \frac{\pi}{2}\right] \\ &= \pi - 2 \sin^{-1} x \end{aligned}$$

114 (a)

Since, $-\frac{\pi}{2} < \sin^{-1} x \leq \frac{\pi}{2}$

$$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \leq i \leq 20$$

$$\Rightarrow x_i = 1, 1 \leq i \leq 20$$

$$\text{Thus, } \sum_{i=1}^{20} x_i = 20$$

115 (a)

1 rad $>$ 45°

$$\Rightarrow \tan 1^\circ > \tan 45^\circ \Rightarrow \tan 1 > 1$$

$$\text{Also, } \tan^{-1} 1 = \frac{\pi}{4} < 1$$

$$\text{Hence, } \tan 1 > \tan^{-1} 1$$

116 (c)

$$\begin{aligned} \alpha + \beta &= \sin^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{3} + \cos^{-1} \frac{1}{3} \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \\ \text{Also, } \alpha &= \frac{\pi}{3} + \sin^{-1} \frac{1}{3} < \frac{\pi}{3} + \sin^{-1} \frac{1}{2} \end{aligned}$$

As $\sin \theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$$\therefore \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Rightarrow \beta > \frac{\pi}{2} > \alpha$$

$$\Rightarrow \alpha < \beta$$

117 (d)

$$\begin{aligned} &2 \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) \\ &= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right) \\ &= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) \\ &= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}\right) \\ &= \tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4} \end{aligned}$$

118 (c)

$$\begin{aligned} &\tan\left[\frac{1}{2} \sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2} \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right] \\ &= \tan\left[\frac{1}{2} \cdot 2 \tan^{-1} a + \frac{1}{2} \cdot 2 \tan^{-1} a\right] \\ &= \tan(2 \tan^{-1} a) \\ &= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right] \\ &= \frac{2a}{1-a^2} \end{aligned}$$

119 (c)

Let $S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$

$$\therefore T_n \cot^{-1} 2n^2$$

$$= \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

$$\therefore S_n = \sum_{n=1}^{\infty} \{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$$

$$= \tan^{-1} \infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

120 (a)

Given, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}}\right) = \frac{\pi}{2}$$

$$\begin{aligned} \Rightarrow \frac{(a+b)x}{x^2 - ab} \\ &= \tan \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{(a+b)x}{x^2-ab} &= \frac{1}{0} \\ \Rightarrow x^2-ab &= 0 \\ \Rightarrow x &= \sqrt{ab} \end{aligned}$$

121 (b)

We have, $\Sigma x_1 = \sin 2\beta$, $\Sigma x_1 x_2 = \cos 2\beta$, $\Sigma x_1 x_2 x_3 = \cos \beta$ and $x_1 x_2 x_3 x_4 = -\sin \beta$

$$\begin{aligned} \therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4 &= \tan^{-1} \left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4} \right) \\ &= \tan^{-1} \left(\frac{\sin 2\beta - \cos \beta}{1 - \cos 2\beta - \sin \beta} \right) \\ &= \tan^{-1} \left(\frac{(2 \sin \beta - 1) \cos \beta}{\sin \beta (2 \sin \beta - 1)} \right) \\ &= \tan^{-1}(\cot \beta) \\ &= \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \beta \right) \right) = \frac{\pi}{2} - \beta \end{aligned}$$

122 (d)

$$\begin{aligned} \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \sin 2x}{5 + 3 \cos 2x} \right) &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{\frac{6 \tan x}{1 + \tan^2 x}}{5 + \frac{3(1 - \tan^2 x)}{1 + \tan^2 x}} \right) \\ &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{6 \tan x}{8 + 2 \tan^2 x} \right) \\ &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \tan x}{4 + \tan^2 x} \right) \\ &= \tan^{-1} \left(\frac{\frac{\tan x}{4} + \frac{3 \tan x}{4 + \tan^2 x}}{1 - \frac{3 \tan^2 x}{4(4 + \tan^2 x)}} \right) \left(\text{as } \left| \frac{\tan x}{4} \cdot \frac{3 \tan x}{4 + \tan^2 x} \right| < 1 \right) \\ &= \tan^{-1} \left(\frac{16 \tan x + \tan^3 x}{16 + \tan^2 x} \right) \\ &= \tan^{-1}(\tan x) = x \end{aligned}$$

124 (c)

Let $\cos^{-1} x = \theta$. Then, $x = \cos \theta$

Also,

$$-1 \leq x \leq 0 \Rightarrow -1 \leq \cos \theta \leq 0 \Rightarrow \frac{\pi}{2} \leq \theta \leq \pi$$

Now,

$$\begin{aligned} \cos^{-1}(2x^2 - 1) &= \cos^{-1}(\cos 2\theta) \\ &= \cos^{-1}(\cos(2\pi - 2\theta)) \\ &= 2\pi - 2\theta \left[\because \frac{\pi}{2} \leq \theta \leq \pi \Rightarrow \pi \leq 2\theta \leq 2\pi \right. \\ &\quad \left. \Rightarrow 0 \leq 2\pi - 2\theta \leq \pi \right] \\ &= 2\pi - 2 \cos^{-1} x \end{aligned}$$

125 (b)

$$\begin{aligned} \therefore \cos^{-1} \frac{3}{5} - \sin^{-1} \frac{4}{5} &= \cos^{-1} x \\ \Rightarrow \sin^{-1} \frac{4}{5} - \sin^{-1} \frac{4}{5} &= \cos^{-1} x \end{aligned}$$

$$\begin{aligned} \Rightarrow \cos^{-1} x = 0 &\Rightarrow x = \cos 0 = 1 \\ \therefore x &= 1 \end{aligned}$$

126 (d)

We have,

$$\begin{aligned} \sec^{-1} x = \operatorname{cosec}^{-1} y &\Rightarrow \cos^{-1} \frac{1}{x} = \sin^{-1} \frac{1}{y} \\ \therefore \cos^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{y} &= \sin^{-1} \frac{1}{y} + \cos^{-1} \frac{1}{y} = \frac{\pi}{2} \end{aligned}$$

128 (c)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$\begin{aligned} -1 \leq x \leq -\frac{1}{2} &\Rightarrow -1 \leq \sin \theta \leq -\frac{1}{2} \Rightarrow -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{6} \Rightarrow -\frac{3\pi}{2} \leq 3\theta \leq -\frac{\pi}{2} \end{aligned}$$

Now,

$$\begin{aligned} \sin^{-1}(3x - 4x^3) &= \sin^{-1}(\sin 3\theta) \\ &= \sin^{-1}(-\pi - 3\theta) \\ &= -\pi - 3\theta \left[-\frac{3\pi}{2} \leq 3\theta \leq -\frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \leq -\pi - 3\theta \leq \frac{\pi}{2} \right] \\ &= -\pi - 3 \sin^{-1} x \end{aligned}$$

129 (d)

We have,

$$\begin{aligned} \frac{\tan \frac{6\pi}{15} - \tan \frac{\pi}{15}}{1 + \tan \frac{6\pi}{15} \tan \frac{\pi}{15}} &= \tan \frac{\pi}{3} \\ \Rightarrow \tan \frac{6\pi}{15} - \tan \frac{\pi}{15} &= \sqrt{3} + \sqrt{3} \tan \frac{6\pi}{15} \tan \frac{\pi}{15} \\ \Rightarrow \tan \frac{2\pi}{5} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{2\pi}{5} \tan \frac{\pi}{15} &= \sqrt{3} \end{aligned}$$

130 (a)

$$\begin{aligned} \tan \left\{ \cos^{-1} \left(-\frac{2}{7} \right) - \frac{\pi}{2} \right\} &= \tan \left\{ \pi - \cos^{-1} \left(\frac{2}{7} \right) - \frac{\pi}{2} \right\} \\ &= \tan \left\{ \frac{\pi}{2} - \cos^{-1} \left(\frac{2}{7} \right) \right\} \\ &= \tan \left\{ \sin^{-1} \frac{2}{7} \right\} \\ &= \tan \left\{ \tan^{-1} \left(\frac{2}{3\sqrt{5}} \right) \right\} = \frac{2}{3\sqrt{5}} \end{aligned}$$

131 (a)

$$\begin{aligned} \sin \left[\sin^{-1} \left(\frac{1}{3} \right) + \sec^{-1}(3) \right] &+ \cos \left[\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1}(2) \right] \\ &= \sin \left[\sin^{-1} \left(\frac{1}{3} \right) + \cos^{-1} \left(\frac{1}{3} \right) \right] \\ &+ \cos \left[\tan^{-1} \left(\frac{1}{2} \right) + \cot^{-1} \left(\frac{1}{2} \right) \right] \end{aligned}$$

$$= \sin \frac{\pi}{2} + \cos \frac{\pi}{2}$$

$$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

$$= \frac{\pi}{2} \text{ and } \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$= 1$$

132 (a)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} < \tan \theta < \frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$$

Now,

$$\tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = \tan^{-1}(\tan 3\theta)$$

$$\Rightarrow \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = 3\theta \left[\because -\frac{\pi}{2} < 3\theta < \frac{\pi}{2} \right]$$

$$\Rightarrow \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = 3 \tan^{-1} x$$

133 (b)

Let $\cos^{-1} \left(\frac{4}{5} \right) = \theta$. Then, $\cos \theta = \frac{4}{5}$

$$\therefore \sin \left(\frac{1}{2} \cos^{-1} \frac{4}{5} \right) = \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}}$$

$$= \frac{1}{\sqrt{10}}$$

134 (a)

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

$$\Rightarrow \frac{3x + 2x}{1 - 6x^2} = \frac{\pi}{4}$$

$$\Rightarrow 5x = 1 - 6x^2$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow x = -1, \frac{1}{6}$$

But when $x = -1$,

$$\tan^{-1} 2x = \tan^{-1}(-2) < 0$$

$$\text{And } \tan^{-1} 3x = \tan^{-1}(-3) < 0$$

This value will not satisfy the given equation

$$\text{Hence, } x = \frac{1}{6}$$

135 (c)

$$\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} = \sin^{-1} \frac{4}{5} + \tan^{-1} \frac{2 \left(\frac{1}{3} \right)}{1 - \left(\frac{1}{3} \right)^2}$$

$$= \sin^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{4} = \sin^{-1} \frac{4}{5} + \cos^{-1} \frac{4}{5} = \frac{\pi}{2}$$

$$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

136 (a)

Given equation is

$$2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$$

Which is not possible as $\cos^{-1} x \in [0, \pi]$.

137 (c)

$$\cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left(\cos \frac{5\pi}{3} \right)$$

$$= \cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left[\sin \left(\frac{\pi}{2} - \frac{5\pi}{3} \right) \right]$$

$$= \frac{5\pi}{3} + \frac{\pi}{2} - \frac{5\pi}{3} = \frac{\pi}{2}$$

Alternate

$$\text{Since, } \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$$

$$\therefore \cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left(\sin \frac{5\pi}{3} \right) = \frac{\pi}{2}$$

138 (d)

$$\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) - \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} = 30^\circ$$

139 (c)

We have,

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$$

$$\Rightarrow \sin \{ \sin^{-1} x + \sin^{-1}(1 - x) \} = \sin(\cos^{-1} x)$$

$$\Rightarrow x\sqrt{1 - (1 - x)^2} + \sqrt{1 - x^2}(1 - x) = \sqrt{1 - x^2}$$

$$\Rightarrow x\sqrt{1 - (1 - x)^2} = x\sqrt{1 - x^2}$$

$$\Rightarrow x = 0 \text{ or } 2x - x^2 = 1 - x^2 \Rightarrow x = 0 \text{ or } x = \frac{1}{2}$$

141 (d)

$$\text{Given, } 5 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + 7 \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$- 4 \tan^{-1} \left(\frac{2x}{1-x^2} \right) - \tan^{-1} x = 5\pi$$

$$\Rightarrow 5(2 \tan^{-1} x) + 7(2 \tan^{-1} x) - 4(2 \tan^{-1} x) - \tan^{-1} x = 5\pi$$

$$\Rightarrow 15 \tan^{-1} x = 5\pi$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{3}$$

$$\therefore x = \sqrt{3}$$

142 (c)

$$\because -\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}, -\frac{\pi}{2} \leq \sin^{-1} y \leq \frac{\pi}{2}$$

$$\text{And } -\frac{\pi}{2} \leq \sin^{-1} z \leq \frac{\pi}{2}$$

$$\text{Given that, } \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Which is possible only when

$$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\text{Or } x = y = z = 1$$

$$\text{Put } p = q = 1$$

$$\text{Then } f(2) = f(1)f(1) = 2 \cdot 2 = 4$$

$$\text{And put } p = 1, q = 2$$

$$\text{Then, } f(3) = f(1)f(2) = 2 \cdot 2^2 = 8$$

$$\therefore x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x + y + z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$

$$= 1 + 1 + 1 - \frac{3}{1 + 1 + 1}$$

$$= 3 - 1 = 2$$

143 (a)

$$\text{Given, } \tan^{-1}\left(\frac{1}{\sqrt{\cos \alpha}}\right) - \tan^{-1}(\sqrt{\cos \alpha}) = x$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1 + \frac{1}{\sqrt{\cos \alpha}} \cdot \sqrt{\cos \alpha}}\right) = x$$

$$\Rightarrow \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} = \tan x$$

$$\Rightarrow \frac{2\sqrt{\cos \alpha}}{1 - \cos \alpha} = \cot x$$

$$\Rightarrow \operatorname{cosec} x = \frac{1 + \cos \alpha}{1 - \cos \alpha}$$

$$\Rightarrow \sin x = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$\Rightarrow \sin x = \frac{2 \sin^2\left(\frac{\alpha}{2}\right)}{2 \cos^2\left(\frac{\alpha}{2}\right)} = \tan^2\left(\frac{\alpha}{2}\right)$$

144 (b)

$$\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{1}{\sqrt{\frac{41}{16} - 1}}$$

$$\begin{aligned} & \left[\because \operatorname{cosec}^{-1} x \right. \\ & \left. = \tan^{-1} \frac{1}{\sqrt{x^2 - 1}} \right] \\ & = \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{4}{5} \\ & = \tan^{-1} \left(\frac{\frac{1}{9} + \frac{4}{5}}{1 - \frac{1}{9} \cdot \frac{4}{5}} \right) \\ & = \tan^{-1} \left(\frac{41}{41} \right) = \frac{\pi}{4} \end{aligned}$$

145 (a)

$$\text{We have, } \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right)$$

$$= \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{(1 + (m^2 + m + 1))(m^2 - m + 1)} \right)$$

$$\begin{aligned} & = \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\ & = \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)] \\ & = (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + \\ & \quad (\tan^{-1} 13 - \tan^{-1} 7) + \dots + \\ & \quad [\tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)] \\ & = \tan^{-1} \frac{n^2 + n + 1 - 1}{1 + (n^2 + n + 1) \cdot 1} \\ & = \tan^{-1} \left(\frac{n^2 + n}{2 + n^2 + n} \right) \end{aligned}$$

146 (b)

$$\therefore \cos^{-1} \frac{3}{5} - \sin^{-1} \frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \sin^{-1} \frac{4}{5} - \sin^{-1} \frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \cos^{-1} x = 0 \Rightarrow x = \cos 0 = 1$$

$$\therefore x = 1$$

147 (a)

$$\text{Given, } \cot(\cos^{-1} x) = \sec\left(\tan^{-1} \frac{a}{\sqrt{b^2 - a^2}}\right)$$

$$\therefore \cot\left(\cot^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)\right)$$

$$= \sec\left(\sec^{-1} \frac{b}{\sqrt{b^2 - a^2}}\right)$$

$$\Rightarrow \frac{x}{\sqrt{1 - x^2}} = \frac{b}{\sqrt{b^2 - a^2}}$$

$$\Rightarrow x^2(b^2 - a^2) = b^2 - b^2 x^2$$

$$\Rightarrow x^2(2b^2 - a^2) = b^2$$

$$\Rightarrow x = \frac{b}{\sqrt{2b^2 - a^2}}$$

148 (b)

$$\text{Given, } \sin^{-1} x - \cos^{-1} x = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6} \quad \dots(i)$$

$$\text{But } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$\sin^{-1} x = \frac{\pi}{3} \text{ and } \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2} \text{ is the unique solution.}$$

149 (d)

$$\text{We have, } \theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$$

$$= \frac{\pi}{2} - \tan^{-1} x = \cot^{-1} x$$

$$\text{Since, } 0 \leq x \leq 1, \text{ therefore } \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$$

150 (a)

$$\therefore \cot^{-1} x + \sin^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\begin{aligned} \Rightarrow \tan^{-1} \frac{1}{x} + \tan^{-1} \frac{1}{2} &= \tan^{-1} 1 \\ \Rightarrow \tan^{-1} \frac{1}{x} &= \tan^{-1} 1 - \tan^{-1} \frac{1}{2} \\ \Rightarrow \tan^{-1} \frac{1}{x} &= \tan^{-1} \left(\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \right) \\ \Rightarrow \tan^{-1} \frac{1}{x} &= \tan^{-1} \frac{1}{3} \\ \Rightarrow x &= 3 \end{aligned}$$

151 (b)

$$\sin \left(\frac{1}{2} \cos^{-1} \frac{4}{5} \right)$$

$$\text{Now, put } \frac{4}{5} = \cos 2\theta$$

$$\therefore \sin \left(\frac{1}{2} \times 2\theta \right)$$

$$\begin{aligned} &= \sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} \\ &= \sqrt{\frac{1 - \frac{4}{5}}{2}} \\ &= \sqrt{\frac{1}{5 \times 2}} \\ &= \frac{1}{\sqrt{10}} \end{aligned}$$

152 (b)

$$\text{Given, } \sin^{-1} \left(\frac{3}{x} \right) = \frac{\pi}{2} - \sin^{-1} \left(\frac{4}{x} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{3}{x} \right) = \cos^{-1} \left(\frac{4}{x} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{3}{x} \right) = \sin^{-1} \left(\frac{\sqrt{x^2 - 16}}{x} \right)$$

$$\Rightarrow \frac{3}{x} = \frac{\sqrt{x^2 - 16}}{x}$$

$$\Rightarrow x = \pm 5$$

$$\therefore x = 5$$

[$\because -5$ not satisfies the given equation]

153 (b)

$$\because 0 \leq \cos^{-1} x \leq \pi$$

$$\text{And } 0 < \cot^{-1} x < \pi$$

$$\text{Given, } [\cot^{-1} x] + [\cos^{-1} x] = 0$$

$$\Rightarrow [\cot^{-1} x] = 0 \text{ and } [\cos^{-1} x] = 0$$

$$\Rightarrow 0 < \cot^{-1} x < 1 \text{ and } 0 \leq \cos^{-1} x < 1$$

$$\therefore x \in (\cot 1, \infty) \text{ and } x \in (\cos 1, 1)$$

$$\Rightarrow x \in (\cot 1, 1)$$

154 (b)

$$\begin{aligned} 3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} \\ = \frac{\pi}{3} \end{aligned}$$

On putting $x = \tan \theta$, we get

$$\begin{aligned} 3 \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) - 4 \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) \\ + 2 \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) = \frac{\pi}{3} \end{aligned}$$

$$\Rightarrow 3 \sin^{-1}(\sin 2\theta)$$

$$- 4 \cos^{-1}(\cos 2\theta)$$

$$+ 2 \tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

$$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3}$$

$$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \tan \frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}}$$

155 (b)

$$\text{Here, } T_n = \cot^{-1} \left(n^2 + \frac{3}{4} \right)$$

$$= \tan^{-1} \left(\frac{4}{4n^2 + 3} \right)$$

$$= \tan^{-1} \left(\frac{1}{1 + (n + \frac{1}{2})(n - \frac{1}{2})} \right)$$

$$= \tan^{-1} \left[\frac{(n + \frac{1}{2}) - (n - \frac{1}{2})}{1 + (n + \frac{1}{2})(n - \frac{1}{2})} \right]$$

$$= \tan^{-1} \left(n + \frac{1}{2} \right) - \tan^{-1} \left(n - \frac{1}{2} \right)$$

$$\therefore S_\infty = T_\infty^{-1} - \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{2} - \tan^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow S_\infty = \cot^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow S_\infty = \tan^{-1}(2)$$

156 (d)

$$2 \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{7} \right)$$

$$= \tan^{-1} \left[\frac{2 \left(\frac{1}{3} \right)}{1 - \frac{1}{9}} \right] + \tan^{-1} \left(\frac{1}{7} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{7} \right)$$

$$= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} \right)$$

$$= \tan^{-1} \left(\frac{25}{25} \right) = \frac{\pi}{4}$$

157 (c)

The given equation is satisfied only when $x = 1$,
 $y = -1, z = 1$

158 (d)

$$\text{Let } \cot^{-1} x = \theta \Rightarrow x = \cot \theta$$

$$\text{Now, } \operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + x^2}$$

$$\Rightarrow \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\sqrt{1 + x^2}}$$

$$\Rightarrow \theta = \sin^{-1} \frac{1}{\sqrt{1 + x^2}}$$

$$\therefore \sin(\cot^{-1} x) = \sin\left(\sin^{-1} \frac{1}{\sqrt{1 + x^2}}\right)$$

$$= \frac{1}{\sqrt{1 + x^2}} = (1 + x^2)^{-1/2}$$

159 (c)

$$\therefore \tan^{-1} x - \tan^{-1} y = 0 \Rightarrow x = y$$

$$\text{Also, } \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow 2 \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2}$$

$$\text{Hence, } x^2 + xy + y^2 = 3x^2 = \frac{3}{2}$$

160 (a)

$$\text{Let } \tan^{-1} x = \theta. \text{ Then, } x = \tan \theta$$

$$\text{Also, } -1 < x < 1 \Rightarrow -1 < \tan \theta < 1 \Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

Now,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta)$$

$$= 2\theta \quad \left[\because -\frac{\pi}{4} < \theta < \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2} \right]$$

$$= 2 \tan^{-1} x$$

161 (c)

$$\text{Given that, } \theta = \tan^{-1} a \text{ and } \phi = \tan^{-1} b$$

$$\text{And } ab = -1$$

$$\therefore \tan \theta \tan \phi = ab = -1$$

$$\Rightarrow \tan \theta = -\cot \phi$$

$$\Rightarrow \tan \theta = \tan\left(\frac{\pi}{2} + \phi\right)$$

$$\Rightarrow \theta - \phi = \frac{\pi}{2}$$

162 (b)

$$\text{Let } \cot^{-1} \frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot \phi$$

$$\Rightarrow \sin \phi = \frac{1}{\sqrt{1 + \cot^2 \phi}} = \frac{2}{\sqrt{5}}$$

$$\text{Let } \cos^{-1} x = \theta \Rightarrow \sec \theta = \frac{1}{x}$$

$$\Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{x^2} - 1}$$

$$\Rightarrow \tan \theta = \frac{\sqrt{1-x^2}}{x}$$

$$\text{Now, } \tan(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$$

$$\Rightarrow \tan\left(\tan^{-1} \frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1} \frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \sqrt{(1-x^2)5} = 2x$$

On squaring both sides, we get

$$(1-x^2)5 = 4x^2$$

$$\Rightarrow 9x^2 = 5$$

$$\Rightarrow x = \pm \frac{\sqrt{5}}{3}$$

163 (b)

$$\text{We have, } \sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$$

$$\text{or } x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$$

$$\text{or } x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$$

$$\text{or } (x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$$

$$\text{or } x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 +$$

$$4x^2y^2z^2 - 4y^2z^2 = 0$$

$$\text{or } x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$$

$$\therefore k = 2$$

164 (c)

$$\text{Let } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$\therefore \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= 3 \tan^{-1} x - 2 \tan^{-1} x$$

$$= \tan^{-1} x$$

165 (d)

$$\text{Let } \alpha = \cos^{-1} \sqrt{p}, \beta = \cos^{-1} \sqrt{1-p}$$

$$\text{And } \gamma = \cos^{-1} \sqrt{1-q}$$

$$\Rightarrow \cos \alpha = \sqrt{p}, \cos \beta = \sqrt{1-p}$$

$$\text{And } \cos \gamma = \sqrt{1-q}$$

$$\text{Therefore, } \sin \alpha = \sqrt{1-p}, \sin \beta = \sqrt{p} \text{ and } \sin \gamma = \sqrt{q}$$

The given equation may be written as

$$\alpha + \beta + \gamma = \frac{3\pi}{4}$$

$$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$$

$$\Rightarrow \cos(\alpha + \beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$$

$$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}$$

$$= -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\sqrt{q}\right)$$

$$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1-q = q$$

$$\Rightarrow q = \frac{1}{2}$$

166 (b)

Let α, β are the roots of given equation $6x^2 - 5x + 1 = 0$

$$\Rightarrow \alpha + \beta = \frac{5}{6} \text{ and } \alpha\beta = \frac{1}{6}$$

$$\begin{aligned} \therefore \tan^{-1} \alpha + \tan^{-1} \beta &= \tan^{-1} \left(\frac{\alpha + \beta}{1 - \alpha\beta} \right) \\ &= \tan^{-1} \left(\frac{\frac{5}{6}}{1 - \frac{1}{6}} \right) = \tan^{-1}(1) = \frac{\pi}{4} \end{aligned}$$

167 (a)

$$\begin{aligned} \text{Since, } \alpha &= \sin^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{1}{3} \right) \\ &= \sin^{-1} \left(\frac{4}{5} \sqrt{1 - \frac{1}{9}} + \frac{1}{3} \sqrt{1 - \frac{16}{25}} \right) \\ \Rightarrow \alpha &= \sin^{-1} \left(\frac{8\sqrt{2}}{15} + \frac{3}{15} \right) = \sin^{-1} \left(\frac{8\sqrt{2} + 3}{15} \right) \end{aligned}$$

$$\text{Since, } \frac{8\sqrt{2} + 3}{15} < 1$$

$$\therefore \alpha < \frac{\pi}{2}$$

$$\begin{aligned} \text{Now, } \beta &= \cos^{-1} \left(\frac{4}{5} \right) + \cos^{-1} \left(\frac{1}{3} \right) \\ \Rightarrow \beta &= \frac{\pi}{2} - \sin^{-1} \left(\frac{4}{5} \right) + \frac{\pi}{2} - \sin^{-1} \left(\frac{1}{3} \right) \\ &= \pi - \left(\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{1}{3} \right) \\ &= \pi - \alpha \\ \Rightarrow \beta &> \alpha \quad (\because \alpha < \frac{\pi}{2}) \end{aligned}$$

168 (c)

$$\begin{aligned} \because [\sin^{-1} x] &> [\cos^{-1} x] \\ \Rightarrow x &> 0 \end{aligned}$$

$$\text{Here, } [\cos^{-1} x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$$

$$\text{and, } [\sin^{-1} x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$$

$$\therefore x \in [\sin 1, 1)$$

$$\because \left[\frac{x}{2} \right] = 1$$

Or we say that $x \in [\sin 1, 1]$

169 (c)

We have,

$$\begin{aligned} \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 \\ &= \tan^{-1} 1 + \pi + \tan^{-1} \left(\frac{2+3}{1-2 \times 3} \right) \\ &= \tan^{-1} 1 + \pi + \tan^{-1}(-1) = \pi \end{aligned}$$

170 (d)

$$\text{We have, } (\sin^{-1} x)^3 + (\cos^{-1} x)^3$$

$$\begin{aligned} &= (\sin^{-1} + \cos^{-1} x)^3 \\ &\quad - 3 \sin^{-1} x \cos^{-1} x (\sin^{-1} x \\ &\quad + \cos^{-1} x) \end{aligned}$$

$$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x) \frac{\pi}{2}$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right)$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x \right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16} \right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[\left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \right] - \frac{3\pi^3}{32}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2} \left(\sin^{-1} x - \frac{\pi}{4} \right)^2$$

$$\therefore \text{The least value is } \frac{\pi^3}{32}$$

$$\text{Since, } \left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \leq \left(\frac{3\pi}{4} \right)^2$$

$$\therefore \text{The greatest value is } \frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$$

171 (d)

$$\text{Given, } \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{3}{4} \right) = \tan^{-1} \left(\frac{x}{3} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{3} + \frac{3}{4}}{1 - \frac{1}{3} \times \frac{3}{4}} \right) = \tan^{-1} \left(\frac{x}{3} \right)$$

$$\Rightarrow \frac{13}{9} = \frac{x}{3} \Rightarrow x = \frac{13}{3}$$

173 (b)

$$\tan^{-1} \frac{x}{y} - \tan^{-1} \frac{x-y}{x+y}$$

$$= \tan^{-1} \frac{x}{y} - \tan^{-1} \left[\frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} \right]$$

$$= \tan^{-1} \frac{x}{y} - \tan^{-1} 1 + \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} \frac{x}{y} + \cot^{-1} \frac{x}{y} - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

174 (c)

Given, two angles of triangle are $\tan^{-1} 2$ and $\tan^{-1} 3$.

Let third angle be θ . Then,

$$\tan^{-1} 2 + \tan^{-1} 3 + \theta = 180^\circ$$

$$\Rightarrow \tan^{-1} \left(\frac{2+3}{1-2 \times 3} \right) = 180^\circ - \theta$$

$$\Rightarrow \frac{5}{-5} = \tan(180^\circ - \theta) = -\tan \theta$$

$$\Rightarrow \tan \theta = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

175 (c)

$$8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, -\frac{5}{2}$$

$$\therefore -1 < -\frac{1}{4} < 1 \text{ and } -\frac{5}{2} < -1$$

$\therefore \sin^{-1}\left(-\frac{1}{4}\right)$ exists but $\sin^{-1}\left(-\frac{5}{2}\right)$ does not exist.

$\sec^{-1}\left(-\frac{5}{2}\right)$ exists but $\sec^{-1}\left(-\frac{1}{4}\right)$ does not exist.

$\tan^{-1}\left(-\frac{1}{4}\right)$ and $\tan^{-1}\left(-\frac{5}{2}\right)$ both exist.

176 (b)

We have, $\Sigma x_1 = \sin 2\beta$, $\Sigma x_1 x_2 = \cos 2\beta$, $\Sigma x_1 x_2 x_3 = \cos \beta$ and $x_1 x_2 x_3 x_4 = -\sin \beta$

$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$$= \tan^{-1} \left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4} \right)$$

$$= \tan^{-1} \left(\frac{\sin 2\beta - \cos \beta}{1 - \cos 2\beta - \sin \beta} \right)$$

$$= \tan^{-1} \left(\frac{(2 \sin \beta - 1) \cos \beta}{\sin \beta (2 \sin \beta - 1)} \right)$$

$$= \tan^{-1}(\cot \beta)$$

$$= \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \beta \right) \right) = \frac{\pi}{2} - \beta$$

177 (d)

$$\cos(2 \cos^{-1} x + \sin^{-1} x)$$

$$= \cos[2(\cos^{-1} x + \sin^{-1} x) - \sin^{-1} x]$$

$$= \cos(\pi - \sin^{-1} x) = -\cos(\sin^{-1} x)$$

$$= -\cos \left[\sin^{-1} \left(-\frac{1}{5} \right) \right] \quad \left(\because x = \frac{1}{5} \right)$$

$$= -\cos \left(\cos^{-1} \frac{2\sqrt{6}}{5} \right)$$

$$= -\frac{2\sqrt{6}}{5}$$

178 (a)

$$\therefore \cot^{-1} x + \sin^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{x} + \tan^{-1} \frac{1}{2} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} 1 - \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} \left(\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \right)$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} \frac{1}{3}$$

$$\Rightarrow x = 3$$

179 (b)

We have,

$$\cos(2 \tan^{-1} x) = \frac{1}{2}$$

$$\Rightarrow 2 \tan^{-1} x = \frac{\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

180 (c)

$$\text{Given that, } \sin^{-1} \frac{1}{3} + \sin^{-1} \frac{2}{3} = \sin^{-1} x$$

$$\Rightarrow \sin^{-1} \left(\frac{1}{3} \sqrt{1 - \frac{4}{9}} + \frac{2}{3} \sqrt{1 - \frac{1}{9}} \right) = \sin^{-1} x$$

$$\Rightarrow \sin^{-1} \left(\frac{1}{3} \cdot \frac{\sqrt{5}}{3} + \frac{2}{3} \cdot \frac{\sqrt{8}}{3} \right) = \sin^{-1} x$$

$$\Rightarrow \sin^{-1} \left(\frac{\sqrt{5} + 4\sqrt{2}}{9} \right) = \sin^{-1} x$$

$$\therefore x = \left(\frac{\sqrt{5} + 4\sqrt{2}}{9} \right)$$

181 (c)

Since, $\tan^{-1} x$ and $\cot^{-1} x$ exists for all $x \in \mathbb{R}$ and

$\cos^{-1}(2-x)$ exists, if $-1 \leq 2-x \leq 1$

$$\therefore \tan^{-1} x - \cot^{-1} x = \cos^{-1}(2-x)$$

Is possible only if $1 \leq x \leq 3$.

Thus the solution of given equation is $[1, 3]$.

182 (a)

$$\text{Since, } 0 \leq \cos^{-1} \left(\frac{x^2}{2} + \sqrt{1-x^2} \sqrt{1-\frac{x^2}{4}} \right) \leq \frac{\pi}{2}$$

Because $\cos^{-1} x$ is in first quadrant when x is positive

$$\text{And } \cos^{-1} \frac{x}{2} - \cos^{-1} x \geq 0$$

$$\text{So, } \cos^{-1} \frac{x}{2} \geq \cos^{-1} x$$

$$\text{Also, } \left| \frac{x}{2} \right| \leq 1, |x| \leq 1 \Rightarrow |x| \leq 1$$

183 (b)

We have,

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \left(\frac{8}{31} \right)$$

$$\Rightarrow \tan^{-1} \left\{ \frac{2x}{1-(x^2-1)} \right\} = \tan^{-1} \left(\frac{8}{31} \right)$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 8x^2 + 62x - 16 = 0 \Rightarrow (4x-1)(x+8) = 0$$

$$\Rightarrow x = \frac{1}{4}, -8$$

184 (b)

$$3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2}$$

$$= \frac{\pi}{3}$$

On putting $x = \tan \theta$, we get

$$3 \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) - 4 \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$+ 2 \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) = \frac{\pi}{3}$$

$$\begin{aligned} &\Rightarrow 3 \sin^{-1}(\sin 2\theta) \\ &\quad - 4 \cos^{-1}(\cos 2\theta) \\ &\quad + 2 \tan^{-1}(\tan 2\theta) = \frac{\pi}{3} \\ &\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3} \\ &\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3} \\ &\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \\ &\Rightarrow x = \tan \frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}} \end{aligned}$$

185 (b)

Given that, $x^2 + y^2 + z^2 = r^2$
 Now, $\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$

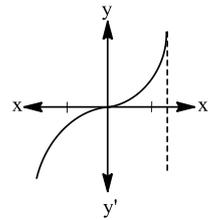
$$\begin{aligned} &= \tan^{-1}\left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \left(\frac{x^2+y^2+z^2}{r^2}\right)}\right] \\ &= \tan^{-1}\left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \frac{r^2}{r^2}}\right] \\ &= \tan^{-2} \infty = \frac{\pi}{2} \end{aligned}$$

186 (d)

We have, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$\begin{aligned} &= (\sin^{-1} + \cos^{-1} x)^3 \\ &\quad - 3 \sin^{-1} x \cos^{-1} x (\sin^{-1} x \\ &\quad + \cos^{-1} x) \\ &= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x) \frac{\pi}{2} \\ &= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x\right) \\ &= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2 \\ &= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x\right] \\ &= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right] \\ &= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32} \\ &= \frac{\pi^3}{32} + \frac{3\pi}{2} \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \\ &\therefore \text{The least value is } \frac{\pi^3}{32} \\ &\text{Since, } \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \left(\frac{3\pi}{4}\right)^2 \\ &\therefore \text{The greatest value is } \frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8} \end{aligned}$$

187 (d)



Hence, the line $x = 1$ is a tangent to the function.

188 (c)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$ and $\sqrt{1-x^2} = \cos \theta$
 Now,

$$\begin{aligned} -1 \leq x \leq -\frac{1}{\sqrt{2}} &\Rightarrow -1 \leq \sin \theta \leq -\frac{1}{\sqrt{2}} \Rightarrow -\frac{\pi}{2} \leq \theta \\ &\leq -\frac{\pi}{4} \\ \therefore \sin^{-1}(2x\sqrt{1-x^2}) &= \sin^{-1}(\sin 2\theta) \\ &= \sin^{-1}(-\sin(\pi + 2\theta)) \\ &= \sin^{-1}(\sin(-\pi - 2\theta)) \\ &= -\pi - 2\theta \left[\because -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \leq -\pi - 2\theta \leq 0 \right] \\ &= -\pi - 2 \sin^{-1} x \end{aligned}$$

189 (a)

$$\begin{aligned} \theta &= \tan^{-1} \frac{a(a+b+c)}{bc} \\ &\quad + \tan^{-1} \frac{b(a+b+c)}{ca} \\ &\quad + \tan^{-1} \frac{c(a+b+c)}{ab} \end{aligned}$$

Let $s^2 = \frac{a+b+c}{abc}$
 Hence, $\theta = \tan^{-1} \sqrt{a^2 s^2} + \tan^{-1} \sqrt{b^2 s^2} + \tan^{-1} \sqrt{c^2 s^2}$

$$\begin{aligned} &= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs) \\ &= \tan^{-1} \left[\frac{as + bs + cs - abc s^3}{1 - abs^2 - acs^2 - bcs^2} \right] \\ \text{Hence, } \tan \theta &= \frac{s[a+b+c] - abc s^3}{1 - (ab+bc+ca)s^2} \\ &= \frac{s[(a+b+c) - (a+b+c)]}{1 - s^2(ab+bc+ca)} = 0 \end{aligned}$$

190 (a)

We have,
 $\theta \in [4\pi, 5\pi] \Rightarrow -4\pi + \theta \in [0, \pi]$
 Also,
 $\cos(-4\pi + \theta) = \cos(4\pi - \theta) = \cos \theta$
 $\therefore \cos^{-1}(\cos \theta) = \cos^{-1}\{\cos(-4\pi + \theta)\}$
 $= -4\pi + \theta$

191 (c)

Given, $\sin^{-1} x = 2 \sin^{-1} a$

Since, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2} \leq 2 \sin^{-1} a \leq \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \leq \sin^{-1} a \leq \frac{\pi}{4}$$

$$\Rightarrow \sin\left(-\frac{\pi}{4}\right) \leq a \leq \sin\frac{\pi}{4}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$$

$$\therefore |a| \leq \frac{1}{\sqrt{2}}$$

192 (b)

We have,

$$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$$

$$\Rightarrow \sin^{-1} 2x = \frac{\pi}{3} - \sin^{-1} x$$

$$\Rightarrow 2x = \sin\left(\frac{\pi}{3} - \sin^{-1} x\right)$$

$$\Rightarrow 2x = \frac{\sqrt{3}}{2} \cos(\sin^{-1} x) - \frac{1}{2} \sin(\sin^{-1} x)$$

$$\Rightarrow 2x = \frac{\sqrt{3}}{2} \times \sqrt{1-x^2} - \frac{x}{2}$$

$$\Rightarrow \frac{5x}{2} = \frac{\sqrt{3}}{2} \sqrt{1-x^2}$$

$$\Rightarrow 25x^2 = 3 - 3x^2$$

$$\Rightarrow x = \pm \frac{1}{2} \sqrt{\frac{3}{7}} \Rightarrow x = \frac{1}{2} \sqrt{\frac{3}{7}} \quad [\because \text{RHS} > 0 \therefore x > 0]$$

193 (c)

Since, $2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow -\frac{\pi}{2} \leq 2 \sin^{-1} x \leq \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{4} \leq \sin^{-1} x \leq \frac{\pi}{4}$$

$$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

194 (c)

$$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$$

$$\Rightarrow \cos^{-1}(xy - \sqrt{1-y^2}\sqrt{1-x^2}) = \pi - \cos^{-1} z$$

$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1} z)$$

$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$

$$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$$

On squaring both sides, we get

$$x^2y^2 + z^2 + 2xyz - 1 - x^2 - y^2 + x^2y^2$$

$$\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz$$

195 (b)

$$\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$$

$$= \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$$

$$= \cot \tan^{-1} \left[\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{1}{2}} \right]$$

$$= \cot \left[\tan^{-1} \left(\frac{17}{6} \right) \right]$$

$$= \frac{6}{17}$$

196 (b)

We have, $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

$$\text{or } x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$$

$$\text{or } x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$$

$$\text{or } (x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$$

$$\text{or } x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 + 4x^2y^2z^2 - 4y^2z^2 = 0$$

$$\text{or } x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$$

$$\therefore k = 2$$

197 (b)

Given, $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$$\Rightarrow \tan^{-1} \left(\frac{x+y+z-xyz}{1-xy-yz-zx} \right) = \pi$$

$$\Rightarrow \frac{x+y+z-xyz}{1-xy-yz-zx} = 0$$

$$\Rightarrow x+y+z = xyz$$

198 (b)

Since, 1 radian = $\frac{7\pi}{22}$

$$\therefore 12 \text{ radian} = \frac{7\pi}{22} \times 12 = \frac{42\pi}{11} = 4\pi - \frac{2\pi}{11}$$

And 14 radian = $\frac{7\pi}{22} \times 14 = \frac{49\pi}{11}$

$$= 4\pi + \frac{5\pi}{11}$$

$$\therefore \cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$$

$$= \cos^{-1} \cos \left(4\pi - \frac{2\pi}{11} \right)$$

$$- \sin^{-1} \left[\sin \left(4\pi + \frac{5\pi}{11} \right) \right]$$

$$= \cos^{-1} \cos \left(\frac{2\pi}{11} \right) - \sin^{-1} \left(\sin \frac{5\pi}{11} \right)$$

$$= 4\pi - 12 - (14 - 4\pi) = 8\pi - 26$$

199 (b)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$\frac{1}{2} \leq x \leq 1 \Rightarrow \frac{1}{2} \leq \sin \theta \leq 1 \Rightarrow \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$$

$$\leq 3\theta \leq \frac{3\pi}{2}$$

Now,

$$\begin{aligned}
& \sin^{-1}(3x - 4x^3) \\
&= \sin^{-1}(\sin 3\theta) \\
&= \sin^{-1}(\sin(\pi - 3\theta)) \\
&= \pi - 3\theta \left[\because \frac{\pi}{2} \leq 3\theta \leq \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} \leq \pi - 3\theta \leq \frac{\pi}{2} \right] \\
&= \pi - 3 \sin^{-1} x
\end{aligned}$$

200 (c)

$$\begin{aligned}
\cos(4095^\circ) &= \cos(45 \times 90^\circ + 45^\circ) \\
&= -\sin 45^\circ \\
&= -\sin \frac{\pi}{4} \\
&= \sin\left(-\frac{\pi}{4}\right) \\
\therefore \sin^{-1}\{\cos(4095^\circ)\} \\
&= \sin^{-1} \sin\left(-\frac{\pi}{4}\right) \\
&= -\frac{\pi}{4}
\end{aligned}$$

201 (d)

We have,

$$\begin{aligned}
\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} &= \tan^{-1} \left\{ \frac{1/2 + 2/9}{1 - 1/4 \times 2/9} \right\} \\
&= \tan^{-1} \left(\frac{1}{2} \right)
\end{aligned}$$

202 (c)

$$\begin{aligned}
& \text{Since, } -\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2} \\
& \therefore \sin^{-1} \alpha = \frac{\pi}{2}, \sin^{-1} \beta = \frac{\pi}{2} \text{ and } \sin^{-1} \gamma = \frac{\pi}{2} \\
& \therefore \alpha = \beta = \gamma = 1 \\
& \text{Thus, } \alpha\beta + \alpha\gamma + \gamma\beta = 3
\end{aligned}$$

203 (d)

We know that,

$$\begin{aligned}
\tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\
&= \frac{\frac{x\sqrt{3}}{2k-x} - \frac{2x-k}{k\sqrt{3}}}{1 + \frac{x\sqrt{3}}{2k-x} \cdot \frac{2x-k}{k\sqrt{3}}} = \frac{1}{\sqrt{3}} \\
&\Rightarrow A - B = 30^\circ
\end{aligned}$$

204 (c)

$$\begin{aligned}
& \text{Given that, } \angle A = \tan^{-1} 2, \angle B = \tan^{-1} 3 \\
& \text{We know that, } \angle A + \angle B + \angle C = \pi \\
& \Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi \\
& \Rightarrow \tan^{-1} \left(\frac{2+3}{1-2 \times 3} \right) + \angle C = \pi \\
& \Rightarrow \tan^{-1}(-1) + \angle C = \pi \\
& \Rightarrow \frac{3\pi}{4} + \angle C = \pi \\
& \Rightarrow \angle C = \frac{\pi}{4}
\end{aligned}$$

205 (a)

We have,

$$\begin{aligned}
(\tan^{-1} x)^2 + (\cot^{-1} x)^2 &= \frac{5\pi^2}{8} \\
\Rightarrow (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) \\
&= \frac{5\pi^2}{8} \\
\Rightarrow \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} \tan^{-1} x + 2(\tan^{-1} x)^2 &= \frac{5\pi^2}{8} \\
\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} &= 0 \\
\Rightarrow \tan^{-1} x = -\frac{\pi}{6}, \frac{3\pi}{4} \Rightarrow \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1
\end{aligned}$$

206 (d)

$$\begin{aligned}
4 \tan^{-1} \frac{1}{5} &= 2 \left[2 \tan^{-1} \frac{1}{5} \right] \\
&= 2 \tan^{-1} \frac{\frac{2}{5}}{1 - \frac{1}{25}} = 2 \tan^{-1} \frac{5}{12} \\
&= \tan^{-1} \frac{\frac{10}{12}}{1 - \frac{25}{144}} \\
&= \tan^{-1} \frac{120}{119} \\
\text{So, } 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239} &= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239} \\
&= \tan^{-1} \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \times \frac{1}{239}} \\
&= \tan^{-1} \frac{(120 \times 239) - 119}{(119 \times 239) + 120} \\
&= \tan^{-1} \frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}
\end{aligned}$$

207 (b)

$$\begin{aligned}
& \text{Given, } \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6} \\
& \Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x \right) - \cos^{-1} x = \frac{\pi}{6} \\
& \Rightarrow \cos^{-1} x = \frac{\pi}{6} \\
& \Rightarrow x = \frac{\sqrt{3}}{2}
\end{aligned}$$

208 (d)

$$\begin{aligned}
& \therefore f(x) = ax + b \\
& \therefore f'(x) = a > 0 \\
& \Rightarrow f(x) \text{ is an increasing function.} \\
& \therefore f(-1) = 0 \text{ and } f(1) = 2 \\
& \text{Or } -a + b = 0 \\
& \text{and } a + b = 2 \\
& \text{then, } a = b = 1 \\
& \Rightarrow f(x) = x + 1 \\
& \text{Now, } \cot [\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18] \\
&= \cot \left\{ \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{1}{8} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
&= \cot \left\{ \tan^{-1} \left(\frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \cdot \frac{1}{8}} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{15}{35} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{3}{11} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{\frac{3}{11} + \frac{1}{18}}{1 - \frac{3}{11} \cdot \frac{1}{18}} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{65}{195} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{1}{3} \right) \right\} \\
&= \cot(\cot^{-1} 3) = 3 = 1 + 2 = f(2)
\end{aligned}$$

209 (d)

$$\begin{aligned}
\cos \left(\frac{33\pi}{5} \right) &= \cos \left(6\pi + \frac{3\pi}{5} \right) = \cos \frac{3\pi}{5} \\
&= \sin \left(\frac{\pi}{2} - \frac{3\pi}{5} \right) = \sin \left(-\frac{\pi}{10} \right) \\
&= \sin^{-1} \sin \left(-\frac{\pi}{10} \right) = -\frac{\pi}{10}
\end{aligned}$$

210 (c)

Given equation is

$$\begin{aligned}
&\cos^{-1} x + \cos^{-1} 2x + \pi = 0 \\
\Rightarrow &\cos^{-1} x + \cos^{-1} 2x = -\pi \\
\Rightarrow &\cos^{-1} (x \cdot 2x - \sqrt{1-x^2} \sqrt{1-4x^2}) = -\pi \\
\Rightarrow &2x^2 - \sqrt{1-x^2} \sqrt{1-4x^2} = -1 \\
\Rightarrow &(1+2x^2) = \sqrt{1-x^2} \sqrt{1-4x^2}
\end{aligned}$$

On squaring both sides, we get

$$\begin{aligned}
1 + 4x^2 + 4x^2 &= (1-x^2)(1-4x^2) \\
\Rightarrow 1 + 4x^4 + 4x^2 &= 1 - 5x^2 + 4x^4 \\
\Rightarrow 9x^2 &= 0 \\
\Rightarrow x &= 0
\end{aligned}$$

But $x = 0$ is not satisfied the given equation.

\therefore The number of real solution is zero.

211 (c)

Let $\cos^{-1} \left(\frac{\sqrt{5}}{3} \right) = \alpha$. Then,

$$\cos \alpha = \frac{\sqrt{5}}{3}, \text{ where } 0 < \alpha < \frac{\pi}{2}$$

Now,

$$\tan \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\Rightarrow \tan \frac{\alpha}{2} = \sqrt{\frac{1 - \sqrt{5}/3}{1 + \sqrt{5}/3}}$$

$$\Rightarrow \tan \frac{\alpha}{2} = \sqrt{\frac{3 - \sqrt{5}}{3 + \sqrt{5}}} = \sqrt{\frac{(3 - \sqrt{5})^2}{9 - 5}} = \frac{1}{2}(3 - \sqrt{5})$$

$$\therefore \tan \left\{ \frac{1}{2} \cos^{-1} \left(\frac{\sqrt{5}}{3} \right) \right\} = \frac{3 - \sqrt{5}}{2}$$

212 (c)

$$\begin{aligned}
\sin \left[2 \cos^{-1} \frac{\sqrt{5}}{3} \right] &= \sin \left[\cos^{-1} \left\{ 2 \cdot \left(\frac{\sqrt{5}}{3} \right)^2 - 1 \right\} \right] \\
&[\because 2 \cos^{-1} x = \cos^{-1} (2x^2 - 1)] \\
&= \sin \left[\cos^{-1} \left(\frac{1}{9} \right) \right] \\
&= \sin \left[\sin^{-1} \sqrt{1 - \left(\frac{1}{9} \right)^2} \right] \\
&[\because \cos^{-1} x = \sin^{-1} (\sqrt{1-x^2})] \\
&= \frac{4\sqrt{5}}{9}
\end{aligned}$$

213 (c)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$x < -\frac{1}{\sqrt{3}} \Rightarrow \tan \theta < -\frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{6}$$

Now,

$$\tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right)$$

$$= \tan^{-1} (\tan 3\theta)$$

$$= \tan^{-1} (\tan(\pi + 3\theta)) = \pi + 3\theta = \pi + 3 \tan^{-1} x$$

214 (c)

$$\begin{aligned}
&\cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right) \\
&= \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}
\end{aligned}$$

215 (d)

Given, $\sin \left[\sin^{-1} \left(\frac{1}{5} \right) + \cos^{-1} x \right] = 1$

$$\therefore \sin^{-1} \left(\frac{1}{5} \right) + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \left(\frac{1}{5} \right) = \sin^{-1} x$$

$$\Rightarrow x = \frac{1}{5}$$

216 (c)

Given that, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$

$$\because 0 \leq \cos^{-1} x \leq \pi$$

Similarly, $0 \leq \cos^{-1} y \leq \pi$

And $0 \leq \cos^{-1} z \leq \pi$

Here, $\cos^{-1} x \cos^{-1} y = \cos^{-1} z = \pi$

$$\Rightarrow x = y = z = \cos \pi = -1$$

$$\therefore xy + yz + zx$$

$$= (-1)(-1) + (-1)(-1)$$

$$+ (-1)(-1)$$

$$= 1+1+1=3$$

217 (a)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$0 \leq x \leq \infty \Rightarrow 0 \leq \theta < \frac{\pi}{2} \Rightarrow 0 \leq 2\theta < \pi$$

Now,

$$\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = \cos^{-1}(\cos 2\theta)$$

$$= 2\theta \quad [\because 0 \leq \theta < \frac{\pi}{2} \Rightarrow 0 \leq 2\theta < \pi]$$

$$= 2 \tan^{-1} x$$

218 (d)

$$\begin{aligned} \cos[2 \tan^{-1}(-7)] &= \cos \left[\cos^{-1} \left(\frac{1-49}{1+49} \right) \right] \\ &= \cos \left[\pi - \cos^{-1} \left(\frac{48}{50} \right) \right] \\ &= -\cos \cos^{-1} \left(\frac{48}{50} \right) \\ &= -\frac{24}{25} \end{aligned}$$

219 (d)

We have,

$$\begin{aligned} \sin \left(4 \tan^{-1} \frac{1}{3} \right) &= 2 \sin \left(2 \tan^{-1} \frac{1}{3} \right) \cos \left(2 \tan^{-1} \frac{1}{3} \right) \\ &= 2 \sin \left(\tan^{-1} \frac{3}{4} \right) \cos \left(\tan^{-1} \frac{3}{4} \right) \\ &= 2 \sin \left(\sin^{-1} \frac{3}{5} \right) \cos \left(\cos^{-1} \frac{4}{5} \right) = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25} \end{aligned}$$

And,

$$\begin{aligned} \cos \left(2 \tan^{-1} \frac{1}{7} \right) &= \cos \left(\tan^{-1} \frac{7}{24} \right) = \cos \left(\cos^{-1} \frac{24}{25} \right) \\ &= \frac{24}{25} \end{aligned}$$

Hence, the value of given expression is 0

220 (c)

Given that, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$

$$\because 0 \leq \cos^{-1} x \leq \pi$$

Similarly, $0 \leq \cos^{-1} y \leq \pi$

And $0 \leq \cos^{-1} z \leq \pi$

Here, $\cos^{-1} x \cos^{-1} y = \cos^{-1} z = \pi$

$$\Rightarrow x = y = z = \cos \pi = -1$$

$$\therefore xy + yz + zx$$

$$\begin{aligned} &= (-1)(-1) + (-1)(-1) \\ &\quad + (-1)(-1) \end{aligned}$$

$$= 1+1+1=3$$

221 (b)

Given expression

$$\begin{aligned} &= \tan \left[\tan^{-1} \frac{a_2 - a_1}{1 + a_1 a_2} \right. \\ &\quad \left. + \tan^{-1} \frac{a_3 - a_2}{1 + a_2 a_3} + \dots + \tan^{-1} \frac{a_n - a_{n-1}}{1 + a_n a_{n-1}} \right] \end{aligned}$$

$$\begin{aligned} &= \tan[\tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \\ &\quad \tan^{-1} a_2 + \dots + \tan^{-1} a_n - \tan^{-1} a_{n-1}] \\ &= \tan[\tan^{-1} a_n - \tan^{-1} a_1] = \frac{a_n - a_1}{1 + a_1 a_n} \\ &= \frac{(n-1)d}{1 + a_1 a_n} \end{aligned}$$

222 (a)

$$\begin{aligned} \sin \left(2 \sin^{-1} \sqrt{\frac{63}{65}} \right) &= \sin \left(\sin^{-1} 2 \sqrt{\frac{63}{65}} \sqrt{1 - \frac{63}{65}} \right) \\ &= \sin \left(\sin^{-1} \frac{2\sqrt{126}}{65} \right) = \frac{2\sqrt{126}}{65} \end{aligned}$$

223 (b)

$$\begin{aligned} \sin^{-1} x - \cos^{-1} x &= \frac{\pi}{6} \\ \Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x \right) - \cos^{-1} x &= \frac{\pi}{6} \\ \Rightarrow 2 \cos^{-1} x &= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \\ \Rightarrow \cos^{-1} x &= \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2} \end{aligned}$$

224 (c)

We know that

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right) = 2 \tan^{-1} x \text{ for all } x \in [-1, 1]$$

And,

$$\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = 2 \tan^{-1} x \text{ for all } x \in [0, \infty)$$

$$\therefore \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = 4 \tan^{-1} x \text{ for all } x \in [0, 1]$$

225 (c)

$$\begin{aligned} &\tan^{-1} \left(\frac{c_1 - y}{c_1 y + x} \right) \\ &\quad + \tan^{-1} \left(\frac{c_2 - c_1}{1 + c_2 c_1} \right) \\ &\quad + \tan^{-1} \left(\frac{c_3 - c_2}{1 + c_3 c_2} \right) + \dots + \tan^{-1} \frac{1}{c_n} \\ &= \tan^{-1} \left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y} \cdot \frac{1}{c_1}} \right) \\ &\quad + \tan^{-1} \left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}} \right) \\ &\quad + \tan^{-1} \left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}} \right) + \dots + \tan^{-1} \frac{1}{c_n} \\ &= \tan^{-1} \frac{x}{y} - \tan^{-1} \frac{1}{c_1} \\ &\quad + \tan^{-1} \frac{1}{c_1} - \tan^{-1} \frac{1}{c_2} + \tan^{-1} \frac{1}{c_2} \\ &\quad - \tan^{-1} \frac{1}{c_3} + \dots + \tan^{-1} \frac{1}{c_{n-1}} - \tan^{-1} \frac{1}{c_n} + \tan^{-1} \frac{1}{c_n} \end{aligned}$$

$$= \tan^{-1}\left(\frac{x}{y}\right)$$

226 (c)

We have, $\tan^{-1} a + \tan^{-1} b = \sin^{-1} 1 - \tan^{-1} c$

$$\Rightarrow \tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left\{ \frac{a + b + c - abc}{1 - (ab + bc + ca)} \right\} = \frac{\pi}{2}$$

$$\Rightarrow ab + bc + ca = 1$$

227 (d)

$$\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$$

$$= \cos \left[\tan^{-1} \left\{ \sin \left(\sin^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right\} \right]$$

$$= \cos \left[\tan^{-1} \frac{1}{\sqrt{1+x^2}} \right]$$

$$= \cos \left[\cos^{-1} \sqrt{\frac{1+x^2}{2+x^2}} \right]$$

$$= \sqrt{\frac{1+x^2}{2+x^2}}$$

228 (b)

$$\because 0 \leq \cos^{-1} x \leq \pi$$

And $0 < \cot^{-1} x < \pi$

Given, $[\cot^{-1} x] + [\cos^{-1} x] = 0$

$$\Rightarrow [\cot^{-1} x] = 0 \text{ and } [\cos^{-1} x] = 0$$

$$\Rightarrow 0 < \cot^{-1} x < 1 \text{ and } 0 \leq \cos^{-1} x < 1$$

$$\therefore x \in (\cot 1, \infty) \text{ and } x \in (\cos 1, 1)$$

$$\Rightarrow x \in (\cot 1, 1)$$

229 (d)

Given, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$

And we know that $0 \leq \cos^{-1} x \leq \pi$

\therefore We know

$$\cos^{-1} x = \pi, \cos^{-1} y = \pi, \cos^{-1} z = \pi$$

$$\therefore x = y = z = \cos \pi = -1$$

$$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1) = 3$$

230 (c)

We have,

$$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \tan^{-1}\left(\frac{1}{1-x}\right)$$

$$\Rightarrow 1+x = \frac{1}{1-x} \Rightarrow 1-x^2 = 1 \Rightarrow x = 0$$

232 (a)

Given equation is

$$2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$$

Which is not possible as $\cos^{-1} x \in [0, \pi]$.

233 (a)

We know that $|\sin^{-1} x| \leq \frac{\pi}{2}$

$$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

$$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow x = y = z = \sin \frac{\pi}{2} = 1$$

$$\therefore x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$$

$$= 3 - \frac{9}{3} = 0$$

234 (d)

We have,

$$\sin(\sin^{-1} 1/5 + \cos^{-1} x) = 1$$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x \Rightarrow \sin^{-1} \frac{1}{5} = \sin^{-1} x \Rightarrow x = \frac{1}{5}$$

235 (d)

$$\because f(x) = ax + b$$

$$\therefore f'(x) = a > 0$$

$\Rightarrow f(x)$ is an increasing function.

$$\therefore f(-1) = 0 \text{ and } f(1) = 2$$

Or $-a + b = 0$

and $a + b = 2$

then, $a = b = 1$

$$\Rightarrow f(x) = x + 1$$

Now, $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$

$$= \cot \left\{ \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{1}{8} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\}$$

$$= \cot \left\{ \tan^{-1} \left(\frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \cdot \frac{1}{8}} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\}$$

$$= \cot \left\{ \tan^{-1} \left(\frac{15}{35} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\}$$

$$= \cot \left\{ \tan^{-1} \left(\frac{3}{11} \right) + \tan^{-1} \left(\frac{1}{18} \right) \right\}$$

$$= \cot \left\{ \tan^{-1} \left(\frac{\frac{3}{11} + \frac{1}{18}}{1 - \frac{3}{11} \cdot \frac{1}{18}} \right) \right\}$$

$$\begin{aligned}
&= \cot \left\{ \tan^{-1} \left(\frac{65}{195} \right) \right\} \\
&= \cot \left\{ \tan^{-1} \left(\frac{1}{3} \right) \right\} \\
&= \cot(\cot^{-1} 3) = 3 = 1 + 2 = f(2)
\end{aligned}$$

236 (d)

$$\begin{aligned}
\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} &= \tan^{-1} \frac{2x}{1-x^2} \\
\Rightarrow 2 \tan^{-1} a - 2 \tan^{-1} b &= 2 \tan^{-1} x \\
\Rightarrow \tan^{-1} \frac{a-b}{1+ab} &= \tan^{-1} x \\
\Rightarrow x &= \frac{a-b}{1+ab}
\end{aligned}$$

237 (c)

We have,

$$\begin{aligned}
\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5} &= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} \\
&= \tan^{-1} 1 = \frac{\pi}{4}
\end{aligned}$$

238 (c)

$$\begin{aligned}
&\tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) \right] \\
&= \tan \left[\frac{1}{2} \cdot 2 \tan^{-1} a + \frac{1}{2} \cdot 2 \tan^{-1} a \right] \\
&= \tan(2 \tan^{-1} a) \\
&= \tan \left[\tan^{-1} \left(\frac{2a}{1-a^2} \right) \right] \\
&= \frac{2a}{1-a^2}
\end{aligned}$$

239 (a)

$$\begin{aligned}
\text{Given, } \tan^{-1} x + \tan^{-1} y &= \frac{\pi}{4} \\
\therefore \tan^{-1} \left(\frac{x+y}{1-xy} \right) &= \frac{\pi}{4} \\
\Rightarrow \frac{x+y}{1-xy} &= 1 \\
\Rightarrow x+y+xy &= 1
\end{aligned}$$

240 (a)

$$\begin{aligned}
\text{Let } \cos^{-1} x &= \theta. \text{ Then, } x = \cos \theta \\
\text{Also, } 0 \leq x \leq 1 &\Rightarrow 0 \leq \cos \theta \leq 1 \Rightarrow 0 \leq \theta \leq \frac{\pi}{2} \\
\text{Now,} \\
\cos^{-1}(2x^2 - 1) &= \cos^{-1}(2 \cos^2 \theta - 1) \\
&= \cos^{-1}(\cos 2\theta) \\
&= 2\theta = 2 \cos^{-1} x \quad [\because 0 \leq 2\theta \leq \pi]
\end{aligned}$$

241 (a)

$$\begin{aligned}
\theta &= \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}} \\
&\quad + \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}} \\
&\quad + \tan^{-1} \sqrt{\frac{c(a+b+c)}{ab}}
\end{aligned}$$

$$\text{Let } s^2 = \frac{a+b+c}{abc}$$

$$\begin{aligned}
\text{Hence, } \theta &= \tan^{-1} \sqrt{a^2 s^2} + \tan^{-1} \sqrt{b^2 s^2} + \tan^{-1} \sqrt{c^2 s^2} \\
&= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs) \\
&= \tan^{-1} \left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - acs^2 - bcs^2} \right]
\end{aligned}$$

$$\begin{aligned}
\text{Hence, } \tan \theta &= \frac{[s(a+b+c) - abcs^2]}{[1 - (ab+bc+ca)s^2]} \\
&= \frac{[s[(a+b+c) - (a+b+c)]]}{[1 - s^2(ab+bc+ca)]} = 0
\end{aligned}$$

242 (a)

$$\begin{aligned}
\text{Given, } \sin^{-1} x + \sin^{-1} y &= \frac{\pi}{2} \\
\therefore \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y &= \frac{\pi}{2} \\
\Rightarrow \cos^{-1} x + \cos^{-1} y &= \frac{\pi}{2}
\end{aligned}$$

243 (c)

$$\begin{aligned}
&\tan^{-1} \left(\frac{c_1 - y}{c_1 y + x} \right) \\
&\quad + \tan^{-1} \left(\frac{c_2 - c_1}{1 + c_2 c_1} \right) \\
&\quad + \tan^{-1} \left(\frac{c_3 - c_2}{1 + c_3 c_2} \right) + \dots + \tan^{-1} \frac{1}{c_n} \\
&= \tan^{-1} \left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y} \cdot \frac{1}{c_1}} \right) \\
&\quad + \tan^{-1} \left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}} \right) \\
&\quad + \tan^{-1} \left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}} \right) + \dots + \tan^{-1} \frac{1}{c_n} \\
&= \tan^{-1} \frac{x}{y} - \tan^{-1} \frac{1}{c_1} \\
&\quad + \tan^{-1} \frac{1}{c_1} - \tan^{-1} \frac{1}{c_2} + \tan^{-1} \frac{1}{c_2} \\
&\quad - \tan^{-1} \frac{1}{c_3} + \dots + \tan^{-1} \frac{1}{c_{n-1}} - \tan^{-1} \frac{1}{c_n} + \tan^{-1} \frac{1}{c_n} \\
&= \tan^{-1} \left(\frac{x}{y} \right)
\end{aligned}$$

244 (d)

We have,

$$\begin{aligned} & \cos\{\tan^{-1}(\tan 2)\} \\ &= \cos\{\tan^{-1}(\tan(2 - \pi))\} = \cos(2 - \pi) \\ &= \cos(\pi - 2) = -\cos 2 \end{aligned}$$

245 (c)

$$\text{We have, } \tan^{-1} \frac{x-1}{x+2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \left[\frac{2x(x+2)}{x^2 + 4 + 4x - x^2 + 1} \right] = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$$

$$\Rightarrow 2x^2 + 4x = 4x + 5$$

$$\Rightarrow x = \pm \sqrt{\frac{5}{2}}$$

246 (a)

Given series can be rewritten as

$$\sum_{r=1}^{\infty} \tan^{-1} \left(\frac{1}{1+r+r^2} \right)$$

$$\text{Now, } \tan^{-1} \left(\frac{1}{1+r+r^2} \right)$$

$$= \tan^{-1} \left(\frac{r+1-r}{1+r(r+1)} \right)$$

$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$

$$\therefore \sum_{r=1}^n [\tan^{-1}(r+1) - \tan^{-1}(r)]$$

$$= \tan^{-1}(n+1) - \tan^{-1}(1)$$

$$= \tan^{-1}(n+1) - \frac{\pi}{4}$$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

247 (c)

$$\text{Here, } x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$$

$$\text{But } -1 \leq (x^2 - 2x + 2) \leq 1$$

Which is possible only when

$$x^2 - 2x + 2 = 1$$

$$\Rightarrow x = 1$$

$$\text{Then, } a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$$

$$\Rightarrow a + \frac{\pi}{2} + 0 = 0$$

$$\Rightarrow a = -\frac{\pi}{2}$$

248 (c)

$$\text{Given that, } \tan^{-1} x - \tan^{-1} y = \tan^{-1} A$$

$$\Rightarrow \tan^{-1} \left(\frac{x-y}{1+xy} \right) = \tan^{-1} A$$

$$\text{Hence, } A = \frac{x-y}{1+xy}$$

249 (a)

$$\therefore \tan^{-1} \left(\frac{a}{x} \right) + \tan^{-1} \left(\frac{b}{x} \right) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} \right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} = \tan \frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$$

$$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

250 (d)

$$\cos^{-1} \frac{1}{2} + 2 \sin^{-1} \frac{1}{2} = \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{2\pi}{3}$$

251 (d)

We have,

$$4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$$

$$= 2 \tan^{-1} \left(\frac{2/5}{1 - 1/25} \right) - \tan^{-1} \frac{1}{239}$$

$$= 2 \tan^{-1}(5/12) - \tan^{-1} 1/239$$

$$= \tan^{-1} \left(\frac{2(2/12)}{(1 - 5/12)^2} \right) - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \left(\frac{120/119 - 1/239}{1 + 120/119 \times 1/239} \right)$$

$$= \tan^{-1} \left(\frac{28569}{28569} \right) = \tan^{-1}(1) = \frac{\pi}{4}$$

252 (c)

$$\text{Since, } 2 \sin^{-1} = \sin^{-1}(2x\sqrt{1-x^2})$$

$$\text{Range of right hand side is } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\Rightarrow -\frac{\pi}{2} \leq 2 \sin^{-1} x \leq \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{4} \leq \sin^{-1} x \leq \frac{\pi}{4}$$

$$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$$

254 (b)

Sum of two given angles is

$$= \cot^{-1} 2 + \cot^{-1} 3$$

$$= \tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{3} \right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\text{So, the third angle is } \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

255 (a)

Roots of equation $x^2 - 9x + 8 = 0$ are 1 and 8

$$\text{Let } y = [\sin^2 \alpha + \sin^4 \alpha + \sin^6 \alpha + \dots \infty] \log_e 2$$

$$\Rightarrow y = \frac{\sin^2 \alpha}{1 - \sin^2 \alpha} \log_e 2 = \tan^2 \alpha \log_e 2$$

$$\Rightarrow y = \log_e 2^{\tan^2 \alpha}$$

$$\Rightarrow e^y = 2^{\tan^2 \alpha}$$

According to question,

$$2^{\tan^2 \alpha} = 8 = 2^3 \Rightarrow \tan^2 \alpha = 3$$

$$\Rightarrow \tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\therefore \sin^{-1} \left(\sin \frac{2\pi}{3} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} = \alpha$$

256 (a)

Let $\cos^{-1} x = \theta$. Then, $x = \cos \theta$

$$\text{Also, } \frac{1}{2} \leq x \leq 1 \Rightarrow \frac{1}{2} \leq \cos \theta \leq 1 \Rightarrow 0 \leq \theta \leq \frac{\pi}{3}$$

Now,

$$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta)$$

$$= 3\theta = 3 \cos^{-1} x \quad \left[\begin{array}{l} \because 0 \leq \theta \leq \frac{\pi}{3} \\ \Rightarrow 0 \leq 3\theta \leq \pi \end{array} \right]$$

257 (b)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$ and $\sqrt{1-x^2} = \cos \theta$

Now,

$$\sin^{-1} \left(2x \sqrt{1-x^2} \right)$$

$$= \sin^{-1}(\sin 2\theta) = 2\theta, \text{ if } -\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$$

$$= 2 \sin^{-1} x, \text{ if } -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \text{ i.e. if } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$\therefore \sin^{-1}(2x\sqrt{1-x^2}) - 2 \sin^{-1} x = 0, \text{ if } -\frac{1}{\sqrt{2}} \leq x \leq$$

$$\frac{1}{\sqrt{2}}$$

258 (c)

$$\because [\sin^{-1} x] > [\cos^{-1} x]$$

$$\Rightarrow x > 0$$

$$\text{Here, } [\cos^{-1} x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$$

$$\text{and, } [\sin^{-1} x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$$

$$\therefore x \in [\sin 1, 1)$$

$$\therefore \left[\frac{x}{2} \right] = 1$$

Or we say that $x \in [\sin 1, 1]$

259 (a)

$$\text{We have, } 1 \leq \sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x \leq \frac{\pi}{2}$$

$$\Rightarrow \sin 1 \leq \cos^{-1} \sin^{-1} \tan^{-1} x \leq 1$$

$$\Rightarrow \cos \sin 1 \geq \sin^{-1} \tan^{-1} x \geq \cos 1$$

$$\Rightarrow \sin \cos \sin 1 \geq \tan^{-1} x \geq \sin \cos 1$$

$$\Rightarrow \tan \sin \cos \sin 1 \geq x \geq \tan \sin \cos 1$$

$$\therefore x \in [\tan \sin \cos 1, \tan \sin \cos \sin 1]$$

260 (d)

$$\text{Given, } \tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$$

$$\therefore \tan^{-1} x + 2 \tan^{-1} \frac{1}{x} = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \left(\frac{2 \left(\frac{1}{x} \right)}{1 - \left(\frac{1}{x} \right)^2} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \left(\frac{2x}{x^2 - 1} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1} \left(\frac{x + \frac{2x}{x^2 - 1}}{1 - \frac{2x^2}{x^2 - 1}} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \frac{x(x^2 + 1)}{-1(x^2 + 1)} = -\sqrt{3}$$

$$\Rightarrow x = \sqrt{3}$$

261 (d)

$$\tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \sin 2x}{5 + 3 \cos 2x} \right)$$

$$= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{\frac{6 \tan x}{1 + \tan^2 x}}{5 + \frac{3(1 - \tan^2 x)}{1 + \tan^2 x}} \right)$$

$$= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{6 \tan x}{8 + 2 \tan^2 x} \right)$$

$$= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \tan x}{4 + \tan^2 x} \right)$$

$$= \tan^{-1} \left(\frac{\frac{\tan x}{4} + \frac{3 \tan x}{4 + \tan^2 x}}{1 - \frac{3 \tan^2 x}{4(4 + \tan^2 x)}} \right) \left(\text{as } \left| \frac{\tan x}{4} \cdot \frac{3 \tan x}{4 + \tan^2 x} \right| < 1 \right)$$

$$= \tan^{-1} \left(\frac{16 \tan x + \tan^3 x}{16 + \tan^2 x} \right)$$

$$= \tan^{-1}(\tan x) = x$$

262 (a)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$-\frac{1}{2} \leq x \leq \frac{1}{2} \Rightarrow -\frac{1}{2} \leq \sin \theta \leq \frac{1}{2} \Rightarrow -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$$

Now,

$$\sin^{-1}(3x - 4x^3)$$

$$= \sin^{-1}(\sin 3\theta)$$

$$= 3\theta \quad \left[\because -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} \leq 3\theta \leq \frac{\pi}{2} \right]$$

$$= 3 \sin^{-1} x$$

263 (c)

We have,

$$\tan \theta + \tan \left(\frac{\pi}{3} + \theta \right) + \tan \left(\frac{-\pi}{3} + \theta \right) = K \tan 3\theta$$

$$\Rightarrow \tan \theta + \tan(60 + \theta) + \tan(-60 + \theta) = K \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{\sqrt{3} + \tan \theta}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = K \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = K \tan 3\theta$$

$$\Rightarrow \frac{3(3 \tan \theta - \tan^3 \theta)}{(1 - 3 \tan^2 \theta)} = K \tan 3\theta$$

$$\Rightarrow 3 \tan 3\theta = K \tan 3\theta \Rightarrow K = 3$$

264 (d)

Let $\cos^{-1} x = \theta$. Then, $x = \cos \theta$

Also,

$$-1 \leq x \leq 0 \Rightarrow -1 \leq \cos \theta \leq 0 \Rightarrow \frac{\pi}{2} \leq \theta \leq \pi$$

Now,

$$\begin{aligned} \cos^{-1}(2x^2 - 1) &= \cos^{-1}(\cos 2\theta) \\ &= \cos^{-1}(\cos(2\pi - 2\theta)) \end{aligned}$$

$$\Rightarrow \cos^{-1}(2x^2 - 1)$$

$$= 2\pi - 2\theta \left[\begin{array}{l} \because \frac{\pi}{2} \leq \theta \leq \pi \Rightarrow \pi \leq 2\theta \leq 2\pi \\ \Rightarrow 0 \leq 2\pi - 2\theta \leq \pi \end{array} \right]$$

$$\Rightarrow \cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1} x$$

265 (c)

We have,

$$\alpha + \beta = \pi$$

Also,

$$\alpha = \sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3} + \sin^{-1} \frac{1}{3}$$

$$\Rightarrow \alpha < \frac{\pi}{3} + \sin^{-1} \frac{1}{2} \quad [$$

$\because \sin^{-1} x$ is increasing on $[-1, 1]$]

$$\Rightarrow \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\therefore \alpha + \beta = \pi \Rightarrow \beta > \frac{\pi}{2}. \text{ Thus, } \alpha < \beta$$

266 (a)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$-1 \leq x \leq 1 \Rightarrow -1 \leq \tan \theta \leq 1 \Rightarrow -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$$

Now,

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta \quad \left[\because -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2} \right]$$

$$= 2 \tan^{-1} x$$

267 (a)

$$\begin{aligned} \sin \left[3 \sin^{-1} \left(\frac{1}{5} \right) \right] &= \sin \left[\sin^{-1} \left\{ 3 \left(\frac{1}{5} \right) - 4 \left(\frac{1}{5} \right)^3 \right\} \right] \\ &= \frac{3}{5} - \frac{4}{125} = \frac{71}{125} \end{aligned}$$

268 (a)

$$\text{Since, } -\frac{\pi}{2} < \sin^{-1} x \leq \frac{\pi}{2}$$

$$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \leq i \leq 20$$

$$\Rightarrow x_i = 1, 1 \leq i \leq 20$$

$$\text{Thus, } \sum_{i=1}^{20} x_i = 20$$

269 (d)

Given, $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$

$$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(1+x^2)}} \right)$$

$$= \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow \frac{1}{\sqrt{1+(1+x^2)}} = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow 1+x^2+2x+1 = x^2+1$$

$$\Rightarrow x = -\frac{1}{2}$$

270 (b)

$$\therefore \tan \left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \left(\frac{a}{b} \right) \right] + \tan \left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \left(\frac{a}{b} \right) \right]$$

$$= \tan \left[\frac{\pi}{4} + \phi \right] + \tan \left[\frac{\pi}{4} - \phi \right]$$

$$\left[\text{put } \frac{1}{2} \cos^{-1} \left(\frac{a}{b} \right) = \phi \Rightarrow \cos 2\phi = \frac{a}{b} \right]$$

$$= \frac{1 + \tan \phi}{1 - \tan \phi} + \frac{1 - \tan \phi}{1 + \tan \phi}$$

$$= \frac{2(1 + \tan^2 \phi)}{1 - \tan^2 \phi}$$

$$= \frac{2}{\cos 2\phi} = \frac{2b}{a}$$

271 (b)

$$\tan^{-1} \frac{x}{y} - \tan^{-1} \frac{x-y}{x+y}$$

$$= \tan^{-1} \frac{x}{y} - \tan^{-1} \left[\frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} \right]$$

$$= \tan^{-1} \frac{x}{y} - \tan^{-1} 1 + \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} \frac{x}{y} + \cot^{-1} \frac{x}{y} - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

272 (c)

$$\text{Here, } x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$$

$$\text{But } -1 \leq (x^2 - 2x + 2) \leq 1$$

Which is possible only when

$$x^2 - 2x + 2 = 1$$

$$\Rightarrow x = 1$$

$$\text{Then, } a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$$

$$\Rightarrow a + \frac{\pi}{2} + 0 = 0$$

$$\Rightarrow a = -\frac{\pi}{2}$$

273 (d)

$$\cos^{-1} \left(-\frac{1}{2} \right) - 2 \sin^{-1} \left(\frac{1}{2} \right)$$

$$+ 3 \cos^{-1} \left(-\frac{1}{\sqrt{2}} \right) + 4 \tan^{-1}(-1)$$

$$\begin{aligned}
&= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) \\
&\quad + 4\tan^{-1}(1) \\
&= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4 \cdot \frac{\pi}{4} \\
&= \frac{\pi}{3} + 3 \cdot \frac{3\pi}{4} + \pi = \frac{43\pi}{12}
\end{aligned}$$

274 (b)

$$\begin{aligned}
\theta &= \sin^{-1}x + \cos^{-1}x - \tan^{-1}x = \frac{\pi}{2} - \tan^{-1}x \\
&\quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \right]
\end{aligned}$$

$$\Rightarrow \theta = \cot^{-1}x$$

Since, $1 \leq x < \infty$, therefore $0 \leq \theta \leq \frac{\pi}{4}$

275 (b)

$$\text{Given, } 4\sin^{-1}x + \cos^{-1}x = \pi$$

$$\Rightarrow 4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$$

$$\Rightarrow 3\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{2}$$

276 (d)

$$\cos\left(\frac{33\pi}{5}\right) = \cos\left(6\pi + \frac{3\pi}{5}\right) = \cos\frac{3\pi}{5}$$

$$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \sin\left(-\frac{\pi}{10}\right)$$

$$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

277 (b)

Given expression

$$\begin{aligned}
&= \tan\left[\tan^{-1}\frac{a_2 - a_1}{1 + a_1a_2}\right. \\
&\quad \left. + \tan^{-1}\frac{a_3 - a_2}{1 + a_2a_3} + \dots + \tan^{-1}\frac{a_n - a_{n-1}}{1 + a_{n-1}a_n}\right]
\end{aligned}$$

$$= \tan[\tan^{-1}a_2 - \tan^{-1}a_1 + \tan^{-1}a_3 - \tan^{-1}a_2 + \dots + \tan^{-1}a_n - \tan^{-1}a_{n-1}]$$

$$= \tan[\tan^{-1}a_n - \tan^{-1}a_1] = \frac{a_n - a_1}{1 + a_1a_n}$$

$$= \frac{(n-1)d}{1 + a_1a_n}$$

278 (a)

$$\because \tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} = \tan\frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$$

$$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

280 (b)

$$\text{Given, } \tan\left\{\sec^{-1}\left(\frac{1}{x}\right)\right\} = \sin(\tan^{-1}2)$$

$$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{1+2^2}}\right)$$

$$\left[\because \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} \right]$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow 4x^2 = 5(1-x^2)$$

$$\Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}$$

282 (b)

$$\begin{aligned}
\text{Given, } (\sqrt{3} - i) &= (a + ib)(c + id) \\
&= (ac - bd) + i(ad + bc)
\end{aligned}$$

On comparing the real and imaginary part on both sides, we get

$$ac - bd = \sqrt{3}$$

$$\text{And } ad + bc = 1$$

$$\begin{aligned}
\text{Now, } \tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right) \\
= \tan^{-1}\left(\frac{bc + ad}{ac - bd}\right)
\end{aligned}$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= n\pi + \frac{\pi}{6}, n \in I$$

283 (b)

$$\text{Given, } \tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$$

$$\text{Let } x = \tan\theta$$

$$\therefore \tan^{-1}\left(\frac{1 - \tan\theta}{1 + \tan\theta}\right) = \frac{1}{2}\tan^{-1}(\tan\theta)$$

$$\Rightarrow \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \theta\right)\right\} = \frac{1}{2}\tan^{-1}(\tan\theta)$$

$$\Rightarrow \frac{\pi}{4} - \theta = \frac{\theta}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\therefore x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

284 (c)

$$\text{Let } S_\infty = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \dots$$

$$\therefore T_n \cot^{-1}2n^2$$

$$= \tan^{-1}\frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

$$\therefore S_n = \sum_{n=1}^{\infty} \{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$$

$$= \tan^{-1}\infty - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

286 (d)

$$\begin{aligned}\sin^{-1}\left\{\tan\left(\frac{-5\pi}{4}\right)\right\} &= \sin^{-1}\left\{-\tan\left(\pi + \frac{\pi}{4}\right)\right\} \\ &= \sin^{-1}\left(-\tan\frac{\pi}{4}\right) \\ &= \sin^{-1}\left(-\sin\frac{\pi}{2}\right) \\ &= -\frac{\pi}{2}\end{aligned}$$

287 (a)

$$\begin{aligned}\therefore \tan^{-1}\left(\frac{1}{1+r+r^2}\right) &= \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right) \\ &= \tan^{-1}(r+1) - \tan^{-1}(r) \\ \therefore \sum_{r=0}^n [\tan^{-1}(r+1) - \tan^{-1}(r)] \\ &= \tan^{-1}(n+1) - \tan^{-1}(0) \\ &= \tan^{-1}(n+1) \\ \Rightarrow \sum_{r=0}^{\infty} \tan^{-1}\left(\frac{1}{1+r+r^2}\right) &= \tan^{-1}(\infty) = \frac{\pi}{2}\end{aligned}$$

288 (c)

Let $\cos^{-1}x = \theta$. Then, $x = \cos\theta$

Also,

$$-1 \leq x \leq -\frac{1}{2} \Rightarrow -1 \leq \cos\theta \leq -\frac{1}{2} \Rightarrow \frac{2\pi}{3} \leq \theta \leq \pi$$

Now,

$$\begin{aligned}\cos^{-1}(4x^3 - 3x) \\ &= \cos^{-1}(\cos 3\theta) \\ &= \cos^{-1}(\cos(2\pi - 3\theta)) \\ &= \cos^{-1}(\cos(3\theta - 2\pi)) \\ &= 3\theta - 2\pi \left[\because \frac{2\pi}{3} \leq \theta \leq \pi \Rightarrow 0 \leq 3\theta - 2\pi \leq \pi \right] \\ &= 3\cos^{-1}x - 2\pi\end{aligned}$$

289 (c)

Given that, $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}A$$

$$\text{Hence, } A = \frac{x-y}{1+xy}$$

290 (b)

We have, $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\Rightarrow \frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x$$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

291 (c)

Clearly, $x(x+1) \geq 0$ and $x^2 + x + 1 \leq 1$

$$\Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0, -1$$

When $x = 0$,

$$\text{LHS} = \tan^{-1}0 + \sin^{-1}1 = \frac{\pi}{2}$$

When $x = -1$,

$$\text{LHS} = \tan^{-1}0 + \sin^{-1}\sqrt{1-1+1}$$

$$= 0 + \sin^{-1}(1) = \frac{\pi}{2}$$

Thus, the number of solution is 2

292 (b)

We have,

$$\cos\left\{\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right\} = \cos\frac{\pi}{2} = 0$$

293 (c)

The given equation is satisfied only when $x = 1$,

$$y = -1, z = 1$$

294 (c)

Given, $\sin^{-1}(1-x) = \frac{\pi}{2} + 2 \sin^{-1}x$

$$\Rightarrow 1-x = \sin\left(\frac{\pi}{2} + 2 \sin^{-1}x\right)$$

$$\Rightarrow 1-x = \cos(2 \sin^{-1}x)$$

$$\Rightarrow 1-x = \cos(2 \cos^{-1}\sqrt{1-x^2})$$

$$\Rightarrow 1-x = \cos\{\cos^{-1}(1-2x^2)\}$$

$$\Rightarrow 1-x = 1-2x^2$$

$$\Rightarrow x = 0, \frac{1}{2}$$

$$\Rightarrow x$$

$$= 0 \left[\because x = \frac{1}{2} \text{ does not satisfy the given equation} \right]$$

295 (d)

We have,

$$\cos^{-1}\left(\frac{15}{17}\right) + 2 \tan^{-1}\left(\frac{1}{5}\right)$$

$$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{1-1/25}{1+1/25}\right)$$

$$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{12}{13}\right)$$

$$= \cos^{-1}\left\{\frac{15}{17} \times \frac{12}{13} - \sqrt{1-\left(\frac{15}{17}\right)^2} \sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$$

$$= \cos^{-1}\left(\frac{140}{221}\right)$$

296 (c)

Let $\cot^{-1}x = \theta$. Then, $x = \cot\theta$

$$\text{Also, } x < 0 \Rightarrow \cot\theta < 0 \Rightarrow \frac{\pi}{2} < \theta < \pi$$

Now,

$$\tan^{-1}\left(\frac{1}{x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan\theta)$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\tan(\pi - \theta))$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan(\theta - \pi))$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \theta$$

$$-\pi \left[\frac{\pi}{2} < \theta < \pi \Rightarrow -\frac{\pi}{2} < \theta - \pi < 0 \right]$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x - \pi$$

297 (d)

$$\text{Let } \alpha = \cos^{-1} \sqrt{p}, \beta = \cos^{-1} \sqrt{1-p}$$

$$\text{And } \gamma = \cos^{-1} \sqrt{1-q}$$

$$\Rightarrow \cos \alpha = \sqrt{p}, \cos \beta = \sqrt{1-p}$$

$$\text{And } \cos \gamma = \sqrt{1-q}$$

$$\text{Therefore, } \sin \alpha = \sqrt{1-p}, \sin \beta = \sqrt{p} \text{ and } \sin \gamma = \sqrt{q}$$

The given equation may be written as

$$\alpha + \beta + \gamma = \frac{3\pi}{4}$$

$$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$$

$$\Rightarrow \cos(\alpha + \beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$$

$$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p} = -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\sqrt{q}\right)$$

$$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1-q = q$$

$$\Rightarrow q = \frac{1}{2}$$

298 (c)

$$\begin{aligned} \tan^{-1} \frac{m}{n} &= \frac{m}{n} - \tan^{-1} \frac{m-n}{m+n} \\ &= \tan^{-1} \frac{m}{n} - \tan^{-1} \frac{\frac{m}{n} - 1}{1 + \frac{m}{n}} \\ &= \tan^{-1} \frac{m}{n} - \tan^{-1} \frac{m}{n} + \tan^{-1}(1) = \frac{\pi}{4} \end{aligned}$$

299 (c)

$$\begin{aligned} \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] &= \cos \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ &= \cos \cos^{-1} \sqrt{1 - \frac{3}{4}} \\ &= \cos \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \end{aligned}$$

300 (d)

$$\cos^{-1} x, \sin^{-1} x \text{ are real, if } -1 \leq x \leq 1$$

$$\text{But } \cos^{-1} x > \sin^{-1} x$$

$$\Rightarrow 2 \cos^{-1} x > \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{4}$$

$$\therefore \cos(\cos^{-1} x) < \cos \frac{\pi}{4}$$

$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

The common value are $-1 \leq x < \frac{1}{\sqrt{2}}$

301 (a)

$$\begin{aligned} \cot^{-1} \frac{xy+1}{x-y} + \cot^{-1} \frac{yz+1}{y-z} + \cot^{-1} \frac{zx+1}{z-x} \\ = \cot^{-1} y - \cos^{-1} x \\ \quad + \cot^{-1} z \\ \quad - \cot^{-1} y + \cot^{-1} x - \cot^{-1} z \\ = 0 \end{aligned}$$

302 (a)

$$\begin{aligned} \tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\} \\ = \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\} \\ = \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\} = \tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\} \\ = \tan\left\{\tan^{-1}\left(\frac{2}{7}\right)\right\} = \frac{2}{7} \end{aligned}$$

303 (a)

$$\begin{aligned} \sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x} \\ = [\sin^{-1} x + \cos^{-1} x] + \left[\sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right)\right] \\ = \frac{\pi}{2} + \frac{\pi}{2} = \pi \end{aligned}$$

304 (b)

We know that

$$2 \tan^{-1} x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{ if } x > 1$$

$$\therefore x = \sin(2 \tan^{-1} 2)$$

$$\Rightarrow x = \sin\left\{\pi + \tan^{-1}\left(\frac{4}{1-4}\right)\right\}$$

$$\begin{aligned} \Rightarrow x &= \sin\left(\pi - \tan^{-1}\frac{4}{3}\right) = \sin\left(\tan^{-1}\frac{4}{3}\right) \\ &= \sin\left(\sin^{-1}\frac{4}{5}\right) = \frac{4}{5} \end{aligned}$$

And,

$$y = \sin\left(\frac{1}{2} \tan^{-1} \frac{4}{5}\right)$$

$$\Rightarrow y = \sin \frac{\theta}{2}, \text{ where } \theta = \tan^{-1} \frac{4}{3} \text{ i.e. } \tan \theta = \frac{4}{3}$$

$$\Rightarrow y = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - 3/5}{2}} = \frac{1}{\sqrt{5}}$$

Clearly, $x = 1 - y^2$ or, $y^2 = 1 - x$

306 (c)

Given, $\tan^{-1} \sqrt{x(x+1)} = \frac{\pi}{2} - \sin^{-1} \sqrt{x^2 + x + 1}$

$$\Rightarrow \cos^{-1} \frac{1}{\sqrt{(x^2+x)^2+1}} = \cos^{-1} \sqrt{x^2+x+1}$$

$$\Rightarrow \frac{1}{\sqrt{(x^2+x)^2+1}} = \sqrt{x^2+x+1}$$

$$\Rightarrow 1 = (x^2+x+1)[(x^2+x)^2+1]$$

$$\Rightarrow (x^2+x)^3 + (x^2+x)^2 + (x^2+x) + 1 = 1$$

$$\Rightarrow (x^2+x)\{(x^2+x)^2 + (x^2+x) + 1\} = 0$$

$$\Rightarrow x^2+x = 0$$

$$\Rightarrow x = 0, -1$$

307 (b)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$-\infty < x \leq 0 \Rightarrow -\infty < \tan \theta \leq 0 \Rightarrow -\frac{\pi}{2} < \theta \leq 0$$

$$\Rightarrow -\pi < 2\theta \leq 0$$

Now,

$$\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$= \cos^{-1}(\cos 2\theta)$$

$$= \cos^{-1}(\cos(-2\theta))$$

$$= -2\theta = -2 \tan^{-1} x \quad [\because 0 \leq -2\theta < \pi]$$

308 (c)

$$\tan(\sin^{-1} x) = \tan \left(\tan^{-1} \frac{x}{\sqrt{1-x^2}} \right), x \in (-1, 1)$$

$$= \frac{x}{\sqrt{1-x^2}}$$