Session : 2025-26 **AS PER NEW NTA SYLLABUS**

Total Questions : 335

MATHS (QUESTION BANK)

3.MATRICES

Single Correct Answer Type

1. If n is a natural number. Then $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^n$, is

   a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even

   b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is odd

   c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is a natural number

   d) None of these

2. $\begin{bmatrix} 0 & a \\ b & O \end{bmatrix}^4 = I$,then

   a) $a = 1 = 2b$ b) $a = b$ c) $a = b^2$ d) $ab = 1$

3. If a square matrix A is such that $AA^T = I = A^T A$, then $|A|$ is equal to

   a) 0 b) ± 1 c) ± 2 d) None of these

4. If k is a scalar and I is a unit matrix of order 3, then adj (kI) is equal to

   a) $k^3 I$ b) $k^2 I$ c) $-k^3 I$ d) $-k^2 I$

5. If A is a singular matrix, then A adj (A) is a

   a) Scalar matrix b) Zero matrix c) Identity matrix d) Orthogonal matrix

6. If $A = \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix}$ and $A = \begin{bmatrix} \cos^2 \beta & \cos \alpha \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix}$ are two matrices such that the product AB is null matrix, then $\alpha - \beta$ is

   a) 0 b) Multiple of π

   c) An odd multiple of $\pi/2$ d) None of the above

7. Consider the following statements:

   1. If A and B are two square matrices of same order and commute, then $(A + B)(A - B) = A^2 - B^2$
   2. If A and B are two square matrices of same order, then $(AB)^n = A^n B^n$
   3. If A and B are two matrices such that $AB = A$ and $BA = B$, then A and B are idempotent

   Which of these is/are not correct?

   a) Only (1) b) (2) and (3) c) (3) and (1) d) All of these

8. If $A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{bmatrix}$, the rank of A is equal to

   a) 0 b) 1 c) 2 d) 3

9. If A and B are 3×3 matrices such that $AB + B$ and $BA = A$, then

   a) $A^2 = A$ and $B^2 \neq B$ b) $A^2 \neq A$ and $B^2 = B$ c) $A^2 = A$ and $B^2 = B$ d) $A^2 \neq A$ and $B^2 \neq B$

10. The inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is

    a) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 & 1 \\ 4 & 3 & 8 \\ 3 & 4 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

11. If A is a skew-symmetric matrix and n is a positive integer, then A^n is

a) A symmetric matrix
b) Skew-symmetric matrix
c) Diagonal matrix
d) None of these

12. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0,is
a) Less than 4 b) 5 c) 6 d) At least 7

13. If A=$\begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then trace of matrix A is
a) 17 b) 25 c) 3 d) 12

14. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}$, then $(A + B)^{-1}$ is equal to
a) Is a skew-symmetric matrix
b) $A^{-1} + B^{-1}$
c) Does not exist
d) None of the above

15. If $f(x) = x^2 - 5x, A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then$f$ (A) is equal to
a) $\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$

16. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is
a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

17. If the rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is 1, than the value of a is
a) -1 b) 2 c) -6 d) 4

18. The value of λ such that $x + 3y + \lambda z = 0, 2x + 4y - z = 0, x + 5y - 2z = 0$ has a non-trivial solution is
a) -1 b) 0 c) 1 d) 2

19. If $A = [a_{ij}]_{m\times n}$ is a matrix and B is a non-singular square submatrix of order r, then
a) Rank of A is r
b) Rank of A is greater than r
c) Rank of A is less than r
d) None of these

20. From the matrix equation $AB = AC$ we can conclude $B = C$ provided that
a) A is singular b) A is non-singular c) A is symmetric d) A is square

21. The values of α for which the system of equations
$x + y + z = 1$
$x + 2y + 4z = \alpha$
$x + 4y + 10z = \alpha^2$
Is consistent, are
a) $1, -2$ b) $-1, 2$ c) $1, 2$ d) None of these

22. If A is a non-singular matrix such that $A^3 = A + I$, then the inverse of $B = A^6 - A^5$ is
a) A b) A^{-1} c) -A d) $-A^{-1}$

23. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then the value of $a^3 + b^3 + c^3$ is
a) 1 b) 2 c) 3 d) 4

24. If A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then $A^2 - 4\,A$ is equal to

a) $2I_3$ b) $3I_3$ c) $4I_3$ d) $5I_3$

25. If $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$,then $A^4=$

a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$ c) $\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$ d) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

26. The simultaneous equations $Kx+2y-z=1,(K-1)y-2z=2$ and $(K+2)z=3$ have only one solution when

a) $K=-2$ b) $K=-1$ c) $K=0$ d) $K=1$

27. The system of equation,

$x+y+z=6$

$x+2y+3z=10$

And $x+2y+\lambda z=\mu$

Has no solution, if

a) $\lambda=3,\mu=10$ b) $\lambda\neq 3,\mu=10$ c) $\lambda\neq 3,\mu\neq 10$ d) $\lambda=3,\mu\neq 10$

28. If a square matrix A is such that $AA^T=I=A^TA$, then$|A|$is equal to

a) 0 b) ± 1 c) ± 2 d) None of these

29. Let $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$, then $AB=O$, if

a) $\theta=n\phi, n=0,1,2,\ldots$ b) $\theta+\phi=n\pi, n=0,1,2,\ldots$ c) $\theta=\phi+(2n+1)\frac{\pi}{2}, n=0,1,2,\ldots$ d) $\theta=\phi+n\frac{\pi}{2}, n=0,1,2,\ldots$

30. If A,B are two square matrices such that $AB=A$ and $BA=B$, then

a) A,B are idempotent b) Only A is idempotent c) Only B is idempotent d) None of these

31. If $\begin{bmatrix}x-y-z\\ -y+z\\ z\end{bmatrix}=\begin{bmatrix}0\\5\\3\end{bmatrix}$, than the values of x, y and z are respectively

a) 5, 2, 2 b) 1, -2, 3 c) 0, -3, 3 d) 11, 8, 3

32. Inverse of the matrix $\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$ is

a) $\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$ b) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$ c) $\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$ d) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

33. If $A=\begin{bmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{bmatrix}$, $10\,B=\begin{bmatrix}4 & 2 & 2\\-5 & 0 & \alpha\\1 & -2 & 3\end{bmatrix}$and B is the inverse of A, then the value of α is

a) 2 b) 0 c) 5 d) 4

34. $A=\begin{bmatrix}0 & 3 & 3\\-3 & 0 & -4\\-3 & 4 & 0\end{bmatrix}$and $B=\begin{bmatrix}x\\y\\z\end{bmatrix}$, then $B'(AB)$is

a) Null matrix b) Singular matrix c) Unit matrix d) Symmetric matrix

35. If $A=\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{bmatrix}$, then A^{-1} is

a) $\begin{bmatrix}1/a & 0 & 0\\0 & 1/b & 0\\0 & 0 & 1/c\end{bmatrix}$

b) $\begin{bmatrix}-1/a & 0 & 0\\0 & -1/b & 0\\0 & 0 & -1/c\end{bmatrix}$

c) $\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & 1/c\end{bmatrix}$

d) None of these

36. If $A = \begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $a^2 + b^2 = 1$,then adj(A) is equal to

(Here, A^T is the transpose of A)

a) A^{-1} b) A^T c) A d) $-A$

37. The rank of a null matrix is

a) 0 b) 1 c) Does not exist d) None of these

38. If A and B are matrices of the same order, then $(A + B)^2 = A^2 + 2\,AB + B^2$ is possible, iff

a) $AB = I$ b) $BA = I$ c) $AB = BA$ d) None of these

39. If A=$\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

a) 2 b) $-\frac{1}{2}$ c) 1 d) $\frac{1}{2}$

40. If the system of equations

$ax + ay - z = 0$

$bx - y + bz = 0$

and $-x + cy + cz = 0$

Has a non-trivial solution, then the value of

$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$ is

a) 0 b) 1 c) 2 d) 3

41. The system of equation

$3x - y + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + \lambda z = -3$

Has at least one solution, if

a) $\lambda = -5$ b) $\lambda = 5$ c) $\lambda = 3$ d) $\lambda = -13$

42. If A is a skew-symmetric matrix and n is an even positive integer, then A^n is

a) A symmetric matrix

b) A skew-symmetric matrix

c) A diagonal matrix

d) None of these

43. If $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then$|\text{adj } A|$is equal to

a) 0 b) 9 c) $\frac{1}{9}$ d) 81

44. If A is skew-symmetric matrix of order 3, then matrix A^3 is

a) Skew-symmetric matrix b) Symmetric matrix

c) Diagonal matrix d) None of the above

45. If $[1 \;\; x \;\; 1] \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix}$=0, then the value of x is

a) 0 b) $\frac{2}{3}$ c) $\frac{5}{4}$ d) $-\frac{4}{5}$

46. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$, then $(A + B)^{-1} =$

a) Is a skew-symmetric matrix

b) $A^{-1} + B^{-1}$

c) Does not exist

d) None of these

47. Let A be an orthogonal non-singular matrix of order n, then the determinant of matrix $'A - I_n'$ ie, $|A - I_n|$ is equal to

a) $|I_n - A|$ b) $|A|$ c) $|A||I_n - A|$ d) $(-1)^n|A||I_n - A|$

48. The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

a) Unitary b) Orthogonal c) Nilpotent d) Involutory

49. The inverse of a symmetric matrix is

a) Symmetric b) Skew-symmetric c) Diagonal matrix d) None of these

50. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are

a) 1, 3, 6 b) 1, 2, 4 c) 4, 5, 6 d) 2, 4, 6

51. If matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k}\text{adj } A$, then k is

a) 7 b) -7 c) $\frac{1}{7}$ d) 11

52. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

a) 0 b) $2^9 - 1$ c) 168 d) 2

53. If A is an invertible matrix of order n, then the determinant of adj (A) is equal to

a) $|A|^n$ b) $|A|^{n+1}$ c) $|A|^{n-1}$ d) $|A|^{n+2}$

54. adj $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$, then $[a\ \ b]$ is equal to

a) [-4 1] b) [-4 -1] c) [4 1] d) [4 -1]

55. If $A = [x\ y\ z], B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Then, $B'(AB) = O$, if

a) $[ax^2 + by^2 + cz^2 + 2gxy + 2fyz + 2czx] = O$ b) $[ax^2 + cy^2 + bz^2 + xy + yz + zx] = O$

c) $[ax^2 + by^2 + cz^2 + 2hxy + 2by + 2cz] = O$ d) $[ax^2 + by^2 + cz^2 + 2gzx + 2hxy + 2fyz] = O$

56. The multiplicative inverse of matrix $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & -1 \\ -7 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -4 & -1 \\ 7 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & -7 \\ 7 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

57. The rank of matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$ is

a) 4 b) 3 c) 2 d) 1

58. The solution of $(x, y, z)$the equation

$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ is (x, y, z)

a) (1, 1, 1) b) (0, -1, 2) c) (-1, 2, 2) d) (-1, 0, 2)

59. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$

60. Let A, B and C be $n \times n$ matrices. Which one of the following is a correct statements?

a) If $AB = AC$, then $B = C$ b) If $A^3 + 2A^2 + 3A + 5I = O$; then A is invertible

c) If $A^2 = O$, then $A = O$ d) None of the above

61. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then $|3\ AB|$ equals

a) -9 b) -81 c) -27 d) 81

62. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ is

a) 1 b) 2 c) 3 d) 4

63. If $A + 1 = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then$(A = I).(A - I)$is equal to

a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ d) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$

64. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i, then $|A| =$

a) nk b) $n + k$ c) n^k d) k^n

65. If A $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such that $|A|$=0 and $A^2 - (a - d)A + kI = 0$, then k is equal to

a) $b\ + c$ b) $a\ + d$ c) $ab\ +\ cd$ d) Zero

66. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T$ is equal to

a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ d) None of these

67. If A is a matrix such that there exists a square submatrix of order r which is non-singular and every square submatrix of order $r + 1$ or more is singular, then

a) rank $(A) = r + 1$ b) rank $(A) = r$ c) rank $(A) > r$ d) rank $(A) \geq r + 1$

68. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A =$

a) I b) $14I$ c) 0 d) None of these

69. The order of $[x\ y\ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

a) 3×1 b) 1×1 c) 1×3 d) 3×3

70. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2\ B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then $A =$

a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$ c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ d) None of these

71. If A is singular matrix, then A adj (A)

a) Is a scalar matrix b) Is a zero matrix

c) Is an identity matrix d) Is an orthogonal matrix

72. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then X is equal to

a) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$

73. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, which of the following is correct?

a) $A^3 + 3A^2 - I = 0$ b) $A^3 - 3A^2 - I = 0$ c) $A^3 + 2A^2 - I = 0$ d) $A^3 - A^2 + I = 0$

74. The matrices $P = \begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}; Q = \frac{1}{9}\begin{bmatrix} 2 & 2 & 1 \\ 12 & -5 & m \\ -8 & 1 & 5 \end{bmatrix}$ are such that $PQ = I$, an identity matrix. Solving the equation $\begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, the value of y comes out to be -3. Then, the value of m is equal to

a) 27 b) 7 c) -27 d) -7

75. If A is an invertible matrix, then which of the following is correct

a) A^{-1} is multivalued b) A^{-1} is singular c) $(A^{-1})^T \neq (A^T)^{-1}$ d) $|A| \neq 0$

76. If $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then the matrix A equals

a) $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ b) $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ d) $\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}$

77. If A is any square matrix, then $\det(A - A^T)^T$ is equal to

a) 0 b) 1
c) Can be 0 or a perfect square d) Cannot be determined

78. If $O(A) = 2 \times 3, O(B) = 3 \times 2$, and $O(c) = 3 \times 3$, Which one of the following is not defined?

a) $CB + A'$ b) BAC c) $C(A + B')'$ d) $C\ (A + B')$

79. Suppose A is a matrix of order 3 and $B = |A|A^{-1}$ .If $|A| = -5$, then $|B|$is equal to

a) 1 b) -5 c) -1 d) 25

80. $A = \begin{bmatrix}-2 & 4\\-1 & 2\end{bmatrix}$, the A^2 is equal to

a) Null matrix b) Unit matrix c) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ d) $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$

81. The system of simultaneous equations $kx + 2y - z = 1$ $(k-1)y - 2z = 2$ and $(k+2)z = 3$ has a unique solution, if k equals

a) -2 b) -1 c) 0 d) 1

82. If $A = \begin{bmatrix}-1 & 2\\2 & -1\end{bmatrix}$ and $B = \begin{bmatrix}3\\1\end{bmatrix}$, $Ax = B$, then X is equal to

a) $[0 \quad 7]$ b) $\frac{1}{3}\begin{bmatrix}5\\7\end{bmatrix}$ c) $\frac{1}{3}[5 \quad 7]$ d) $\begin{bmatrix}5\\7\end{bmatrix}$

83. For the equations $x + 2y + 3z = 1, 2x + y + 3z = 2$ and $5x + 5y + 9z = 4$

a) There is only one solution b) There exists infinitely many solutions
c) There is no solution d) None of the above

84. If $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ and $n \in N$, then A^n is equal to

a) $2^n\ A$ b) $2^{n-1}\ A$ c) $n\ A$ d) None of these

85. If $A = \begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}, B = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$, then $(B^{-1}A^{-1})^{-1} =$

a) $\begin{bmatrix}2 & -2\\2 & 3\end{bmatrix}$ b) $\begin{bmatrix}3 & -2\\2 & 2\end{bmatrix}$ c) $\frac{1}{10}\begin{bmatrix}2 & 2\\-2 & 3\end{bmatrix}$ d) $\frac{1}{10}\begin{bmatrix}3 & 2\\-2 & 2\end{bmatrix}$

86. If A is a skew symmetric matrix of order n and C is a column matrix of order $n \times 1$, then C^TAC is

a) An identity matrix of order n b) An identity matrix of order1
c) A zero matrix of order 1 d) None of the above

87. If A is a square matrix, then $A + A^T$ is

a) Non-singular matrix b) Symmetric matrix
c) Skew-symmetric matrix d) Unit matrix

88. If A=$[a_{ij}]_{2\times2}$, where $a_{ij} = i + j$, then A is equal to

a) $\begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\1 & 2\end{bmatrix}$ c) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ d) $\begin{bmatrix}2 & 3\\3 & 4\end{bmatrix}$

89. If $P = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 4 & 5\end{bmatrix}\begin{bmatrix}-1 & -2\\-2 & 0\\0 & -4\end{bmatrix}\begin{bmatrix}-4 & -5 & -6\\0 & 0 & 1\end{bmatrix}$, then$P_{22}$is equal to

a) 40 b) -40 c) -20 d) 20

90. Which of the following is/are incorrect?

(i) adjoint of a symmetric matrix is symmetric
(ii) adjoint of a unit matrix is a unit matrix
(iii) $A(\text{adj } A) = (\text{adj } A)A = |A|\ I$
(iv) adjoint of a diagonal matrix is a diagonal matrix

a) (i) b) (ii) c) (iii) and (iv) d) None of these

91. If A and B are 3×3 matrices such that $AB = A$ and $BA = B$, then

a) $A^2 = A$ and $B^2 \neq B$ b) $A^2 \neq A$ and $B^2 = B$

c) $A^2 = A$ and $B^2 = B$ d) $A^2 \neq A$ and $B^2 \neq B$

92. Let A be a skew-symmetric matrix of even order, then $|A|$

a) Is a square b) Is not a square c) Is always zero d) None of these

93. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$, $B = (\text{adj } A)$ and $C = 5A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to

a) 5 b) 25 c) -1 d) 1

94. If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix, C, then B is

a) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

95. The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -2 & -2 & b \end{bmatrix}$ is a singular matrix, if b is equal to

a) -3 b) 3 c) 0 d) For any value of b

96. If A and B are square matrices of order 3×3 , then which of the following is true?

a) $AB = O \Rightarrow A = O$ or $B = O$ b) det (2AB)=8 det (A) det (B)

c) $A^2 - B^2 = (A + B)(A - B)$ d) det $(A + B)$=det (A)+det (B)

97. If $A = [a_{ij}]_{n\times n}$ be a diagonal matrix with diagonal element all different and $B = [b_{ij}]_{n\times n}$ be some another matrix. Let $AB = [C_{ij}]_{n\times n}$, then c_{ij} is equal to

a) $a_{jj}\, b_{ij}$ b) $a_{ii}b_{ij}$ c) $a_{ij}b_{ij}$ d) $a_{ij}b_{ji}$

98. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ be a matrix, then (determinant of A)$\times$ (adjoint of inverse A) is equal to

a) $O_{3\times 3}$ b) $\begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ c) I_3 d) $\begin{bmatrix} 3 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & -3 \end{bmatrix}$

99. The rank of $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is

a) 0 b) 1 c) 2 d) 3

100. Let a, b, c are positive real numbers. The following system of equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, in x, y and z has

a) Infinite solutions b) Unique solution

c) No solution d) Finite number of solutions

101. If $A = [a_{ij}]_{m\times n}$ is a matrix of rank r and B is a square submatrix of order $r + 1$, then

a) B is invertible

b) B is not invertible

c) B may or may not be invertible

d) None of these

102. If A is square matrix, A', is its transpose, then $\frac{1}{2}\ (A - A')$is

a) A symmetric matrix b) A skew-symmetric matrix

c) A unit matrix d) An elementary matrix

103. Inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$is

a) $\frac{1}{10}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ b) $\frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ d) $\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

104. Let A be a matrix of rank r. Then,

a) rank $(A^T) = r$ b) rank $(A^T) < r$ c) rank $(A^T) > r$ d) None of these

105. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

106. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to

a) $-(3A^2 + 2A + 5)$ b) $3A^2 + 2A + 5$ c) $3A^2 - 2A - 5$ d) None of these

107. Let $A = [a_{ij}]_{n \times n}$ be a square matrix, and let c_{ij} be cofactor of a_{ij} in A. If $C = [c_{ij}]$, then

a) $|C| = |A|$ b) $|C| = |A|^{n-1}$ c) $|C| = |A|^{n-2}$ d) None of these

108. The system of equations $x + y + z = 0, 2x + 3y + z = 0$ and $x = 2y = 0$ has

a) A unique solution; $x = 0, y = 0, z = 0$ b) Infinite solutions
c) No solutions d) Finite number of non-zero solutions

109. If $2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$, then X is equal to

a) $\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ \frac{7}{2} & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 2 \\ \frac{7}{2} & 1 \end{bmatrix}$ d) None of these

110. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then the values of x and y are

a) $x = -\frac{1}{11}, y = \frac{2}{11}$ b) $x = -\frac{1}{11}, y = -\frac{2}{11}$ c) $x = \frac{1}{11}, y = \frac{2}{11}$ d) $x = \frac{1}{11}, y = -\frac{2}{11}$

111. Let A and B be two symmetric matrices of same order. Then, the matrix $AB - BA$ is

a) A symmetric matrix b) A skew-symmetric matrix
c) A null matrix d) The identity matrix

112. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix} a, B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ and adj $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the values of x and y are respectively

a) $(1, 1)$ b) $(-1, 1)$ c) $(1, 0)$ d) None of these

113. Let p is a non-singular matrix such that $1 + p + p^2 + \ldots + p^n = O$ (O denotes the null matrix), then p^{-1} is

a) p^n b) $-p^n$ c) $-(1 + p + \ldots + p^n)$ d) None of these

114. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then the value of $x + y + z$ is

a) 3 b) 0 c) 2 d) 1

115. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the matrix reflection in the line

a) $x = 1$ b) $x + y = 1$ c) $y = 1$ d) $x = y$

116. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

a) $a = 1, b = 1$ b) $a = \sin 2\theta, b = \cos 2\theta$
c) $a = \cos 2\theta, b = \sin 2\theta$ d) None of the above

117. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then adj A is equal to

a) A b) A' c) $3A$ d) $3A'$

118. Let the homogeneous system of linear equations $px + y + z = 0, x + qy + z = 0$, and $x + y + rz = 0$, where $p, q, r \neq 1$, have a non-zero solution, then the value of $\frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r}$ is

a) -1 b) 0 c) 2 d) 1

119. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then B is equal to

a) $\cos^2\frac{\theta}{2} \cdot A$ b) $\cos^2\frac{\theta}{2} \cdot A^T$ c) $\cos^2\theta \cdot I$ d) $\sin^2\frac{\theta}{2} \cdot A$

120. The values of x, y, z in order, if the system of equations $3x + y + 2z = 3, 2x - 3y - z = -3,\ x + 2y + z = 4$ has unique solution, are

a) 2, 1, 5 b) 1, 1, 1 c) $1, -2, -1$ d) $1, 2, -1$

121. Matrix A is such that $A^2 = 2A - I$, where I is the indentity matrix, then for $n \geq 2, A^n$ is equal to

a) $nA - (n-1)I$ b) $nA - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

122. Matrix M_r is defined as $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}, r \in N$ value of $\det(M_1) + \det(M_2) + \det(M_3) + \ldots + \det(M_{2007})$ is

a) 2007 b) 2008 c) 2008^2 d) 2007^2

123. The number of solutions of the system of equations $x_2 - x_3 = 1, -x_1 + 2x_3 = -2, x_1 - 2, x_1 - 2x_2 = 3$ is

a) Zero b) One c) Two d) Infinite

124. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i, then trace of A is equal to

a) nk b) $n + k$ c) n/k d) None of these

125. If $D = \text{diag}[d_1, d_2, d_3, \ldots, d_n]$, where $d_i \neq 0 \forall\ i = 1, 2, \ldots, n$ then D^{-1} is equal to

a) O b) I_n

c) diag $[d_1^{-1}, d_2^{-1}, \ldots, d_n^{-1}]$ d) None of the above

126. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then $\lim\limits_{n \to \infty} \frac{1}{n} A^n$ is

a) $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ d) None of these

127. The system of equations $2x + y - 5 = 0, x - 2y + 1 = 0, 2x - 14y - a = 0$, is consistent. Then, a is equal to

a) 1 b) 2 c) 5 d) None of these

128. The system of equation

$ax + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

Has no solution, if α is

a) 1 b) Not-2 c) Either-2 or1 d) -2

129. A matrix $A = |a_{ij}|$ is an upper triangular matrix, if

a) It is a square matrix and $a_{ij} = 0, i < j$

b) It is a square matrix and $a_{ij} = 0, i > j$

c) It is not a square matrix and $a_{ij} = 0, i > j$

d) It is not a square matrix and $a_{ij} = 0, i < j$

130. If A=$\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to

a) -1 b) 0 c) 1 d) 2

131. $A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$ and $A^{-1} = \lambda$ (adj A), then λ equal to

a) $-\frac{1}{6}$ b) $\frac{1}{3}$ c) $-\frac{1}{3}$ d) $\frac{1}{6}$

132. If $A = [a_{ij}]$ is a 4×4 matrix and C_{ij} is the cofactor of the element a_{ij} in $|A|$, then the expression $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}$ is equal to

a) 0 b) -1 c) 1 d) $|A|$

133. For what value of λ,the system of equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = 10$ is consistent?

a) 1 b) 2 c) -1 d) 3

134. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then A^{100} is equal to

a) $2^{100}A$ b) $2^{99}A$ c) $100\,A$ d) $299\,A$

135. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is

a) $\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$ b) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$ c) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$ d) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

136. Which of the following is correct?
   a) Determinant is square matrix
   b) Determinant is a number associated to a matrix
   c) Determinant is a number associated to a square matrix
   d) None of these
137. If $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, J=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then B equals
   a) $I\cos\theta+J\sin\theta$ b) $I\sin\theta+J\cos\theta$ c) $I\cos\theta-J\sin\theta$ d) $-I\cos\theta+J\sin\theta$
138. What must be the matrix X if $2X+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$?
   a) $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ b) $\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}$ c) $\begin{bmatrix}2 & 6\\ 4 & -2\end{bmatrix}$ d) $\begin{bmatrix}2 & -6\\ 4 & -2\end{bmatrix}$
139. A and B be 3×3 matrices. Then, $AB=O$ implies
   a) $A=O$ and $B=O$
   b) $|A|=O$ and $|B|=O$
   c) Either $|A|=O$ or $|B|=O$
   d) $A=O$ or $B=O$
140. Let $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, D=\begin{bmatrix}3\\ 5\\ 11\end{bmatrix}$ and $A=\begin{bmatrix}1 & -1 & -2\\ 2 & 1 & 1\\ 4 & -1 & -2\end{bmatrix}$, if $X=A^{-1}D$, then X is equal to
   a) $\begin{bmatrix}1\\ 0\\ 2\end{bmatrix}$ b) $\begin{bmatrix}\frac{8}{3}\\ \frac{-1}{3}\\ 0\end{bmatrix}$ c) $\begin{bmatrix}-\frac{8}{3}\\ 1\\ 0\end{bmatrix}$ d) $\begin{bmatrix}\frac{8}{3}\\ \frac{1}{3}\\ -1\end{bmatrix}$
141. If A and B are matrics such that AB and $A+B$ both are defined, then
   a) A and B can be any two matrices
   b) A and B are square matrices not necessarily of the same order
   c) A, B are square matrices of the same order
   d) Number of columns of A is same as the number of rows of B
142. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz, y=az+cx$, and $z=bx+ay$ have non-zero solution. Then, $a^2+b^2+c^2+2\,abc$ is equal to
   a) 1 b) 2 c) -1 d) 0
143. If I_n is the identity matrix of order n, then rank of I_n is
   a) 1 b) n c) 0 d) None of these
144. If the matrix $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & \lambda\end{bmatrix}$ is singular, then λ is equal to
   a) 3 b) 4 c) 2 d) 5
145. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, then $I+A+A^2+A^3+\cdots\infty$ equals to
   a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}-1 & -2\\ -3 & -4\end{bmatrix}$ c) $\begin{bmatrix}1/2 & -1/3\\ -1/2 & 0\end{bmatrix}$ d) $\begin{bmatrix}-1/4 & 1/3\\ 1/2 & 0\end{bmatrix}$
146. If A is a non-singular square matrix of order n, then the rank of A is
   a) Equal to n b) Less than n c) Greater than n d) None of these
147. If A=$\begin{bmatrix}1 & -2\\ 4 & 5\end{bmatrix}$ and $f(t)=t^2-3t+7$, then $f(A)+\begin{bmatrix}3 & 6\\ -12 & -9\end{bmatrix}$ is equal to
   a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ c) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ d) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$
148. The system of linear equations
   $x+y+z=2$
   $2x+y-z=3$

$3x + 2y + kz = 4$ has a unique solution if

a) $k \neq 0$ b) $-1 < k < 1$ c) $-2 < k < 2$ d) $k = 0$

149. The number of solutions of the system of equations

$2x + y - z = 7, x - 3y + 2z = 1, x + 4y - 3z = 5$ is

a) 0 b) 1 c) 2 d) 3

150. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, the value of X^n is equal to

a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$ c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ d) None of these

151. If I_3 is the identity matrix of order 3, then $(I_3)^{-1} =$

a) 0 b) $3 I_3$ c) I_3 d) Not necessarily exists

152. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar, then $|kA| =$

a) $k^n|A|$ b) $k|A|$ c) $k^{n-1}|A|$ d) None of these

153. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

a) Null matrix b) Unit matrix c) $-A$ d) A

154. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

a) 1 b) -1 c) 4 d) No real values

155. If A is a square matrix such that $A\,(\text{adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $|\text{adj } A| =$

a) 4 b) 16 c) 64 d) 256

156. If ω is a complex cube root of unity and $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then A^{50} is

a) $\omega^2 A$ b) ωA c) A d) 0

157. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals

a) 0 b) -1 c) 2 d) None of these

158. The adjoint of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is

a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ b) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ c) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ d) $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

159. The inverse matrix of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ 1 & -6 & 3 \\ 1 & 2 & -1 \end{bmatrix}$ c) $\frac{1}{2}\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ d) $\frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

160. If $f(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $\{f(\theta)^{-1}\}$ is equal to

a) $f(-\theta)$ b) $f(\theta)^{-1}$ c) $f(2\theta)$ d) None of these

161. If the three linear equations

$x + 4ay + az = 0$

$x + 3by + bz = 0$

$x + 2cy + cz = 0$

Have a non-trivial solution, where $a \neq 0, b \neq 0, c \neq 0$, then $ab + bc$ is equal to

a) $2ac$ b) $-ac$ c) ac d) $-2ac$

162. If A and B are two matrices such that rank of $A = m$ and rank of $B = n$, then

a) rank $(AB) = mn$

b) rank $(AB) \geq$ rank (A)
c) rank $(AB) \geq rank(\mathrm{B})$
d) rank $(AB) \leq \min(\text{rank } A, \text{rank } B)$

163. If A is a non-zero column matrix of order $m \times 1$ and B is a non-zero row matrix of order $1 \times n$, then rank of AB equats
a) 1 b) 2 c) 3 d) 4

164. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A is equal to
a) $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

165. If $A^2 - A + I = 0$, then the inverse of A is
a) $I - A$ b) $A - I$ c) A d) $A + I$

166. If B is an invertible matrix and A is a matrix, then
a) $\text{rank}(BA) = \text{rank}(A)$ b) $\text{rank}(BA) \geq \text{rank}(B)$ c) $\text{rank}(BA) > \text{rank}(A)$ d) $\text{rank}(BA) > \text{rank}(B)$

167. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, $|\text{adj } A|$is equal to
a) 6 b) 16 c) 10 d) None of these

168. $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ is equal to
a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

169. Let $A = [a_{ij}]_{m \times n}$ be a matrix such that $a_{ij} = 1$ for all i, j. Then,
a) rank $(A^T) > 1$ b) rank $(A) = 1$ c) rank $(A) = m$ d) rank $(A) = n$

170. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true?
a) If det (A)=±1, then A^{-1} need not exist
b) If det (A)=±1, then A^{-1} exists but all its entries are not necessarily integers
c) If det $(A)$$\neq \pm1$, then A^{-1}exists and all its entries are non $-$ integers
d) If det (A) =±1,then A^{-1} exists and all its entries are integers

171. Matrix $A = \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for
a) $k = 1$ b) $k = -1$ c) $k = \pm1$ d) None of these

172. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then
a) $a = 1, b = 1$
b) $a = \cos 2\theta, b = \sin 2\theta$
c) $a = \sin 2\theta,\ b = \cos 2\theta$
d) None of these

173. If $x^2 + y^2 + z^2 \neq 0, x = cy + bz, y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc =$
a) 2 b) $a + b + c$ c) 1 d) $ab + bc + ca$

174. If $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$ and $A^2 - 4A = 10I = A$ then k is equal to
a) 0 b) -4 c) 4 and not 1 d) 1 or 4

175. Matrix A such that $A^2 = 2A - I$, where I is the identity matrix. Then, for $n \geq 2, A^n$ is equal to
a) $nA - (n-1)I$ b) $nA - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

176. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$is
a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ d) None of these

177. If A is an orthogonal matrix, then A^{-1} equals
a) A b) A^T c) A^2 d) None of these

178. By elementary transformation method, the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$ is

a) $\begin{bmatrix}-2 & 0 & 1\\ 0 & 3 & -2\\ 1 & -2 & 1\end{bmatrix}$ b) $\begin{bmatrix}2 & 0 & -1\\ 0 & -3 & 2\\ -1 & 2 & -1\end{bmatrix}$ c) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 6\end{bmatrix}$ d) None of these

179. What must be the matrix X, if $2X + \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$?

a) $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ b) $\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}$ c) $\begin{bmatrix}2 & 6\\ 4 & -2\end{bmatrix}$ d) $\begin{bmatrix}2 & -6\\ 4 & -2\end{bmatrix}$

180. If $A = \begin{bmatrix}4 & x+2\\ 2x-3 & x+1\end{bmatrix}$ is symmetric, then $x =$

a) 3 b) 5 c) 2 d) 4

181. If A=$\begin{bmatrix}3 & 3 & 3\\ 3 & 3 & 3\\ 3 & 3 & 3\end{bmatrix}$, A^4 is equal to

a) $27A$ b) $81A$ c) $243A$ d) $729A$

182. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix}1 & \omega^2 & \omega\\ \omega^2 & \omega & 1\\ \omega & 1 & \omega^2\end{bmatrix}$ is a

a) Singular matrix b) Non-symmetric matrix
c) Skew-symmetric matrix d) None of these

183. The values of λ and μ for which of the system of equations $x + y + z = 6, x = 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have infinite number of solutions, are

a) $\lambda = 3, \mu = 10$ b) $\lambda = 3, \mu =\neq 10$ c) $\lambda \neq 3, \mu = 10$ d) $\lambda \neq 3, \mu =\neq 10$

184. If A and B are square matrices of the same order such that $(A + B)$ $(A - B)$=$A^2 - B^2$, than $(ABA^{-1})^2$ is equal to

a) B^2 b) I c) A^2B^2 d) A^2

185. If A is a skew-symmetric matrix, then trace of A is

a) 1 b) -1 c) 0 d) None of these

186. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to

a) 4 b) 5 c) 6 d) 7

187. Let A and B are two square matrices such that $AB = A$ and $BA = B$, then A^2 equals to

a) B b) A c) I d) O

188. A and B are two square matrices of same order and A' denotes the transpose of A, then

a) $(AB) = B'A'$
b) $(AB)' = A'B'$
c) $AB = 0 \Rightarrow |A| = 0$ or $|B| = O$
d) $AB = 0 \Rightarrow A = 0$ or $B = 0$

189. The element in the first row and third column of the inverse of the matrix $\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$ is

a) -2 b) 0 c) 1 d) 7

190. If $A = \begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix} = f(x)$, then A^{-1} is equal to

a) $f(-x)$ b) $f(x)$ c) $-f(x)$ d) $-f(-x)$

191. If $A = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$,then $A^{-1}$is

a) $-A$ b) A c) 1 d) None of these

192. If $A = \begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $A^2 - kA - 5\,I_2 = O$, then the value of k is

a) 3 b) 5 c) 7 d) -7

193. Consider the following statements :

1. There can exist two matrices A, B of order 2×2 such that $AB - BA = I_2$

2. Positive odd integral power of a skew-symmetric matrix is symmetric

a) Only (1) b) Only (2) c) Both of these d) None of these

194. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ c) $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

195. The number of non-trivial solutions of the system $x - y + z = 0, x + 2y - z = 0, 2x + y + 3z = 0$ is

a) 0 b) 1 c) 2 d) 3

196. If $\begin{bmatrix} 1-1 & x \\ 1 \quad x & 1 \\ x-1 & 1 \end{bmatrix}$ has no inverse, then the real value of x is

a) 2 b) 3 c) 0 d) 1

197. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is

a) $\frac{13}{25}$ b) $-\frac{25}{13}$ c) $\frac{5}{13}$ d) $\frac{25}{13}$

198. The rank of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$ is

a) 2 b) 3 c) 1 d) Indeterminate

199. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

a) $\alpha = a^2 + b^2, \beta = ab$

b) $\alpha = a^2 + b^2, \beta = 2\,ab$

c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$

d) $\alpha = 2\,ab, = a^2 + b^2$

200. If $A = [a_{ij}]$ is a scalar matrix, then trace of A is

a) $\sum_i \sum_j a_{ij}$ b) $\sum_i a_{ij}$ c) $\sum_j a_{ij}$ d) $\sum_i a_{ii}$

201. The system of equations $x + 2y + 3z = 1, 2x + y + 3z = 2, 5x + 5y + 9z = 5$ has

a) Unique solution b) Infinite many solution

c) Inconsistent d) None of the above

202. The rank of the matrix $\begin{bmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{bmatrix}$ is 2, then

a) $k = \frac{5}{2}, x = \frac{1}{5}$ b) $k = \frac{5}{2}, x \neq \frac{1}{5}$ c) $k = \frac{1}{5}, x = \frac{5}{2}$ d) None of these

203. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 =$

a) $\begin{bmatrix} 8 & -5 \\ -5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 8 & -5 \\ 5 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 8 & -5 \\ -5 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

204. If ω is a root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then A^{-1} is equal to

a) $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ b) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ d) $\frac{1}{2}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

205. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r, then

a) $r = \min(m, n)$ b) $r < \min(m, n)$ c) $r \leq \min(m, n)$ d) None of these

206. For each real x: $-1 < x < 1$. Let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$, then

a) $A(z) = A(x)A(y)$ b) $A(z) = A(x) - A(y)$ c) $A(z) = A(x)[A(y)]^{-1}$ d) $A(z) = A(x) + A(y)$

207. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then the matrix $A^2(\alpha)$ is

a) $A(2\alpha)$ b) $A(\alpha)$ c) $A(3\alpha)$ d) $A(4\alpha)$

208. If A is a symmetric matrix and $n \epsilon N$, then A^n is

a) Symmetric matrix b) A diagonal matrix
c) Skew-symmetric matrix d) None of the above

209. The inverse matrix of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ 1 & -6 & 3 \\ 1 & 2 & -1 \end{bmatrix}$ c) $\frac{1}{2}\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ d) $\frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

210. If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix, then the symmetric matrix is

a) $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

211. If the system of linear equations $x + 2ay + az = 0, x + 3by + bz = 0$ and $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c

a) Are in AP b) Are in GP
c) Are in HP d) Satisfy $a + 2b + 3c = 0$

212. For what value of k the following system of linear equations will have infinite solutions
$x - y + z = 3, 2x + y - z = 2$
and $-3x + 2ky + 6z = 3$

a) $k \neq 2$ b) $k = 0$ c) $k = 3$ d) $k \in [2, 3]$

213. The product of two orthogonal matrices is

a) Orthogonal b) Involutory c) Unitary d) Idempotent

214. The system of equations $x + y + z = 8, x - y + 2z = 6, 3x + 5y - 7z = 14$ has

a) No solution b) Unique solution
c) Infinitely many solution d) None of the above

215. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

a) -1 b) 1 c) 0 d) No real values

216. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$

a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$

217. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $[F(\alpha)]^{-1}$ is equal to

a) $F(-\alpha)$ b) $F(\alpha^{-1})$ c) $F(2\alpha)$ d) None of these

218. Let for any matrix M, M^{-1} exist which of the following is not true?

a) $|M^{-1}| = |M|^{-1}$ b) $(M^2)^{-1} = (M^{-1})^2$ c) $(M^T)^{-1} = (M^{-1})^T$ d) $(M^{-1})^{-1} = M$

219. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?

a) $AB = BA$ b) Either of A or B is a zero matrix
c) Either of A or B is an identity matrix d) $A = B$

220. $x_1 + 2x_2 + 3x_3 = 2x_1 + 3x_2 + x_3 = 3x_1 + x_2 + 2x_3 = 0$.
This system of equations has

a) Infinite solution b) No solution c) No solution d) Unique solution

221. If A is a 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined, then order of B is

a) 3×4 b) 3×3 c) 4×4 d) 4×3

222. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then the value of X^n is

a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$ c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ d) None of these

223. Let $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. Then, $\left(F(\alpha)\right)^{-1}$ is equal to

a) $F(-\alpha)$ b) $F(\alpha^{-1})$ c) $F(2\,\alpha)$ d) None of these

224. For any square matrix A, AA^T is a

a) Unit matrix b) Symmetric matrix

c) Skew-symmetric matrix d) Diagonal matrix

225. If A is a square matrix or order $n \times n$, then adj(adj A) is equal to

a) $|A|^n A$ b) $|A|^{n-1} A$ c) $|A|^{n-2} A$ d) $|A|^{n-3} A$

226. If a system of the equations $(\alpha+1)^3 x + (\alpha+2)^3 y - (\alpha+3)^3 = 0$, $(\alpha+1)x + (\alpha+2)y - (\alpha+3) = 0$, and $x + y - 1 = 0$ is consistent. What is the value of α?

a) 1 b) 0 c) -3 d) -2

227. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $\lim_{n\to\infty} \frac{1}{n} A^n$ is

a) A null matrix b) An identity matrix c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ d) None of these

228. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$

and, $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$

are two matrices such that the product AB is the null matrix, then $(\alpha - \beta)$ is

a) 0

b) Multiple of π

c) An odd multiple of $\pi/2$

d) None of these

229. If A be a square matrix of order n and if $|A| = D$ and $|\text{adj A}| = D'$, then

a) $DD' = D^2$ b) $DD' = D^{-1}$ c) $DD' = D^n$ d) None of these

230. If $1, \omega, \omega^2$ are the cube roots of unity and if $\begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a- & \omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$, then $a^2 + b^2$ is equal to

a) $1+\omega^2$ b) $\omega^2 - 1$ c) $1+\omega$ d) $(1+\omega)^2$

231. If a square matrix A is orthogonal as well as symmetric, then

a) A is involutory matrix

b) A is idempotent matrix

c) A is a diagonal matrix

d) None of these

232. The real value of k for which the system of equations $2k\,x - 2y + 3z = 0, x + ky + 2z = 0, 2x + kz = 0$, has non-trivial solution is

a) 2 b) -2 c) 3 d) -3

233. If the matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$, then AB

a) $\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

234. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

a) 1 b) -1 c) 4 d) No real values

235. If $E(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then $E(\alpha)E(\beta)$ is equal to

a) $E(0°)$ b) $E(\alpha\beta)$ c) $E(\alpha+\beta)$ d) $E(\alpha-\beta)$

236. If $A=\begin{bmatrix}b & b^2\\ -a^2 & -ab\end{bmatrix}$, then A is

a) Iidempotent b) Involutory c) Nilpotent d) Scalar

237. The matrix $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$ is

a) Unitary b) Orthogonal c) Nilpotent d) Involutory

238. Let $X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$, $A=\begin{bmatrix}1 & -1 & 2\\ 2 & 0 & 1\\ 3 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3\\1\\4\end{bmatrix}$. If $AX=B$, then X is equal to

a) $\begin{bmatrix}1\\2\\3\end{bmatrix}$ b) $\begin{bmatrix}-1\\-2\\3\end{bmatrix}$ c) $\begin{bmatrix}-1\\-2\\-3\end{bmatrix}$ d) $\begin{bmatrix}-1\\2\\3\end{bmatrix}$

239. If A is a skew-symmetric matrix of odd order, then $|\text{adj } A|$ is equal to

a) 0 b) n c) n^2 d) None of these

240. The system of equations $x+3y+2z=0, 3x+y+z=0$ and $2x-2y-z=0$

a) Possesses a trivial solution only b) Possesses a non-zero unique solution

c) Does not have a common non-zero solution d) Has infinitely many solutions

241. Consider the following statements:

1. A square matrix A is hermitian, if $A=A'$

2. Let $A=[a_{ij}]$ be a skew- hermitian matrix, then a_{ij} is purely imaginary

3. All integer powers of a symmetric matrix are symmetric. Which of these is/are correct?

a) (1)and (2) b) (2)and (3) c) (3)and (1) d) (1), (2) and (3)

242. If $a_1, a_2, a_3, a_4, a_5, a_6$ are in AP with common difference $d\neq 0$,then the system of equations $a_1x+a_2\,y=a_3, a_4x+a_5y=a_6$has

a) Infinite number of solutions b) Unique solution

c) No solution d) Cannot say any thing

243. If I_n is the identity matrix of order n, then $(I_n)^{-1}$ is equal to

a) Does not exist b) I_n c) 0 d) nI_n

244. If A is a square matrix, then adj $A^T-(\text{adj } A)^T$ is equal to

a) $2\,|A|$ b) $2\,|A|I$ c) Null matrix d) Unit matrix

245. If $\begin{bmatrix}2 & -1 & 3\\ 1 & 3 & -1\\ 3 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}9\\4\\10\end{bmatrix}$,then $\begin{bmatrix}x\\y\\z\end{bmatrix}$ is equal to

a) $\begin{bmatrix}3\\2\\1\end{bmatrix}$ b) $\begin{bmatrix}2\\3\\1\end{bmatrix}$ c) $\begin{bmatrix}1\\2\\3\end{bmatrix}$ d) $\begin{bmatrix}2\\1\\3\end{bmatrix}$

246. Consider the system of equations

$a_1\,x+b_1\,y+c_1\,z=0$

$a_2\,x+b_2\,y+c_2\,z=0$

$a_3\,x+b_3\,y+c_3\,z=0$

if $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$, then the system has

a) More than two solutions

b) One trivial and one non-trivial solutions

c) No solution

d) Only trivial solution (0,0,0)

247. The number of solutions of the system of equations

$x-y+z=2$

$2x + y - z = 5$
$4x + y + z = 10$ is

a) ∞ b) 1 c) 2 d) 0

248. If $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then which of the following statement is not correct?

a) A is orthogonal matrix
b) A' is orthogonal matrix
c) Determinant $A = 1$
d) A is not invertible

249. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is equal to

a) $2A$ b) A c) $-A$ d) I

250. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is

a) 1 if $a = 6$ b) 2 if $a = 1$ c) 3 if $a = 2$ d) 4 if $a = -6$

251. If $D = \text{diag}\,(d_1, d_2, d_3, \dots, d_n)$, where $d_i \neq 0$ for all $i = 1,2,\dots,n$, then D^{-1} is equal to

a) D b) diag $(d_1^{-1} d_2^{-1}, \dots, d_n^{-1})$ c) I_n d) None of these

252. If $f\,(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f\,(A) is$ equal to

a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

253. If $A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the unit matrix of order 2, then A^2 equals

a) $4A - 3I$ b) $3A - 4I$ c) $A - I$ d) $A + I$

254. Which one of the following is true always for any two non-singular matrices A and B of same order?

a) $AB = BA$
b) $(AB)^t = A^t B^t$
c) $(A+B)(A-B) = A^2 - B^2$
d) $(AB)^{-1} = B^{-1}A^{-1}$

255. The inverse of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1/2 & 0 & 2 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$

256. The values of a for which the system of equations $ax + y + z = 0, x - ay + z = 0$, $x + y + z = 0$ possesses non-zero solution, are given by

a) 1, 2 b) 1, -1 c) 0 d) None of these

257. If A is square matrix, then

a) $A + A^T$ is symmetric
b) AA^T is skew-symmetric
c) $A^T + A$ is skew-symmetric
d) $A^T A$ is skew-symmetric

258. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det [adj(adj A)] is equal to

a) 12^4 b) 13^4 c) 14^4 d) None of these

259. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ c) $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

260. If $A = \begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$, then A^{-1} is

a) $\frac{1}{7}\begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$ b) $\frac{1}{7}\begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$ c) $\frac{1}{7}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$ d) Does not exist

261. If the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is commutative with the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then

a) $a = 0, b = c$ b) $b = 0, c = d$ c) $c = 0, d = a$ d) $d = 0, a = b$

262. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,then AB is equal to

a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

263. Let A be a skew-symmetric matrix of odd order, then $|A|$ is equal to

a) 0 b) 1 c) -1 d) None of these

264. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T\ Q^{2005}\ P$ is

a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

265. If X and Y are 2×2 matrices such that $2X + 3Y = O$ and $X + 2Y = I$, where O and I denote the 2×2 zero matrix and the 2×2 identity matrix, then X is equal to

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

266. Consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

a) Infinite number of solutions b) Exactly 3 solutions

c) A unique solution d) No solution

267. If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, then the value of x is

a) 2 b) 3 c) -4 d) 4

268. If A=$\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is

a) A is a zero matrix b) $A = (-1)I$, where I is a unit matrix

c) A^{-1} does not exist d) $A^2 = I$

269. The inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ is equal to

a) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ d) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$

270. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then AB is equal to

a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

271. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(y) = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$, then $[F(x)G(y)]^{-1}$ is equal to

a) $F(-x)G(-y)$ b) $F(x^{-1})G(y^{-1})$ c) $G(-y)F(-x)$ d) $G(y^{-1})F(x^{-1})$

272. Let A=$\begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and A^{-1}=$xA + yI$, then the value of x and y are

a) $x = \frac{-1}{11}, y = \frac{2}{11}$ b) $x = \frac{-1}{11}, y = \frac{-2}{11}$ c) $x = \frac{1}{11}, y = \frac{2}{11}$ d) $x = \frac{1}{11}, y = \frac{-2}{11}$

273. If A^T, B^T are transpose matrices of the square matrices A , B respectively, then $(AB)^T$ is equal to

a) $A^T B^T$ b) AB^T c) BA^T d) $B^T A^T$

274. If $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then the value of (x, y, z) is

a) $(4, 3, 2)$ b) $(3, 2, 4)$ c) $(2, 3, 4)$ d) None of the above

275. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A is equal to

a) Idempotent b) Involuntary c) Nilpotent d) Scalar

276. For non-singular square matrices A, B and C of the same order, $(AB^{-1}C)^{-1}$ is equal to

a) $A^{-1}BC^{-2}$ b) $C^{-1}B^{-1}A^{-1}$ c) CBA^{-1} d) $C^{-1}BA^{-1}$

277. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if

a) $\lambda \neq -17$ b) $\lambda \neq -18$ c) $\lambda \neq -19$ d) $\lambda \neq -20$

278. If a, b, c are non-zero, then the number of solutions of following system of equation is

$\frac{2x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ ...(i)

$-\frac{x^2}{a^2} + \frac{2y^2}{b^2} - \frac{z^2}{c^2} = 0$ ...(ii)

$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{2z^2}{c^2} = 0$ ...(iii)

a) 6 b) 8 c) 9 d) Infinite

279. If $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$, then $|A|$ is equal to

a) 1 b) 0 c) $\log_a b$ d) $\log_b a$

280. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then A^{-1} is equal to

a) $-\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ b) $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

281. If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to

a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

282. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (\text{adj } A)$, and $C = 5A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to

a) 5 b) 25 c) -1 d) 1

283. For $0 < \theta < \pi$, if $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then

a) $A^T = A$ b) $A^T = -A$ c) $A^2 = I$ d) $A^T = A^{-1}$

284. The values of a for which the system of equations $x + y + z = 0, x + ay + az = 0, x - ay + z = 0$, possesses non-zero solutions, are given by

a) 1, 2 b) 1,-1 c) 1, 0 d) None of these

285. If $x\begin{bmatrix} -3 \\ 4 \end{bmatrix} + y\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$, then

a) $x = -2, y = 1$ b) $x = -9, y = 10$ c) $x = 22, y = 1$ d) $x = 2, y = -1$

286. If A is a square matrix such that $AA^T = I = A^TA$, then A is

a) A symmetric matrix

b) A skew-symmetric matrix

c) A diagonal matrix

d) An orthogonal matrix

287. The inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ is

a) $\frac{1}{11}\begin{bmatrix}1 & 2\\ -3 & 5\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\ -3 & 5\end{bmatrix}$ c) $\frac{1}{13}\begin{bmatrix}-2 & 5\\ 1 & 3\end{bmatrix}$ d) $\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

288. If $A=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$, then $A^5=$

a) $5A$ b) $10A$ c) $16A$ d) $32A$

289. If $A(\theta)=\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}$and $AB=I$, then $(\sec^2\theta)B$ is equal to

a) $A(\theta)$ b) $A\left(\frac{\theta}{2}\right)$ c) $A(-\theta)$ d) $A\left(-\frac{\theta}{2}\right)$

290. If $A=[a_{ij}]$ is a skew-symmetric matrix of order n, then $a_{ii}=$

a) 0 for some i b) 0 for all $i=1,2,\dots,n$ c) 1 for some i d) 1 for all $i=1,2,\dots,n$

291. Let $A=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$, then A^n is equal to

a) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a^n & 0\\ 0 & 0 & a\end{bmatrix}$ b) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$ c) $\begin{bmatrix}a^n & 0 & 0\\ 0 & a^n & 0\\ 0 & 0 & a^n\end{bmatrix}$ d) $\begin{bmatrix}na & 0 & 0\\ 0 & na & 0\\ 0 & 0 & na\end{bmatrix}$

292. If A, B are symmetric matrices of the same order then $AB-BA$ is

a) Symmetric matrix
b) Skew-symmetric matrix
c) Null matrix
d) Unit matrix

293. If A is any $m\times n$ matrix such that AB and BA are both defined., then B is an

a) $m\times n$ matrix b) $n\times m$ matrix c) $n\times n$ matrix d) $m\times m$ matrix

294. If A is a square matrix of order $n\times n$ and k is a scalar, then adj (kA) is equal to

a) k adj A b) k^n adj A c) k^{n-1} adj A d) k^{n+1} adj A

295. $x+ky-z=0, 3x-ky-z=0$ and $x-3y+z=0$ has non-zero solution for k is equal to

a) -1 b) 0 c) 1 d) 2

296. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, $I$is the unit matrix of order 2 and a, b are arbitrary constants, then$(aI+bA)^2$is equal to

a) a^2I-abA b) $a^2I+2abA$ c) a^2I+b^2A d) None of the above

297. If A is an orthogonal matrix, then

a) $|A|=0$ b) $|A|=\pm1$ c) $|A|=\pm2$ d) None of these

298. Given$2x-y+2z=2, x-2y+2z=-4, x+y+\lambda z=4$ then the value of λ such that the given system of equations has no solution, is

a) 3 b) 1 c) 0 d) -3

299. If $A=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & x\end{bmatrix}$ is an idempotent matrix, then x is equal to

a) -5 b) -1 c) -3 d) -4

300. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 3 & 0\\ 0 & 1 & 2\end{bmatrix}$ and adj $A=\begin{bmatrix}6 & -2 & -6\\ -4 & 2 & x\\ y & -1 & -1\end{bmatrix}$, then $x+y=$

a) 6 b) -1 c) 3 d) 1

301. If $\begin{bmatrix}x+y & 2x+z\\ x-y & 2z+w\end{bmatrix}=\begin{bmatrix}4 & 7\\ 0 & 10\end{bmatrix}$, then the value of x, y, z, w are

a) 2, 2, 3, 4 b) 2, 3, 1, 2 c) 3, 3, 0, 1 d) None of these

302. If for a matrix A, $A^2+I=O$, where I is the identity matrix, then A equals

a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}-i & 0\\ 0 & -i\end{bmatrix}$ c) $\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$ d) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

303. If $m[-3\quad 4]+n[4\quad -3]=[10\quad -11]$, then $3m+7n$ is equal to

a) 3 b) 5 c) 10 d) 1

304. If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$ and $A(\text{adj }A)=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the value of k is

a) $\sin x\cos x$ b) 1 c) 2 d) 3

305. If $A=\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}$, then$(A^{-1})^3$ is equal to

a) $\frac{1}{27}\begin{bmatrix}1 & -26\\ 0 & 27\end{bmatrix}$ b) $\frac{1}{27}\begin{bmatrix}-1 & 26\\ 0 & 27\end{bmatrix}$ c) $\frac{1}{27}\begin{bmatrix}1 & -26\\ 0 & -27\end{bmatrix}$ d) $\frac{1}{27}\begin{bmatrix}-1 & -26\\ 0 & -27\end{bmatrix}$

306. Let $M=[a_{uv}]_{n\times n}$ be a matrix, where
$a_{uv}=\sin(\theta_u-\theta_v)+i\cos(\theta_u-\theta_v)$, the M is equal to

a) $\bar{M}$ b) $-M$ c) $\bar{M}^T$ d) $-\bar{M}^T$

307. If the system of homogeneous equations $2x-y+z=0, x-2y+z=0, \lambda x-y+2z=0$ has infinitely many solutions, then

a) $\lambda=5$ b) $\lambda=-5$ c) $\lambda\neq\pm5$ d) None of these

308. Assuming that the sum and product given below are defined, which of the following is not true for matrices?

a) $A+B=B+A$ b) $AB=AC$ does not imply $B=C$
c) $AB=O$ implies $A=O$ or $B=O$ d) $(AB)'=B'A'$

309. If $E(\theta)=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$, then $E(\theta)$ E (∅) is a

a) Unit matrix b) Null matrix c) Diagonal matrix d) None of these

310. Consider the system of equations in x,y,z as
$x\sin 3\theta-y+z=0$
$x\cos 2\theta+4y+3z=0$
and $2x+7y+7z=0$
If this system has a non-trivial solution, then for integer n, values of θ are given by

a) $\pi\left(n+\frac{(-1)^n}{3}\right)$ b) $\pi\left(n+\frac{(-1)^n}{4}\right)$ c) $\pi\left(n+\frac{(-1)^n}{6}\right)$ d) $\frac{n\pi}{2}$

311. If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$, then the values of k,a,b are respectively

a) $-6,-12,-18$ b) $-6,4,9$ c) $-6,-4,-9$ d) $-6,12,18$

312. If A is a non-singular matrix of order 3, then adj (adj A) is equal to

a) A b) A^{-1} c) $\frac{1}{|A|}A$ d) $|A|A$

313. If A is a matrix such that $A^2=A+1$, where I is the unit matrix, then A^5 is equal to

a) $5A+I$ b) $5A+2I$ c) $5A+3I$ d) $5A+4I$

314. If A and B are two matrices such that both $A+B$ and AB are defined, then

a) A and B are of same order b) A is of order $m\times m$ and B is of order $n\times n$
c) Both A and B are of same order $n\times n$ d) A is of order $m\times n$ and B is of order $n\times m$

315. If $U=[2-3\ \ 4], X=[0\ \ 2\ \ 3], V=\begin{bmatrix}3\\2\\1\end{bmatrix}$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then $UV+XY$

a) [20] b) 20 c) [-20] d) -20

316. The matrix $A=\begin{bmatrix}a & 2\\ 2 & 4\end{bmatrix}$ is singular, if

a) $a\neq1$ b) $a=1$ c) $a=0$ d) $a=-1$

317. If $A=\begin{bmatrix}1 & x\\ x^2 & 4y\end{bmatrix}$and $B=\begin{bmatrix}-3 & 1\\ 1 & 0\end{bmatrix}$, adj $A+B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$,
Then values of x and y are

a) 1, 1 b) ±1, 1 c) 1, 0 d) None of these

318. $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, then $A^{-1}$is

a) A b) $-A$ c) adj (A) d) -adj (A)

319. Which of the following is incorrect?

a) $A^2 - B^2 = (A+B)(A-B)$

b) $(A^T)^T = A$

c) $(AB)^n = A^n B^n$ where A, B commute

d) $(A-I)(I+A) = 0 \Leftrightarrow A^2 = I$

320. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$, then $f(A)$ is

a) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ d) None of these

321. If A is an invertible matrix, then det (A^{-1}) is equal to

a) det (A) b) $\dfrac{1}{\det(A)}$ c) 1 d) None of these

322. If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to

a) $3B$ b) $\frac{1}{3}B$ c) $3B^{-1}$ d) $\frac{1}{3}B^{-1}$

323. If the points(x_1, y_1), $(x_2, y_2)$and $(x_3, y_3)$are collinear,

then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than

a) 2 b) 3 c) 1 d) None of these

324. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the unit matrix of order 2, then A^2 equals

a) $4A - 3I$ b) $3A - 4I$ c) $A - I$ d) $A + I$

325. If $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$, then adj A is

a) $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -6 \\ 5 & -7 \end{bmatrix}$ c) $\begin{bmatrix} 7 & -5 \\ 6 & -2 \end{bmatrix}$ d) None of these

326. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ then $A.$(adj A) is equal to

a) A b) $|A|$ c) $|A|I$ d) None of these

327. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \epsilon N$. Then

a) There exist more than one but finite number of B's such that $AB = BA$

b) There exist exactly one B such$AB = BA$

c) There exists infinitely many B's such that $AB = BA$

d) There cannot exist any B such that $AB = BA$

328. If $2A + 3B$=$\begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$, then B is

a) $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$

329. If I is unit matrix of order 10, then the determinant of I is equal to

a) 10 b) 1 c) 1/10 d) 9

330. Let $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$is equal to

a) 80 b) 100 c) -110 d) 92

331. If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$is the sum of a symmetric matrix B and skew-symmetric matrix C, then B is

a) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

332. Let A be a non-singular square matrix. Then, $|\text{adj } A|$ is equal to

a) $|A|^n$ b) $|A|^{n-1}$ c) $|A|^{n-2}$ d) None of these

333. If A=$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}$ $(A = B)^2 = A^2 + B^2$.Then a and b are respectively

a) 1, -1 b) 2, -3 c) -1, 1 d) 3, -2

334. For non-singular square matrices A, B and C of same order, $(AB^{-1}C)^{-1}$ is equal to

a) $A^{-1}BC^{-1}$ b) $C^{-1}B^{-1}A^{-1}$ c) $CB^{-1}A^{-1}$ d) $C^{-1}BA^{-1}$

335. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$, then $A^{-1} =$

a) $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

MATHS (QUESTION BANK)

3.MATRICES

: ANSWER KEY :

1)	a	2)	d	3)	b	4)	b	141)	b	142)	a	143)	b	144)	a
5)	b	6)	c	7)	a	8)	b	145)	c	146)	a	147)	b	148)	a
9)	c	10)	d	11)	d	12)	d	149)	a	150)	d	151)	c	152)	a
13)	a	14)	d	15)	a	16)	b	153)	b	154)	d	155)	b	156)	a
17)	c	18)	a	19)	d	20)	b	157)	a	158)	a	159)	a	160)	a
21)	c	22)	b	23)	d	24)	d	161)	a	162)	d	163)	c	164)	c
25)	a	26)	b	27)	d	28)	b	165)	a	166)	a	167)	c	168)	d
29)	c	30)	a	31)	b	32)	d	169)	b	170)	d	171)	c	172)	b
33)	c	34)	a	35)	a	36)	a	173)	c	174)	c	175)	a	176)	c
37)	c	38)	c	39)	d	40)	c	177)	b	178)	a	179)	a	180)	b
41)	a	42)	a	43)	d	44)	a	181)	d	182)	a	183)	a	184)	a
45)	c	46)	d	47)	c	48)	c	185)	c	186)	c	187)	b	188)	a
49)	a	50)	a	51)	d	52)	a	189)	d	190)	a	191)	b	192)	b
53)	c	54)	c	55)	d	56)	d	193)	d	194)	b	195)	a	196)	d
57)	c	58)	d	59)	a	60)	b	197)	b	198)	a	199)	b	200)	d
61)	a	62)	c	63)	a	64)	d	201)	a	202)	a	203)	d	204)	b
65)	d	66)	b	67)	b	68)	b	205)	c	206)	a	207)	a	208)	a
69)	b	70)	c	71)	b	72)	a	209)	a	210)	a	211)	c	212)	c
73)	b	74)	d	75)	d	76)	a	213)	a	214)	b	215)	a	216)	a
77)	c	78)	d	79)	d	80)	a	217)	a	218)	b	219)	a	220)	c
81)	b	82)	b	83)	a	84)	b	221)	a	222)	d	223)	a	224)	b
85)	a	86)	c	87)	b	88)	d	225)	c	226)	d	227)	a	228)	c
89)	a	90)	d	91)	c	92)	a	229)	c	230)	c	231)	a	232)	a
93)	d	94)	a	95)	d	96)	b	233)	a	234)	d	235)	c	236)	c
97)	b	98)	b	99)	d	100)	b	237)	c	238)	d	239)	a	240)	d
101)	b	102)	b	103)	b	104)	a	241)	b	242)	b	243)	b	244)	c
105)	b	106)	d	107)	b	108)	b	245)	c	246)	a	247)	b	248)	d
109)	c	110)	b	111)	b	112)	a	249)	b	250)	b	251)	b	252)	d
113)	a	114)	a	115)	d	116)	c	253)	a	254)	d	255)	d	256)	b
117)	d	118)	d	119)	b	120)	d	257)	a	258)	c	259)	b	260)	a
121)	a	122)	d	123)	a	124)	a	261)	c	262)	a	263)	a	264)	a
125)	c	126)	a	127)	d	128)	d	265)	c	266)	d	267)	d	268)	d
129)	b	130)	b	131)	a	132)	d	269)	b	270)	a	271)	c	272)	a
133)	d	134)	b	135)	d	136)	c	273)	d	274)	c	275)	c	276)	d
137)	a	138)	a	139)	c	140)	b	277)	a	278)	d	279)	b	280)	a

281)	b	282)	d	283)	d	284)	b	313)	c	314)	c	315)	a	316)	b
285)	a	286)	d	287)	a	288)	c	317)	a	318)	c	319)	a	320)	a
289)	c	290)	b	291)	c	292)	b	321)	b	322)	b	323)	b	324)	a
293)	b	294)	c	295)	c	296)	b	325)	a	326)	c	327)	c	328)	b
297)	b	298)	b	299)	c	300)	a	329)	b	330)	b	331)	a	332)	b
301)	a	302)	b	303)	d	304)	b	333)	a	334)	d	335)	b		
305)	a	306)	d	307)	a	308)	c								
309)	b	310)	c	311)	c	312)	d								

MATHS (QUESTION BANK)

3.MATRICES

: HINTS AND SOLUTIONS :

1 **(a)**

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$. Then,

$A^2 = AA = \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore A^3 = A, A^4 = A^2 = I, A^2 = A^4 A = IA = A$ etc.

Hence, $A^n = \begin{cases} A, \text{if } n \text{ id odd} \\ 0, \text{if } n \text{ is even} \end{cases}$

2 **(d)**

$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$

and $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2$

$= \begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [given]

$\Rightarrow \quad a^2b^2 = 1 \Rightarrow ab = 1$

3 **(b)**

Given matrix A is a square matrix

And $AA^T = I = A^T A$

$\Rightarrow |AA^T| = |I| = |A^T A|$

$\Rightarrow |A||A^T| = 1 = |A^T||A|$

$\Rightarrow |A|^2 = 1 \quad [\because A \cdot A^T = |A|^2]$

$\Rightarrow |A| = \pm 1$

4 **(b)**

Let $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,then, $kI = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

$\Rightarrow \text{adj}(kI) = \begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & K^2 \end{bmatrix} = k^2 I$

5 **(b)**

Given, A is singular$\Rightarrow |A| = 0$

Now, $A(\text{adj}A) = |A|I_n = 0$

$\therefore A(\text{adj } A) = 0$ *ie*, $A(\text{adj } A)$is a zero matrix.

6 **(c)**

Given, AB= O

$\therefore \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix} \times \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin\beta \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \cos\beta\sin\alpha\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad \cos(\alpha - \beta) = 0$

$\Rightarrow \quad \alpha - \beta$ is an odd multiple of $\frac{\pi}{2}$

8 **(b)**

Given, $\quad A \begin{vmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{vmatrix}$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 3R_1$

$\Rightarrow A = \begin{vmatrix} 2 & 4 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

Since the equivalent matrix in echelon from has only one non-zero row,

$\therefore \quad \text{Rank}(A) = 1$

9 **(c)**

$\because AB = B, BA = A$

$\therefore A^2 + B^2 = AA + BB = A(BA) + B(AB)$

$= (AB)A + (BA)B = BA + AB$

$= A + B$

$\Rightarrow A^2 = A$ and $B^2 = B$

10 **(d)**

$|A| = 7(1-0) + 3(-1-0) - 3(0+1) = 1$

Cofactors of matrix A are

$C_{11} = 1, \quad C_{12} = 1, \; C_{13} = 1$

$C_{21} = 3, \quad C_{22} = 4, C_{23} = 3$

$C_{31} = 3, \quad C_{32} = 3, C_{33} = 4$

$\therefore \quad \text{adj }(A) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

11 **(d)**

It is given that A is a skew-symmetric matrix

$\therefore A^T = -A$

$\Rightarrow (A^T)^n = (-A)^n$

$\Rightarrow (A^n)^T = (-1)^n A^n$

$\Rightarrow (A^n)^T = \begin{cases} A^n & \text{if } n \text{ is even} \\ -A^n & \text{if } n \text{ is odd} \end{cases}$

Hence, A^n is skew-symmetric when n is odd

12 **(d)**

Consider $\begin{bmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{bmatrix}$. By placing a_1 in any one of the 6^*

Position and 0 elsewhere. We get 6 non-singular matrices.

Similarly, $\begin{bmatrix} * & * & 1 \\ * & 1 & * \\ 1 & * & * \end{bmatrix}$ gives at least one non-singular.

13 **(a)**

We know that, $\text{Tr}(A) = \sum_{i=1}^{3} a_{ii}$

$\text{Tr}(A) = 1 + 7 + 9 = 17$

14 **(d)**

Now, $A + B = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow |A + B| = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2 - 4 = -2$

Also, $\text{adj}(A + B) = \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$

$\therefore (A + B)^{-1} = -\frac{1}{2}\begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 1 & -1/2 \end{bmatrix}$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix}$ and $B^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix}$

$\therefore A^{-1} + B^{-1} = \begin{bmatrix} 1 & -1 \\ -1/2 & 3/4 \end{bmatrix} + \begin{bmatrix} -1/2 & 1/2 \\ 0 & -1/2 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/4 \end{bmatrix}$

$\therefore (A + B)^{-1} \neq A^{-1} + B^{-1}$

15 **(a)**

$f(A) = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

16 **(b)**

$C_{11} = 1, \quad C_{12} = -2, C_{13} = -2$

$C_{21} = -1, C_{22} = 3, C_{23} = 3$

$C_{31} = 0, C_{32} = -4, C_{33} = -3$

$\therefore \text{adj}(A) = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

17 **(c)**

Since, the rank of given matrix is 1, then

$\begin{bmatrix} 2 & 5 \\ -4 & a-4 \end{bmatrix} = 0$

$\Rightarrow 2a - 8 + 20 = 0$

$\Rightarrow a = -6$

18 **(a)**

Since, given equations have a non-trivial solution

$\therefore \quad \Delta = 0$

$\Rightarrow \begin{vmatrix} 1 & 3 & \lambda \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{vmatrix} = 0$

$\Rightarrow 1(-8+5) - 3(-4+1) + \lambda(10-4) = 0$

$\Rightarrow 6\lambda = -6 \quad \Rightarrow \quad \lambda = -1$

20 **(b)**

Since, $|a| \neq 0$ So, A^{-1} exists

$\therefore AB = AC$

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow B = C$

21 **(c)**

Let $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}, B = \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

For consistent, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0$

$\Rightarrow 1(20-16) - 1(10-4) + 1(4-2) = 0$

$\Rightarrow 4 - 6 + 2 = 0$

$\Rightarrow 0 = 0$

and $(\text{adj } A)B = O$

$\Rightarrow \begin{bmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 - 6\alpha + 2\alpha^2 \\ -6 + 9\alpha - 3\alpha^2 \\ 2 - 3\alpha + \alpha^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2\alpha^2 - 6\alpha + 4 = 0, -3\alpha^2 + 9\alpha - 6 = 0$

and $\alpha^2 - 3\alpha + 2 = 0$

Now, $2\alpha^2 - 6\alpha + 4 = 0$

$\Rightarrow (2\alpha - 2)(\alpha - 2) = 0$

$\Rightarrow \alpha = 1, 2$

Similarly from other equations we also get the same value

22 **(b)**

Given, $B = A^6 - A^5$, where $A^3 = A + I$

$\Rightarrow B = (A^3)^2 - A^3A^2$

$= (A + I)^2 - (A = I)A^2$

$= A^2 + I^2 + 2AI - A^3 - A^2I$

$= I + 2A - (A + I)$

$\Rightarrow \quad B = A$

$\therefore$ Inverse of $B = A^{-1}$

23 **(d)**

Since, $A^TA = I$

$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c62 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \ a^2 + b^2 + c62 = 1$ and $ab + bc + ca = 0$

Now, $(a+b+c)^2 =$

$a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 1 + 2 \cdot 0 = 1$

$\Rightarrow \ a + b + c = 1 \quad ...(i)$

Now, $(a^3 + b^3 + c^3) = (a+b+c)$

$(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$= (a+b+c) + 3$

$\Rightarrow \ a^3 + b^3 + c^3 = 1 + 3 = 4$ [using Eq. (i)]

24 **(d)**

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now,

$$A^{2-}4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5I_3$$

25 **(a)**

We have,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\Rightarrow A^4 = A^2A^2 = I_2I_2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

26 **(b)**

The system of given equation are

$Kx + 2y - z = 1 \qquad ...(i)$

$(K-1)y - 2z = 2 \qquad ...(ii)$

and $\qquad (K+2)z = 3 \qquad ...(iii)$

This system of equations has a unique solution, if

$$\begin{vmatrix} K & 2 & -1 \\ 0 & K-1 & -2 \\ 0 & 0 & K+2 \end{vmatrix} \neq 0$$

$\Rightarrow (K=2)(K)(K-1) \neq 0$

$\Rightarrow \qquad K \neq= 2, 0, 1$

ie, $K = -1$, is a required answer.

27 **(d)**

Given, $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$

For no solution, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow \ 1(2\lambda - 6) - 1(\lambda - 3) + 1(2-2) = 0$

$\lambda - 3 = 0 \ \Rightarrow \ \lambda = 3$

and $\qquad \Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix} \neq 0$

$\Rightarrow 6(6-6) - 1(30 - 3\mu) + 1(20 - 2\mu) \neq 0$

$\Rightarrow \ \mu - 10 \neq 0 \Rightarrow \ \mu \neq 10$

28 **(b)**

$|AA^T| = |I| = |A^TA|$

$\Rightarrow |A||A^T| = 1 = |A^2||A|$

$\Rightarrow |A|^2 = 1 \qquad [\because |A^T| = |A|]$

$\Rightarrow |A| = \pm 1$

29 **(c)**

AB

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \text{Cos}^2\theta\cos^2\phi + \sin\theta\cos\phi\cos\theta\sin\phi \\ \cos^2\phi\cos\theta\sin\theta + \ \sin^2\theta\sin\phi\cos\phi \\ \cos^2\theta\sin\phi\cos\phi + \sin^2\phi\sin\theta\cos\theta \\ \cos\theta\sin\theta\sin\phi\cos\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \sin\phi\cos\theta\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$\because \ AB = O$

$\Rightarrow \ \cos(\theta - \phi) = 0$

$\Rightarrow \ \cos(\theta-\phi) = \cos(2n+1)\dfrac{\pi}{2}$

$\Rightarrow \ \ \theta = (2n+1)\frac{\pi}{2} + \phi,$ where $n = 0, 1, 2, ...$

30 **(a)**

We have,

$A\,B = A$ and $B\,A = B$

Now, $A\,B = A$

$\Rightarrow (A\,B)A = A\,A$

$\Rightarrow A\,(B\,A) = A^2$

$\Rightarrow A\,B = A^2 \qquad [\because B\,A = B]$

$\Rightarrow A = A^2 \qquad [\because AB = A]$

and, $B\,A = B$

$\Rightarrow (B\,A)B = B^2$

$\Rightarrow B(A\,B) = B^2$

$\Rightarrow B\,A = B^2 \qquad [\because AB = A]$

$\Rightarrow B = B^2 \qquad [\because B\,A = B]$

$\therefore A^2 = A$ and $B^2 = B$

$\Rightarrow A$ and B are indempotent matrices

31 **(b)**

From given matrix equation, we have

$$x - y - z = 0$$
$$-y + z = 5$$
$$z = 3$$

$\Rightarrow \qquad x = 1, y = -2, z = 3$

32 **(d)**

Here, cofactors are

$C_{11} = \cos 2\theta, \qquad C_{12} = -\sin 2\theta$

$C_{21} = \sin 2\theta, \qquad C_{22} = \cos 2\theta$

$\therefore |A| = |\cos^2 2\theta + \sin^2 2\theta| = 1$

$$\therefore A^{-1} = \frac{1}{|A|}\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos\theta \end{bmatrix}$$

33 **(c)**

Since, B is the inverse of A.

ie, $\quad B = A^{-1}$

So, $\quad 10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow \; 10A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$

$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$

$\Rightarrow \quad -5+\alpha = 0 \;\Rightarrow\; \alpha = 5$

34 **(a)**

$AB = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3y+3z \\ -3x-4z \\ -3x+4y \end{bmatrix}$

Now, $B'(AB) = [x\ y\ z] \begin{bmatrix} 3y+3z \\ -3x-4z \\ -3x+4y \end{bmatrix}$

$= [3xy + 3zx - 3xy - 4yz - 3xz + 4yz]$

$= [0]$

$\therefore B'(AB)$ is a null matrix.

36 **(a)**

Given, $A = \begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow |A| = a^2 + b^2 = 1$

$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj}\,(A) = \text{adj}(A)$

38 **(c)**

We have,

$(A+B)^2 = A^2 + 2\,AB + B^2$

$\Leftrightarrow (A+B)(A+B) = A^2 + 2\,AB + B^2$

$\Leftrightarrow A^2 + A\,B + B\,A + B^2 = A^2 + 2\,A\,B + B^2$

$\Leftrightarrow A\,B = B\,A$

Hence, $(A+B)^2 = A^2 + 2\,AB + B^2 \Leftrightarrow A\,B = B\,A$

39 **(d)**

$A^{-1} = \frac{1}{2x^2}\begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix} = \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix}$

But $\; A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$\therefore \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \Rightarrow \frac{1}{2x} = 1 \Rightarrow x = \frac{1}{2}$

40 **(c)**

Given,

$\therefore \begin{vmatrix} a & a-1 & 1 \\ b-1 & b & 1 \\ -1 & c & c \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} a & 0 & -(1+a) \\ b & -(1+b) & 0 \\ -1 & c+1 & c+1 \end{vmatrix} = 0$

$\Rightarrow -a(1+b)(1+c) - b[0 + (1+a)(1+c)]$

$-1[0 - (1+a)(1+b)] = 0$

$\Rightarrow -a(1+b)(1+c) - b(1+a)(1+c)$

$+1(1+a)(1+b) = 0$

On dividing by $(1+a)(1+b)(1+c)$, we get

$-\frac{a}{1+a} - \frac{b}{1+b} + \frac{1}{1+c} = 0$

$\Rightarrow -\frac{a}{1+a} + 1 - \frac{b}{1+b} + 1 + \frac{1}{1+c} = 2$

$\Rightarrow \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$

41 **(a)**

The given system of equation has at least one solution, if

$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 0$

$\Rightarrow 3(2\lambda + 15) + 1(\lambda + 18) + 4(5 - 12) = 0$

$\Rightarrow 7\lambda = -35 \;\Rightarrow\; \lambda = -5$

43 **(d)**

Given, $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$C_{11} = 4, C_{12} = 1, C_{13} = -2$

$C_{21} = -2, C_{22} = 4, C_{23} = 1$

$C_{31} = 1, C_{32} = -2, C_{33} = 4$

$\therefore \quad \text{adj}(A) = \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 4 & -2 \\ -2 & 1 & 4 \end{bmatrix}$

$\therefore |\text{adj}\,A| = 4(16+2) + 2(4-4) + (1+8)$

$= 72 + 0 + 9 = 81$

Alternate

$|A| \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix}$

$= 2(4-0) - 1(0-1) = 9$

$\therefore |\text{adj}\,A| = |A|^{3-1} = (9)^2 = 81$

44 **(a)**

Since, $A' = -A$

$\therefore \quad A^3 = AAA$

And $(A^3)' = A'A'A' = -A^3$

Hence, matrix A^3 is a skew-symmetric matrix

45 **(c)**

Given, $\quad [1\ x\ 1] \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$

$\Rightarrow \quad [1\ x\ 1] \begin{bmatrix} x+2-6 \\ 0+5-2 \\ 0+3-4 \end{bmatrix} = 0$

$$\Rightarrow \quad [1\ x\ 1]\begin{bmatrix} x-4 \\ 3 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad x - 4 + 3x - 1 = 0 \Rightarrow x = \frac{5}{4}$$

46 **(d)**

We have, $A + b = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

Clearly, $A + B$ is a symmetric matrix

Now,

$$(A+B)^{-1} = -1\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 4 & -4 \\ -2 & 3 \end{bmatrix}, B^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\therefore A^{-1} + B^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & -1 \end{bmatrix}$$

We observe that $(A + b)^{-1}$ is a symmetric matrix and $(A + B)^{-1} \neq A^{-1} + B^{-1}$

47 **(c)**

$\because \ A \cdot A^T = I_n$

$\Rightarrow \ A - I_n = A - AA^T = A(I_n - A^T)$

$\Rightarrow \ |A - I_n| = |A(I_n - A^T)|$

$= |A||I_n - A^T|$

$= |A||I_n - A|$

48 **(c)**

$$A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} + \frac{1}{2} & \frac{1}{2} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$Matrix A is nilpotent of order 2.

49 **(a)**

Let A be a symmetric matrix. Then,

$A\,A^{-1} - I$

$\Rightarrow (A\,A^{-1})^T = I$

$\Rightarrow (A^{-1})^T\,A^T = I$

$\Rightarrow (A^{-1})^T = (A^T)^{-1}$

$\Rightarrow (A^{-1})^T = (A)^{-1} \qquad [\because A^T = A]$

$\Rightarrow A^{-1}$ is a symmetric matrix

50 **(a)**

Since, given matrix is a triangular matrix, so its characteristic roots are the diagonal elements.

Hence, required roots are 1, 3, 6.

51 **(d)**

Given that, $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

and $A^{-1} = \frac{1}{k}\text{adj } A$

$\Rightarrow \ k = |A|$

$$= \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$= 3(2+1) - 2(1-0) + 4(1-0)$

$= 9 - 2 + 4 = 11$

52 **(a)**

Since, $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is linear equation in three variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

53 **(c)**

Since, A is invertible matrix of order n, then the determinant of adj $A = |A|^{n-1}$

54 **(c)**

Given, $\text{adj}\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$ ...(i)

Cofactors of $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$ are

$C_{11} = 5, C_{12} = 1, C_{13} = -2$

$C_{21} = 4, C_{22} = 1, C_{23} = -2$

$C_{31} = -2, C_{32} = 0, C_{33} = 1$

$$\Rightarrow \begin{bmatrix} 5 & 4 & -2 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$$

On comparing the corresponding elements, we get

$a = 4, \qquad b = 1$

$\therefore \quad [a \quad b] = [4 \quad 1]$

55 **(d)**

$$AB = [x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$= [xa + yh + zg \quad xh + yb + zf \quad xg + yf + zc]$

Now, $\ ABC = O$

$$\Rightarrow [xa + yh + zg \quad xh + yb + zf \quad xg + yf + zc]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = O$$

$\Rightarrow [ax^2 + hxy + gxz + hxy + y^2b + fzy + gxz + yfz + z^2c]$

$= O$

$\Rightarrow [ax^2 + by^2 + cz^2 + 2gzx + 2hxy + 2\,fyz] = O$

56 **(d)**

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Let B be the multiplicative inverse of A, then

$AB = I$

$\Rightarrow \ B = A^{-1}$

$= \frac{1}{8-7}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

57 **(c)**

Let $A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$

Now, we take a submatrix of order 3×3

$B = \begin{bmatrix} 4 & 1 & 0 \\ 3 & 0 & 1 \\ 6 & 0 & 2 \end{bmatrix}$

$|B| = -1(6-6) = 0$

Now, we take a submatrix of order 2×2.

ie, $C = \begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}$

$|C| = 0 - 3 \neq 0$

$\therefore$ Rank of matrix A is 2.

58 **(d)**

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

Now, $X = A^{-1}B$

$= \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

59 **(a)**

$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 21+4+10 \\ 27+8+5 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$

60 **(b)**

Since, A, B and C are $n \times n$ matrices ans, if $A^3 + 2A^2 + 3A + 5I = O$, then A is invertible.

61 **(a)**

We have,

$|A| = -1, |B| = 3$

$\therefore |3\,AB| = 3^3\,|AB| = 3^3|A|\,|B| = 3 \times -1 \times 3$
$= -9$

62 **(c)**

Let $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. Then,

$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ By applying $R_4 \to R_4 - R_3 - R_2 - R_1$

$\Rightarrow A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ By applying $R_2 \to R_2 - 2\,R_1$, $R_3 \to R_3 - 3\,R_1$

We observe that the leading minor of the third order of this matrix is non-zero i.e.

$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & -3 \\ 0 & -4 & -8 \end{vmatrix} = -12 \neq 0$. Hence, rank $(A) = 3$

63 **(a)**

Given, $\quad A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$

$\therefore \quad A - I = A + I - 2I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix}$

$\therefore (A+I)(A-I) = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$

64 **(d)**

We have,

$A = k\,I_n \Rightarrow |A| = |k\,I_n| = k^n\,|I_n| = k^n$

65 **(d)**

Given, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix}$

$\therefore A^2 - (a+d)A + kI = 0$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix} - \begin{bmatrix} a^2 + ad & ab + bd \\ ac + dc & ad + d^2 \end{bmatrix}$
$+ \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} bc - ad + k & 0 \\ 0 & bc - ad + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On equating, we get

$bc - ad + k = 0$

$\Rightarrow \quad k = ad - bc \quad$ ...(i)

Also, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\Rightarrow \quad ad - bc = 0$

$\therefore$ From Eq. (i), $k = 0$

66 **(b)**

Given that, $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\therefore \; AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

$\Rightarrow \; (AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

68 **(b)**

We have,

$A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$

$\therefore A^2 - 5A = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow A^2 - 5A = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = 14\ I$

71 **(b)**

Given that A is a singular matrix

$\therefore\ |A| = 0$

$\because\ A \text{ adj } A = |A| = 0$

$\therefore A$ adj A is a zero matrix

72 **(a)**

$\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$

73 **(b)**

Given, $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore\ A^2\begin{bmatrix} 2 & 3 & 1 \\ 5 & 6 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 12 & 4 \end{bmatrix}$

Hence, $A^3 - 3A^2 - I = O$

74 **(d)**

$\because\ PQ = I\ \Rightarrow\ P^{-1} = Q$

Now, the system in matrix notation is $PX = B$

$\therefore\ X = P^{-1}B = QB$

$\Rightarrow\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 2 & 2 & 1 \\ 13 & -5 & m \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$

$\Rightarrow\ y = \frac{1}{9}(13 - 5 + 5m)$

$\Rightarrow\ -27 = 8 + 5m$ (given $y = -3$)

$\therefore\ m = -7$

75 **(d)**

Since A is invertible. Therefore, $|A| \neq 0$

Thus, option (d) is correct.

76 **(a)**

We have,

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}\right)$

$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\right)$

$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

77 **(c)**

If A is a square matrix, then $A - A^T$ is a skew-symmetric matrix, then $|A - A^T|$ is '0' or a perfect square as A is of odd order or even order

78 **(d)**

O $(A') = 3 \times 2, O(B') = 2 \times 3$

(a) $CB + A'$

Now, order of CB

=(order of C is 3×3)(order of B is 3×2)

= order of CB is 3×2

Since, $O(A') = 3 \times 2$

$\therefore$ Matrix $CB + A'$ can be determined.

(b) $O(BA) = 3 \times 3$

and $O(C) = 3 \times 3$

$\therefore$ Matrix BAC can be determined.

(c) $O(A + B') = 2 \times 3$

$\Rightarrow\ O(A + B')' = 3 \times 2$

and $O(C) = 3 \times 3$

$\therefore$ Matrix $C(A + B')$ can be determined.

(d) $O(A + B') = 2 \times 3$

And $O(C) = 3 \times 3$

$\therefore$ Matrix $C(A + B')$ cannot be determined.

79 **(d)**

Given A is a matrix of order 3 and $B = |A|A^{-1}, |A| = -5$

$\therefore B = |A|\frac{(\text{adj } A)}{|A|}\ \Rightarrow\ B = (\text{adj})\ A)$

$\Rightarrow\ |B| = |A|^{3-1} = 25$

80 **(a)**

$\because\ A^2 = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore\ A^2$ is null matrix.

81 **(b)**

For unique solution,

$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0$

$\Rightarrow\ k(k-1)(k+2) \neq 0$

$\Rightarrow\ k \neq 0, 1$ or -2

82 **(b)**

Given, $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\therefore\ A^{-1} = -\frac{1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Also, $AX = B$

$\Rightarrow\ X = A^{-1}B = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 + 2 \\ 6 + 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

83 **(a)**

$$\Delta=\begin{bmatrix}1&2&3\\2&1&3\\5&5&9\end{bmatrix}$$
$= 1(9-15)-2(18-15)+3(10-5)$
$= -6-6+15$
$= 3 \neq 0$

Hence, the system of equations has a unique solution.

84 **(b)**

It is given that

$A=\begin{bmatrix}1&1\\1&1\end{bmatrix}$

$\therefore A^2=\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}2&2\\2&2\end{bmatrix}=2\,A$

$\Rightarrow A^3=2\,(A\,A)=2\,A^2=2(2\,A)=2^2\,A$

Continuing in this manner, we have $A^n=2^{n-1}\,A$

85 **(a)**

We have,

$(B^{-1}\,A^{-1})^{-1}=(A^{-1})^{-1}\,(B^{-1})^{-1}$ $[\because (PQ)^{-1} = Q^{-1}\,P^{-1}]$

$\Rightarrow (B^{-1}\,A^{-1})^{-1}=AB$

$\Rightarrow (B^{-1}\,A^{-1})=\begin{bmatrix}2&2\\-3&2\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$\Rightarrow (B^{-1}\,A^{-1})^{-1}=\begin{bmatrix}2&-2\\2&3\end{bmatrix}$

Hence, option (a) is correct

86 **(c)**

Here, C, A and C^T are matrix of order $n\times 1, n\times n$ and $1\times n$ respectively.

Let $C^TAC=k$

Then, $(C^T\,AC)^T=C^TA^T(C^T)^T$
$=C^TA^TC=C^T(-A)C$
$=-C^TAC=-k$

$\Rightarrow k=-k \Rightarrow k=0$

$\Rightarrow C^TAC$ is null matrix.

Which shows that C^TAC is a zero matrix of order1.

87 **(b)**

Since, $A+A^T$ is a square matrix

$\therefore (A+A^T)^T=A^T+(A^T)^T=A^T+A$

Hence, $A+A^T$ is symmetric matrix

88 **(d)**

$A=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}=\begin{bmatrix}1+1&1+2\\2+1&2+2\end{bmatrix}=\begin{bmatrix}2&3\\3&4\end{bmatrix}$

89 **(a)**

Given, $P=\begin{bmatrix}1&2&3\\2&3&4\\3&4&5\end{bmatrix}\begin{bmatrix}-1&-2\\-2&0\\0&-4\end{bmatrix}\begin{bmatrix}-4&-5&-6\\0&0&1\end{bmatrix}$

$=\begin{bmatrix}1&2&3\\2&3&4\\3&4&5\end{bmatrix}\begin{bmatrix}4&5&4\\8&10&12\\0&0&-4\end{bmatrix}$

$\therefore\ P_{22}=[2\ 3\ 4]\begin{bmatrix}5\\10\\0\end{bmatrix}$

$=10+30=40$

91 **(c)**

Given, $AB=A \Rightarrow B=I$

And $BA=B \Rightarrow A=I$

$\because B=I$ and $A=I$

$\Rightarrow B^2=B$ and $A^2=A$

92 **(a)**

Let $A=\begin{bmatrix}0&x&y&z\\-x&0&a&b\\-y&-a&0&c\\-z&-b&-c&0\end{bmatrix}$ be a skew-symmetric matrix. Then,

$|A|=\begin{vmatrix}0&x&y&z\\-x&0&a&b\\-y&-a&0&c\\-z&-b&-c&0\end{vmatrix}=(cx-by+az)^2$

93 **(d)**

Since, $A=\begin{bmatrix}1&-1&1\\0&2&-3\\2&1&0\end{bmatrix}$

$\therefore B=\text{adj } A=\begin{bmatrix}3&1&1\\-6&-2&3\\-4&-3&2\end{bmatrix}$

$\Rightarrow \text{adj } B=\begin{bmatrix}5&-5&5\\0&10&-15\\10&5&0\end{bmatrix}$

$\Rightarrow |\text{adj } B|=\begin{bmatrix}5&-5&5\\0&10&-15\\10&5&0\end{bmatrix}=625$

Given that, $C=5A$

$\Rightarrow |C|=5^3|A|=125\begin{vmatrix}1&-1&1\\0&2&-3\\2&1&2\end{vmatrix}=625$

Hence, $\frac{|\text{adj }(B)|}{|C|}=\frac{625}{625}=1$

94 **(a)**

Given, $A=\begin{bmatrix}6&8&5\\4&2&3\\9&7&1\end{bmatrix}$

amd symmetric matrix $B=\frac{A+A'}{2}$

$\therefore B=\frac{1}{2}\left\{\begin{bmatrix}6&8&5\\4&2&3\\9&7&1\end{bmatrix}+\begin{bmatrix}6&4&9\\8&2&7\\5&3&1\end{bmatrix}\right\}=\begin{bmatrix}6&6&7\\6&2&5\\7&5&1\end{bmatrix}$

95 **(d)**

The matrix $\begin{bmatrix}5&10&3\\-2&-4&6\\-1&-2&b\end{bmatrix}$ is singular, if

$\begin{bmatrix}5&10&3\\-2&-4&6\\-2&-2&b\end{bmatrix}=0$

$\Rightarrow -1(60+12)+2(30+6)+b(-20+20)=0$

$\Rightarrow -72+72+0b=0$

Hence, the given matrix is singular for any value of b

96 **(b)**

$\det(2AB) = 2^3 \det(A)\det(B)$

$= 8\det(A)\det(B)$

97 **(b)**

$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$ (In general)

And in a diagonal matrix non-diagonal elements are zero *ie*,

$$a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ a_{ij}, & \text{if } i = j \end{cases}$$

So, $c_{ij} = a_{ii} b_{ij}$

98 **(b)**

Here, $|A| = (1)(9) - 2(-11) - 3(6)$

$= 9 + 22 - 18 = 33$

Since, $A^{-1}\text{adj}(A^{-1}) = |A^{-1}| I_3$

$\Rightarrow A^{-1}\text{adj}(A^{-1}) = (|A|^{-1}) I_3$

$\Rightarrow A \cdot A^{-1}\text{adj}(A^{-1}) = (|A|)^{-1} A I_3$

$\Rightarrow \text{adj}(A^{-1}) = (|A|)^{-1} A$

$\Rightarrow |A|\text{adj}(A^{-1}) = A$ (But $|A| \neq 0$)

99 **(d)**

$$\text{Let } A \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$$

$\therefore |A| = 1(1+1) + 1(1-1) + 1(1+1) = 4 \neq 0$

$\therefore$ Rank of matrix A is 3

100 **(b)**

Let $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y$ and $\frac{z^2}{c^2} = Z$, then given equation will be

$X + Y - Z = 1, X - Y + Z = 1, -X + Y + Z = 1$

$$\text{Here, } A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Now, $|A| = -4 \neq 0$

Therefore, the given system of equation has unique solution.

102 **(b)**

$$\left[\frac{1}{2}(A - A')\right]' = \frac{1}{2}(A - A')' = \frac{1}{2}(A' - A)$$

$$= -\frac{1}{2}(A - A')$$

Hence, it is a skew-symmetric matrix.

103 **(b)**

$$\because \text{ adj } A = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

$$\text{and } |A| = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 10$$

$$\therefore A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

105 **(b)**

$$\text{Let } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

106 **(d)**

We have,

$3A^3 + 2A^2 + 5A + I = 0$

$\Rightarrow I = -3A^3 - 2A^2 - 5A$

$\Rightarrow I A^{-1} = (-3A^3 - 2A^2 - 5A)A^{-1}$

$\Rightarrow A^{-1} = -3A^2 - 2A - 5I$

108 **(b)**

The given system of equations are

$x + y + z = 0$,

$2x + 3y + z = 0$ and $x + 2y = 0$

$$\text{Here, } \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0-2) - 1(0-1) + (4-3)$$

$= -2 + 1 + 1 = 0$

$\therefore$ This system has infinite solutions

109 **(c)**

$$2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 4 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & 2 \\ 7/2 & 1 \end{bmatrix}$$

111 **(b)**

Given, $A = A', B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\therefore AB - BA$ is a skew-symmetric matrix.

113 **(a)**

Given that, p is a non-singular matrix such that

$1 + p + p^2 + \ldots + p^n = O$

$\Rightarrow (1 + p)(1 + p + p^2 + \ldots + p^n) = O$

$\Rightarrow 1 - p^{n+1} = O$

$\Rightarrow p^{n+1} = 1$

$\Rightarrow p^n \times p^1 = 1$

$\Rightarrow p^n = 1/p$

$\therefore p^{-1} = p^n$

114 **(a)**

$$\text{Given, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25+0-25 \\ -25+0+65 \\ 50+0+30 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow\ x = 0, y = 1, z = 2$

$\therefore\ x + y + z = 0 + 1 + 2 = 3$

115 **(d)**

$\because \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$

Then, $X = y$ and $Y = x$

ie, $y = x$

116 **(c)**

Given $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}.\frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1-\tan^2\theta}{1+\tan^2\theta} & \frac{2\tan\theta}{1+\tan^2\theta} \\ \frac{2\tan\theta}{1+\tan^2\theta} & \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$\Rightarrow\ a = \cos 2\theta,\ \ b = \sin 2\theta$

117 **(d)**

$$\text{Given, } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\therefore\ B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = (B)' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A'$$

118 **(d)**

Given equations are

$px + y + z = 0, x + qy + z = 0, x + y + rz = 0$

Since, the system have a non-zero solution, then

$$\begin{bmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{bmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$and $C_3 \to C_3 - C_2$

$$\Rightarrow \begin{vmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)\begin{vmatrix} \frac{p}{1-p} & 1 & 0 \\ \frac{1}{1-q} & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)\left[\frac{p}{1-p}(1) - 1\left(-\frac{1}{1-q} - \frac{1}{1-r}\right)\right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow\ \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

119 **(b)**

Given, $AB = I\ \Rightarrow\ B = A^{-1}$

Now, $A^{-1} = \frac{\text{adj } A}{|A|}$

$$= \frac{\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}}{1+\tan^2\frac{\theta}{2}}$$

$$= \frac{A^T}{\sec^2\frac{\theta}{2}} = \cos^2\frac{\theta}{2}A^T$$

120 **(d)**

Given equations are

$3x + y + 2z = 3\quad$ ...(i)

$2x - 3y - z = -3\quad$ ...(ii)

and $x + 2y + z = 4\quad$ ...(iii)

$$\text{Let } A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore\ |A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= 3\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

$= 3(-3+2) - 1(2+1) + 2(4+3)$

$= -3 - 3 + 14 = 8$

$$\text{adj}.A = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}^T$$

$$= \begin{vmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{vmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

Now, $X = A^{-1}B$

$= \frac{1}{8}\begin{bmatrix}-1 & 3 & 5\\ -3 & 1 & 7\\ 7 & -5 & -11\end{bmatrix}\begin{bmatrix}3\\ -3\\ 4\end{bmatrix}$

$= \frac{1}{8}\begin{bmatrix}-3-9+20\\ -9-3+28\\ 21+15-44\end{bmatrix} = \frac{1}{8}\begin{bmatrix}8\\ 16\\ -8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$

$\Rightarrow x = 1, y = 2, z = -1$

121 **(a)**

$A^2 = 2A - I$

$\therefore A^2A = 2AA - IA$

$= 2A^2 - A = 2(2A - I) - A$

$\Rightarrow A^3 = 3A - 2I$

$\Rightarrow A^3.A = 3AA - 2IA = 3(2A - I) - 2A$

$\Rightarrow A^4 = 4A - 3I$

Similarly, $A^n = nA - (n - I)I$

122 **(d)**

$\det(M_r) = \begin{bmatrix} r & r-1\\ r-1 & r\end{bmatrix} = 2r - 1$

$\sum_{r=1}^{2007} \det(M_r) = 2\sum_{r=1}^{2007} r - 2007$

$= 2 \times \frac{2007 \times 2008}{2} - 2007 = (2007)^2$

123 **(a)**

Let $A = \begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 2\\ 1 & -2 & 0\end{bmatrix}$

$\therefore |A| = \begin{vmatrix}0 & 1 & -1\\ -1 & 0 & 2\\ 1 & -2 & 0\end{vmatrix}$

$= 0\begin{vmatrix}0 & 2\\ -2 & 0\end{vmatrix} - 1\begin{vmatrix}-1 & 2\\ 1 & 0\end{vmatrix} - 1\begin{vmatrix}-1 & 0\\ 1 & -2\end{vmatrix}$

$= 0 + 2 - 2 = 0$

$\Rightarrow |A| = 0$

Now, $(\text{adj } A)B = \begin{bmatrix}4 & 2 & 2\\ 2 & 1 & 1\\ 2 & 1 & -1\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$

$= \begin{bmatrix}4-4+6\\ 2-2+3\\ 2-2-3\end{bmatrix} = \begin{bmatrix}6\\ 3\\ -3\end{bmatrix} \neq O$

$\therefore$ This system of equation is inconsistent, so it has no solution

125 **(c)**

Given, $D = \text{diag}\,(d_1, d_2, d_3, \dots, d_n)$

$\Rightarrow D^{-1} = \text{diag}(d_1^{-1}, d_2^{-1}, \dots, d_n^{-1})$

126 **(a)**

We have,

$a = \begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$

$\Rightarrow A^n = \begin{bmatrix}1 & na\\ 0 & 1\end{bmatrix}$ [Using PMI]

$\Rightarrow \frac{1}{n}A^n = \begin{bmatrix}\frac{1}{n} & a\\ 0 & \frac{1}{n}\end{bmatrix} \Rightarrow \lim_{n\to\infty}\frac{1}{2}A^n = \begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$

127 **(d)**

The given system of equations are

$2x + y - 5 = 0$ ...(i)

$x - 2y + 1 = 0$ ...(ii)

and $2x - 14y - a = 0$ ...(iii)

This system is consistent.

$\therefore \begin{bmatrix}2 & 1 & -5\\ 1 & -2 & 1\\ 2 & -14 & -a\end{bmatrix} = 0$

$\Rightarrow 2(2a + 14) — 1(-a - 2) - 5(-14 + 4) = 0$

$\Rightarrow 4a + 28 + a + 2 + 50 = 0$

$\Rightarrow 5a = -80 \quad \Rightarrow \quad a = -16$

128 **(d)**

The system of given equations has no solution,

if $\begin{vmatrix}\alpha & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common $(\alpha + 2)$from C_1, we get

$(\alpha + 2)\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow (\alpha + 2)\begin{vmatrix}1 & 1 & 1\\ 0 & \alpha - 1 & 0\\ 0 & 0 & \alpha - 1\end{vmatrix} = 0$

$\Rightarrow (\alpha + 2)(\alpha - 1)^2 = 0$

$\Rightarrow \alpha = 1, -2$

But α =1 makes given three equations same. So, the system of equation have infinite solution. So, answer is $\alpha = -2$ for which the system of equations has no solution

130 **(b)**

Given, $A = \begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}$

$\therefore A^2 = \begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x^2 + 1 & x\\ x & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow x^2 + 1 = 1, x = 0$

$\Rightarrow x = 0$

131 **(a)**

Given that, $A^{-1} = \lambda\,(\text{adj } A)$

On comparing with $A^{-1} = \frac{1}{|A|\text{adj}} A$ we get

$\lambda = \frac{1}{|A|}$

Now, $|A| = \begin{vmatrix}0 & 3\\ 2 & 0\end{vmatrix} = 0 - 6 = -6$

$\Rightarrow\ \lambda = -\dfrac{1}{6}$

132 **(d)**

$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14} = |A|$

133 **(d)**

Given equation are $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \lambda z = 10$

Since, it is consistent.

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow\ 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) = 0$

$\Rightarrow\ \lambda - 3 = 0 \ \Rightarrow\ \lambda = 3$

134 **(b)**

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

$\therefore\ A^4 = 2A.2A = 4A^2 = 4 \times 2A = 2^3A$

Similarly, $A^8 = 2^7A$

$\Rightarrow\ A^{100} = 2^{99}A$

135 **(d)**

Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$\therefore\ |A| = \cos^2 2\theta + \sin^2 2\theta = 1$

and $\operatorname{adj} A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{1}{1}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

138 **(a)**

Give equation can be written as,

$2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow\ X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

139 **(c)**

We have,

$A\,B = 0$

$\Rightarrow |A\,B| = 0$

$\Rightarrow |A|\,|B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Then, $A\,B = O$. But $A \neq O, B \neq O$

140 **(b)**

Given, $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix}$

Now, $A^{-1}D = \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 8 \\ -1 \\ 0 \end{bmatrix}$

$\Rightarrow\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/3 \\ -1/3 \\ 0 \end{bmatrix}$

142 **(a)**

Given equations are $x - cy - bz = 0$

$cx - y + az = 0$ and $bx + ay - z = 0$

For non-zero solution

$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$\Rightarrow\ a^2 + b^2 + c^2 + 2abc = 1$

143 **(b)**

We have,

Det $(I_n) = 1(\neq 0) \Rightarrow$ rank $(I_n) = n$

144 **(a)**

The given matrix A is singular, if

$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$

$\Rightarrow\ 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow\ 56\lambda - 128 - 36\lambda + 48 + 20 = 0$

$\Rightarrow\ 20\lambda = 60$

$\Rightarrow\ \lambda = 3$

145 **(c)**

Let $B = I + A + A^2 + A^3 \ldots \infty$

$\Rightarrow AB = A + A^2 + A^3 + \cdots \infty$

$\Rightarrow B - AB = I$

$\Rightarrow B(I - A) = I$

$\Rightarrow B = (I - A)^{-1}$

$\Rightarrow B = \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}^{-1} = -\dfrac{1}{6}\begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$

146 **(a)**

Since A is non-singular matrix

$\therefore |A| \neq 0 \Rightarrow \operatorname{rank}(A) = n$

147 **(b)**

$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$

Now, $f(A) = A^2 - 3A + 7$

$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$

$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

148 **(a)**

The given system of equations will have a unique solution, if

$$\begin{vmatrix}1 & 1 & 1\\2 & 1 & -1\\3 & 2 & k\end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

149 **(a)**

Given, $2x + y - z = 7$ ...(i)

$x - 3y + 2z = 1$ ...(ii)

and $x + 4y - 3z = 5$ ...(iii)

From Eqs.(i) and (ii), we get

$5x - y = 15$ ...(iv)

From Eqs. (i) and (iii)

$5x - y = 16$ ...(v)

Eqs. (iv) and (v) shows that they are parallel and solution does not exist.

150 **(d)**

We have,

$X = \begin{bmatrix}3 & -4\\1 & -1\end{bmatrix} \Rightarrow X^2 = \begin{bmatrix}5 & -8\\2 & -3\end{bmatrix}$

Clearly for $n = 2$, the matrices in options (a), (b), (c) do not tally with $\begin{bmatrix}5 & -8\\2 & -3\end{bmatrix}$

152 **(a)**

We have,

$A = [a_{ij}]\ \therefore |k\,A| = k^n|A|$

153 **(b)**

$$A^2 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\a & b & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\a & b & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I$$

154 **(d)**

Given that, $A = \begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}$

$= \begin{bmatrix}\alpha^2 + 0 & 0 + 0\\\alpha + 1 & 0 + 1\end{bmatrix}$

$= \begin{bmatrix}\alpha^2 & 0\\\alpha + 1 & 1\end{bmatrix}$

Also, $B = A^2$ (given)

$\Rightarrow \begin{bmatrix}1 & 0\\5 & 1\end{bmatrix} = \begin{bmatrix}\alpha^2 & 0\\\alpha + 1 & 1\end{bmatrix}$

Clearly this is not satisfied by any real value of α

155 **(b)**

We have,

$$A\,(\text{adj}\,A) = \begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix}$$

$\Rightarrow |A|\,I = 4\,I$ $[\because A(\text{adj}\,A) = |A|\,I]$

$\Rightarrow |A| = 4$

$\Rightarrow |\text{adj}\,A| = |A|^2$ $[|\text{adj}\,A| = |A|^{n-1}]$

156 **(a)**

Given, $A = \begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix}$

$A^2 = \begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix}\begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix} = \begin{bmatrix}\omega^2 & 0\\0 & \omega^2\end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix}\omega^2 & 0\\0 & \omega^2\end{bmatrix}\begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix} = \begin{bmatrix}\omega^3 & 0\\0 & \omega^3\end{bmatrix}$

Similarly, $A^{50} = \begin{bmatrix}\omega^{50} & 0\\0 & \omega^{50}\end{bmatrix}$

$= \begin{bmatrix}(\omega^2)^{16}\omega^2 & 0\\0 & (\omega^3)^{16}\omega^2\end{bmatrix}$

$= \begin{bmatrix}\omega^2 & 0\\0 & \omega^2\end{bmatrix}$

$= \omega^2 A$

157 **(a)**

We have,

$AB = I_3$

$\Rightarrow \begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0 & x + y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow x + y = 0$

158 **(a)**

Let $A = \begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$

$\therefore$ $\text{adj(A)} = \begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}$

159 **(a)**

$\because |A| = 0 - 1(1 - 9) + 2(1 - 6)$

$= 8 - 10 = -2 \neq 0$

$\text{adj}\,(A) = \begin{bmatrix}-1 & 1 & -1\\8 & -6 & 2\\-5 & 3 & -1\end{bmatrix}$

$\therefore A^{-1} = \frac{1}{-2}\begin{bmatrix}-1 & 1 & -1\\8 & -6 & 2\\-5 & 3 & -1\end{bmatrix} = \begin{bmatrix}1/2 & -1/2 & 1/2\\-4 & 3 & -1\\5/2 & -3/2 & 1/2\end{bmatrix}$

160 **(a)**

$|f(\theta)| = 1(\cos^2\theta + \sin^2\theta) = 1$

Now, $\text{adj}\{f(\theta)\} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}$

$\therefore \{f(\theta)\}^{-1} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix} = f(-\theta)$

161 **(a)**

Given, $x + 4ay + az = 0$ ...(i)

$x + 3by + bz = 0$ ...(ii)

And $x + 2cy + cz = 0$ ...(iii)

For non-trivial solution

$$\begin{vmatrix}1 & 4a & a\\1 & 3b & b\\1 & 2c & c\end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b-4a & b-a \\ 0 & 2c-4a & c-a \end{vmatrix} = 0$$
$\Rightarrow 1[(3b-4a)(c-a) - 2(b-a)(c-2a)] = 0$
$\Rightarrow 3bc - 3ab - 4ac + 4a^2$
$\qquad - 2(bc - 2ab - ac + 2a^2) = 0$
$\Rightarrow \quad bc + ab - 2ac = 0$
$\Rightarrow \quad ab + bc = 2ac$

162 **(d)**
We know that
rank $(A\,B) \le \text{rank}(A)$
and, rank $(A\,B) \le$ rank (B)
$\therefore$ rank $(A\,B) \le \min(\text{rank}\,A, \text{rank}\,B)$

163 **(c)**
Let $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ and $B = [b_{11}\; b_{12}\; b_{13}\; \cdots b_{1n}]$ be two non-zero column and row matrices respectively
We have, $A\,B =$
$$\begin{bmatrix} a_{11}\,b_{11} & a_{11}\,b_{12} & a_{11}\,b_{13} & \cdots a_{11}\,b_{1\,n} \\ a_{21}\,b_{11} & a_{21}\,b_{12} & a_{21}\,b_{13} & a_{21}\,b_{1\,n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}\,b_{11} & a_{m1}\,b_{12} & a_{m1}\,b_{13} & a_{m1}\,b_{1\,n} \end{bmatrix}$$
Since A and B are non-zero matrices. Therefore, the matrix AB will also be a non-zero matrix. The matrix AB will have at least one non-zero element obtained by multiplying corresponding non-zero elements of A and B. All the two-rowed minors of A obviously vanish. But, A is a non-zero matrix. Hence, rank $(A = 1)$

164 **(c)**
$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$
$$= -\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

165 **(a)**
If A is any square matrix, then
$AA^{-1} = I$ and $A^{-1}I = A^{-1}$
Since, $\quad A^2 - A + I = O$
$\Rightarrow \quad A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$
$\Rightarrow \quad (A^{-1}A)\,A - (A^{-1}A) + A^{-1} = 0$
$\Rightarrow \quad A - 1 + A^{-1} = 0 \Rightarrow A^{-1} = I - A$

166 **(a)**
Since, B is invertible, therefore B^{-1} exists
Now, rank $(A) = \text{rank}[(AB)B^{-1}] \le \text{rank}(AB)$
But $\quad \text{rank}(AB) \le \text{rank}(A)$
$\therefore \quad$ rank $(AB) = \text{rank}(A)$

167 **(c)**
Given, $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ of order $n = 2$
$\therefore |\text{adj}(A)| = |A|^{2-1} = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix} = 10$

168 **(d)**
$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

169 **(b)**
Let A denote the matrix every element of which is unity. Then, all the 2-rowed minors of A obviously vanish. But A is a non-null matrix. Hence, rank of A is 1

170 **(d)**
As $\quad \det(A) = \pm 1, A^{-1}$ exists
and $\quad A^{-1} = \frac{1}{\det(A)}(\text{adj}\,A) = \pm(\text{adj}\,A)$
All entries in adj (A) are integers.
$\therefore A^{-1}$ has integer entries.

171 **(c)**
Since, A is invertible
$$\therefore \; |A| \ne 0 \;\Rightarrow\; \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix} \ne 0$$
$\Rightarrow \; 1(1-0) + k(0-k) \ne 0$
$\Rightarrow \; 1 - k^2 \ne 0 \;\; \Rightarrow k \ne \pm 1$

172 **(b)**
We have,
$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \frac{1}{1+\tan^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
$$\Rightarrow \frac{1}{1+\tan^2\theta} \begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \frac{1-\tan^2\theta}{1+\tan^2\theta} & \frac{-2\tan\theta}{1+\tan^2\theta} \\ \frac{2\tan\theta}{1+\tan^2\theta} & \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
$\Rightarrow a = \cos 2\theta, b = \sin 2\theta$

173 **(c)**
We have,
$x^2 + y^2 + z^2 \ne 0$
$\Rightarrow$ At least one of x, y, z is non-zero
Now,
$x = cy + bz, y = az + cx, z = bx + ay$
$\Rightarrow x - cy - bz = 0$
$cx - y + az = 0$

$bx + zy - z = 0$

As at least one of x, y, z is non-zero. Therefore, the above system of equations has non-trivial solutions

$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

174 **(c)**

$A^2 - 4A + 10I = A$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} - 4\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} - \begin{bmatrix} 4 & -12 \\ 8 & 4K \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 9-3k \\ -6+2k & 4+k^2-4K \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow 9 - 3k = -3, -6 + 2k = 2$ ...(i)

and $4 + k^2 - 4k = k$

$\Rightarrow k^2 - 5k + 4 = 0 \Rightarrow k = 4,1$

But $k = 1$ is not satisfied the Eq (i).

175 **(a)**

Given, $A^2 = 2A - I$

Now, $A^3 = A^2 \cdot A = 2A^2 = -IA$

$= 2A^2 - A = 2(2A - I) - A$

$= 3A - 2I = 3A - (3-1)I$

… … … … …

… … … … …

$A^n = nA - (n-1)I$

176 **(c)**

We have, $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

177 **(b)**

It is given that A is an orthogonal matrix

$\therefore A\,A^T = I = A^T A \Rightarrow A^{-1} = A^T$

178 **(a)**

Let $A = IA$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - 2R_2$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A$

Applying $R_2 \to -R_2$ and $R_2 \to R_2 - 2R_3$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2 - 3R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$

$\therefore \ A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

179 **(a)**

Given that, $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$

$\Rightarrow \ 2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow \ 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow \ X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

180 **(b)**

Since the given matrix is symmetric

$\therefore (A)_{12} = (A)_{21} \Rightarrow x + 2 = 2\,x - 3 \Rightarrow x = 5$

181 **(d)**

Given, $A = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} . 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} 9A$

$\therefore \quad A^4 = A^2 . A^2 = 9A . 9A = 81.9A = 729A$

182 **(a)**

Now, $\begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix}$

$= 1(\omega^3 - 1) - \omega^2(\omega^4 - \omega) + \omega(\omega^2 - \omega^2)$

$= 1(1-1) - \omega^2(\omega - \omega) + 0$

$= 0$

Hence, matrix A is singular

183 **(a)**

Given system of equations are

$x + y + z = 6, x + 2y + 3z = 10$

and $\quad x + 2y + \lambda z = \mu$

The given system of equations has infinite number of solutions, if any tow equations will be same *ie*, the last two equations will be same, if $\lambda = 3, \mu = 10$.

184 **(a)**

Given, $(A + B)(A - B) = A^2 - B^2$

$\Rightarrow \quad A^2 - AB + BA - B^2 = A^2 - B^2$

$\Rightarrow \quad AB = BA$

Now, $(ABA^{-1})^2 = (BAA^{-1})^2 = B^2$

185 **(c)**

Since diagonal elements of a skew –symmetric matrix are all zeros i.e. $a_{ii} = 0$ for all i

$$\therefore \text{tr}\,(A) = \sum_{i=1}^{n} a_{ii} = 0$$

186 **(c)**

$\because\ P63 = P(I - P) \quad \because P^2 = I - P)$

$= PI - P^2 = PI - (I - P)$

Now, $P^4 = P \cdot P^3$

$\Rightarrow\ P^4 = P(2P - I)$

$\Rightarrow\ P^4 = 2P^2 - P$

$\Rightarrow\ P^4 = 2I - 2P - P$

$\Rightarrow\ P^4 = 2I - 3P$

And $P^5 = P(2I - 3P)$

$\Rightarrow\ P^5 = 2P - 3(I - P)$

$\Rightarrow\ P^5 = 5P - 3I$

Also, $P^6 = P(5P - 3I)$

$\Rightarrow\ P^6 = 5P^2 - 3P$

$\Rightarrow\ P^6 = 5(I - P) - 3P$

$\Rightarrow\ P^6 = 5I - 8P$

So, $n = 6$

Alternate Solution

$\because\ P^n = 5I - 8P$

$= 5(I - P) - 3P$

$= P(5P - 3I)\ (\because\ P^2 = I - P)$

$= P(2P - 3P^2)$

$= P^2(2I - 3P)$

$= P^2[2(I - P) - P]$

$= P^2[2P^2 - P]$

$= P^3[2P - I]$

$= P^4\,[I - P]$

$= P^4 \cdot P^2 = P^6$

$\Rightarrow\ n = 6$

187 **(b)**

$A^2 = A.A = AB.A$

$= A.BA = AB = A$

189 **(d)**

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore\ |A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

and adj $A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

hence, $A^{-1} = \frac{1}{|A|}$ adj $A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

so, required element $= A_{13}^{-1} = 7$

190 **(a)**

$\because\ |A| = 1$

and $A^c = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and adj $A = (A^c)' = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{\text{adj}\,A}{|A|} = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-x)$

191 **(b)**

$\because |A| = 1(0 - 1) = -1$

$\therefore$ Cofactors of A are

$C_{11} = 0, C_{12} = 0, C_{13} = -1$

$C_{21} = 0, C_{22} = -1, C_{23} = 0$

$C_{31} = -1, C_{32} = 0, C_{33} = 0$

$\therefore\ A^{-1} = \dfrac{1}{-1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = A$

192 **(b)**

We have,

$A^2 - 5\,I_2 - \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\,A$

$\therefore k = 5$

194 **(b)**

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

$\therefore\ AX = B$

$\Rightarrow\ X = A^{-1}B$

Here, $A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix}$

$\therefore\ X = \dfrac{1}{6}\begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

$= \dfrac{1}{6}\begin{bmatrix} 0 + 6 + 0 \\ 0 + 0 + 12 \\ 0 - 6 - 12 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

195 **(a)**

The given system of equations can be rewritten as matrix from $AX = B$ as

$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = 1(6 + 1) + 1(3 + 2) + 1(1 - 4)$

$= 7 + 5 - 3 = 9 \neq 0$

Since, $|A \neq 0|$. So, the given system of equations has only trivial solution. So, there is no non-trivial solution.

196 **(d)**

If matrix has no inverse it means the value of determinant should be zero.

$\therefore \quad \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$

If we put $x = 1$, then column Ist and IIIrd are identical.

197 **(b)**

Since, $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix

$\therefore \quad \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$

$\Rightarrow \ (2+x)(5-2) - 3(-5-2x) + 4(1+x) = 0$

$\Rightarrow \ 6 + 3x + 15 + 6x + 4 + 4x = 0$

$\Rightarrow \ 13x + 25 = 0 \ \Rightarrow \ x = -\dfrac{25}{13}$

198 **(a)**

We have,

$A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$

$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -4 & 2 \end{bmatrix}$ Applying $R_2 \to 2R_2 + R_3$

$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix}$ Applying $C_3 \to C_3 - 2C_2$, $C_4 \to C_4 + C_2$

$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Applying $R_2 \leftrightarrow R_3$

Clearly, $\begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} \neq 0$ and every minor of order 3 is zero

Hence, rank of A is 2

199 **(b)**

We have,

$A^2 = AA = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2\,ab \\ 2\,ab & a^2+b^2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta^2 = 2\,ab$

200 **(d)**

In a square matrix, the trace of A is defined as the sum of the diagonal elements

Hence, trace of $A = \displaystyle\sum_{i=1}^{n} a_{ii}$

201 **(a)**

Given system of equations is $x + 2y + 3z = 1$, $2x + y + 3z = 2$ and $5x + 5y + 9z = 5$

Now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix}$

$= 1(9-15) - 2(18-15) + 3(10-5)$

$= -6 - 6 + 15$

$= 3 \neq 0$

Hence, it has unique solution

202 **(a)**

Let $\Delta = \begin{vmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{vmatrix}$

Applying $R_1 \to R_1 + R_3$

$\Rightarrow \quad \Delta = \begin{vmatrix} 10 & 5 & 2 \\ 5 & k & 1 \\ 6 & 3 & 1+x \end{vmatrix}$

Applying $C_1 \to C_1 - 2C_2$

$\Rightarrow \quad \Delta = \begin{vmatrix} 0 & 5 & 2 \\ 5-2k & k & 1 \\ 0 & 3 & 1+x \end{vmatrix}$

$\Rightarrow \quad (5-2k)(5+5x-6) = 0$

$\Rightarrow \quad k = \dfrac{5}{2}, \quad x = \dfrac{1}{5}$

204 **(b)**

Since, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \dfrac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

205 **(c)**

It is a direct consequence of the definition of rank

206 **(a)**

Now, $A(x)A(y) = (1-x)^{-1} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix} (1-y)^{-1} \begin{bmatrix} 1 & -y \\ -y & 1 \end{bmatrix}$

$= [(1+xy) - (x+y)]^{-1} \begin{bmatrix} 1+xy & -(x+y) \\ -(x+y) & 1+xy \end{bmatrix}$

$= \left(1 - \dfrac{x+y}{1+xy}\right)^{-1} \begin{bmatrix} 1 & -\dfrac{x+y}{1+xy} \\ -\dfrac{x+y}{1+xy} & 1 \end{bmatrix}$

$= A(z)$

207 **(a)**

$A^2(\alpha) = \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 2\cos\alpha\sin\alpha \\ -2\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} = A(2\alpha)$

208 **(a)**

Since, A is symmetric matrix, therefore $A^T = A$

Now, $(A^n)^T = (A^T)^n = A^n$

Hence, A^n is a symmetric matrix.

209 **(a)**

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore\ |A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

$= 0\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= -(1-9) + 2(1-6) = 8 - 10 = -2$

and $\text{Adj } A = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$= -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

210 **(a)**

We know that

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Clearly, $\frac{1}{2}(A + A^T)$ is a symmetric matrix and

$\frac{1}{2}(A - A^T)$ is a skew-symmetric matrix

Now,

$\frac{1}{2}(A + A^T) = \frac{1}{0}\left\{\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}\right\}$

$\Rightarrow \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

211 **(c)**

Since, the system of linear equations has a non-zero solution , then

$\begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{bmatrix} = 0$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 4c - 2a & c - a \end{vmatrix} = 0$

$\Rightarrow \quad (3b - 2a)(c - a) - (4c - 2a)(b - a) = 0$

$\Rightarrow \quad 3bc - 3ba - 2ac + 2a^2$

$= 4bc - 2ab - 4ac + 2a^2$

$\Rightarrow \quad 2ac = bc + ab$

On dividing by abc both sides ,we get

$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \quad a, b, c$ are in HP.

212 **(c)**

Given system of equations is

$x - y + z = 3$

$2x + y - z = 2$

and $-3x - 2ky + 6z = 3$

$\therefore$ The given system will have infinite solutions.

$\therefore \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -3 & -2k & 6 \end{vmatrix} = 0$

$\Rightarrow \quad 6k - 18 = 0 \Rightarrow k = 3$

213 **(a)**

The product of two orthogonal matrix is an orthogonal matrix

214 **(b)**

Given system of equations can be rewritten as

$AX = B$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 14 \end{bmatrix}$

$\therefore \quad |A| = 1(7 - 10) - 1(-7 - 6) + 1(5 + 3)$

$= -3 + 13 + 8 = 18 \neq 0$

$\therefore$ Given system has unique solution.

215 **(a)**

Given, equations $(x + ay = 0, az + y = 0, ax + z = 0)$ has infinite soluations.

$\therefore$ Using Crames's rule, its determinant=0

$\Rightarrow \quad \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} 0$

$\Rightarrow \quad 1 + a^3 = 0 \quad \Rightarrow \quad a = -1$

217 **(a)**

Given that, $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad F(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore\ F(\alpha)F(-\alpha)$

$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \left[\begin{matrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \\ 0 & 0 \end{matrix}\right.$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow\ [F(\alpha)]^{-1} = F(-\alpha)$

218 **(b)**

By using inverse of matrix, we know

$|M^{-1}| = |M|^{-1}$holds true

$(M^T)^{-1} = (M^{-1})^T$ holds true

and $(M^{-1})^{-1} = M$ holds true

but $(M^2)^{-1} = (M^{-1})^{-2}$ not true

219 **(a)**

Since, $A^2-B^2=(A-B)(A+B)$
$=A^2-B^2+AB-BA$
$\Rightarrow\quad AB=BA$

220 **(c)**
Given, $x_1+2x_2+3x_3=0$ ...(i)
$2x_1+3x_2+x_3=0$ ...(ii)
$3x_1+x_2+2x_3=0$ ...(iii)
$$\Rightarrow\quad \Delta=\begin{vmatrix}1&2&3\\2&3&1\\3&1&2\end{vmatrix}$$
$=1(6-1)-2(4-3)+3(2-9)$
$=-18$
Then, this is system has the unique solution.

222 **(d)**
$$X^2=X\cdot X=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}\begin{bmatrix}3&-4\\1&-1\end{bmatrix}=\begin{bmatrix}5&-8\\2&-3\end{bmatrix}$$
For $n=2$, no option is satisfied
Hence, option (d) is correct

223 **(a)**
We have,
$F(\alpha)F(-\alpha)$
$$=\begin{bmatrix}\cos\alpha&-\sin\alpha&1\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}\begin{bmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$$
$$\Rightarrow F(\alpha)F(-\alpha)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$
$\Rightarrow F(-\alpha)=[F(\alpha)]^{-1}$

224 **(b)**
We have, $(AA^T)^T=(A^T)^TA^T=AA^T$
$\therefore\ AA^T$ is symmetric matrix

225 **(c)**
For any square matrix X, we have
$X(\text{adj }X)=|X|\,I_n$
Taking $X=\text{adj }A$, we have
$(\text{adj }A)\big(\text{adj }(\text{adj }A)\big)=|\text{adj }A|\,I_n$
$\Rightarrow \text{adj }A\big(\text{adj }(\text{adj }A)\big)=|A|^{n-1}I_n$ $[\because |\text{adj }A|=|A|^{n-1}]$
$\Rightarrow (A\text{ adj }A)\big(\text{adj }(\text{adj }A)\big)=|A|^{n-1}A$ $[\because A\,I_n=A]$
$\Rightarrow (|A|\,I_n)\big(\text{adj }(\text{adj }A)\big)=|A|^{n-1}A$
$\Rightarrow \text{adj }(\text{adj }A)=|\text{A}|^{n-2}\,A$

226 **(d)**
Given equations are
$(\alpha+1)^3x+(\alpha+2)^3y-(\alpha+3)^3=0$
$(\alpha+1)x+(\alpha+2)y-(\alpha+3)=0$
and $x+y-1=0$
Since, this system of equations is consistent.
$$\therefore\ \begin{vmatrix}(\alpha+1)^3&(\alpha+2)^3&-(\alpha+3)^3\\(\alpha+1)&(\alpha+2)&-(\alpha+3)\end{vmatrix}=0$$
Applying $C_2\to C_2-C_1$ and $C_3\to C_3+C_1$
$$\Rightarrow\begin{bmatrix}(\alpha+1)^3 & (\alpha+2)^3 \ -(\alpha+1)^3 & (\alpha+1)^3-\ (\alpha+3)^3\\(\alpha+1) & (\alpha+2)\ -(\alpha+1) & -(\alpha+3)+\ (\alpha+1)\end{bmatrix}=0$$
$$\Rightarrow\begin{bmatrix}(\alpha+1)^3&3\alpha^2+9\alpha+7&-6\alpha^2-24\alpha-26\\(\alpha+1)&1&-2\\1&0&0\end{bmatrix}$$
$=0$
$\Rightarrow\quad -2(3\alpha^2+9\alpha+7)+6\alpha^2+24\alpha+26=0$
$\Rightarrow 6\alpha+12=0\quad\Rightarrow\alpha=-2$

227 **(a)**
We have,
$$A^n=\begin{bmatrix}\cos n\,\theta&\sin n\,\theta\\-\sin n\,\theta&\cos n\,\theta\end{bmatrix}$$
$$\Rightarrow\frac{1}{n}A^n=\begin{bmatrix}\frac{\cos n\,\theta}{n}&\frac{\sin n\,\theta}{n}\\\frac{-\sin n\,\theta}{n}&\frac{\cos n\,\theta}{n}\end{bmatrix}$$
$$\Rightarrow\lim_{n\to\infty}\frac{1}{n}A^n=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

228 **(c)**
We have,
$AB=O$
$$\Rightarrow\begin{bmatrix}\cos^2\alpha&\cos\alpha\sin\alpha\\\cos\alpha\sin\alpha&\sin^2\alpha\end{bmatrix}\begin{bmatrix}\cos^2\beta&\cos\beta\sin\\\cos\beta\sin\beta&\sin^2\beta\end{bmatrix}$$
$=O$
$$\Rightarrow\begin{bmatrix}\cos\alpha\cos\beta\cos(\alpha-\beta)&\cos\alpha\sin\beta\cos(\alpha-\beta)\\\cos\beta\sin\alpha\cos(\alpha-\beta)&\sin\alpha\sin\beta\cos(\alpha-\beta)\end{bmatrix}$$
$=O$
$\Rightarrow\cos(\alpha-\beta)=0$
$\Rightarrow\alpha-\beta$ is an odd multiple of $\frac{\pi}{2}$

229 **(c)**
$|A||\text{adj }A|=|A|^n$ for order n
$\Rightarrow\ DD'=D^n$

230 **(c)**
Given, $\begin{bmatrix}1+\omega&2\omega\\-2\omega&-b\end{bmatrix}+\begin{bmatrix}a&-\omega\\3\omega&2\end{bmatrix}=\begin{bmatrix}0&\omega\\\omega&1\end{bmatrix}$
$\Rightarrow\quad\begin{bmatrix}1+\omega+a&\omega\\\omega&2-b\end{bmatrix}=\begin{bmatrix}0&\omega\\\omega&1\end{bmatrix}$
$\Rightarrow\quad 1+\omega+a=0, 2-b=1$
$\Rightarrow\quad a=-1-\omega, b=1$
$\therefore\quad a^2+b^2=(-1-\omega)^2+1^2$
$=1+\omega^2+2\omega+1^2$
$=0+\omega+1$ $\quad(\because\ 1+\omega+\omega^2=0)$
$=1+\omega$

232 **(a)**
Given, $2kx-2y+3z=0, x+ky+2z=0, 2x+kz=0$
For non-trivial solution

$$\begin{vmatrix} 2k & -2 & 3 \\ 1 & k & 2 \\ 2 & 0 & k \end{vmatrix} = 0$$

$\Rightarrow 2k(k^2 - 0) + 2(k - 4) + 3(0 - 2k) = 0$

$\Rightarrow \quad 2k^3 - 4k - 8 = 0$

$\Rightarrow (k - 2)(2k^2 + 4k + 4) = 0$

$\Rightarrow \quad k = 2$

233 **(a)**

$$AB = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 + 0 + 15 & -2 + 2 + 0 \\ 4 + 0 + 0 & -4 + 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$$

234 **(d)**

$$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = B \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

235 **(c)**

We have,

$E(\alpha) E(\beta)$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = E(\alpha + \beta)$$

Hence, option (c) is correct.

237 **(c)**

We have, $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} + \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\therefore$ Matrix A is nilpotent

238 **(d)**

Since, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

Now, $|A| = 1(0 - 2) + 1(2 - 3) + 2(4 - 0) = 5$

$$\therefore \quad A^{-1} = \frac{1}{5} \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

Now, $A^{-1} B = \frac{1}{5} \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

239 **(a)**

Since, A is a skew-symmetric matrix. Therefore,

$A^T = -A \quad \Rightarrow \quad |A^T| = |-A|$

$\Rightarrow \; |A| = (-1)^n |A|$

Also, n is odd

$\therefore \; 2|A| = 0 \; \Rightarrow |A| = 0$

Thus, $|\text{adj } A| = |A|^2 = 0$

240 **(d)**

Given System of equations are

$x + 3y + 2z = 0$

$3x + y + z = 0$

and $\; 2x - 2y - z = 0$

Now, $\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & -1 \end{vmatrix}$

$= 1(-1 + 2) - 3(-3 - 2) + 2(-6 - 2)$

$= 1 + 15 - 16$

$= 0$

Since, determinant is zero, then it has infinitely many solutions.

242 **(b)**

Let$\Delta = \begin{bmatrix} a_1 & a_2 \\ a_4 & a_5 \end{bmatrix}$

$= a_1 a_5 - a_2 a_4$

$= a_1(a_1 + 4d) - (a_1 + d)(a_1 + 3d)$

$= a_1^2 + 4a_1 d - a_1^2 - 4a_1 d - 3d^2 = -3d^2 \neq 0$

Hence, given system of equations has unique solution.

243 **(b)**

$$\because \quad I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}, |I_n| = 1$$

adj $(I_n) = I_n$

$\therefore \; (I_n)^{-1} = I_n$

244 **(c)**

We know that

$(\text{adj } A)^T = \text{adj } A^T$

$\Rightarrow \text{adj } A^T - (\text{adj } A)^T = O$ (Null matrix)

245 **(c)**

Given, $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$

It is of the form $AX = B$ ...(i)

$|A| = 2(3 + 2) + 1(1 + 3) + 3(2 - 9) = -7$

$$\therefore \quad \text{adj}\,(A) = \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{-7}\begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$$

From Eq.(i), $X = -\frac{1}{7}\begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -7 \\ -14 \\ -21 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 2, z = 3$

247 **(b)**

$$A = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{vmatrix}$$

$= 1(1+1) + 1(2+4) + 1(2-4) = 6 \neq 0$

Hence, it has unique solution.

248 **(d)**

Given, A= $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, $|A| = \cos^2\theta + \sin^2\theta = 1 \neq 0$.

$\therefore A$ is invertible.

249 **(b)**

$|A| = -1$ and adj $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$

250 **(b)**

$$A = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & a+6 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$$

[using $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - 2R_3$]

$$= \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix} \quad [\text{using} R_1 \to R_1 + R_2]$$

When $a = -6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -2 & -5 \end{vmatrix}$ $\quad \therefore \rho(A) = 1$

When $a = 6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -12 \\ 1 & -2 & 7 \end{vmatrix}$, $\quad \therefore \rho(A) = 2$

When $a = 1$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -7 \\ 1 & -2 & 2 \end{vmatrix}$, $\quad \therefore \rho(A) = 2$

When $a = 2$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & -2 & 3 \end{vmatrix}$ $\quad \therefore \rho(A) = 2$

251 **(b)**

Let $D = \begin{bmatrix} d_1 & 0 & 0 \cdots & 0 \\ 0 & d_2 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0 \cdots & d_n \end{bmatrix}$

Then, $|D| = d_1\, d_2 \cdots d_n$

Now, Cofactor of $D_{11} = d_2\, d_3 \cdots d_n$

Cofactor of $D_{22} = d_1\, d_3 \cdots d_n$ etc

And, Cofactor of $D_{ij} = 0$ when $i \neq j$

$\therefore D^{-1} = \frac{1}{|D|}$ adj D

$$= \frac{1}{d_1\, d_2 \cdots d_n}\begin{bmatrix} d_2\, d_3 \cdots d_n & 0 & 0 \cdots & 0 \\ 0 & d_2 d_3 \cdots d_n & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0 \cdots & d_1\, d_2 \cdots \end{bmatrix}$$

$$\therefore D^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & & \frac{1}{d_n} \end{bmatrix} = \text{diag}\,(s_1^{-1}\, d_2^{-1} \cdots d_n^{-1})$$

252 **(d)**

$\because A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$

$\therefore f(A) = A^2 + 4A - 5$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

253 **(a)**

$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

Again now, $4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$\therefore \qquad A^2 = 4A - 3I$

254 **(d)**

$\because (AB)^{-1} = B^{-1}A^{-1}$

255 **(d)**

$|A| = -8$

$\text{adj}(A) = \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$

$A^{-1} = \frac{1}{-8}\begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$

256 **(b)**

Since given system of equations possesses a non-zero solution.

$\therefore \qquad \Delta = \begin{bmatrix} a & 1 & 1 \\ 1 & -a & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0$

$\Rightarrow \quad a(-a-1) - 1(1-1) + 1(1+a) = 0$

$\Rightarrow a^2 = 1 \quad \Rightarrow \quad a = \pm 1$

257 **(a)**

Now, $(A + A^T)^T = A^T + (A^T)^T = A^T + A$

$\therefore A + A^T$ is symmetric matrix.

258 **(c)**

$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} = 14$

$\because\ \left(\text{adj}(\text{adj}\,A)\right) = |A|^{n-2}A = 14^{3-2}A = 14\,A$

$\therefore\ |\text{adj}\,(\text{adj}\,A)| = |14A| = 14^3|A| = 14^4$

259 **(b)**

Given, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \Rightarrow$

$\begin{bmatrix} x+y+z \\ x-2y-2z \\ x+3y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

On Comparing both sides, we get

$x+y+z=0$ ...(i)

$x-2y-2z=3$ ...(ii)

and $x+3y+z=4$ ...(iii)

On solving Eqs. (i), (ii) and (iii) , we get

$x=1, y=2$ and $z=-3$

260 **(a)**

$|A| = \begin{vmatrix} 1 & 2 \\ -4 & -1 \end{vmatrix}$

$= -1+8 = 7$

adj $A = \begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$

261 **(c)**

It is given that

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$

$\Rightarrow a+c=a, a+b=b+d, c+d=d$

$\Rightarrow c=0$ and $a=d$

262 **(a)**

$AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

263 **(a)**

Let A be a skew-symmetric matrix of odd order $(2\,n+1)$ say. Since A is skew-symmetric

$\therefore A^T = -A$

$\Rightarrow |A^T| = |-A|$

$\Rightarrow |A^T| = (-1)^{2\,n+1}|A|$

$\Rightarrow |A^T| = -|A|$

$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

264 **(a)**

As, $PP^T = \begin{bmatrix} \sqrt{3/2} & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow PP^T = I$ or $P^T = P^{-1}$ ...(i)

As, $Q = PAP^T$

$\therefore P^TQ^{2005}P = P^T[PAP^T)(PAP)^T)\ldots 2005 \text{ times}]P$

$= \frac{(P^TP)A(P^TP)A(P^TP)\ldots(P^TP)A(P^T\,P)}{2005\ times}$

$= IA^{2005} = A^{2005}$

$\therefore\ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$...and so on

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow P^TQ^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

265 **(c)**

Given, $2X+3Y=0$ ... (i)

and $X+2Y=I$ ...(ii)

where $O=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On solving Eqs. (i) and (ii), we get

$X = -3I = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

266 **(d)**

Subtracting the addition of first two equations from third equation, we get

$0 = -5$ which is an absurd result.

267 **(d)**

Given $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$

$|A| = \begin{vmatrix} x & -2 \\ 3 & 7 \end{vmatrix} = 7x+6$

$\therefore\ A^{-1} = \frac{1}{7x+6}\begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$

But given $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$

$\therefore\ \frac{7}{7x+6} = \frac{7}{34}$

$\Rightarrow\ 7x+6=34 \Rightarrow 7x=28 \Rightarrow x=4$

268 **(d)**

(a) It is clear that A is not a zero matrix.

(b) $(-1)I = -1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

ie, $(-1)I \neq A$

(c) $|A| = 0\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since, $|A| \neq 0$ so A^{-1} exists.

(d) $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^2 = I$

270 **(a)**

Since, $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

271 **(c)**

We have,

$[F(x)\,G(y)]^{-1} = [G(y)]^{-1}\,[f(x)]^{-1}$

$\Rightarrow [F(x)\,G(y)]^{-1} = G(-y)\,F(-x)$

272 **(a)**

$$A^{-1} = \frac{1}{1+10}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$$

Also, $A^{-1} = xA + yI$

$$\Rightarrow \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$
$$\Rightarrow x + y = \frac{1}{11}, 2x = \frac{-2}{11}$$
$$\Rightarrow x = \frac{-1}{11}, \; y = \frac{2}{11}$$

273 **(d)**

Now, $(AB)^T = B^T A^T$

274 **(c)**

On comparing corresponding elements, we get

$x + y + z = 9$

$x + y = 5$

and $y + z = 7$

On solving these, we get $x = 2, y = 3, z = 4$

$\Rightarrow (x, y, z) = (2,3,4)$

275 **(c)**

$$A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$
$$= \begin{bmatrix} = a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$
$$\Rightarrow A^2 = 0$$

$\therefore A$ is nilpotent matrix of order 2.

276 **(d)**

Since A,B and C are non-singular matrices, then

$(AB^{-1}C)^{-1} = C^{-1}(AB^{-1})^{-1}$

$= C^{-1}((B^{-1})^{-1}\,A^{-1}) = C^{-1}BA^{-1}$

277 **(a)**

Given matrix is invertible

$$\Rightarrow \begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \neq 0$$

$\Rightarrow \lambda\;(0-1) + 1(-6+1) + 4(-3-0) \neq 0$

$\Rightarrow \quad -\lambda - 5 - 12 \neq 0$

$\Rightarrow \lambda \neq -17$

278 **(d)**

From Eqs. (ii) and (iii), we get

$$\frac{3y^2}{b^2} - \frac{3z^2}{c^2} = 0$$
$$\Rightarrow \frac{z^2}{c^2} = \frac{y^2}{b^2}$$

On putting this value in Eq. (i), we get

$$\frac{2x^2}{a^2} - \frac{2y^2}{b^2} = 0$$
$$\Rightarrow \frac{x^2}{a62} = \frac{y^2}{b^2}$$

$\therefore \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} = k^2$ (say)

$\Rightarrow x = \pm ka, y = \pm kb, z = \pm kc, \forall k \in R$

279 **(b)**

We have, $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$

$\therefore |A| = 1 - \log_a b \log_b a = 1 - 1 = 0$

280 **(a)**

$|A| = 4 - 6 = -2$

adj $(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\therefore A^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

281 **(b)**

Since, $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$\therefore PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$
$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix}$$
$$= \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

282 **(d)**

$$B = \text{adj}(A) = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

Therefore, adj(B)=$\begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$

Now, $|\text{adj } B| = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 625$

and $|C|$=125$|A| = 125\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 5 & 0 \end{vmatrix} = 625$

$\therefore \frac{|\text{adj}(B)|}{|C|} = \frac{625}{625} = 1$

Alternate

$|A| = 1(0+3) + 1(0+6) + (0-4)$

Now, adj B = adj(adj A)

$= |A|A = 5A$

$$\therefore \frac{|\text{adj } B|}{|C|} = \frac{|5A|}{|5A|} = 1$$

283 **(d)**

Given, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore \quad A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A^T$

284 **(b)**

The given system of equations posses non-zero solutions,

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a \\ 1-a & 1 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a-1 \\ 0 & -a-1 & 0 \end{vmatrix} = 0$

$\Rightarrow \quad 1\big(0 - (a^2 - 1)\big) = 0$

$\Rightarrow a^2 = 1 \Rightarrow \quad a = \pm 1$

285 **(a)**

Given, $x \begin{bmatrix} -3 \\ 4 \end{bmatrix} + y \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$

$\therefore -3x + 4y = 10 \quad$ ...(i)

and $\quad 4x + 3y = -5 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = -2, y = 1$

287 **(a)**

$|A| = 5 + 6 = 11$

and $\quad$ adj $A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{11} \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$

288 **(c)**

We know that, if

$A^n = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = \text{diag}\,[d_1\ d_2\ d_3]$

Then,

$A^n = \begin{bmatrix} d_1^n & 0 & 0 \\ 0 & d_2^n & 0 \\ 0 & 0 & d_3^n \end{bmatrix} = \text{diag}\,[d_1^n\ d_2^n\ d_3^n]$

$\therefore A^5 = \begin{bmatrix} 2^5 & 0 & 0 \\ 0 & 2^5 & 0 \\ 0 & 0 & 2^5 \end{bmatrix} = 16\,A$

289 **(c)**

$AB = I \Rightarrow \quad B = A^{-1}$

$= \frac{1}{1 + \tan^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$\Rightarrow \quad (\sec^2\theta)\,\mathrm{B} = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \mathrm{A}(-\theta)$

290 **(b)**

It is given that $A = [a_{ij}]$ is a skew-symmetric matrix

$a_{ij} = -a_{ji}$ for all i, j

$\Rightarrow a_{ii} = -a_{ii}$ for all i

$\Rightarrow 2\,a_{ii} = 0$ for all $i \Rightarrow a_{ii} = 0$ for all i

291 **(c)**

We know that, if $A = \text{diag.}\,(d_1\ d_2, \dots \dots.., d_n)$ is a diagonal matrix, then for any $k \in N$

$A^k = \text{diag}\,(d_1^k, d_2^k, \dots, d_n^k)$

Here, $A = \text{diag.}\,(a, a, a)$

$\therefore A^n = \text{diag}\,(a^n, a^n, a^n) = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

292 **(b)**

We have,

$(A\,B - B\,A)^T = (A\,B)^T - (B\,A)^T$

$\qquad = B^T\,A^T - A^T\,B^T$

$\Rightarrow (A\,B - B\,A)^T = B\,A - A\,B \qquad [\because A^T = A, B^T = B]$

$\Rightarrow (A\,B - B\,A)^T = -(A\,B - B\,A)$

So, $A\,B - B\,A$ is skew-symmetric matrix

293 **(b)**

Since $A\,B$ exists

$\therefore$ No. of rows in $B =$ No. of columns in A

$\Rightarrow$ No. of rows in $B = n$

Also, $B\,A$ exists

$\Rightarrow$ No. of columns in $B =$ No. of rows in A

$\Rightarrow$ No. of columns in $= m$

Hence, B is of order $n \times m$

294 **(c)**

We have,

$(k\,A)(\text{adj}\,k\,A) = |k\,A|\,I_n$

$\Rightarrow k(A\,\text{adj}\,k\,A) = k^n|A|\,I_n \qquad [\because |k\,A| = k^n|A|]$

$\Rightarrow A(\text{adj}\,k\,A) = k^{n-1}|A|\,I_n$

$\Rightarrow A\,\text{adj}\,(k\,A) = k^{n-1}A(\text{adj}\,A) \quad [\because A\,\text{adj}\,A = |A|\,I_n]$

$\Rightarrow A\,\text{adj}\,(k\,A) = A(k^{n-1}\text{adj A})$

$\Rightarrow A^{-1}\big(A\,\text{adj}\,(k\,A)\big) = A^{-1}\big(A\,(k^{n-1}\text{adj}\,A)\big)$

$\Rightarrow (A^{-1}A)(\text{adj}\,k\,A) = (A^{-1}\,A)(k^{n-1}\,\text{adj}\,A)$

$\Rightarrow I(\text{adj}\,k\,A) = I(k^{n-1}\,\text{adj}\,A)$

$\Rightarrow \text{adj}\,k\,A = k^{n-1}\,(\text{adj}\,A)$

295 **(c)**

It has a non- zero solution if

$\begin{vmatrix} 1 & k & -1 \\ 3-k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad -6k + 6 = 0$

$\Rightarrow \qquad k = 1$

296 **(b)**

$(aI + bA)^2 = (aI + bA)(aI + bA)$

$= a^2I^2 + aI(bA) + bA(aI) + (bA)^2$

Now, $I^2 = I$ and $IA = A$

$\therefore (aI + bA)^2 = a^2I + 2abA + b^2(A^2)$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore (aI + bA)^2 = a^2I + 2abA$

297 **(b)**

Since A is orthogonal matrix

$\therefore A\,A^T = I = A^T\,A$

$\Rightarrow |A\,A^T| = |I| = |A^T\,A|$

$\Rightarrow |A||A^T| = 1 = |A^T||A|$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

298 **(b)**

Since, given system of equations has no solution, $\Delta = 0$ and any one amongst $\Delta x, \Delta y, \Delta z$ is non-zero.

Where $\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

And $\Delta z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & \lambda \end{vmatrix} = 6 \neq 0$

$\Rightarrow \quad \lambda = 1$

299 **(c)**

Since, A is an idempotent matrix, therefore $A^2 = A$

$\Rightarrow \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$

On comparing, $16 + 4x = 4$

$\Rightarrow \quad x = -3$

300 **(a)**

We have,

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and adj $A = \begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & 1 \end{bmatrix}$

Clearly, $|A| = 6 - 8 + 4 = 2$

$\therefore A\,(\text{adj}\,A) = |A|\,I$

$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2y-2 & 0 & 2x-18 \\ 0 & 2 & 3x-12 \\ 2y-4 & 0 & x-2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\Rightarrow 2y - 2 = 2, 2y - 4 = 0, 2x - 8 = 0, 3x - 12$
$= 0, x - 2 = 2$

$\Rightarrow x = 4, y = 2 \Rightarrow x + y = 6$

301 **(a)**

Since, $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$

$\Rightarrow \; x + y = 4 \quad$ ...(i)

$x - y = 0 \quad$ ...(ii)

$2x + z = 7 \quad$ ...(iii)

and $2z + w = 10 \quad$ ...(iv)

On solving these equations, we get

$x = 2, y = 2, z = 3, w = 4$

302 **(b)**

We have,

$A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow A^2 = -I \Rightarrow A^2 + I = O$

303 **(d)**

$m[-3 \;\; 4] + n[4 \;\; -3] = [10 - 11]$

$\Rightarrow [-3m + 4n \;\; 4m - 3n] = [10 - 11]$

$\Rightarrow \quad -3m + 4n = 10 \quad$ ...(i)

and $\quad 4m - 3n = -11 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$n = 1, \;\; m = -2$

Now, $\quad 3m + 7n = 3(-2) + 7(1) = 1$

304 **(b)**

We know that

$A(\text{adj}\,A) = |A|\,I$

If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then $|A| = 1$

$\therefore A\,(\text{adj}\,A) = I \Rightarrow KI = I \Rightarrow k = 1$

305 **(a)**

$\because |A| = 3, \text{adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$\therefore \; A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow \quad (A^{-1})^3 = \frac{1}{27}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}^3$

$= \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$

306 **(d)**

Given, $M = [a_{uv}]_{n\times n}$

$= [\sin(\theta_u - \theta_v) + i\cos(\theta_u - \theta_v)]$

$\Rightarrow \quad \overline{M} = [\sin(\theta_u - \theta_v) - i\cos(\theta_u - \theta_v)]$

$\Rightarrow \quad (\overline{M})^T = [\sin(\theta_v - \theta_u) - i\cos(\theta_v - \theta_u)]$

$= [-\sin(\theta_u - \theta_v) - i\cos(\theta_u - \theta_v)]$

$= -[\sin(\theta_u - \theta_v) + i\cos(\theta_u - \theta_v)]$

$= -M$

307 **(a)**

If given system of equations have infinitely many solutions, then

$\begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2(-4 + 1) + 1(2 - \lambda) + 1(-1 + 2\lambda) = 0$

$\Rightarrow -6 + 2 - \lambda - 1 + 2\lambda = 0$

$\Rightarrow \qquad \lambda - 5 = 0$

$\Rightarrow \qquad \lambda = 5$

308 **(c)**

If $AB = O$, then A and B may be equal to O individually. It is not necessary in any condition

309 **(b)**

$$E(\theta)E(\emptyset) = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \times \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(2n+1)\frac{\pi}{2} \\ \cos\theta\sin\phi\cos(2n+1)\frac{\pi}{2} \\ \cos\theta\sin\phi\cos(2n+1)\frac{\pi}{2} \\ \sin\theta\sin\phi(2n+1)\frac{\pi}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \left[\because \cos(2n+1)\frac{\pi}{2} = 0\right]$$

310 **(c)**

Given, $x\sin 3\theta - y + z = 0$

$x\cos 2\theta + 4y + 3z = 0$ and $2x + 7y + 7z = 0$

For non-trivial solution.

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta(28 - 21) - \cos 2\theta(-7 - 7) + 2(-3 - 4) = 0$

$\Rightarrow 7\sin 3\theta + 14\cos 2\theta - 14 = 0$

$\Rightarrow 7(3\sin\theta - 4\sin^3\theta) + 14(1 - 2\sin^2\theta) - 14 = 0$

$\Rightarrow -28\sin^3\theta - 28\sin^2\theta + 21\sin\theta = 0$

$\Rightarrow -7\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\Rightarrow \sin\theta(2\sin\theta + 3)(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0, \sin\theta = \frac{1}{2} \quad \left(\because \sin\theta \neq -\frac{3}{2}\right)$

$\Rightarrow \theta = n\pi, n\pi + (-1)^n\frac{\pi}{6}$

311 **(c)**

$\because kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow 2k = 3a, 3k = 2b, -4k = 24$

$\Rightarrow a = \frac{2k}{3}, b = \frac{3k}{2}, k = -6$

$\therefore a = -4, b = -9, k = -6$

312 **(d)**

$\because$ adj (adj A) $= |A|^{n-2}A$

Here $n = 3$

$\Rightarrow$ adj (adj A) $= |A|A$

313 **(c)**

$A^5 = A^2A^2A = (A + I)(A + I)A$

$= (A^2 + 2AI + I^2)A$

$= (A + I + 2A + I)A = (3A + 2I)A$

$= 3A^2 + 2IA = 3(A + I) + 2IA$

$= 3A + 3I + 2A = 5A + 3I$

314 **(c)**

Matrices $A + B$ and AB are defined only if both A and B are of same order $n \times n$.

315 **(a)**

$UV + XY = [2 - 3 \quad 4]\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + [0 \quad 2 \quad 3]\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$= 6 - 6 + 4] + [0 + 4 + 12] = [4] + [16] = [20]$

316 **(b)**

For matrix $A = \begin{bmatrix} a & 2 \\ 2 & 4 \end{bmatrix}$ to be singular,

$\begin{vmatrix} a & 2 \\ 2 & 4 \end{vmatrix} = 0$

$\Rightarrow 4a - 4 = 0$

$\Rightarrow a = 1$

317 **(a)**

adj $(A) = \begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix}$

$\therefore$ adj $(A) + B = \begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4y - 3 & -x + 1 \\ -x^2 + 1 & 1 + 0 \end{bmatrix}$

$\Rightarrow 4y - 3 = 1 \Rightarrow y = 1$

and $-x + 1 = 0 \Rightarrow x = 1$

318 **(c)**

$|A| = 1.(\cos^2\alpha + \sin^2\alpha) = 1$

Now, $A^{-1} = \frac{1}{|A|}$ adj(A) = adj(A)

319 **(a)**

We have,

$(A + B)(A - B) = A^2 - AB + BA - B^2$

So, option (a) is correct.

320 **(a)**

Given, $(1 - x)f(x) = 1 + x$

$\Rightarrow (I - A)f(A) = (I + A)$ ($\because$ Put $x = A$)

$\Rightarrow f(A) = (I - A)^{-1}(I + A)$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)$

$\Rightarrow f(A) = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$$= \frac{\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}{-4}$$

$$= \frac{\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}}{-4} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

321 **(b)**

We have,

$AA^{-1} = I$

$\Rightarrow \det(AA^{-1} = \det(I)$

$\Rightarrow \det(A)\det(A^{-1}) = 1$

$\begin{bmatrix} \because \det(A\,B) = \det(A)\det(B) \\ \text{and } \det(I) - 1) \end{bmatrix}$

$\Rightarrow \det(A^{-1}) = \dfrac{1}{\det(A)}$

322 **(b)**

Since $\quad AB = 3I$

$\Rightarrow \quad A^{-1}AB = 3IA^{-1}$

$\Rightarrow \quad B = 3A^{-1}$

$\Rightarrow \quad A^{-1} = \dfrac{B}{3}$

323 **(b)**

We have, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{vmatrix} = 0$

[using $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$]

$\because$ The given points$(x_1, y_1), (x_2, y_2)$and (x_3, y_3) are collinear, therefore the rank of matrix is always greater than 0 and less than 3.

324 **(a)**

$\because \; A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

And $\quad 4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$\therefore \; A^2 = 4A - 3I$

325 **(a)**

Given, $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$

$\therefore$ adj $A \begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$

326 **(c)**

Given, $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow |A| = 1$

$\therefore A$ adj $(A) = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|I$

327 **(c)**

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

And $\; BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

If $AB = BA$, than $a = b$

Hence, $AB = BA$ is possible for infinitely many values of B's.

328 **(b)**

We have,

$2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ ...(i)

and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$ ...(ii)

On multiplying Eq.(ii) by 2 and then subtracting Eq.(i) from Eq.(ii), we get

$B = 2\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

329 **(b)**

Determinant of unit matrix of any order is 1

330 **(b)**

$AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix}$

$\Rightarrow |AB| = 102 - 2$

$= 100$

331 **(a)**

We have, $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$and $A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

$\therefore$Symmetric matrix, $B = \frac{A + A'}{2}$

$= \dfrac{1}{2}\left\{\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right\}$

$= \dfrac{1}{2}\begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$

332 **(b)**

Since A is non-singular. Therefore, A^{-1} exists

Now, A (adj A) $= |A|I =$ (adj A)A

$\Rightarrow |A||\text{adj } A| = |A|^n = |\text{adj } A||A|$

$\Rightarrow |\text{adj } A| = |A|^{n-1} \qquad [\because |A| \neq 0]$

333 **(a)**

$\because A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix} = \begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix}$

$\Rightarrow (A+B)^2 = \begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix}\begin{bmatrix} 2 & -1+a \\ 6 & -1+b \end{bmatrix}$

$\begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix}$

and $\quad A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Also, $\quad B^2 = \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}\begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}$

$= \begin{bmatrix} 1+4a & a+ab \\ 4+4b & 4a+b^2 \end{bmatrix}$

Given,$(A+B)^2 = A^2 + B^2$

$$\therefore \begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1+4a & a+ab \\ 4+4b & 4a+b^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+6a & -1+a-b+ab \\ 6+6b & -5+6a-2b+b^2 \end{bmatrix} = \begin{bmatrix} 4a & a+ab \\ 4+4b & -1+4a+b^2 \end{bmatrix}$$

On comparing both sides, we get

$-2+6a = 4a$ and $6+6b = 4+4b$

$\Rightarrow a = 1$ and $b = -1$

334 **(d)**

$(AB^{-1}C)^{-1} = C^{-1}(B^{-1})^{-1}A^{-1} = C^{-1}BA^{-1}$